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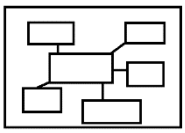
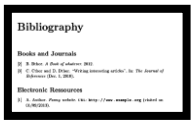
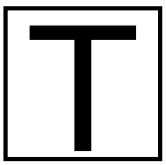
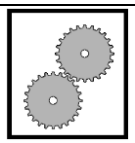
MATHEMATICS

GRADE 12

SPRING CLASSES 2026

TEACHER AND LEARNER CONTENT MANUAL

ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

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SECTION 2: Analytical Geometry	33 - 48

SECTION 1: FUNCTIONS

POLICY GUIDELINES & OUTCOMES

GRADE 12

1. Definition of a *function*.
2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function.
3. Determine and sketch graphs of the inverses of the functions defined by

   $y = ax + q;\ y = ax^2$

   $y = b^x; (b > 0, b \neq 1)$

Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases /decreases.

Examples:

1. Consider the function f where $f(x) = 3x - 1$.

   1.1 Write down the domain and range of f. (K)

   1.2 Show that f is a one-to-one relation. (R)

   1.3 Determine the inverse function f^{-1}. (R)

   1.4 Sketch the graphs of the functions f, f^{-1} and $y = x$ line on the same set of axes. What do you notice? (R)

2. Repeat Question 1 for the function $f(x) = -x^2$ and $x \leq 0$. (C)

Caution:

1. Do not confuse the inverse function f^{-1} with the reciprocal $\frac{1}{f(x)}$. For example,

   for the function where $f(x) = \sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while $f^{-1}(x) = x^2$ for $x \geq 0$.

2. Note that the notation $f^{-1}(x) = \ldots$ is used only for one-to-one relation and must not be used for inverses of many-to-one relations, since in these cases the inverses are not functions.

1. Revision of the exponential function and the exponential laws and graph of the function defined by $y = b^x$ where $b > 0$ and $b \neq 1$
2. Understand the definition of a logarithm: $y = \log_b x \Leftrightarrow x = b^y$, where $b > 0$ and $b \neq 1$.
3. The graph of the function define $y = \log_b x$ for both the cases $0 < b < 1$ and $b > 1$.

Comment:

The four logarithmic laws that will be applied, only in the context of real-life problems related to finance, growth and decay, are:

$$\log A^n = n \log A$$

They follow from the basic exponential laws (term 1 of grade 10).

- Manipulation involving the logarithmic laws will not be examined.

Caution:

1. Make sure learners know the difference between the two functions defined by $y = b^x$ and $y = x^b$ where b is a positive (constant) real number.
2. Manipulation involving the logarithmic laws will not be examined.

Examples:

1. Solve for x: $75(1{,}025)^{x-1} = 300$ (R)
2. Let $f(x) = a^x$, $a > 0$.

   2.1 Determine a if the graph of f goes through the point $(2; \frac{25}{16})$. (R)

   2.2 Determine the function f^{-1}. (R)

   2.3 For which values of x is $f^{-1}(x) > -1$? (C)

   2.4 Determine the function h if the graph of h is the reflection of the graph of f through the y-axis. (C)

   2.5 Determine the function k if the graph of k is the reflection of the graph of f through the x-axis. (C)

   2.6 Determine the function p if the graph of p is obtained by shifting the graph of f two units to the left. (C)

   2.7 Write down the domain and range for each of the functions f, f^{-1}, h, k and p. (R)

   2.8 Represent all these functions graphically. (R)

(SOURCE: CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS) FET PHASE GRADES (10 - 12) MATHEMATICS)

KEY CONCEPTS

	WORD	DESCRIPTION / DEFINITION
1	Relation	Is described by a rule that links any 2 set of variables $(x \text{ and } y)$, by linking each element in the first set to at least one element in the 2nd set.
2	Function	Is a relation that links each element in the 1st set (domain) to only one element in the 2nd set (range). ✓ One to one mapping ✓ Many to one mapping
3	Domain	The elements of the 1st set (the x -values) in a relation.
4	Range	The elements of the 2nd set (the y -values), called the images of the relation.
5	Vertical line test	Is a line (ruler) moved parallel to the y-axis, from left to right, it helps to determine if a relation is a function; if such a line intersects /cuts the graph at one place at a time, it **is a function**. If it cuts the graph more than once, the relation **is not a function**.
6	Horizontal line test	Is the line (ruler) moved parallel to the x-axis, up and down. If such a line intersects/cuts the function at one place at a time, then the graph is a one-to-one function. If the ruler at any stage cuts the graph in more than one place, the graph is a many-to-one function.
7	Asymptote	Is a vertical or a horizontal straight line on a graph, which represents a value which is excluded from the graph, as it makes the value undefined. **A graph never intersects its asymptotes**
8	Axis of symmetry	Is a straight line, around which a graph, or part of a graph, forms a mirror image/ reflection of itself. A parabola and a hyperbola have an axis of symmetry.
9	Translation	Move point up and down by adding or subtracting the y-value; or move graph left or right by adding or subtracting the x-value
10	Zero points of function	Where the $y = 0 \quad (x - \text{intercepts})$ and $x = 0 \quad (y - \text{intercept})$
11	Minimum	The lowest possible y-value of the function
12	Maximum	The highest possible y-value of the function
13	Increasing	The function is increasing when the value of y increases as x is increasing. **THE GRAPH GOES UP**
14	Decreasing	The function is decreasing when the value of y decrease as x is increasing. **THE GRAPH GOES DOWN**

SUMMARY NOTES

CHARACTERISTICS OF EACH FUNCTION

Straight Line	Parabola	Hyperbola	Exponential
$y = mx + c$ m … **gradient** and c … **y-intercept**	$y = a(x+p)^2 + q$ **Axis of symmetry with equation** $x = -p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y = \frac{a}{x+p} + q$ **Vertical asymptote with equation** $x = -p$ **Horizontal asymptote with equation** $y = q$	$y = a.b^{x+p} + q$ $b > 0$ **and** $b \neq 1$ **Horizontal asymptote with equation** $y = q$
$m < 0$ **… graph is decreasing** $m > 0$ **… graph is increasing**	$a < 0$ **… graph faces downwards(concave down) and has a minimum turning point** $a > 0$ **… graph faces upwards (concave up) and has a maximum turning point**	$a < 0$ **… graph is on the second and the fourth quadrant** $a > 0$ **… graph is on the first and the third quadrant**	$a < 0$ **… graph is below the asymptote** $a > 0$ **… graph is above the asymptote**
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$	**Domain:** $x \in R,\ x \neq -p$ **Range:** $y \in R,\ y \neq q$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$
$y - y_1 = m(x - x_1)$	$y = ax^2 + bx + c$ **Axis of symmetry:** $x = \frac{-b}{2a}$ $y = a(x - x_1)(x - x_2)$ x_1 **and** x_2 **are x-intercepts**	**Axis of symmetry/lines of symmetry:** $\left\{\begin{matrix} y = x + c \\ y = -x + c \end{matrix}\right\}$ **substitute point of intersection of asymptotes** **OR** $\left\{\begin{matrix} y = (x-p) + q \\ y = -(x-p) + q \end{matrix}\right\}$	

TRANSFORMATIONS

Change	Effect
$h(x) = f(x) + k$	Vertical shift up by k units
$h(x) = f(x) - k$	Vertical shift down by k units
$h(x) = f(x - k)$	Horizontal shift to the right by k units
$h(x) = f(x + k)$	Horizontal shift to the left by k units
$h(x) = -f(x)$	Reflection in the x – axis, values of x remain unchanged and • change in the sign of y values
$h(x) = f(-x)$	Reflection in the y – axis, values of y remain unchanged • change in the sign of x – values

GRAPH INTERPRETATION

Notation	Region
$( )\ ;\ <\ or\ >$	denotes excluding critical / end values.
$[\ ]\ ;\ \leq\ or\ \geq$	denotes including critical / end values.
$f(x) > 0$	where the graph is above the $x-$ axis.
$f(x) < 0$	where the graph is below the $x-$ axis.
$f(x).g(x) > 0$	where both graphs are either below or above the $x-$ axis.
$f(x).g(x) < 0$	Where both graphs are in different sides of the $x-$axis. i.e One graph above the $x-$ axis and the other one below the $x-$ axis.
$f(x) = g(x)$	where the graphs of f and g intersect.
$f(x) > g(x)$	where the graph of f is above the graph of g.
$f(x) < g(x)$	where the graph of f is below the graph of g.
$\frac{f(x)}{g(x)} > 0$	where both graphs are either below or above the $x-$ axis. But in th case, always remember that $g(x) \neq 0$.

$f'(x) = 0$	where the graph has a local minimum/ local maximum
$f'(x) > 0$	where the graph is increasing
$f'(x) < 0$	where the graph is increasing

LINEAR FUNCTION (STRAIGHT LINE GRAPHS)

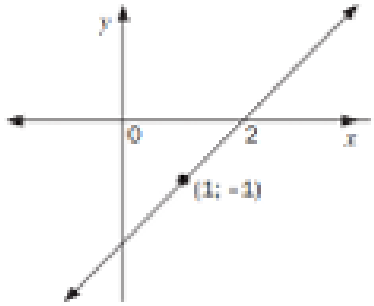
QUESTION 1

Sketch the graph of $y = 2x - 4$ and determine the domain and range of the function, and state if the function is increasing or decreasing.

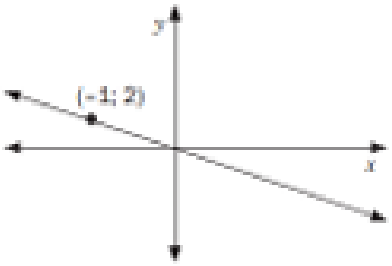
QUESTION 2

Determine the equations of the following graphs:

1.



