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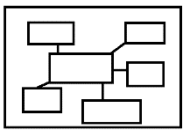
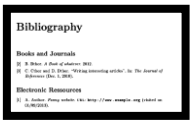
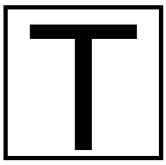
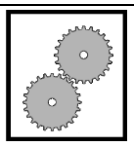
# TECHNICAL MATHEMATICS

## GRADE 12

## SPRING CLASSES 2026

# TEACHER AND LEARNER CONTENT MANUAL

## ICON DESCRIPTION

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|  BIBLIOGRAPHY |  TERMINOLOGY |  WORKED EXAMPLES |  STEPS |

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## SECTION 1 : CUBIC FUNCTION

### 1. SKETCHING CUBIC FUNCTION

You must be able to sketch the cubic function, giving the special attention to the following key features:

- Intercepts (both $x$ and $y$ intercepts)
- Turning points (stationary points)

Clearly indicating the intercepts with the axis as well as the turning points, every time you are asked to sketch a cubic function.



Follow the following steps when finding the key points of the cubic functions :

- $y$ intercept

  Let $x = 0$ and simplify.

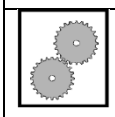
- $x$ intercept

  Let $y = 0$ and solve a third-degree polynomial by inspection, long division or synthetic method. The latter will require you to use FACTOR THEOREM to find the divisor (factor).

  NOTE: there can be one, two or three x intercepts depending on the given function.

- Stationary / turning points of a function $f(x)$
  - Determine the derivative $f'(x)$
  - Equate the derivative to zero, $f'(x) = 0$
  - Solve for $x$, then substitute the obtained values of x into the original function $f(x)$ to obtain the corresponding y values.

Plot the points and join them.



**WORKED EXAMPLE 1**

Given the function $f(x) = -2x^3 + 12x^2 - 18x + 8$:

1. Determine the $y$ intercept of $f$.
2. Show that $x - 1$ is a factor of $f$.
3. Hence, determine the $x - int$ of $f$.
4. Determine the coordinates of the turning points.
5. Sketch the graph of $f$, clearly indicate the intercepts with the axis as well as the turning points.

Solutions

1. $f(0) = -2(0)^{3} + 12(0)^{2} - 18(0) + 8$

$= 8$

$(0\,;8)$

2. $f(1) = -2(1)^{3} + 12(1)^{2} - 18(1) + 8$

$= 0$ $(1\,;0)$

3. $0 = (x-1)(-2x^{2} + 10x - 8)$

$x - 1 = 0$ or $-2x^{2} + 10x - 8 = 0$

$x = 1$ or $x^{2} - 5x + 4 = 0$

$x = 1$ or $(x-4)(x-1) = 0$

$x = 1$ or $x = 4$ or $x = 1$

$(1\,;0)$ and $(4:0)$

4. $f'(x) = -6x^{2} + 24x - 18$

$0 = -6x^{2} + 24x - 18$

$0 = x^{2} - 4x + 3$

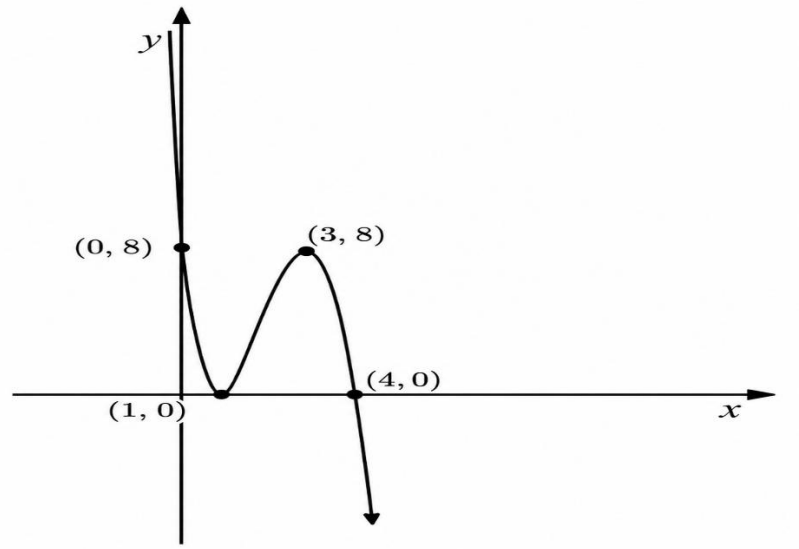
$0 = (x-3)(x-1)$

$x = 3$ or $x = 1$

$f(3) = 8$ and $f(1) = 0$

$(3:8)$ and $(1;0)$

5.



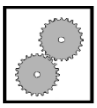
## 2. FINDING THE EQUATION OF CUBIC FUNCTION

2.1 When given 3 $x-intercepts$ and another point, then use:

$$y = a(x - x_1)(x - x_2)(x - x_3)$$

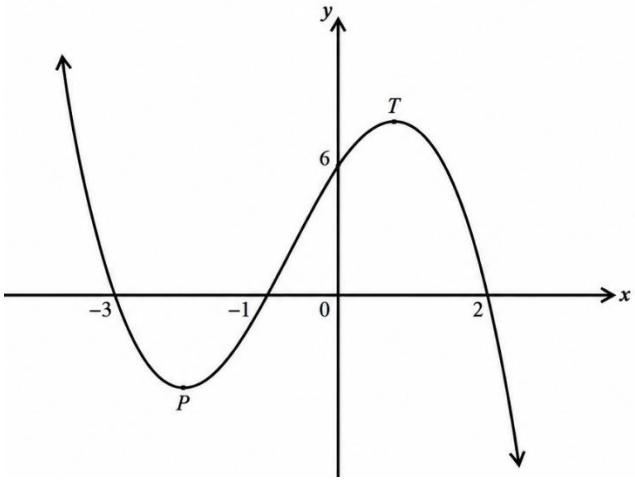
2.2 If the graph has a two $x$-intercepts, use the equation:

$$y = a(x - x_1)(x - x_2)^2$$

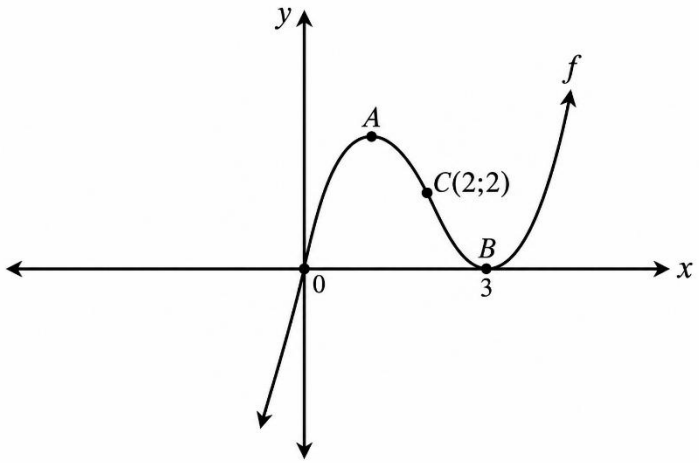


**WORKED EXAMPLE 2**

1. The sketch below represents the graph of $f$ defined by $f(x) = -x^3 + ax^2 + bx + c$. $P\ and\ T$ are the turning points of $f$. The x-intercepts of the graph are $-3, -1\ and\ 2$. The graph has a y-cut at 6. Determine the values of $a, b\ and\ c$.




2. The sketch below represents the graph of $f$ defined be $f(x) = ax^3 + bx^2 + c$. A and B are the turning points of $f$. B (3;0) is also the $x$ – intercept of $f$. C is a point with coordinates (2;2). The graph passes through the origin.



Solutions

1.

$$y = -(x+3)(x+1)(x-2)$$
$$y = -(x^3 + 2x^2 - 5x - 6)$$
$$= -x^3 - 2x^2 + 5x + 6$$
$$a = -2;\ b = 5;\ c = 6$$

2.

$$y = a(x - x_1)(x - x_2)^2$$
$$2 = a(2-0)(2-3)^2$$
$$2 = a(2)(1)^2$$
$$a = 1$$
$$y = 1(x)(x-3)^2$$
$$f(x) = x^3 - 6x^2 + 9x$$

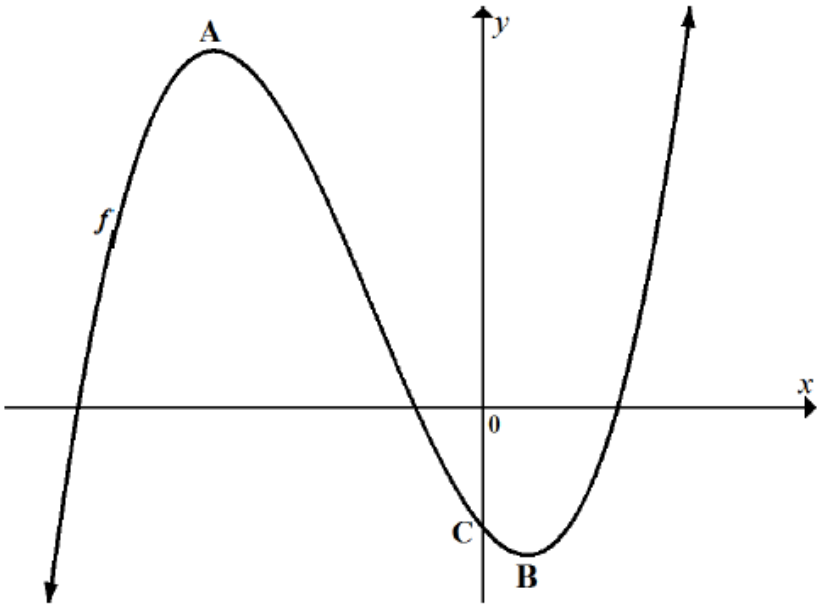
**ACTIVITIES**

**ACTIVITY 1**

**QUESTION 9**

The sketch below shows the graph of $f(x) = x^3 + 5x^2 - 8x - 12$.

The point A is a local maximum and point B is a local minimum, and point C is the y-intercept of $f$.