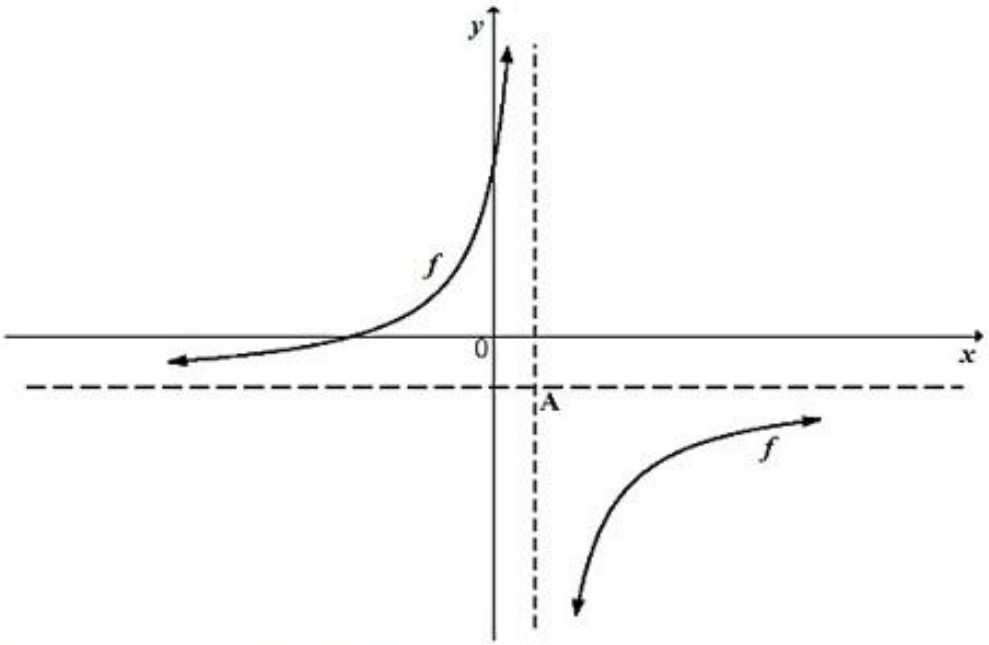
2.



RATIONAL FUNCTIONS (HYPERBOLA)

QUESTION 1

1. The sketch below shows the graph of $f(x) = \frac{-9}{x-1} - 2$.
A is the point of intersection of the asymptotes of f.



1.1 Write down the coordinates of A.

1.2 Determine the coordinates of the x- and y-intercepts of f.

1.3 Write down an equation of the axis of symmetry of f that has a negative gradient.

1.4 Hence, or otherwise, determine the coordinates of a point that lies on f in the fourth quadrant, which is the closest to point A.

1.5 The graph of f is reflected about the x-axis to obtain the graph of g. Write down the equation of g in the form $y = ...$

QUESTION 2

Given: $f(x)=\dfrac{8}{x-2}+3$

2.1 Write down the equations of the asymptotes of f.

2.2 Calculate the x- and y-intercepts of f.

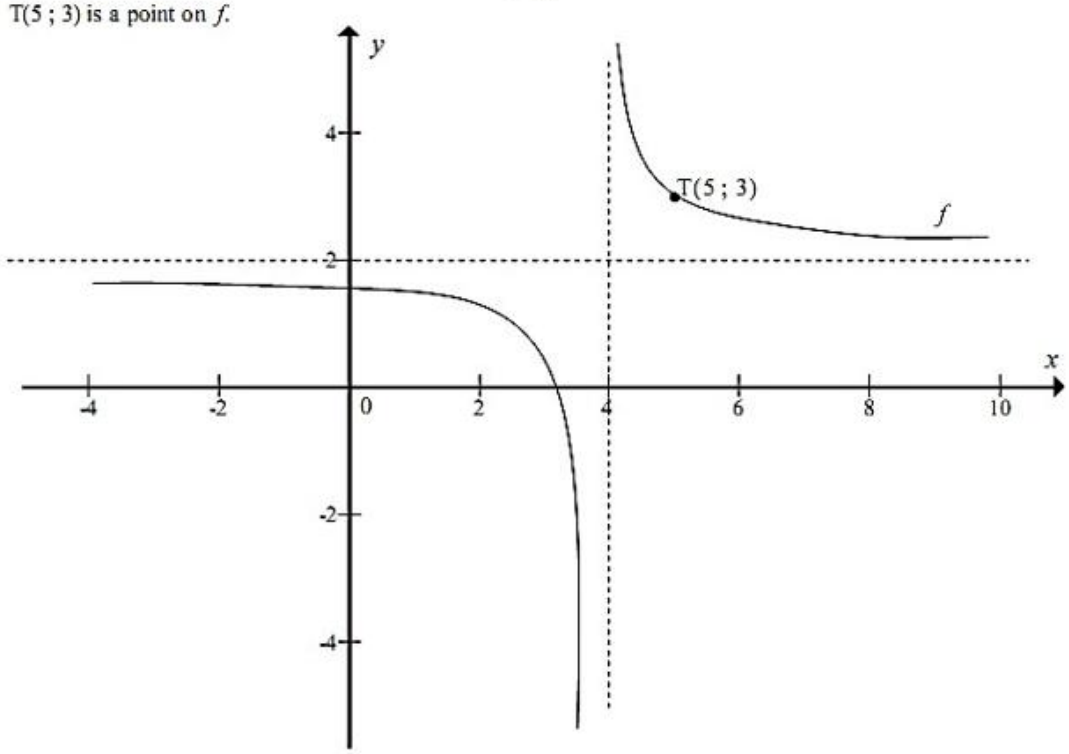
2.3 Sketch the graph of f. Show clearly the intercepts with the axes and the asymptotes.

2.4 If $y=x+k$ is an equation of the line of symmetry of f, calculate the value of k.

QUESTION 3

The diagram below represents the graph of $f(x)=\dfrac{a}{x-p}+q$.

T(5 ; 3) is a point on f.



3.1 Determine the values of a, p and q.

3.2 If the graph of f is reflected across the line having equation $y=-x+c$, the new graph coincides with the graph of $y=f(x)$. Determine the value of c.

QUESTION 4

Given: $f(x)=\dfrac{x+3}{x+1}$

4.1 Calculate the x- and y-intercepts of f.

4.2 Show that $f(x)=\dfrac{2}{x+1}+1$

4.3 Write down the equations of the vertical and horizontal asymptotes of f.

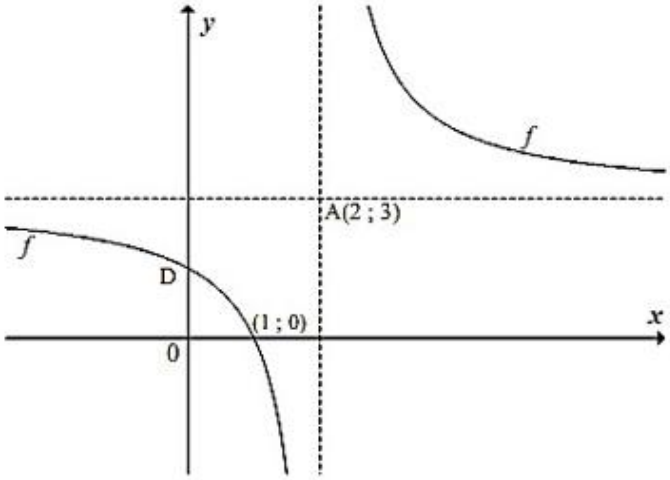
4.4 Draw a sketch graph of $f(x)$ showing clearly the intercepts and asymptotes on the axes.

4.5 Use your graph to solve: $\dfrac{2}{x+1}\geq -1$

QUESTION 5

Given $f(x)=\frac{a}{x-p}+q$. The point A(2 ; 3) is the point of intersection of the asymptotes of f.

The graph of f intersects the x-axis at (1 ; 0).
D is the y-intercept of f.



5.1 Write down the equations of the asymptotes of f.

5.2 Determine an equation of f.

5.3 Write down the coordinates of D.

5.4 Write down an equation of g if g is the straight line joining A and D.

5.5 Write down the coordinates of the other point of intersection of f and g.

QUESTION 6

6.1 Consider the function: $f(x)=\frac{-6}{x-3}-1$

6.1.1 Calculate the coordinates of the y-intercept of f.

6.1.2 Calculate the coordinates of the x-intercept of f.

6.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes.

6.1.4 For which values of x is $f(x)>0$?

6.1.5 Calculate the average gradient of f between $x=-2$ and $x=0$.

QUESTION 7

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbf{R},\ x \neq 5$
- Range: $y \in \mathbf{R},\ y \neq 1$
- Passes through the point $(2\ ;\ 0)$

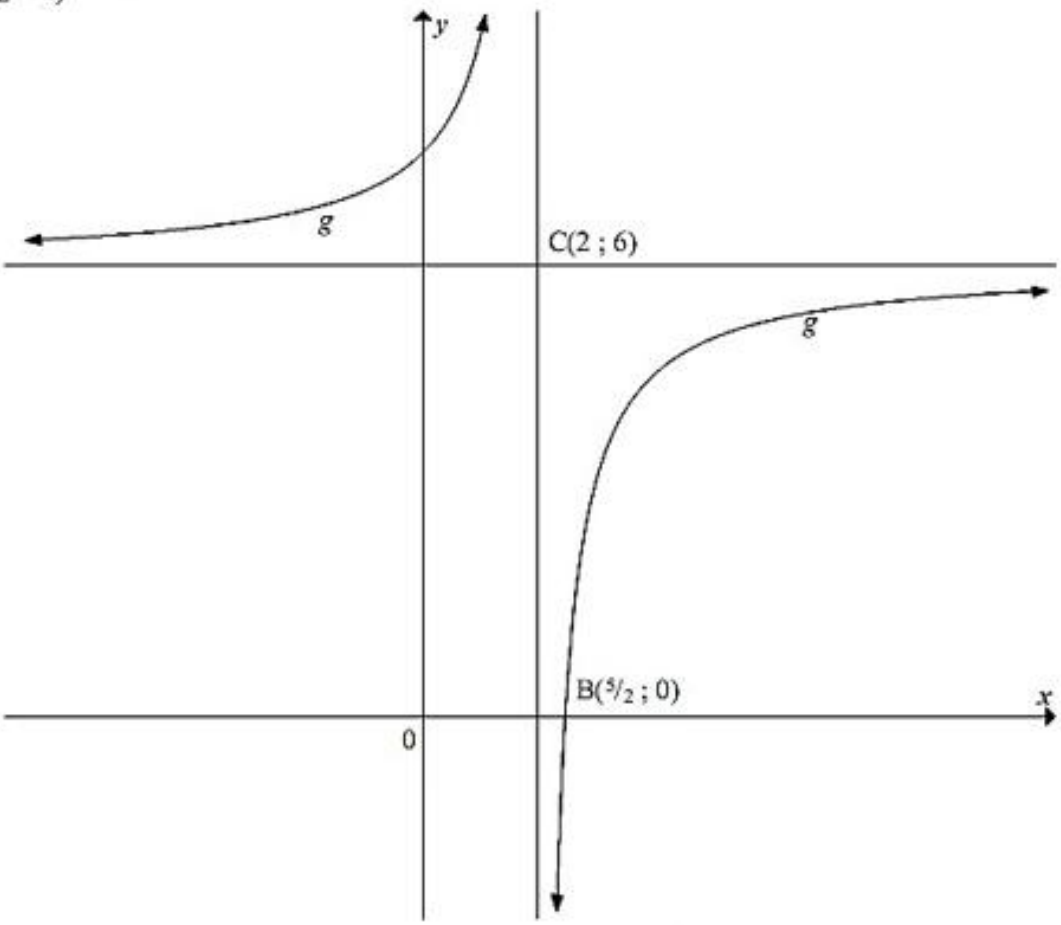
Determine $f(x)$.

QUESTION 8

Sketched below is the graph of $g(x) = \dfrac{a}{x-p} + q$.

C(2 ; 6) is the point of intersection of the asymptotes of g.

$B\left(\dfrac{5}{2}\ ;\ 0\right)$ is the x-intercept of g.



8.1 Determine the equation for g in the form $g(x) = \dfrac{a}{x-p} + q$

8.2 F is the reflection of B across C. Determine the coordinates of F.

QUESTION 9

Consider the function $f(x)=\frac{3}{x-1}-2$.

9.1 Write down the equations of the asymptotes of f.

9.2 Calculate the intercepts of the graph of f with the axes.

9.3 Sketch the graph of f on DIAGRAM SHEET 1.

9.4 Write down the range of $y = -f(x)$.

9.5 Describe, in words, the transformation of f to g if $g(x)=\frac{-3}{x+1}-2$.

QUESTION 10

Consider: $f(x)=\frac{6}{x-2}+3$

10.1 Write down the equations of the asymptotes of the graph of f.

10.2 Write down the domain of f.

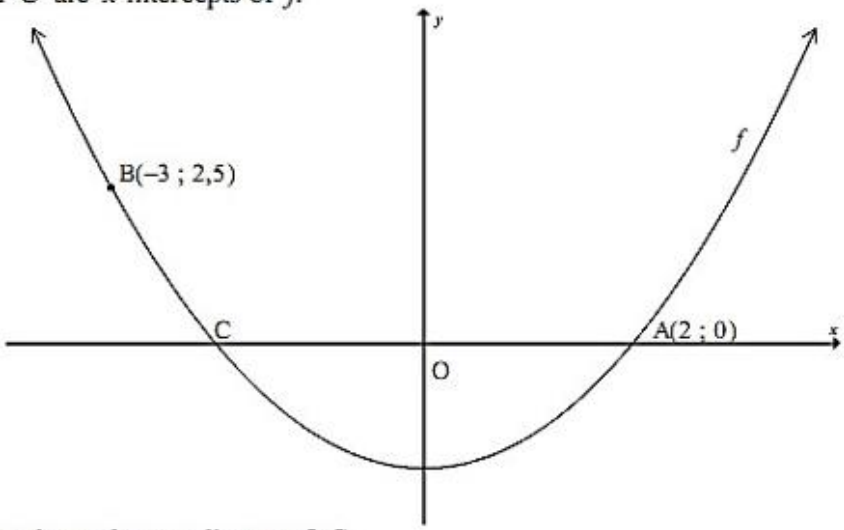
10.3 Draw a sketch graph of f in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes.

10.4 The graph of f is translated to g. Describe the transformation in the form $(x\,;y)\rightarrow\ldots$ if the axes of symmetry of g are $y=x+3$ and $y=-x+1$.

QUADRATIC FUNCTIONS (PARABOLA)

QUESTION 1

The graph of $f(x)=ax^2+q$ is sketched below.
Points A(2 ; 0) and B(– 3 ; 2,5) lie on the graph of f.
Points A and C are x-intercepts of f.



1.1 Write down the coordinates of C.

1.2 Determine the equation of f.

1.3 Write down the range of f.

1.4 Write down the range of h, where $h(x)=-f(x)-2$.

1.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and which passes through the point A.

QUESTION 2

Given: $f(x)=ax^2+c$
f passes through the x-axis at $(d-5)$ and $(d-1)$, where $d \in R$.

2.1 Determine the value of d.

2.2 Determine the values of a and c if it is also given that $f(1)=-9$.

QUESTION 3

3.1 Give the equation of the quadratic function if it is given that:

- The range of f is: $y \geq -4$
- Domain: $x \in \mathbb{R}$
- Zero points are (3 ; 0), (-1 ; 0) and (0 ; -3)

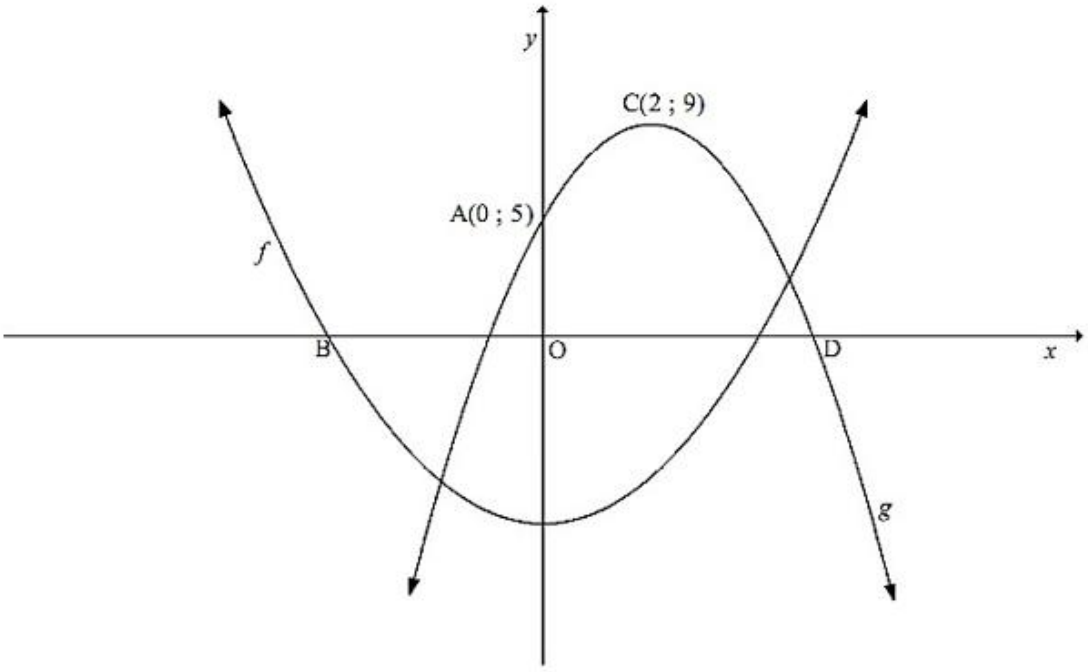
3.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0$, $b < 0$, $c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution.

QUESTION 4

The sketch below represents the graphs of two parabolas, f and g.

$f(x) = \frac{1}{2}x^2 - 8$

The turning point of g is C (2 ; 9) and the y-intercept of g is A (0 ; 5).
B and D are the x-intercepts of f and g respectively.



4.1 Show that $g(x) = -x^2 + 4x + 5$.

4.2 Calculate the average gradient of g between A and C.

4.3 Calculate the length of BD.

4.4 Use the graphs to solve for x, if:

4.4.1 $f(x) \geq 0$

4.4.2 f and g are both strictly increasing

QUESTION 5

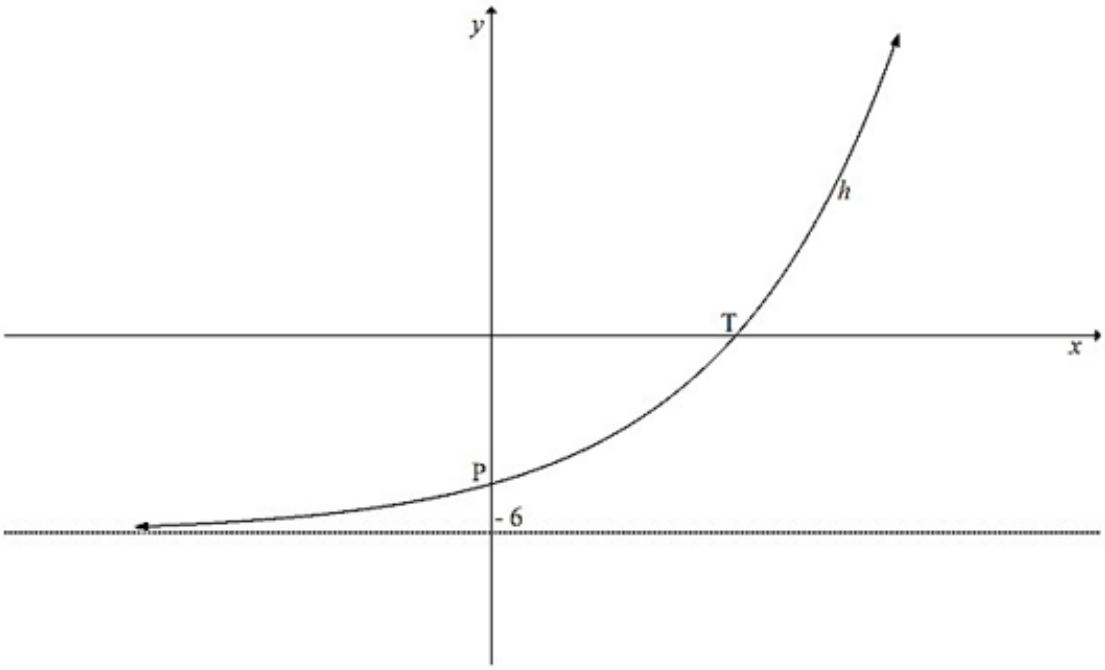
Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- The range of f is $(-\infty;7]$
- $a \neq 0$
- $b < 0$
- One root of f is positive and the other root of f is negative.