9.1 Determine the co-ordinates of C. (2)

9.2 Determine the co-ordinates of the turning points A and B. (6)

9.3 Calculate the value(s) of $x$ for which $f$ strictly increases as $x$ increases. (2)

9.4 If $h(x) = f'(x)$, determine for which value(s) of $x$ will $h(x) \leq 0$. (2)

9.5 Determine the equation of the tangent to $f$ at C. (4)

**[16]**

## ACTIVITY 2

### QUESTION 7

Given: $f(x)=x^3-2x^2-7x-4$

7.1 Write down the $y$-intercept of $f$. (1)

7.2 Show that $x-4$ is a factor of $f$. (2)

7.3 Determine the $x$-intercepts of $f$. (3)

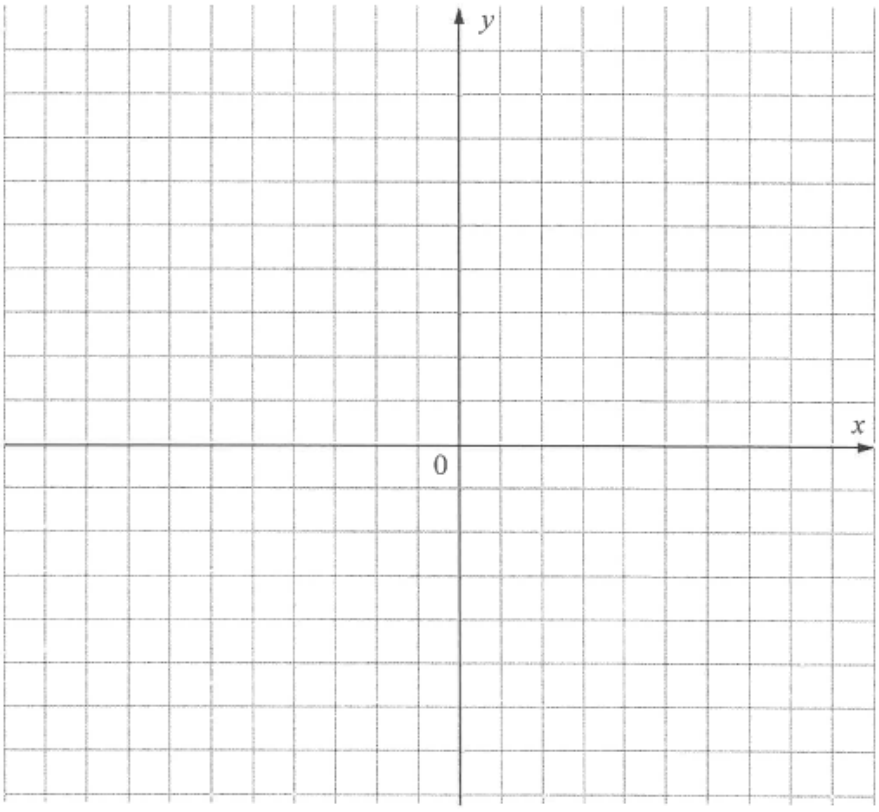
7.4 Determine the coordinates of the turning points of $f$. (5)

7.5 Sketch the graph of $f$ on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes and the turning points. (4)

7.6 Determine the value(s) of $x$ for which the graph of $f$ is decreasing. (2)

**[17]**

**QUESTION 4.1.2**



**ACTIVITY 3**

**QUESTION 7**

Given: $f(x) = -x^3 - 5x^2 + 24x$

| | | |
|---|---|---|
| 7.1 | Write down the $y$-intercept of $f$. | (1) |
| 7.2 | Factorise $f(x)$ completely. | (2) |
| 7.3 | Hence, write down the $x$-intercepts of $f$. | (1) |
| 7.4 | Determine the coordinates of the turning points of $f$. | (5) |
| 7.5 | Sketch the graph of $f$ on the grid provided in the SPECIAL ANSWER BOOK. | (3) |
| 7.6 | Use your graph to determine the values of $x$ for which $x \times f'(x) > 0$ | (3) |

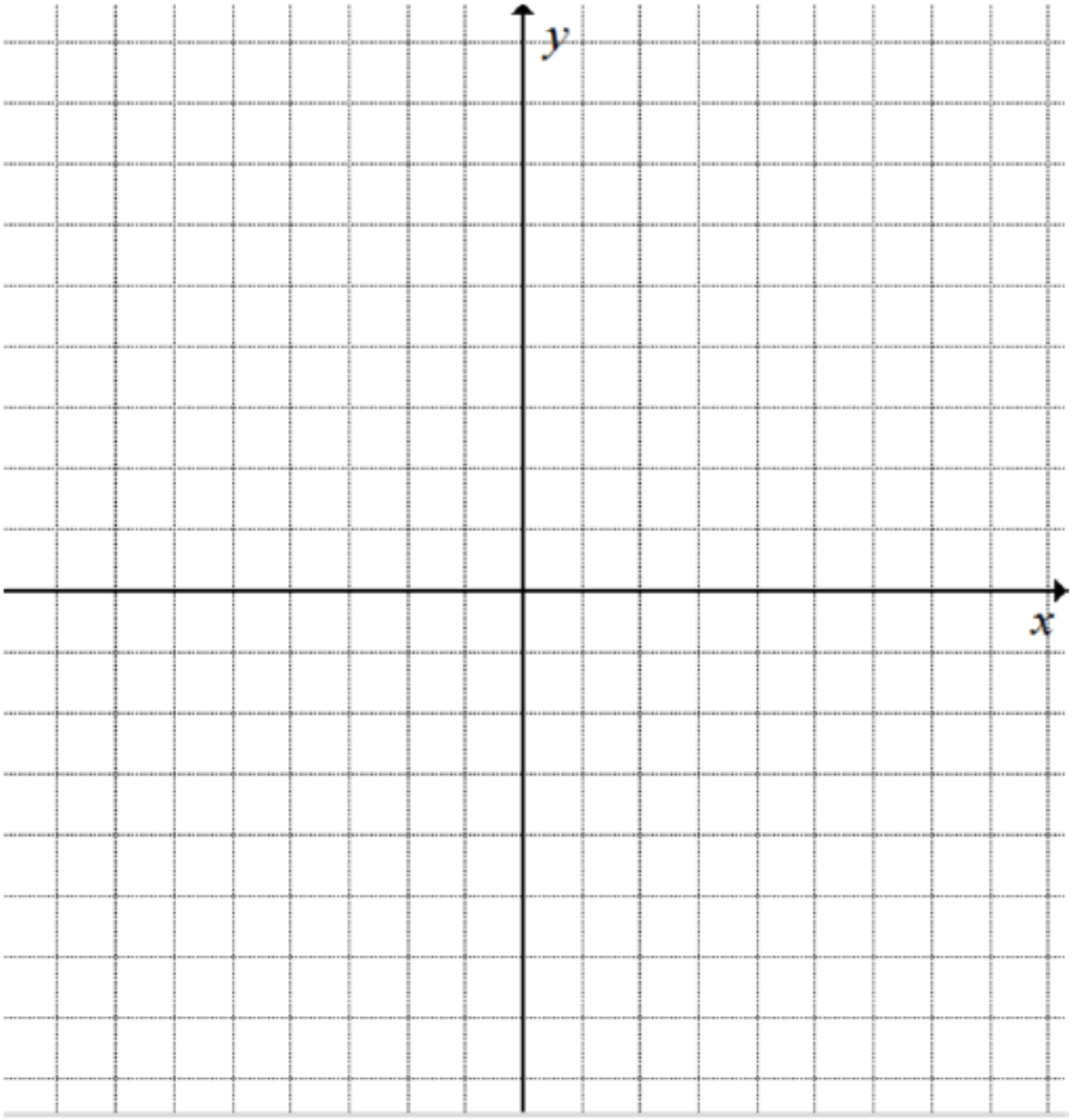
**ACTIVITY 4**

**QUESTION 7**

Given: $f(x) = -x^3 + 6x^2 - 11x + 6$

| | | |
|---|---|---|
| 7.1 | Write down the $y$-intercept of $f$. | (1) |
| 7.2 | Determine the $x$-intercepts of $f$. | (4) |
| 7.3 | Determine the coordinates of the turning points of $f$. | (5) |
| 7.4 | Sketch the graph of $f$ on the ANSWER SHEET provided, showing all the intercepts and the turning points of $f$. | (5) |
| 7.5 | Determine the possible value(s) of $x$ for which $h(x) = -11x + q$ is a tangent to $f$. | (4) |
| | | **[19]** |

**QUESTION 7.4**



## SECTION 2 : INTEGRATION

You should be able to understand the concept of integration as both a definite integral and an indefinite integral and apply standard form of integrals as a converse of differentiation.

### 1. Indefinite integrals

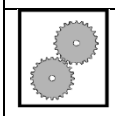
You should be able to integrate functions of the following forms :

$$\int k\,x^n dx = \frac{k\,x^{n+1}}{n+1} + C \quad , \; n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \; x > 0 \text{ and } k \in \mathbb{R} \; ; \; k \neq 0$$

$$\int k\,a^{nx} dx = \frac{k\,a^{nx}}{n \ln a} + C \quad , \; a > 0 \; ; \; a \neq 1 \text{ and } \; k, a \in \mathbb{R} \; ; \; k \neq 0$$

And this equations are provided in the formular sheet.



### WORKED EXAMPLE 3

Determine :

1. $\int 6x^5 \, dx$
2. $\int \frac{1}{2} x \, dx$
3. $\int (2x^3 - 3x^2 - \frac{7}{x}) dx$
4. $\int (3^x + \frac{3}{\pi}) \, dx$
5. $\int \left(\pi x^{\frac{2}{3}}\right) dx$
6. $\int \left(\frac{t}{x} - x^2 . \sqrt{t^2}\right) dt$

SOLUTIONS

1. $\int 6x^5 \, dx = \frac{6x^{5+1}}{5+1} + C$

$= \frac{6x^6}{6} + C$

$= x^6 + C$

2. $\int \frac{1}{2}x\,dx = \frac{1x^{1+1}}{2(1+1)} + C$

$= \frac{x^2}{4} + C$

3.

$$\int\left(2x^3 - 3x^2 - \frac{7}{x}\right)dx$$

$$= \frac{1}{2}x^4 - x^3 - 7\ln x + c$$

4.

$$\int\left(3^x + \frac{3}{\pi}\right)dx$$

$$= \frac{3^x}{\ln 3} + \frac{3}{\pi}x + c$$

5.

$$\int\left(\pi\, x^{\frac{2}{3}}\right)dx$$

$$= \frac{\pi\, x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$

$$= \frac{\pi\ 3x^{\frac{5}{3}}}{5} + C$$

6.