EXPONENTIAL FUNCTIONS

QUESTION 1

Given: $h(x) = a.2^{x-1} + q$. The line $y = -6$ is an asymptote to the graph of h. P is the y-intercept of h and T is the x-intercept of h.



1.1 Write down the value of q.

1.2 If the graph of h passes through the point $\left(-1;-5\frac{1}{4}\right)$, calculate the value of a.

1.3 Calculate the average gradient between the x-intercept and the y-intercept of h.

1.4 Determine the equation of p if $p(x) = h(x-2)$ in the form $p(x) = a.2^{x-1} + q$.

QUESTION 2

Given: $h(x) = 4\left(2^{-x}\right) + 1$

2.1 Determine the coordinates of the y-intercept of h.

2.2 Explain why h does not have an x-intercept.

2.3 Draw a sketch graph of h, clearly showing all asymptotes, intercepts with the axes and at least one other point on h.

2.4 Describe the transformation from h to g if $g(x) = 4\left(2^{-x} + 2\right)$.

QUESTION 3

Consider the function $f(x) = 4^{-x} - 2$.

3.1 Calculate the coordinates of the intercepts of f with the axes.

3.2 Write down the equation of the asymptote of f.

3.3 Sketch the graph of f.

3.4 Write down the equation of g if g is the graph of f shifted 2 units upwards.

3.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.)

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the y-intercept of the graph of f.

4.1.2 Calculate the coordinates of the x-intercept of the graph of f.

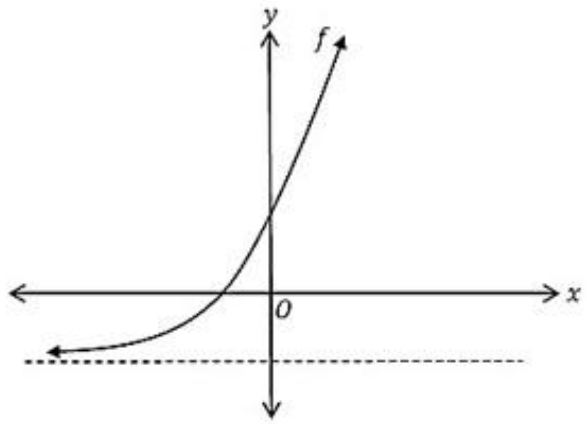
4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes.

4.1.4 Write down the range of f.

QUESTION 5

The sketch below shows the graph of $f(x) = 2 \times a^x - 1$. The point $A(1;5)$ is a point on the graph.



5.1 Show that $a = 3$.

5.2 Determine the y-intercept of f.

5.3 Write down the range of f.

5.4 Determine $f(0{,}23)$, rounded off to three decimal places.

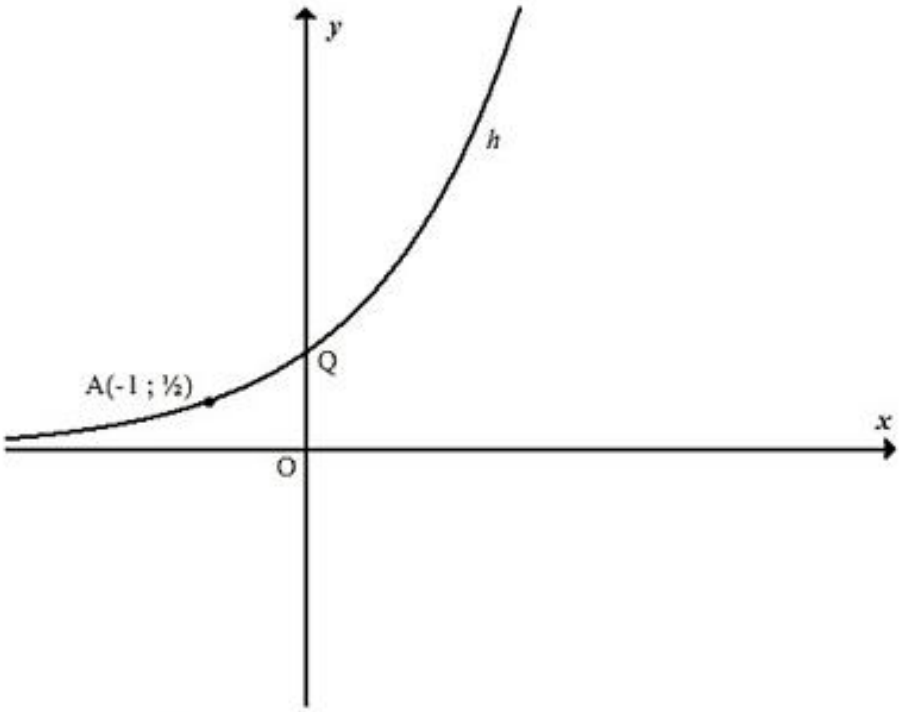
5.5 Write down the equation if f is reflected about the x-axis followed by a translation of 2 units to the left.

INVERSE FUNCTIONS

QUESTION 1

The graph of $h(x) = a^x$ is sketched below.

$A\left(-1;\frac{1}{2}\right)$ is a point on the graph of h.



1.1 Explain why the coordinates of Q are (0 ; 1).

1.2 Calculate the value of a.

1.3 Write down the equation for the inverse function, h^{-1}, in the form $y = ...$

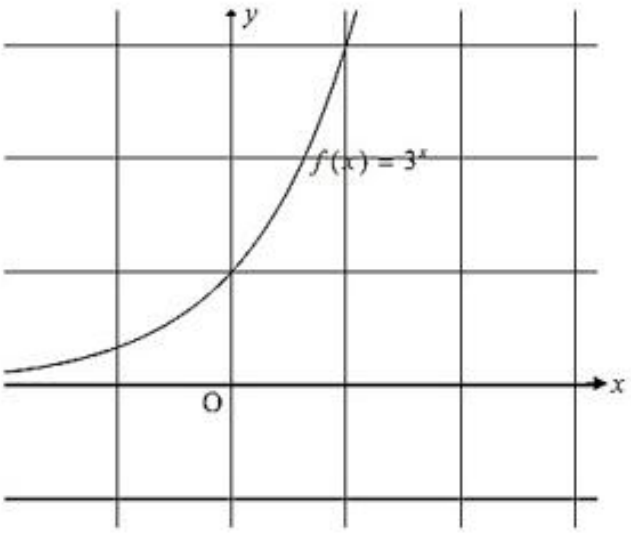
1.4 Draw a sketch graph, on ANSWER BOOK, of h^{-1}. Indicate on this graph the coordinates of two points that lie on this graph.

1.5 Read off from your graph the values of x for which $\log_2 x > -1$.

1.6 If $g(x) = (100).3^x$, determine the value of x for which $h(x) = g(x)$.

QUESTION 2

The graph of $f(x) = 3^x$ is drawn below.



2.1 Write f^{-1} in the form $y = \ldots$

2.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$

2.3 Use your graphs to solve for x if $\log_3(x-2) < 1$.

QUESTION 3

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$.

3.1 Is f an increasing or decreasing function? Give a reason for your answer.

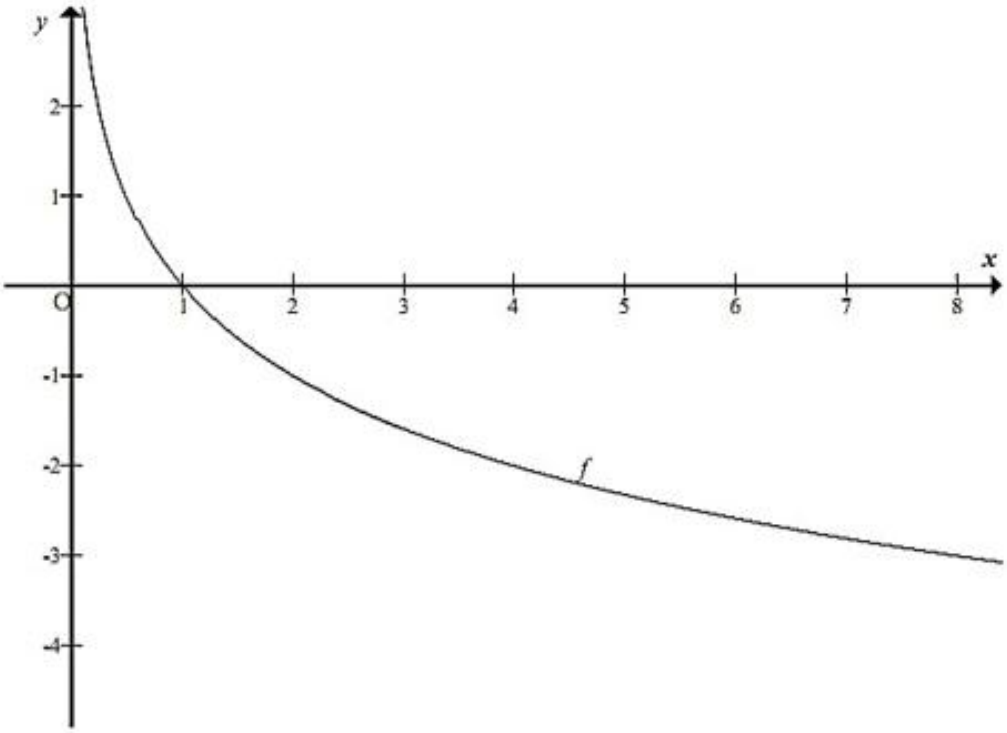
3.2 Determine $f^{-1}(x)$ in the form $y = \ldots$

3.3 Write down the equation of the asymptote of $f(x) - 5$.

3.4 Describe the transformation from f to g if $g(x) = \log_3 x$.

QUESTION 4

Sketched below is the graph of $f(x) = -\log_2 x$.



4.1 Write down the domain of f.

4.2 Write down the equation of f^{-1} in the form $y = \ldots$

4.3 Write down the equation of the asymptote of f^{-1}.

4.4 Explain how, using the graph of f, you would sketch the graphs of:

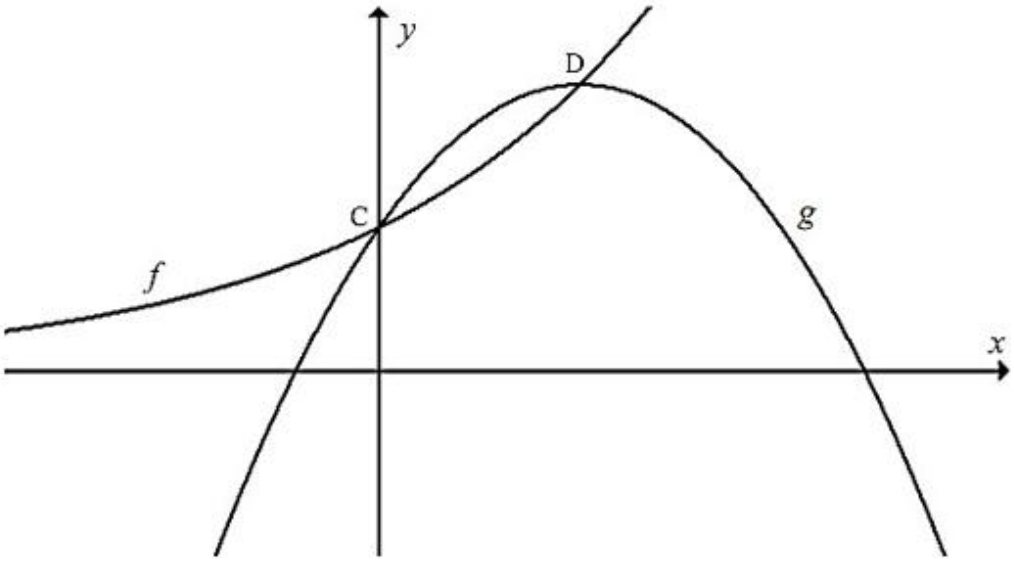
4.4.1 $g(x) = \log_2 x$

4.4.2 $h(x) = 2^{-x} - 5$

4.5 Use the graph of f to solve for x where $\log_2 x < 3$.

QUESTION 5

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + b$, where b is a constant.
The graphs of f and g intersect the y-axis at C. D is the turning point of g.



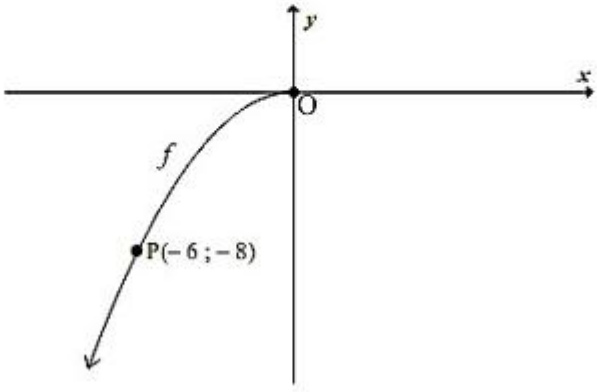
5.1 Write down the coordinates of the turning point of g.

5.2 Write down the equation of $f^{-1}(x)$ in the form $y = ...$

5.3 Sketch the graph of f^{-1}
Indicate the x-intercept and the coordinates of one other point on your graph.

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P(– 6 ; – 8) lies on the graph of f.



6.1 Calculate the value of a.

6.2 Determine the equation of f^{-1}, in the form $y = \ldots$

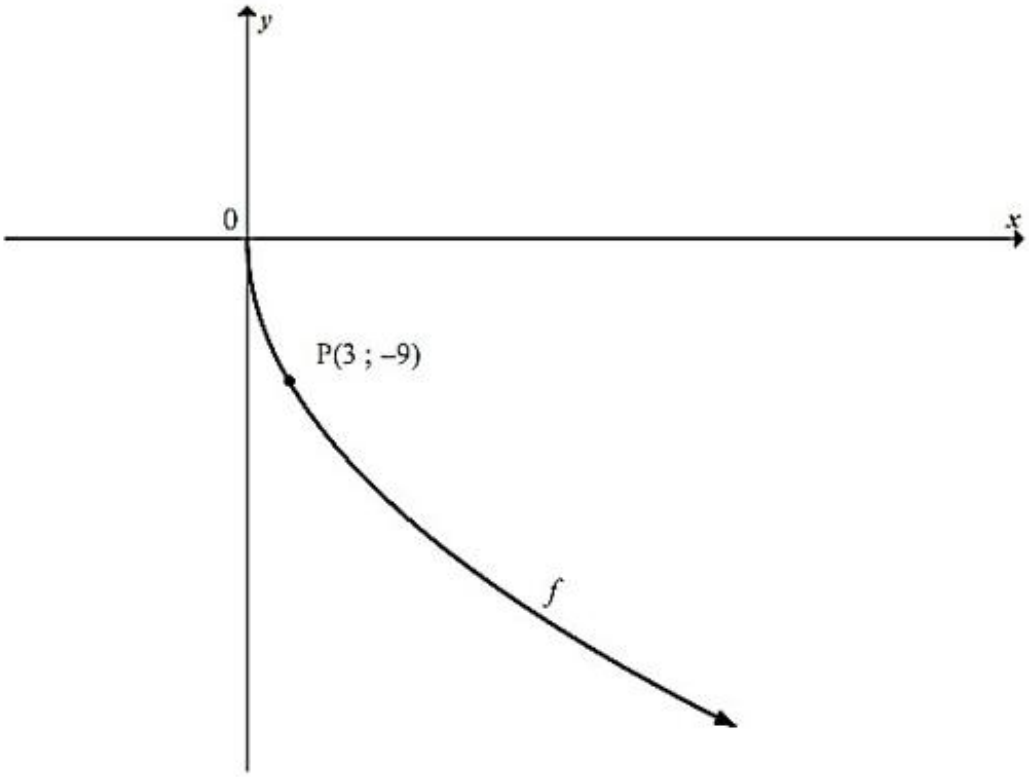
6.3 Write down the range of f^{-1}.

6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from $(0\ ;\ 0)$.

6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axis. Determine the equation of the new function in the form $y = \ldots$

QUESTION 7

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



7.1 Use your graph to determine the values of x for which $f(x) \geq -9$.

7.2 Write down the equation of f^{-1} in the form $y = \ldots$ Include ALL restrictions.

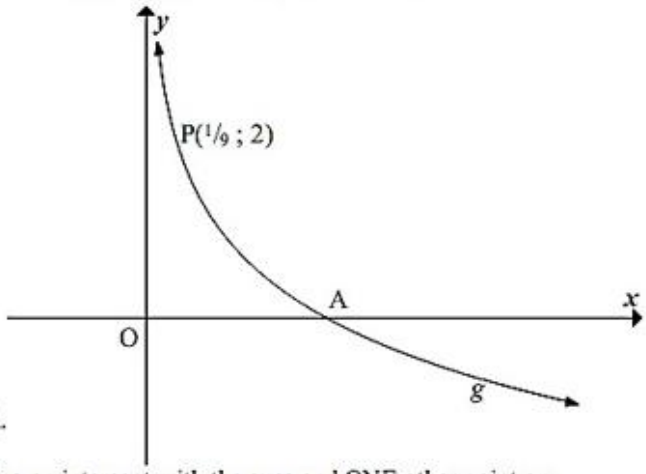
7.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point.

7.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$.

QUESTION 8

Given the graph of $g(x)=\log_{\frac{1}{3}} x$

- A is the x-intercept of g.
- P $\left(\frac{1}{9};2\right)$ is a point on g.



8.1 Write down the coordinates of A.

8.2 Sketch the graph of g^{-1} indicating an intercept with the axes and ONE other point on the graph.

8.3 Write down the domain of g^{-1}.

QUESTION 9

Given: $f(x)=3^x$

9.1 Determine an equation for f^{-1} in the form $f^{-1}(x)=...$

9.2 Sketch, in your ANSWER BOOK, the graphs of f and f^{-1}, showing clearly ALL intercepts with the axes.

9.3 Write down the domain of f^{-1}.

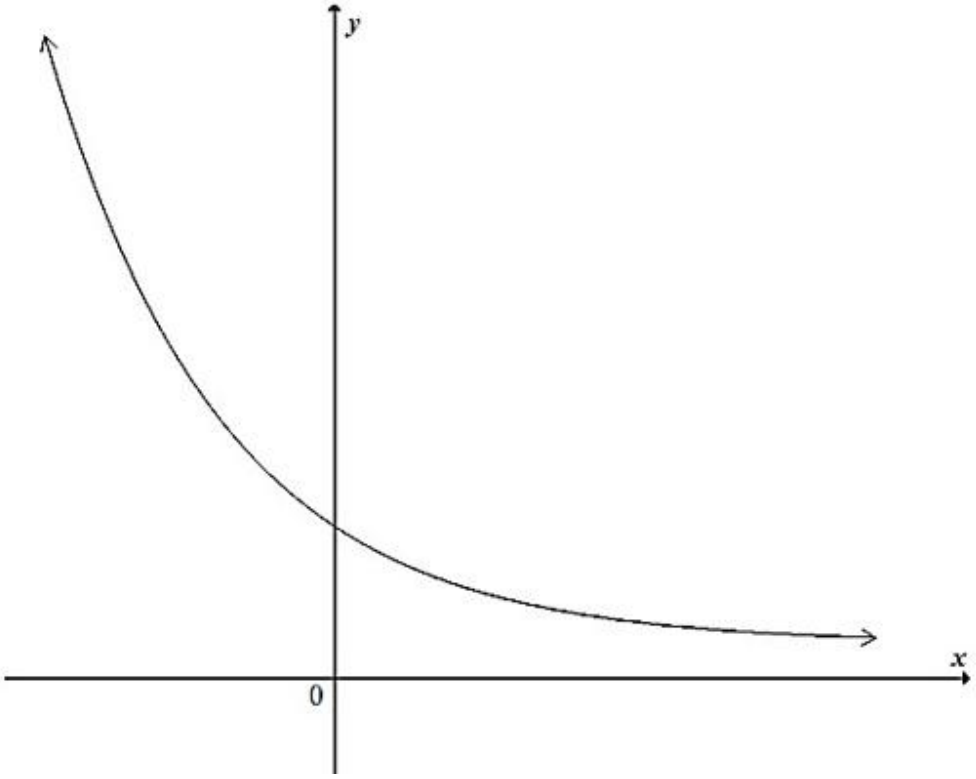
9.4 For which values of x will $f(x).f^{-1}(x)\leq 0$?