$$\int\left(\frac{t}{x} - x^2\sqrt{t^2}\right)dt$$

$$= \int\left(\frac{t}{x} - x^2 t\right)dt$$

$$= \frac{t^2}{2x} - \frac{x^2t^2}{2} + C$$

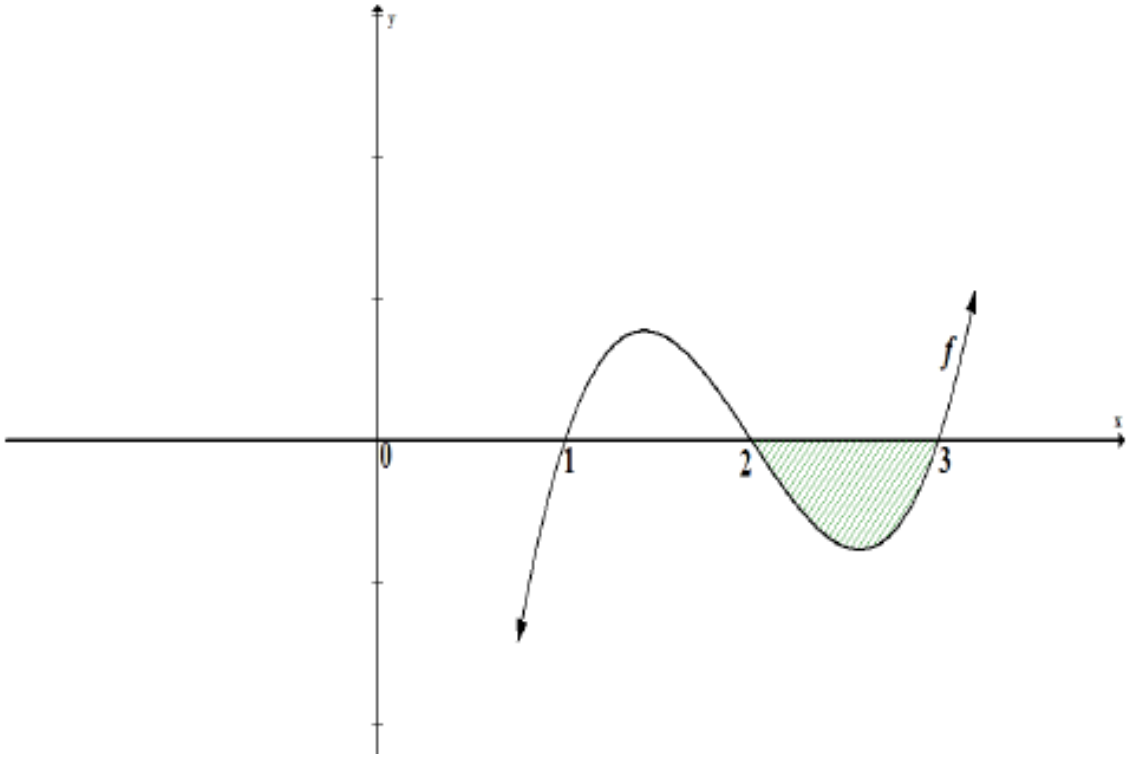
## 2. Definite Integrals

You must be able apply integration to determine the magnitude of an area included by a curve and the $x$-axis or by a curve, the $x$-axis and the ordinates $x = a$ and $x = b$ where $a, b \in Z$. If $f$ and $g$ as continuous functions, then the following properties holds :

| | |
|---|---|
| 1 | $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ |
| 2 | Constants may be factored through the integral sign: $\int_b^a kf(x)\,dx = k\int_b^a f(x)\,dx$ |
| 3 | The integral of a sum (and /or difference) of integrands is the sum (and /or) difference) of the integrals: $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)\,dx + \int_a^b g(x)dx$ |

### WORKED EXAMPLE 4

The diagram below shows the shaded area bounded by the curve defined by $f(x) = x^3 - 6x^2 + 11x - 6$.




Determine the area of the shaded region of the graph of $f$ bounded by the graph and the $x$ −axis between $x = 2$ and $x = 3$.

SOLUTION

$$A_x = -\int_2^3 y\,dx$$

$$= -\left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_2^3$$

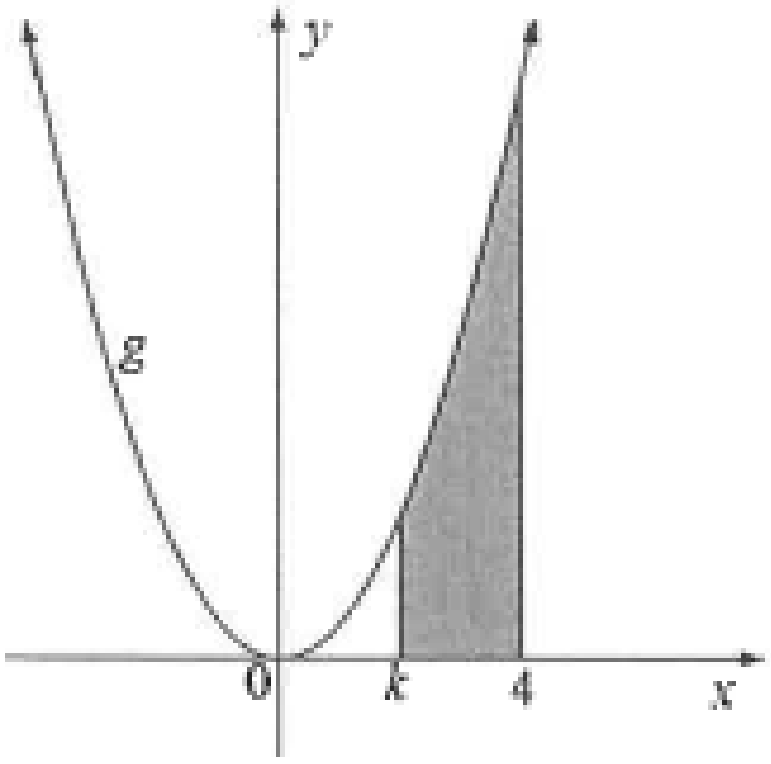
$$= -\left(\frac{1}{4}(3)^4 - 2(3)^3 + \frac{11}{2}(3)^2 - 6(3)\right) - \left(\frac{1}{4}(2)^4 - 2(2)^3 + \frac{11}{2}(2)^2 - 6(2)\right)$$

$$= -\left[\left(-\frac{9}{4}\right) - (-2)\right]$$

$$= \frac{1}{4} = 0{,}25 \text{ square units/}\textit{vierkantseenheid}$$

**WORKED EXAMPLE 5**

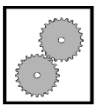
The sketch below represents the area bounded by the function $g$ defined by $g(x) = 3x^2$ and the points where $x = k$ and $x = 4$




Determine the value of $k$ if bounded area is 56 square units.

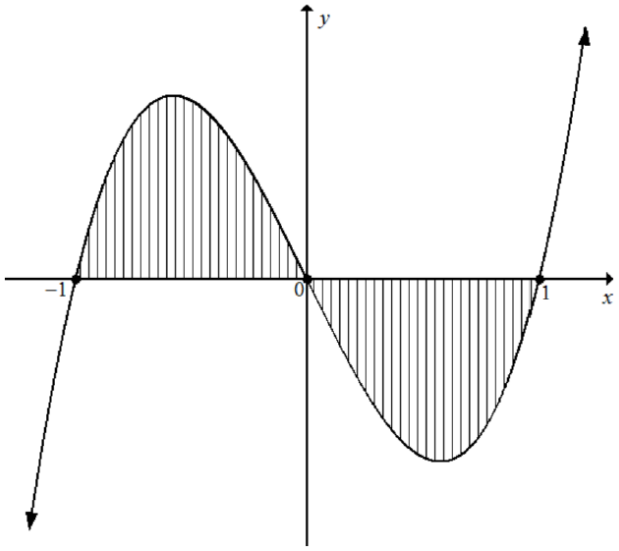
SOLUTION

$$\begin{aligned} \text{A} &= \int_k^4 g(x)\,dx \\ &= \int_k^4 3x^2\,dx \\ &= x^3\Big]_k^4 \\ &= (4)^3 - (k)^3 \\ \therefore\ & (4)^3 - (k)^3 = 56 \\ \therefore\ & k^3 = 8 \\ \therefore\ & k = 2 \end{aligned}$$



**WORKED EXAMPLE 6**

The sketch below shows the shaded area bounded by the curver of the function defined by $f(x) = x^3 - x$ and the $x$ −axis between pointts $x = -1$ and $x = 1$.




Determine (showing ALL calculations) the shaded are bounded by the curve $f$ and the $x$ −axis between the points where $x = -1$ and $x = 1$.

SOLUTION

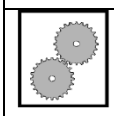
$$\text{Area} = \int_{-1}^{0}\left(x^3 - x\right)dx + \int_{0}^{1}\left(x^3 - x\right)dx$$

$$\text{Area} = \left[\frac{x^{3+1}}{3+1} - \frac{x^{1+1}}{1+1} + C\right]_{-1}^{0} + \left[\frac{x^{3+1}}{3+1} - \frac{x^{1+1}}{1+1} + C\right]_{0}^{1}$$

$$\text{Area} = \left(0 - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right)\right) + \left(\frac{(1)^4}{4} - \frac{(1)^2}{2} - 0\right)$$

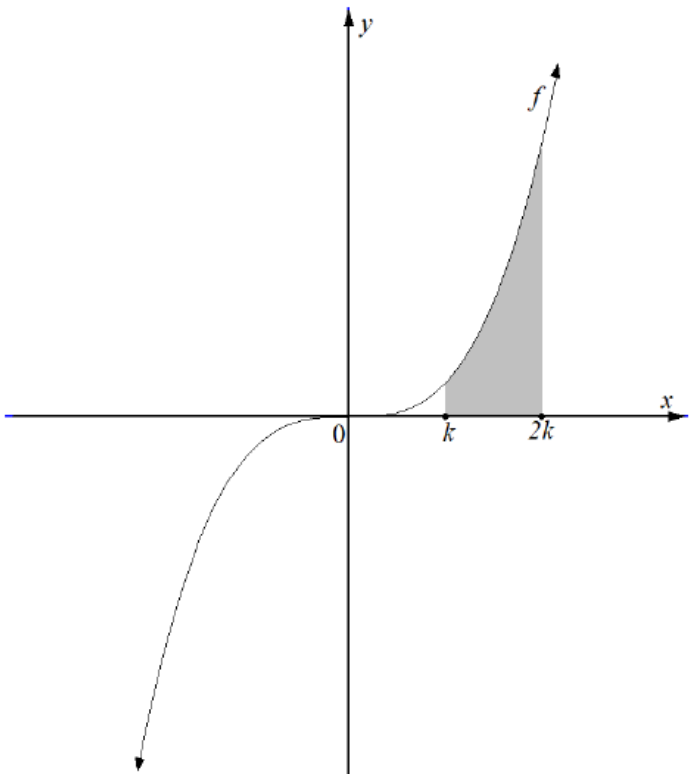
$$\text{Area} = \frac{1}{4} + \frac{1}{4}$$

$$\text{Area} = \frac{1}{2}$$



**WORKED EXAMPLE 7**

The sketch below represents the function defined by $f(x) = 4x^3$. The shaded area that is bounded by the curve of *f,* the *x*-axis and the ordinates $x = k$ and $x = 2k$ is equal to 36 015 square units.