9.5 Write down the range of $h(x)=3^{-x}-4$

9.6 Write down an equation for g if the graph of g is the image of the graph of f after f has been translated two units to the right and reflected about the x-axis.

QUESTION 10

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



10.1 Write down the domain of f.

10.2 Write down the equation of the asymptote of f.

10.3 Write down the equation of f^{-1} in the form $y=...$

10.4 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate the x-intercept and ONE other point.

10.5 Write down the equation of the asymptote of $f^{-1}(x+2)$.

10.6 Prove that: $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$ for all values of x.

QUESTION 11

11.1 Consider the function f where $f(x)=3x-1$.

11.1.1 Write down the domain and range of f.

11.1.2 Determine the inverse function f^{-1}.

11.1.3 Sketch the graphs of the functions f, f^{-1} and $y=x$ line on the same set of axes.

COMBINATIONS

QUESTION 1

Given the following two functions:

$h(x) = \frac{1}{x} + 5$ and $g(x) = x + 5$

1.1 Determine the x-intercept of h.

1.2 Sketch neat graphs of h and g on the same set of axes. Clearly show all intercepts with the axes as well as asymptotes.

1.3 Write down the equation of the vertical asymptote of h.

1.4 Determine the coordinates of the points of intersection of h and g. Show all calculations.

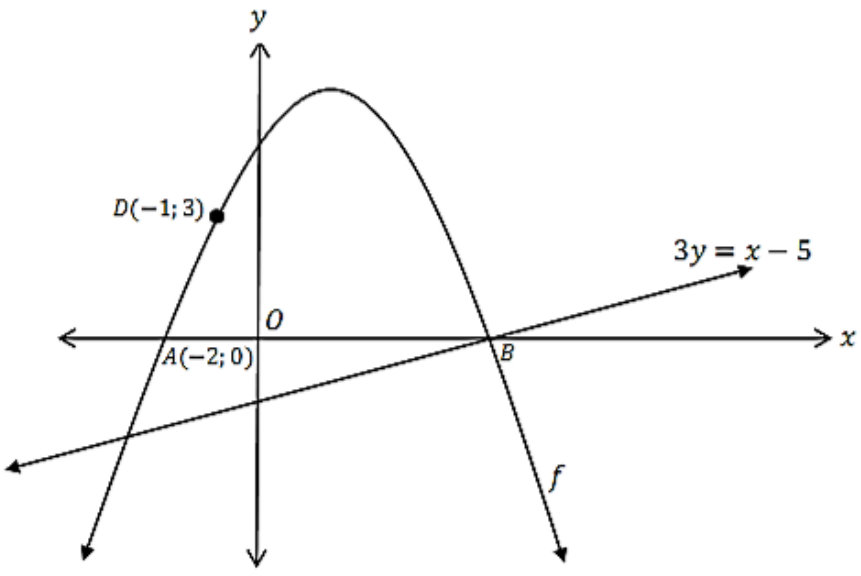
1.5 Write down the equation of f if f is the reflection of g about the line $y = 4$.

1.6 Write down the equation if h is translated so that $(-2; 3)$ is the new point of intersection of the asymptotes.

QUESTION 2

The sketch below shows the graph of the function $f(x) = ax^2 + bx + c$.

The straight line with equation $3y = x - 5$ intersects f at B. The points $A(-2; 0)$ and B are the x-intercepts of f. Point $D(-1; 3)$ is a point on f.



2.1 Determine the coordinates of B.

2.2 Determine the equation of f.

2.3 Determine the coordinates of the turning point of f.

2.4 Point E is a point on the straight line so that DE is parallel to the y-axis. Determine the length of DE.

2.5 Calculate the average gradient between B and D.

2.6 For which value(s) of x is $x \times f(x) \geq 0$?

QUESTION 3

Given $f(x) = 2.3^x - 1$ and $g(x) = \frac{4}{x+3}$

3.1 Write down the equation of the asymptote of f.

3.2 Determine the y-intercept of f. (Give your answer in co-ordinate form)

3.3 Find ONE other point on the graph of f.

3.4 Sketch the graph of f.

3.5 What is the range of f?

3.6 Write down the equations of the asymptotes of g.

3.7 Determine the y-intercept of g.

3.8 Write down the equation of the axis of symmetry of g.

3.9 Sketch the graph of g on its own set of axes.

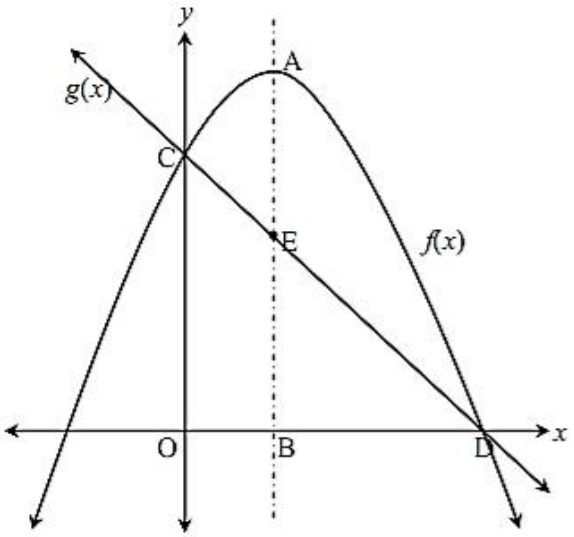
3.10 Determine the average gradient of g between the points x = -2 and x = 1.

QUESTION 4

4.1 4.1.1 Find the equation of the parabola that cuts the x-axis at -2 and 3, and the y-axis at the point, (0 ; -12).

4.1.2 Write your answer in QUESTION 4.1.1 in the form $y = a(x - p)^2 + q$.

4.2 In the diagram below, $f(x) = -x^2 + x + 12$ and $g(x) = mx + c$



4.2.1 Determine the coordinates of C and D.

4.2.2 Determine the values of m and c and hence determine the equation of $g(x)$.

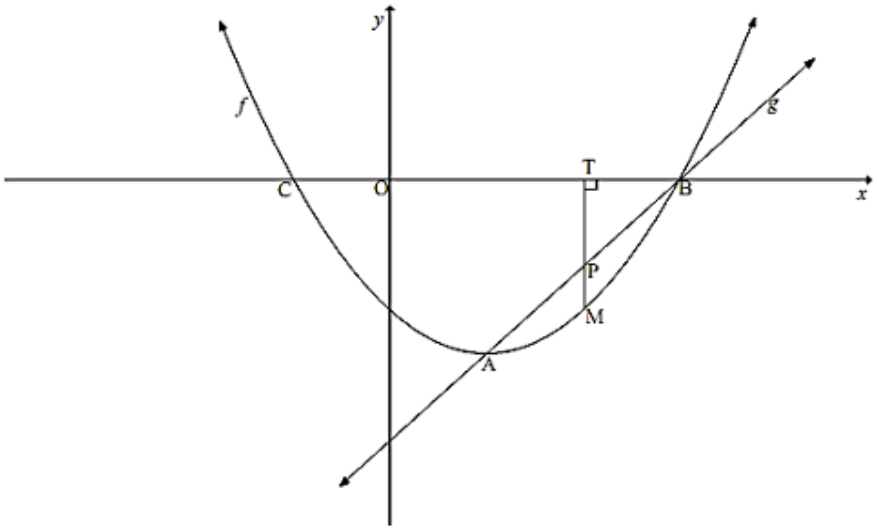
4.2.3 If OB $= \frac{1}{2}$, find the length of AE.

4.2.4 For which values of x is $f(x)$ decreasing?

4.2.5 Write down the range of $f(x)$.

QUESTION 5

The graph of $f(x) = x^2 + bx + c$ and the straight line g are sketched below. A and B are the points of intersection of f and g. A is also the turning point of f. The graph of f intersects the x-axis at B(3 ; 0) and C. The axis of symmetry of f is $x = 1$.



5.1 Write down the coordinates of C.

5.2 Determine the equation of f in the form $y = x^2 + bx + c$.

5.3 Determine the range of f.

5.4 Calculate the equation of g in the form $y = mx + c$.

5.5 For which values of x will:

5.5.1 $f(x) \geq 0$

5.5.2 $\frac{f(x)}{g(x)} > 0$

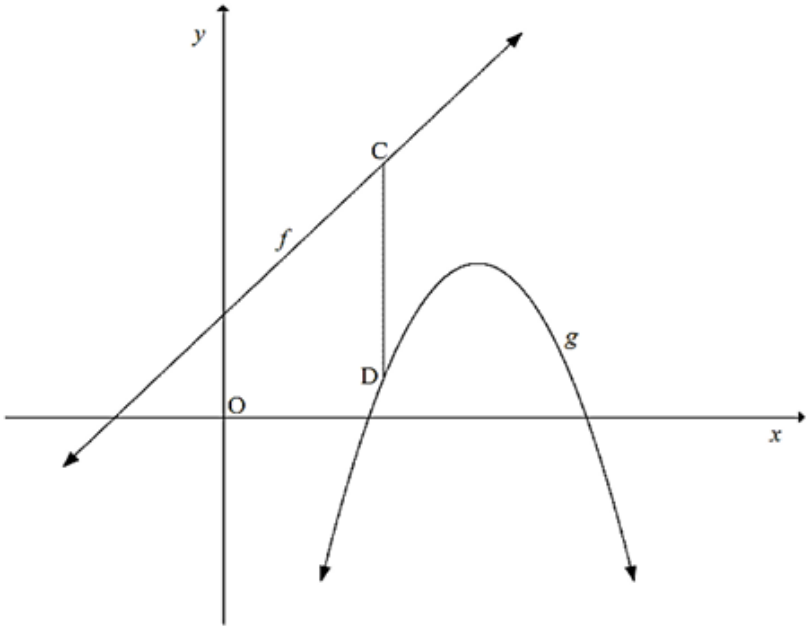
5.5.3 $x.f(x) > 0$

5.6 For what values of p will $x^2 - 2x = p$ have non-real roots?

5.7 T is a point on the x-axis and M is a point on f such that TM $\perp$ x-axis. TM intersects g at P. Calculate the maximum length of PM.