It was claimed to a Technical Mathematics class that the value of $2k$ lies between 10 and 20. Determine, showing ALL calculations, whether the claim is valid.

SOLUTION

$$A = \int_k^{2k} f(x)\,dx$$

$$= \int_k^{2k} 4x^3\,dx$$

$$= \left[ x^4 \right]_k^{2k} \quad \textbf{OR/}\textit{OF} \quad = \left[ \frac{4x^{3+1}}{3+1} \right]_k^{2k}$$

$$(2k)^4 - (k)^4 = 36\,015$$

$$15k^4 = 36\,015$$

$$k^4 = 2\,401$$

$$k = \sqrt[4]{2\,401} = 7$$

$\therefore 2k = 14$

$\therefore 10 < 2k < 20$

$\therefore$ The claim is **VALID** /

**ACTIVITIES**

**ACTIVITY 1**

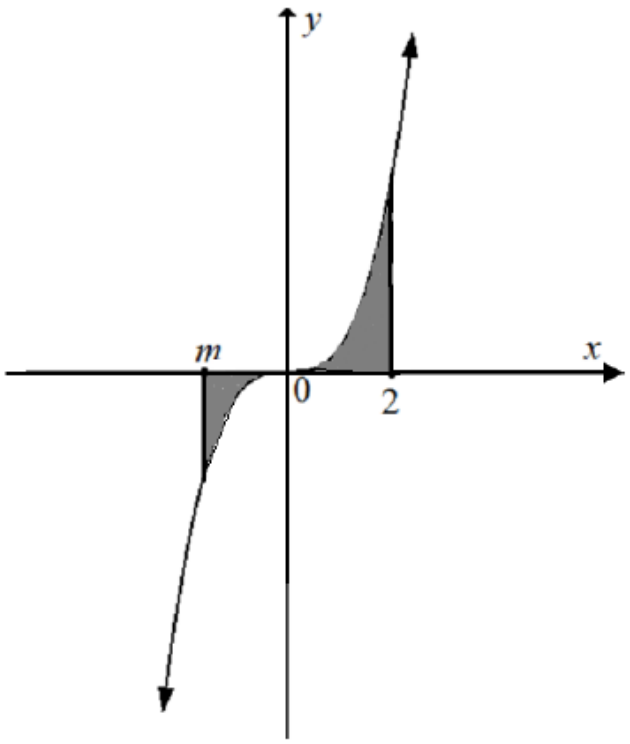
**QUESTION 9**

9.1 Determine the following integrals:

9.1.1 $\int\left(\frac{1}{x}-1\right)dx$ (3)

9.1.2 $\int\left(x^5+6x\right)dx$ (2)

9.2 The sketch below represents the shaded bounded area of the curve of the function defined by $f(x)=x^3$ and the $x$-axis where $x=m$ and $x=2$.




Determine the value of $m$ if the bounded area is 3,75 square units. (7)

**ACTIVITY 2**

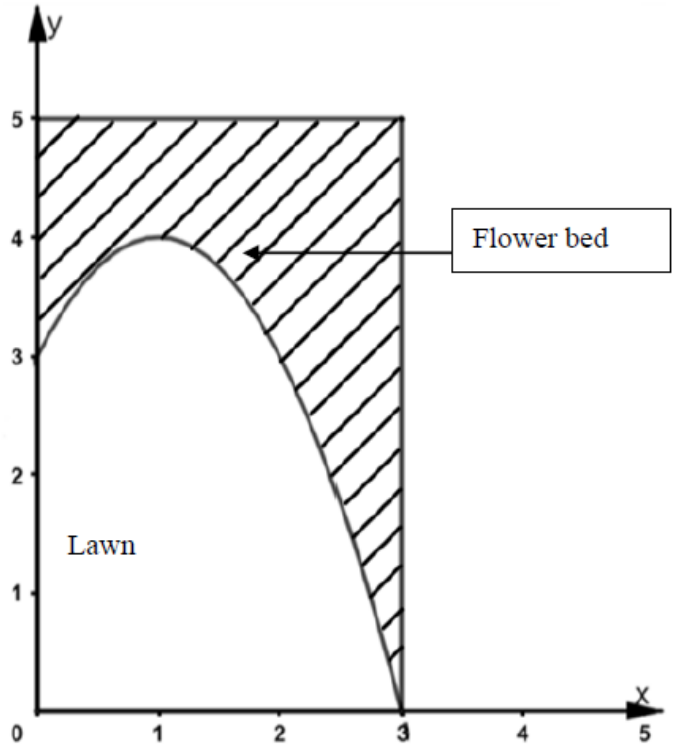
**QUESTION 9**

9.1 Determine the following integrals:

9.1.1 $\int \left(\frac{2}{5x} - 2^{3x} + 4\right) dx$ (3)

9.1.2 $\int \left(\frac{2a^3 - 54}{a - 3}\right) da$ (5)

9.2 A gardener is considering a new design for his client's garden. He has a rectangular piece of lawn measuring 5 m by 3 m, and he wants to dig up a part of it to include a flower bed. He draws a plan of the lawn and flower bed on a piece of graph paper, taking the bottom and left-hand edges as the axes, and chooses a scale such that 1 unit along each axis represents 1 metre on the ground.




The equation of the curved edge of the flower bed is given by:

$y = -x^2 + 2x + 3$

Calculate the area of the flower bed. (5)

**[13]**

**ACTIVITY 3**

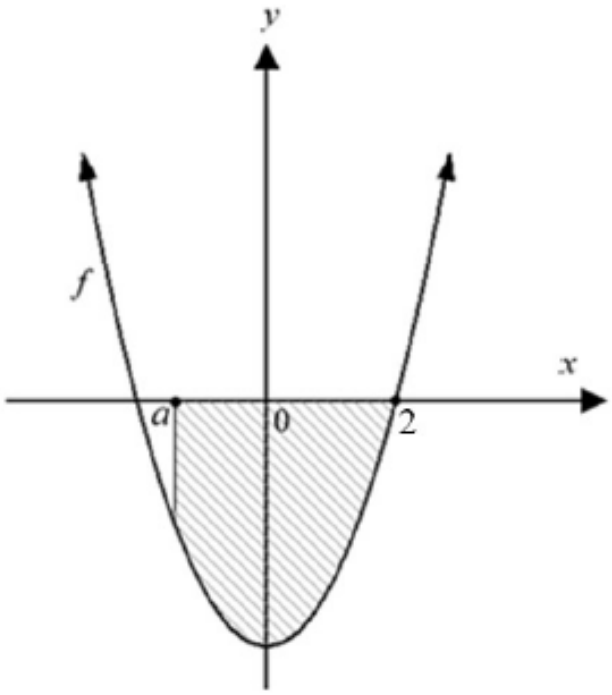
**QUESTION 9**

9.1 Determine the following integrals:

9.1.1 $\int x^{\frac{1}{2}}\,dx$ (2)

9.1.2 $\int\left(x^{-2}-\frac{\pi}{x}\right)dx$ (2)

9.2 The sketch below represents the shaded area bounded by the function defined by $f(x)=5x^2-20x$ and the $x$-axis from $x=a$ to $x=2$




Determine the value of $a$ if the shaded area is 45 square units. (8)

**[12]**

## ACTIVITY 4

### QUESTION 9

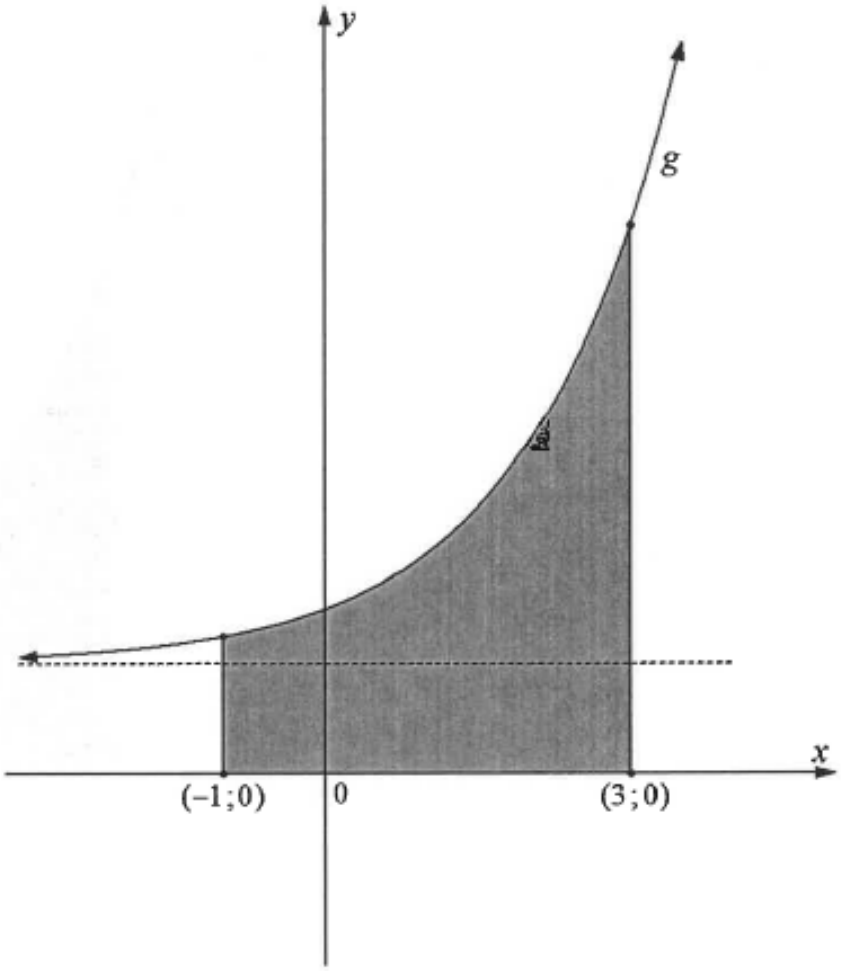
9.1 Determine the following integrals:

9.1.1 $\int x^4 \, dx$ (2)

9.1.2 $\int \left(2\pi + \frac{4}{x}\right) dx$ (2)

9.1.3 $\int \left(2x^{-3}\right)^3 \, dx$ (2)

9.2 The sketch below shows function $g$ defined by $g(x) = 2^x + 2$ and the shaded area bounded by the curve of $g$ and the $x$-axis between $x = -1$ and $x = 3$




Determine the area of the shaded region. Show ALL working details. (6)

**[12]**

## ACTIVITY 5

### QUESTION 9

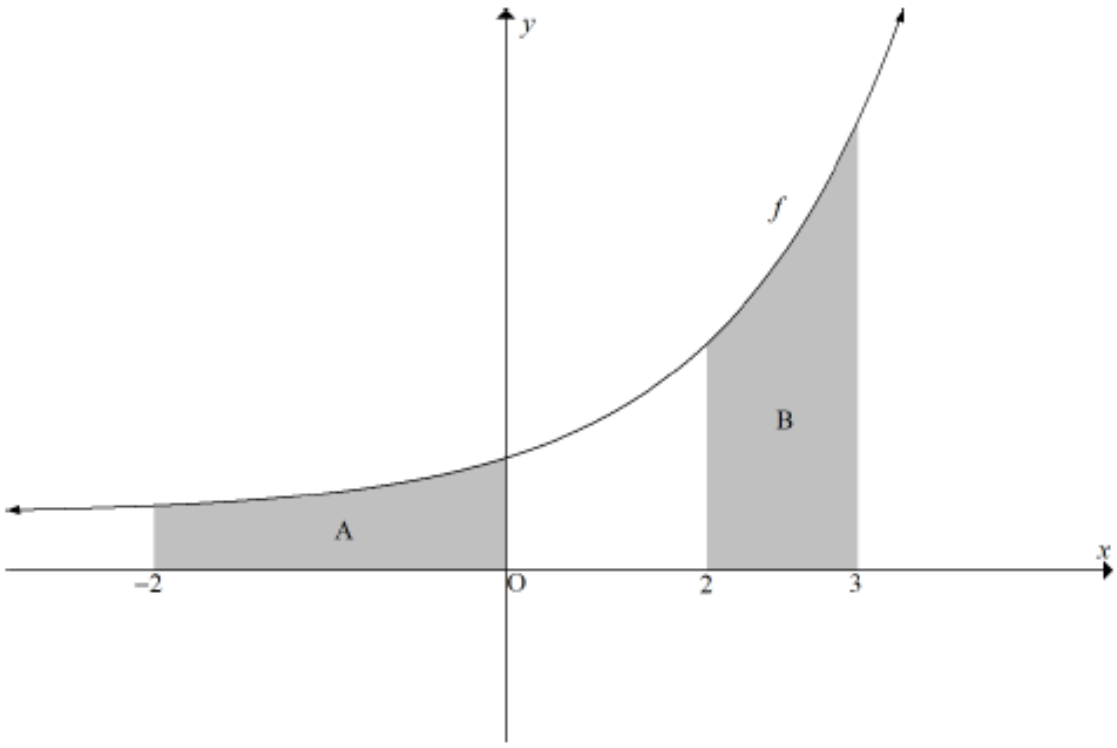
9.1 Determine the following integrals:

9.1.1 $\int -\frac{6}{x}dx$ (2)

9.1.2 $\int (3x-4)(x+2)dx$ (4)

9.2 The sketch below shows function $f$ defined by $f(x) = 2^x$

- A represents the shaded area bounded by the graph of $f$, the $x$-axis and the ordinates $x = -2$ and $x = 0$
- B represents the shaded area bounded by the graph of $f$, the $x$-axis and the ordinates $x = 2$ and $x = 3$



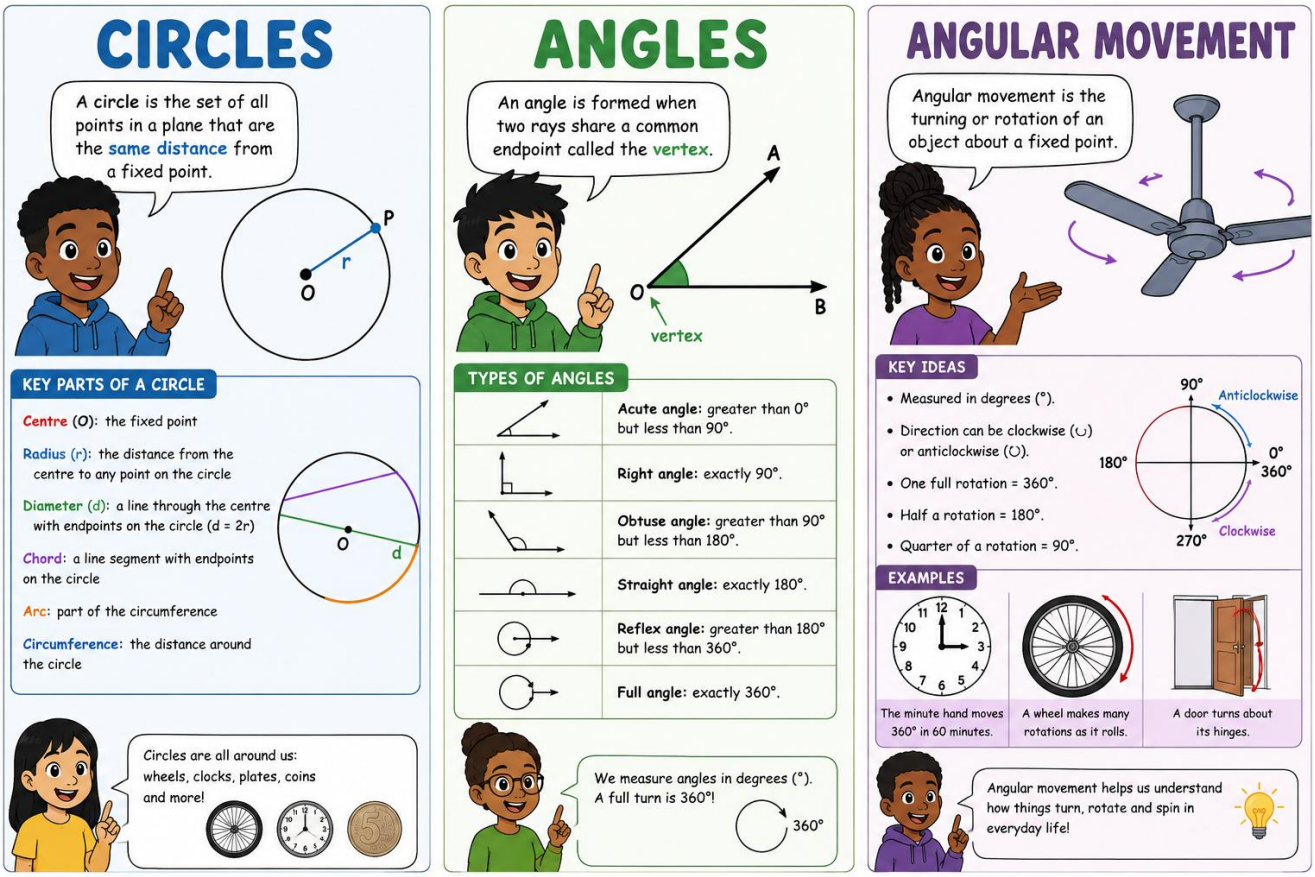

9.2.1 Determine: $\int 2^x\,dx$ (1)

9.2.2 A learner claims that area B is equal to 4 times that of area A.

Verify, showing ALL calculations, whether the learner's claim is VALID. (7)

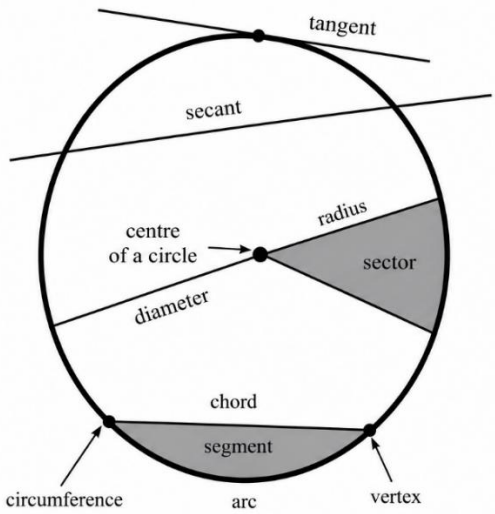
**[14]**

## SECTION 3 : CIRCLE, ANGLES AND ANGULAR MOVEMENT




### 3.1 Parts of the circle

When working with circles, there is some terminology (words) that help us to identify different parts of the circle. These are:




**Radius:**
A line from the centre to any point on the circumference of the circle.

**Diameter:**
A line passing through the centre of the circle. It is double the length of the radius.

**Chord:**
A line with end-points on the circumference.

**Tangent:**
A line touching the circle at only one point.

**Secant:**
A line passing through two points on the circle.

**Centre of a circle:**
The point from which all points on the circle are the same distance apart.

**Circumference:**
The length of the boundary. It is the circle's perimeter (or boundary).

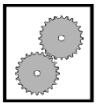
**Vertex:**
The point where two or more line segments (or edges) meet.

### 3.2 Degrees and Radians

A complete revolution is: $360° = 2\pi$ radians

Sometimes we may need to convert between degrees and radians:

- to convert from degrees to radians , multiply the number of degrees by $\frac{\pi\ radians}{180°}$
- to convert from radians to degrees , multiply the number of radians by $\frac{180°}{\pi\ radians}$



**WORKED EXAMPLE 8**

1. Convert the following to radians

    1.1 $30°$

    1.2 $75°$

2. Conver the following to degrees :

    2.1 $\frac{\pi}{3}\ radians$

    2.2 $\frac{2\pi}{9} radians$

Solutions

1.

1.1 $30° \times \frac{\pi\ radians}{180°} = \frac{\pi}{6}\ radians$

1.2 $75° \times \frac{\pi\ radians}{180°} = \frac{5\pi}{12}\ radians$

2

2.1 $\frac{\pi}{3}\ radians \times \frac{180°}{\pi\ radians} = 60°$

2.2 $\frac{2\pi}{9} radians \times \frac{180°}{\pi\ radians} = 40°$

**Radian measures and arc length**

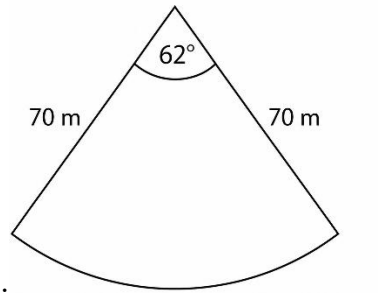
To measure an angle, we mostly use degrees. The circumference of the circle is divided into 360 equal parts. We call each of these equal parts a degree. Another common measure to measure an angle is the radian. The radian uses the relationship between the radius of a circle and the length of an arc of the circle. The distance travelled along the circumference of a circle is called the arc length. Formula: $s = r\theta$

where: $s$= arc length

- $r$= radius
- $\theta$= central angle in radians

**WORKED EXAMPLE 9**

An enclosure is in the shape of a 72° sector of a circle with a radius of 80 m.