QUESTION 6

The sketch below shows the graphs of $f(x) = 2x + 3$ and $g(x) = -2x^2 + 14x + k$.
C is any point on f and D any point on g, such that CD is parallel to the y-axis.
k is a value such that C lies above D.



6.1 Write down a simplified expression for the length of CD in terms of x and k.

6.2 If the minimum length of CD is 5, calculate the value of k.

QUESTION 7

$f(x) = -2x^2 + 2$ and $g(x) = 2^x + 1$ are the defining equations of graphs f and g respectively.

7.1 Write down an equation for the asymptote of g.

7.2 Sketch the graphs of f and g on the same set of axes, clearly showing ALL intercepts with the axes, turning points and asymptotes.

7.3 Write down the range of f.

7.4 Determine the maximum value of h if $h(x) = 3^{f(x)}$.

7.5 What transformation does the graph of $y = f(x)$ undergo in order to obtain the graph of $y = 2x^2 - 2$?

QUESTION 8

Given: $h(x) = 4^x$ and $f(x) = 2(x-1)^2 - 8$.

8.1 Sketch the graphs of h and f on the diagram sheet provided. Indicate ALL intercepts with the axes and any turning points.

8.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes.

8.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph.

8.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$.

SECTION 2: ANALYTICAL GEOMETRY

POLICY GUIDELINES & OUTCOMES

GRADE 10

Represent geometric figures on a Cartesian co-ordinate system.

Derive and apply for any two points $(x_1;y_1)$ and $(x_2;y_2)$ the formulae for calculating the:

1. distance between the two points;
2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and
3. coordinates of the mid-point of the line segment joining the two points.

Example:

Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane.

1.1. Calculate the distance PQ. (K)

1.2 Find the coordinates of R if M $(-1;0)$ is the mid-point of PR. (R)

1.3 Determine the coordinates of S if PQRS is a parallelogram. (C)

1.4 Is PQRS a rectangle? Why or why not? (R)

GRADE 11

Derive and apply:

1. the equation of a line though two given points;
2. the equation of a line through one point and parallel or perpendicular to a given line; and
3. the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line $(0° \leq \theta < 180°)$.

Example:

Given the points $A(2;5)$; $B(-3;-4)$ and $C(4;-2)$ determine:

1. the equation of the line AB; and (R)
2. the size of $B\hat{A}C$. (C)

GRADE 12

1. The equation $(x-a)^2+(y-b)^2=r^2$ defines a circle with radius r and centre $(a;b)$.
2. Determination of the equation of a tangent to a given circle.

Examples:

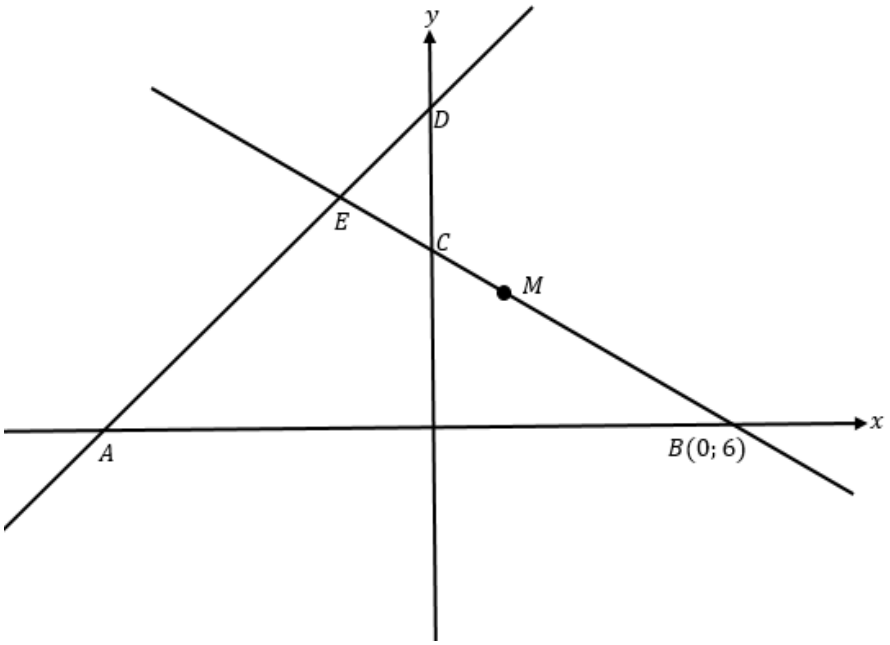
1. Determine the equation of the circle with centre $(-1;2)$ and radius $\sqrt{6}$ (K)
2. Determine the equation of the circle which has the line segment with endpoints $(5;3)$ and $(-3;6)$ as diameter. (R)
3. Determine the equation of a circle with a radius of 6 units, which intersects the x-axis at $(-2;0)$ and the y-axis at $(0;3)$. How many such circles are there? (P)
4. Determine the equation of the tangent that touches the circle defined by $x^2-2x+y^2+4y=5$ at the point $(-2;-1)$. (C)
5. The line with the equation $y=x+2$ intersects the circle defined by $x^2+y^2=20$ at A and B .
   - 5.1 Determine the co-ordinates of A and B . (R)
   - 5.2 Determine the length of chord AB. (K)
   - 5.3 Determine the co-ordinates of M, the midpoint of AB. (K)
   - 5.4 Show that $OM \perp AB$, where O is the origin. (C)
   - 5.5 Determine the equations of the tangents to the circle at the points A and B . (C)
   - 5.6 Determine the co-ordinates of the point C where the two tangents in 5.5 intersect. (C)
   - 5.7 Verify that $CA = CB$. (R)
   - 5.8 Determine the equations of the two tangents to the circle, both parallel to the line with the equation $y=-2x+4$. (P)

(SOURCE: CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS) FET PHASE GRADES (10 - 12) MATHEMATICS)

STRAIGHT LINES

QUESTION 1

In the diagram, A and D are the x-and y-coordinates of line AD respectively. Line BE cuts the x-axis at B and the y-axis at C. The equations of AD and EB are y$= 2x + 10$ and $y = -x + 6$ respectively. M is the midpoint of line BE, AE $= \frac{11}{3}\sqrt{5}$. CB and AD intersect at E$(-4\,;8)$.



1.1 Write down the coordinates of C. (1)

1.2 Calculate the angle of inclination of line BC. (2)

1.3 Calculate the length of line CD. (2)

1.4 Determine the equation of line through M, parallel to AD, in the form y = … (4)

1.5 Calculate the size of $A\hat{E}B$. (4)

1.6 Calculate the coordinates of G, such that ABGE is a parallelogram. (4)

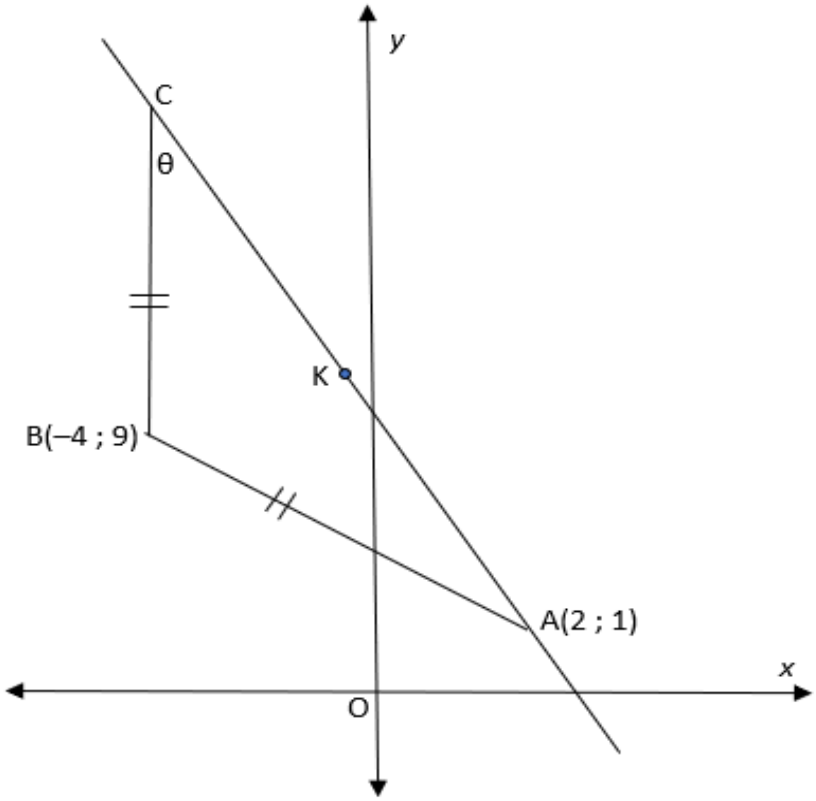
1.7 Calculated the area of parallelogram ABGE. (5)

1.8 If it is given that CD is a diameter of a circle passing through C and D, determine how many units must M be translated so that it becomes the center of the new circle. (4)

QUESTION 2

In the diagram, ABC is an isosceles triangle with A(2 ; 1) and B(–4 ; 9).

AB = BC, and BC is parallel to the y-axis.