Calculate the cost of fencing the enclosure, if fencing costs R360 per metre.

Solution :

To calculate the cost of fencing, first calculate the perimeter.

Perimeter = twice the radius plus the arc length

$$\text{Arc length} = r\theta = 80 \times 72° \times \frac{\pi}{180°} = 100{,}53 \text{ m}$$

$$\text{Perimeter} = 2 \times 80 + 100{,}53 = 261 \text{ m}$$

$$\text{Cost of fencing} = \text{R}360 \times 261$$
$$= \text{R}93\,600{,}00$$

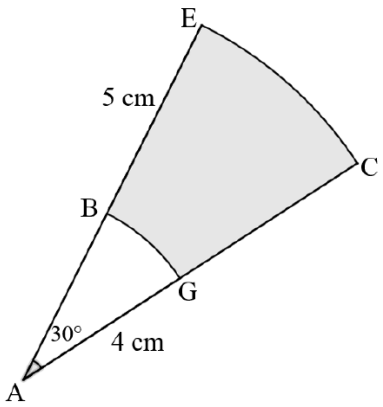
### 3.3 Area of a Sector

A sector is the region enclosed by two radii and an arc.

Formula: $A = \frac{1}{2}r^2\theta$

**WORKED EXAMPLE 10**

In the diagram below EC and BG are arc lengths of circle sectors with different radii, that both subtend an angle of 30°. The radii, AG, of the smaller sector is 4cm and BE is 5cm.




1. Convert $\hat{A}$ to radians.
2. Determine the arc length of BG.
3. Determine the area of sector AEC.
4. Hence, determine the area of the shaded area BECG

Solution

1. $\hat{A} = 30° \times \frac{\pi\ radians}{180°}$

$= \frac{\pi}{6}\ radians$

2. $s = r.\theta$

$= (4)\left(\frac{\pi}{6}\right)$

$BG = \frac{2\pi}{3}\ cm$

3. Area of sector AEC $= \frac{r^2.\theta}{2}$

$= \frac{(9)^2.\left(\frac{\pi}{6}\right)}{2}$

$= \frac{27\pi}{4} cm^2$

4. Area of sector ABG $= \frac{r^2.\theta}{2}$

$= \frac{(4)^2.\left(\frac{\pi}{6}\right)}{2}$

$= \frac{4\pi}{3} cm^2$

Area of BECG $= \frac{27\pi}{4} - \frac{4\pi}{3}$

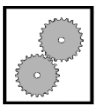
$= \frac{65\pi}{12} cm^2$

**3.4 Circumferential (Linear) Velocity and angular velocity**

Angular velocity ω (omega) can be determined by multiplying the radians in one revolution ($2\pi r$ ) with the revolutions per seconds ($n$). Thus we can say: $\omega = 2\pi n$

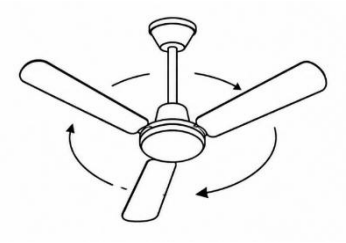
Circumferential velocity is the speed of an object moving along a circular path.

Formula: $v = r\omega$ , Units: metres per second (m/s)

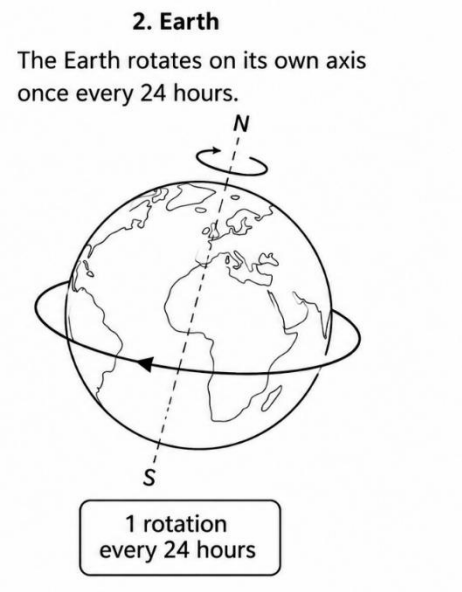


**WORKED EXAMPLE 11**

1. A ceiling fan rotates 30 times per minute. Calculate the angular velocity of the fan in
degrees per hour.



2. The Earth rotates on its own axis once every 24 hours. Calculate the Earth's angular velocity of rotation in degrees per hour.




Solution:

1.

The angular velocity $= 2\pi r \times 30\ radian$

$$= \frac{60\pi\ radians}{min} \times 60 \times \frac{min}{hour}$$

$$= 3600\,\frac{\pi\ radians}{min} \times \frac{min}{hour}$$

$$= \frac{3600\pi\ radians}{hour}\left(\frac{180}{\pi}\right)$$

$$= 648°\ per\ hour$$

2. The complete circle is $2\pi$ radians, therefore, the angular velocity of the Earth's rotation equals $\frac{2\pi}{24}$ radians per hour.

$$= \frac{2\pi}{24}\ \left(\frac{180}{\pi}\right)$$

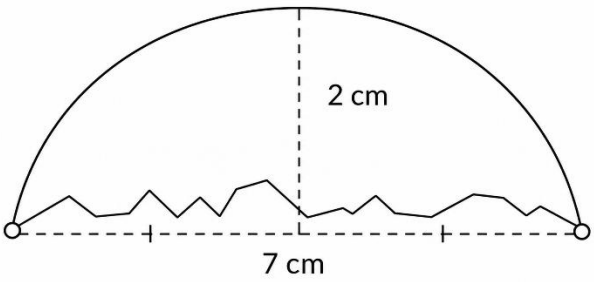
$$= 15°\ per\ hour$$

**3. 5 The relationship between a segment's height, diameter and its width**

We use the following formula to determine the relationship between the diameter, the width and the height of the segment : $4h^2 - 4dh + x^2 = 0;$ where $h$ is the height of the segment, $d$ is the diameter and $x$ is the width of the segment.

**WORKED EXAMPLE 12**

The diagram below shows a broken plate.




Use the given information to calculate the diameter of the original plate.

$h = 2\ cm$ and $x = 7\ cm$

$4h^2 - 4dh + x^2 = 0$

$4(2)^2 - 4d(2) + (7)^2 = 0$

$16 - 8d + 49 = 0$

$65 = 8d$

$d = 8{,}125\ cm$

**SUMMARY**

# ANGULAR VELOCITY, ROTATIONAL FREQUENCY AND CIRCUMFERENTIAL VELOCITY

## 1. ANGULAR VELOCITY

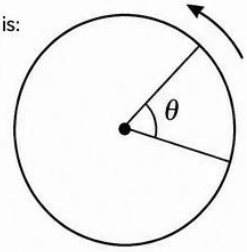
- angular velocity refers to how fast an angle is changing with respect to time. It can also be described as the change in angle over the change in time. The symbol for angular velocity is $\omega$ (omega) and the unit of measurement is **radians per minute**.

**Note:**
For calculations radians per minute must be converted to radians per seconds (rad.s$^{-1}$) since the standard unit of time is seconds.

- the formula for calculating Angular velocity is:

$$\omega = \frac{\theta}{t}$$

where: $\omega$ = angular velocity (rad.s$^{-1}$)
$\theta$ = angle (radians)
$t$ = time (seconds)




## 2. ROTATIONAL FREQUENCY

- rotational frequency is the number of revolutions per second (rev/s). The symbol for rotational frequency is $n$. The formula for rotational frequency is:

$$n = \frac{\text{no of revolutions}}{t}$$

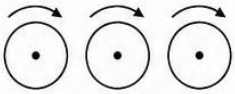
- in one revolution per second, the angular velocity is: $\omega = \frac{\theta}{t} = \frac{2\pi \text{ radians}}{1s} = 2\pi \text{rad.s}^{-1}$



1 revolution

- for $n$ number of revolution: $\omega = n \times 2\pi = 2n\pi$,

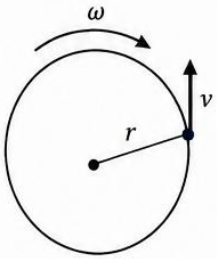
hence $\boxed{\omega = 2n\pi}$



$n$ revolutions

## 3. CIRCUMFERENTIAL (PERIPHERAL) VELOCITY

- circumferential/peripheral velocity refers to how fast the object is travelling with respect to time. Hence the formula for calculating circumferential velocity is $v = \frac{s\,(\text{metres})}{t\,(\text{seconds})} = \text{m/s}$.




- in one revolution per second, the Circumferential velocity is: $v = \frac{s}{t} = \frac{\pi D}{1s} = \pi D \text{ m/s}$
- for $n$ number of revolution $v = n \times \pi D$ or $v = \pi Dn$
- if we further explore the above formula $v = \pi Dn = \pi \times 2r \times n = 2\pi nr = \omega r$ or $v = \omega r$.

### SUMMARY OF FORMULAE

| Quantity | Formula | Unit |
|---|---|---|
| Angular velocity | $\omega = \frac{\theta}{t}$ | rad.s$^{-1}$ |
| Rotational frequency | $n = \frac{\text{no of revolutions}}{t}$ | rev/s |
| Angular velocity | $\omega = 2n\pi$ | rad.s$^{-1}$ |
| Circumferential velocity | $v = \frac{s}{t}$ | m/s |
| Circumferential velocity | $v = \pi Dn$ | m/s |
| Circumferential velocity | $v = 2\pi nr$ | m/s |
| Relationship | $v = \omega r$ | m/s |

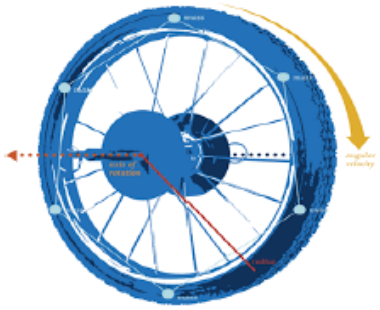
**KEY:**
$\omega$ = angular velocity (rad.s$^{-1}$)
$\theta$ = angle (radians)
$t$ = time (seconds)
$n$ = rotational frequency (rev/s)
$s$ = distance travelled (metres)
$D$ = diameter (metres)
$r$ = radius (metres)
$v$ = circumferential velocity (m/s)

**ACTIVITIES**

**ACTIVITY 1**

**QUESTION 10**

A wheel which turns through an angle of $\theta$ radians in time $t$ seconds is given by $\theta(t) = 256t - 24t^2$.



10.1 Calculate the instantaneous angular velocity after 3 seconds. (4)

10.2 Calculate the instantaneous angular acceleration after 3 seconds if:

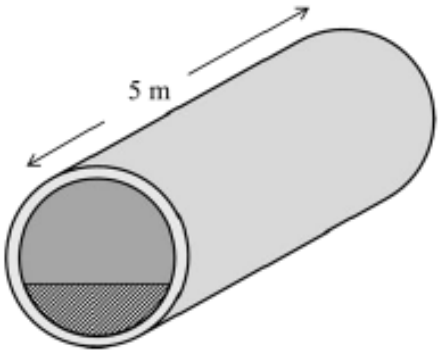
$\theta'(t) = -48t + 256$ (2)

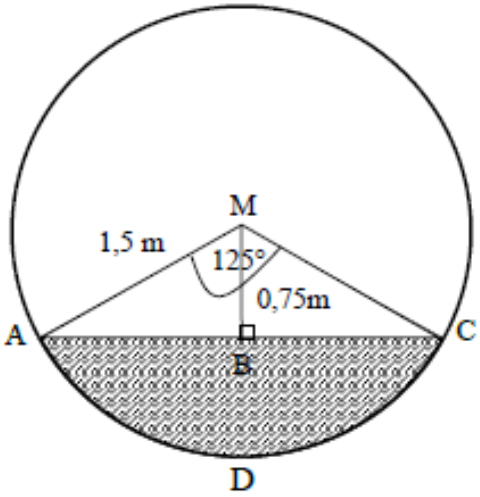
**[6]**

**ACTIVITY 2**

## QUESTION 11

The diagram below, shows a cross section of a water pipe partially filled with water. The radius of the pipe is 1,5 m and the length is 5 m. The segment ABCD represents the water level. M is the centre of the circle, $A\hat{M}C = 125°$ and MB = 0,75 m.




11.1 Calculate the length of AB, correct to ONE decimal place. (2)

11.2 Calculate the area of ΔAMC. (3)

11.3 Convert 125° to radians. (1)

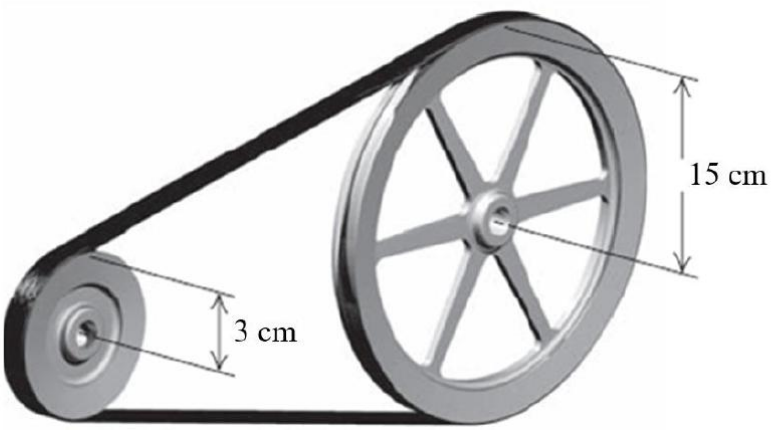
11.4 Calculate the area of sector ADCM. (3)

11.5 Hence calculate the area of segment ABCD. (2)

11.6 Calculate the volume of water in the pipe if it is given that the volume of a prism = area of the base x height. (2)
**[13]**

**ACTIVITY 3**

10.2 Two pulleys are connected by a belt so that one pulley rotates. The linear speeds of the belt and both pulleys are the same. The radius of the smaller pulley is 3 cm, and the radius of the larger pulley is 15 cm. The smaller pulley is turning at 120 revolutions per minute.

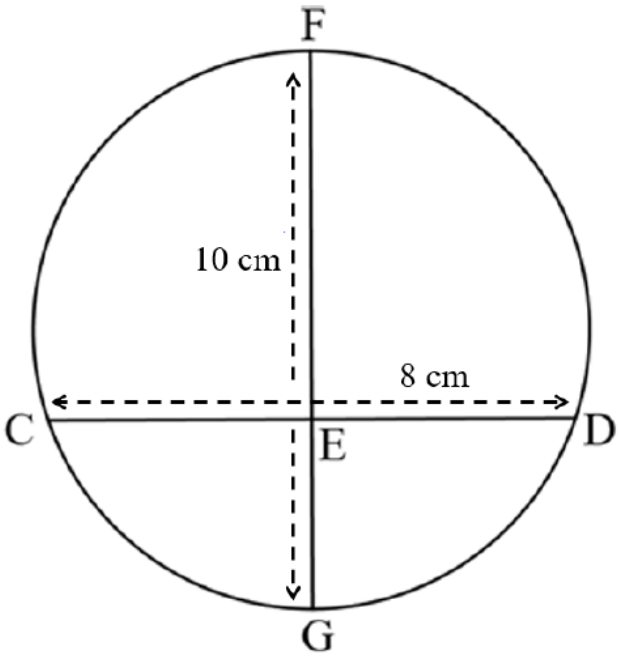



10.2.1 Determine the angular velocity of the smaller pulley. (3)

10.2.2 Hence, determine the linear velocity of the smaller pulley. (3)

10.2.3 Hence, determine the angular velocity of the larger pulley. (3)

10.3 In the diagram below, FG is the diameter of the circle, with a length of 10 cm. CD is a chord of the circle with a length of 8 cm. CD divides the circle into two segments.




Determine the height of the smaller segment. (5)

**[26]**

## ACTIVITY 4

9.2 A point on the circumference of a driver wheel driving a belt moves at a speed of 3 m/s. The diameter of the driver wheel is 0,6 m.

Calculate:

9.2.1 the distance covered by this point in 20 s (2)

9.2.2 the number of revolutions per second (3)

9.2.3 the time it takes the wheel to complete 12 revolutions. (2)

## ACTIVITY 5

### QUESTION 10

A wheel with a diameter 250 mm, has a circumferential velocity of 108 kilometres per hour.

10.1 Convert 108 km/h to m/s. (2)

10.2 Determine the rotational frequency of the wheel in seconds. (5)

10.3 Determine the angular velocity of the wheel in seconds. (3)

10.4 Determine the distance, in km, a point on the wheel will cover in 10 min. (3)

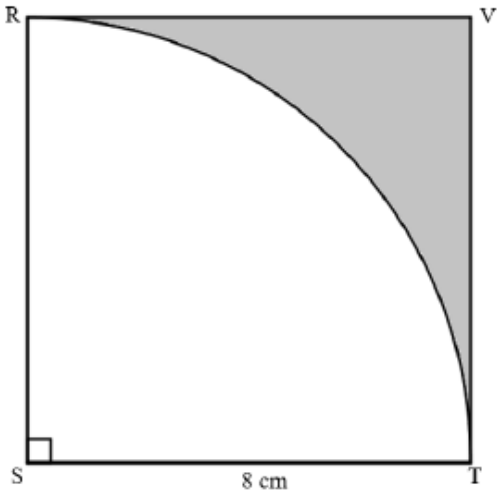
10.5 Determine how long it will take the wheel to make 20 revolutions. (2)

**[15]**

## ACTIVITY 6

### QUESTION 11

11.1 In diagram below, RSTV is a square with sides 8 cm.
RT is an arc of the sector RST. (2)

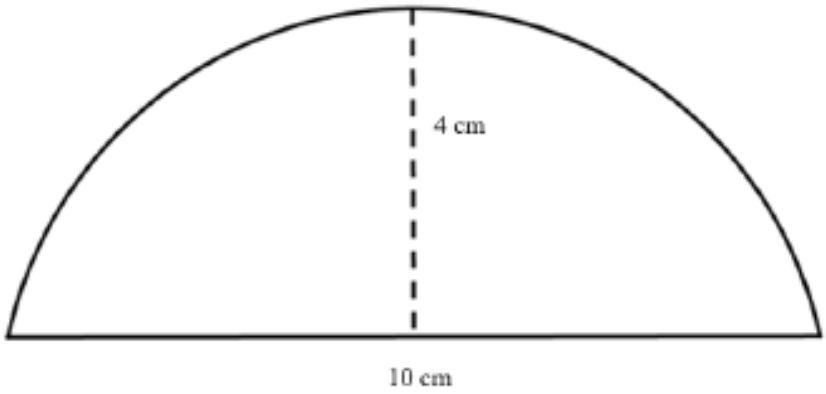



11.1.1 Determine the length of arc RT. (3)

11.1.2 Determine the area of sector RST. (3)

11.1.3 Hence, calculate the area of the shaded area. (3)

11.2 The diagram below is a minor segment of a circle with height 4 cm and chord 10 cm.




Determine the length of the radius of the circle. (5)

**ACTIVITY 7**

**QUESTION 10**

10.1 A drone has propeller blades where each blade has a length of 30 cm in diameter, rotating at 5 280 revolutions per minute.



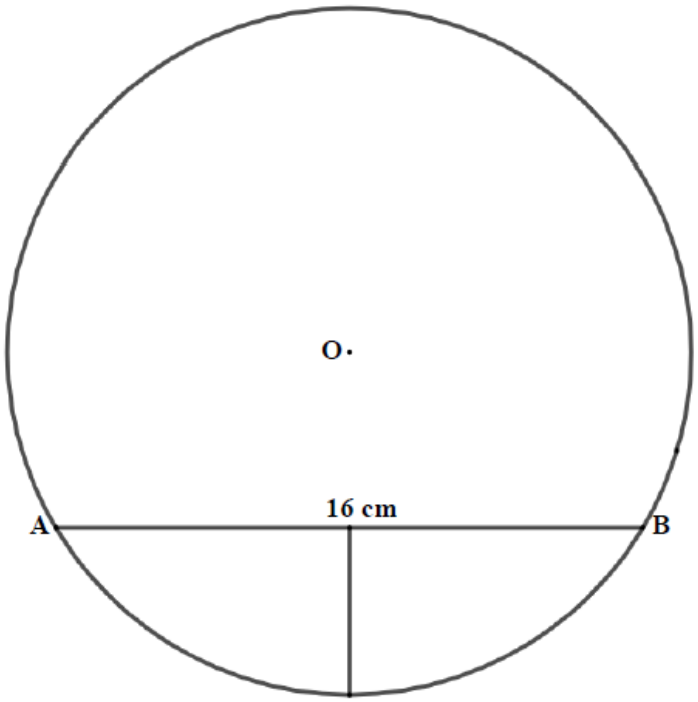

Calculate the following for one rotating blade:

10.1.1 The rotational frequency in revolutions per second (1)

10.1.2 The circumferential velocity in metres per second (3)

10.1.3 The angular velocity in radians per second (2)

10.2 Calculate the larger height of the two segments in a circle with centre O which has a chord AB of 16 cm and a diameter of 18 cm. (5)



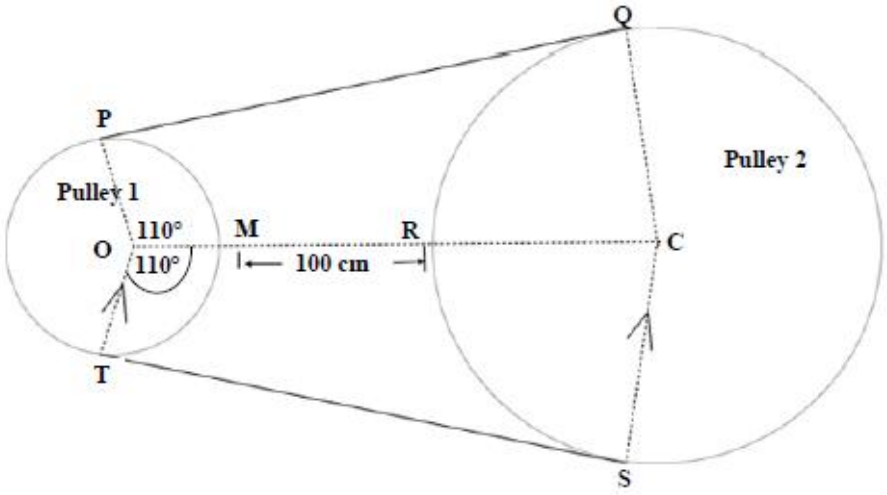
## ACTIVITY 8

### QUESTION 11

A pulley, in mechanics, is a wheel that carries a flexible rope, cord, cable, chain, or belt on its rim. Pulleys are used, singularly or in combination, to transmit energy and motion. Together with a lever, wedge, wheel, axle, and screw, the pulley is considered one of the five simple machines. Below is a picture of a set of pulleys.



Below is a diagram, not drawn to scale, of two pulleys with a belt connecting the two.
MR = 100 cm, TS = 220 cm, TS = PQ, TO ∥ SC and $\hat{TOC} = 110°$.
Pulley 1 has a diameter of 30 cm and Pulley 2 has a diameter of 50 cm.




11.1 Calculate the magnitude of $\hat{SCR}$. (1)

11.2 Determine the length between the two centres of the pulleys. (2)

11.3 Convert 70° to radians. (2)

11.4 Calculate the reflex angle ($\hat{QCS}$) of arc length QS if $\hat{SCR} = \hat{RCQ}$. (2)

11.5 Calculate QS, the arc length of Pulley 2. (3)

11.6 Calculate the total length of the belt. (4)
**[14]**

**ACTIVITY 9**

**QUESTION 10**

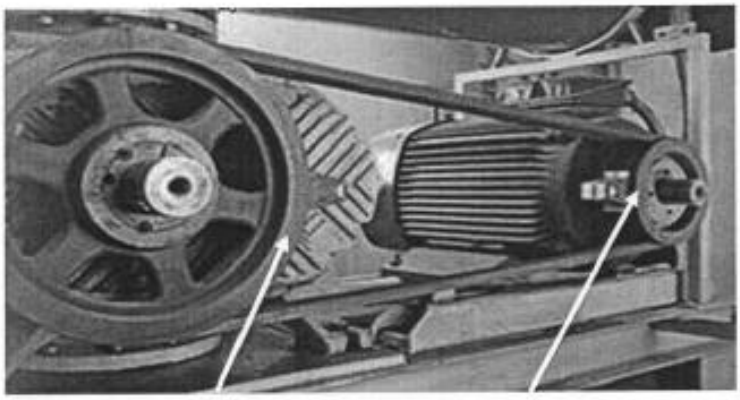
10.1 The picture and diagram below show two circular pulleys, A and B, connected with a belt and moving anti-clockwise.

Pulley A has a radius of 50 cm and pulley B has a radius of 20 cm.

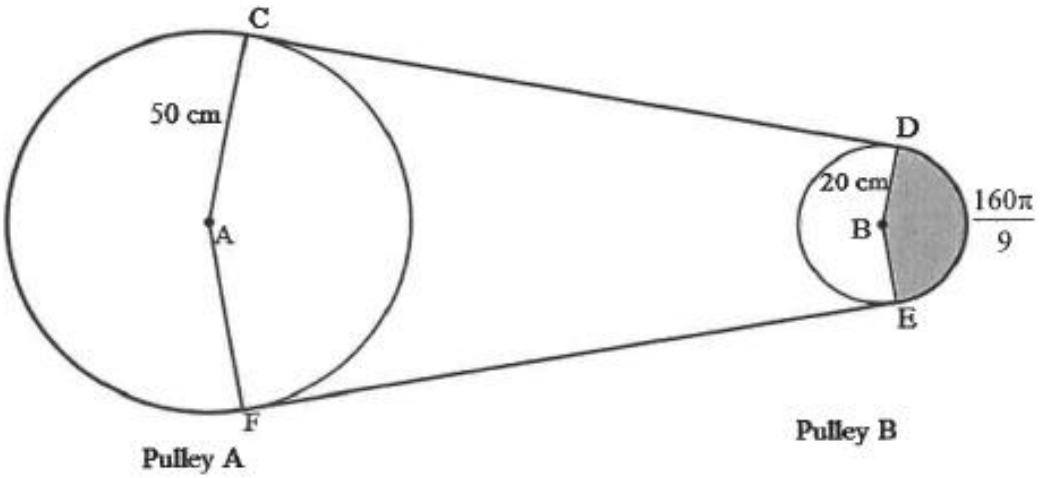
The belt covers $\frac{5}{9}$ of the arch length of pulley A.

The belt forms tangents to the pulleys at points C, D, E and F.

The arch length of DE = $\frac{160\pi}{9}$ cm.



Pulley A Pulley B



10.1.1 Show that reflex $C\hat{A}F = 200°$ (1)

10.1.2 Convert reflex $C\hat{A}F = 200°$ to radians. (1)

10.1.3 Hence, determine the major arc length of CF. (3)

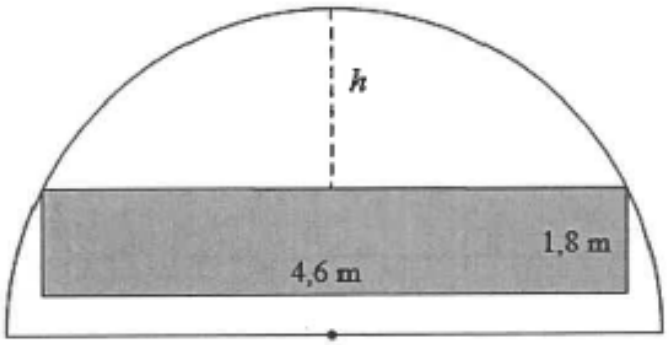
10.1.4 Pulley A rotates at 500 revolutions per minute (r/min).

(a) Calculate the circumferential velocity (cm/min) of a particle on the belt at point F. (3)

(b) Hence, determine, in revolutions per second, the rotational frequency of pulley B. (4)

10.1.5 Determine the area of the shaded minor sector DBE. (3)

10.2 A rectangular advertisement board with a length of 4,6 metres and a breadth of 1,8 metres will be positioned on the semicircular wall as indicated in the diagram below. The height $h$ from the top part of the rectangular board to the top part of the semicircular wall is 0,72 metres longer than the breadth of the rectangular advertisement board.




10.2.1 Determine the value of height $h$. (1)

10.2.2 Hence, calculate the length of the diameter of the semicircular wall. (4)
**[20]**

**ACTIVITY 10**

**QUESTION 10**

10.1 A train moving on a circular track with a diameter of 14 6425 km, takes 50 minutes to complete one revolution.

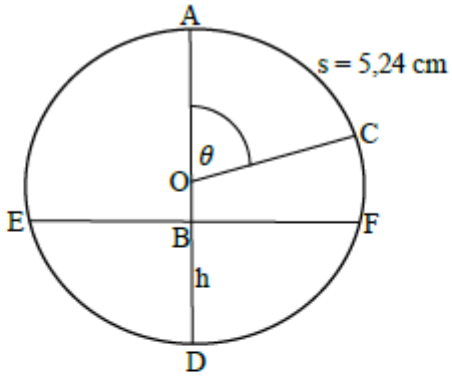
10.1.1 Determine the rotational frequency per minute. (1)

10.1.2 Convert the length of the diameter to metre. (1)

10.1.3 Hence, calculate the circumferential velocity of the train in metres per min. (3)

10.2 A wheel rotates at 15 revolutions per second. Calculate the angular velocity of the wheel in radians per minute. (4)

10.3 The circle below with centre O, has a chord EF of length 80 mm and diameter AD equals 10 cm. Arc AC subtends a central angle $\theta$.




10.3.1 Calculate the height of the minor segment, h (BD), in cm. (5)

10.3.2 If the arc length, AC, of the circle is 5,24 cm, calculate the central angle, $\theta$, to the nearest degree. (4)

10.3.3 Hence, determine the area of the minor sector AOC of the circle. (3)

**[21]**

## FORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b\ , \quad a > 0\ , a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0 \qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In $\Delta ABC$: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \mathrm{cosec}^2\theta$$

$\pi rad = 180^{\circ}$

Angular velocity $= \omega = 2\pi n$ Where $n$ = rotation frequency

Angular velocity $= 360^{\circ} n$ where $n$ = rotation frequency

Circumferencial velocity $= v = \pi Dn$ where D = diameter and $n$ = rotation frequency

Cirfumferential velocity $= v = \omega r$ where $\omega$ = angular velocity and $r$ = radius

Arc length= $s = r\theta$ where $r$ = radius and $\theta$ = central angle in radians

Area of sector = $\frac{rs}{2}$ where $r$ = radius, $s$ = arc length

Area of a sector = $\frac{r^2\theta}{2}$ where $r$ = radius, $s$ = arc length

$\theta$ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where $h$ = height of segment,

$d$ = diameter of circle and $x$ = length of chord

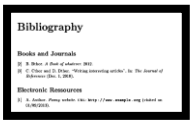
$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where $a$ = width of equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and

$n$ = number of ordinates

**OR**

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where $a$ = width of equal parts, $o_n = n^{th}$

ordinate and $n$ = number of ordinate




# BIBLIOGRAPHY


| | |
|---|---|
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| 3 | Technical mathematics Grade 12 self study guide Book 2 |
| 4 | |