2.1 The length of AB. (2)

2.2 The coordinates of C. (2)

2.3 Coordinates of K, the mid-point of AC. (2)

2.4 The equation of AC in the form $y = mx + c$ (3)

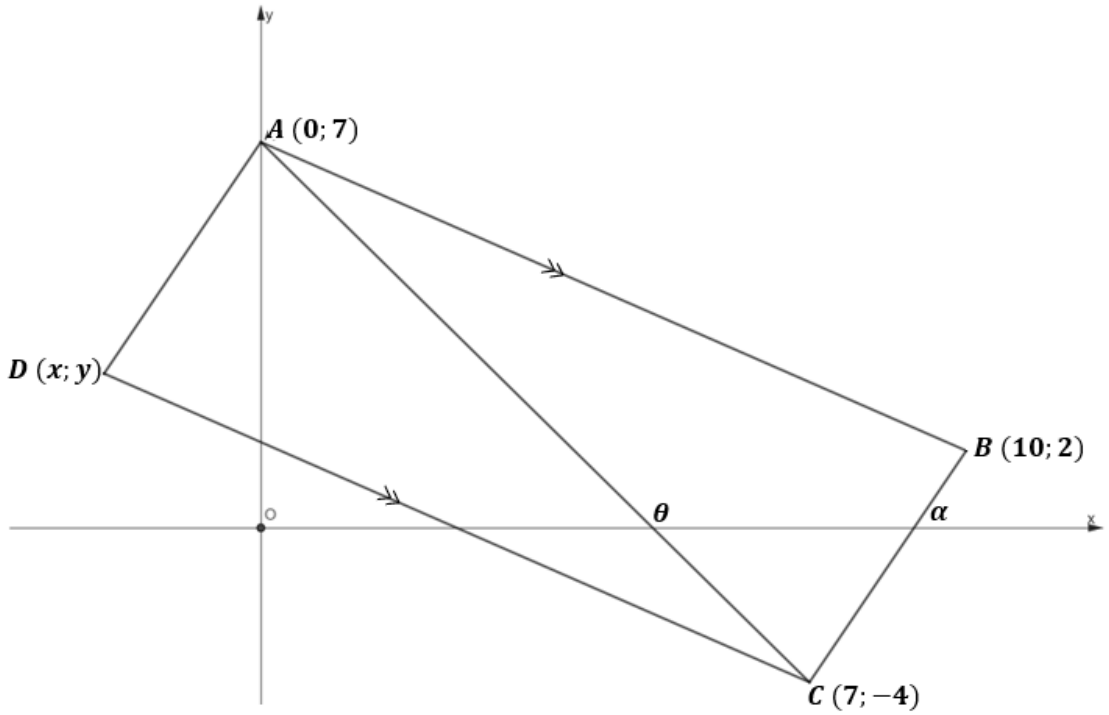
2.5 The size of θ. (3)

2.6 The area of triangle ABC. (4)

2.7 The coordinates of D if ABCD is a rhombus. (2)

QUESTION 3

In the diagram below, the points A (0; 7), B (10; 2), C (7; – 4) and D (x; y) form quadrilateral ABCD. AB ∥ CD, with AC joined. The angle of inclination for AC and BC are θ and α respectively.



Determine:

3.1 The length of AB (leave your answer in simplest surd form.) (3)

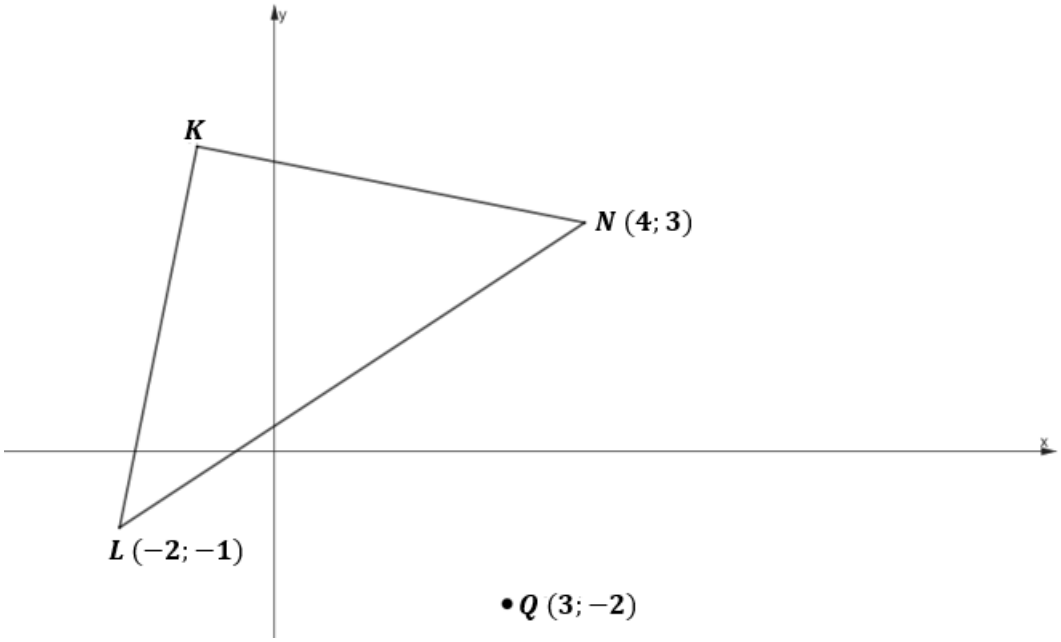
3.2 The gradient of AC. (2)

3.3 The Size of θ, the angle of inclination of AC. (3)

3.4 The magnitude (size) of $\hat{BCD}$. (3)

QUESTION 4

Consider ΔKLN drawn below. KL has the equation $y = 5x + 9$, while KN has the equation $5y + x - 19 = 0$.



4.1 Show that the coordinates of K are (– 1; 4). (3)

4.2 Show that KL ⊥ KN. (3)

4.3 Hence, or otherwise, determine the Area of ΔKLN. (4)

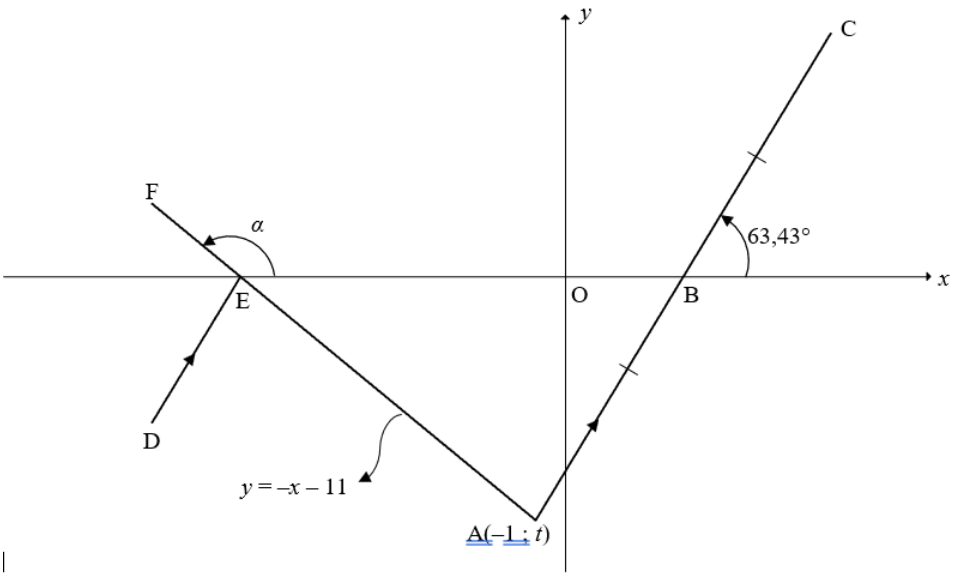
4.4 Determine the equation of the perpendicular bisector of LN. (5)

4.5 If L, N and P (7; y) are collinear, find y. (2)

4.6 Explain with reasons, why LKNQ is a square. (4)

QUESTION 5

In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1 ; t)$ and C. The angles of inclination of AF and AC are α and 63,43° respectively. AF cuts the x-axis in E. D is a point such that DE || AC.



5.1 Calculate the:

5.1.1 Value of t (2)

5.1.2 Size of α (2)

5.1.3 Gradient of AC, to the nearest whole number (2)

5.2 Determine the equation of AC in the form $y = mx + k$. (2)

5.3 Calculate the:

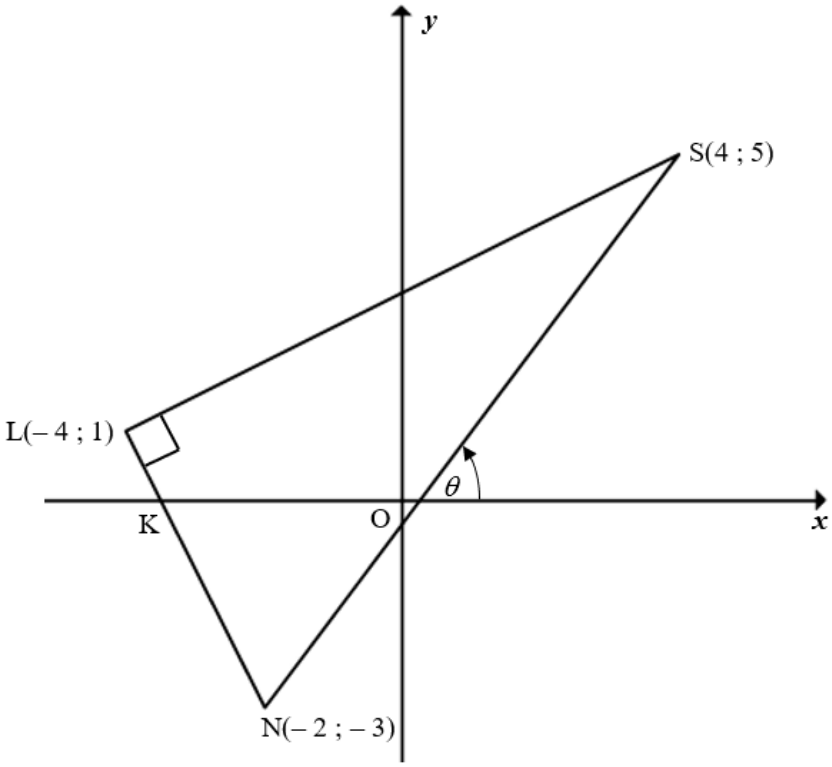
5.3.1 Coordinates of C (3)

5.3.2 Size of FÊD (3)

5.4 G is a point such that EAGC, in that order, is a parallelogram. (4)
Determine the equation of a circle centred at G and passing through the point B.
Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$.

QUESTION 6

In the figure, L$(-4\ ;\ 1)$, S$(4\ ;\ 5)$ and N$(-2\ ;\ -3)$ are the vertices of a triangle having $S\hat{L}N = 90°$. LN intersects the x-axis at K.



6.1 Calculate the length of SL. Leave your answer in surd form.

6.2 Calculate the gradient of SN.

6.3 Calculate the size of θ, the angle of inclination of SN.

6.4 Calculate the size of $L\hat{N}S$.

6.5 Determine the equation of the line which passes through L and is parallel to SN. Write your answer in the form $y = mx + c$.

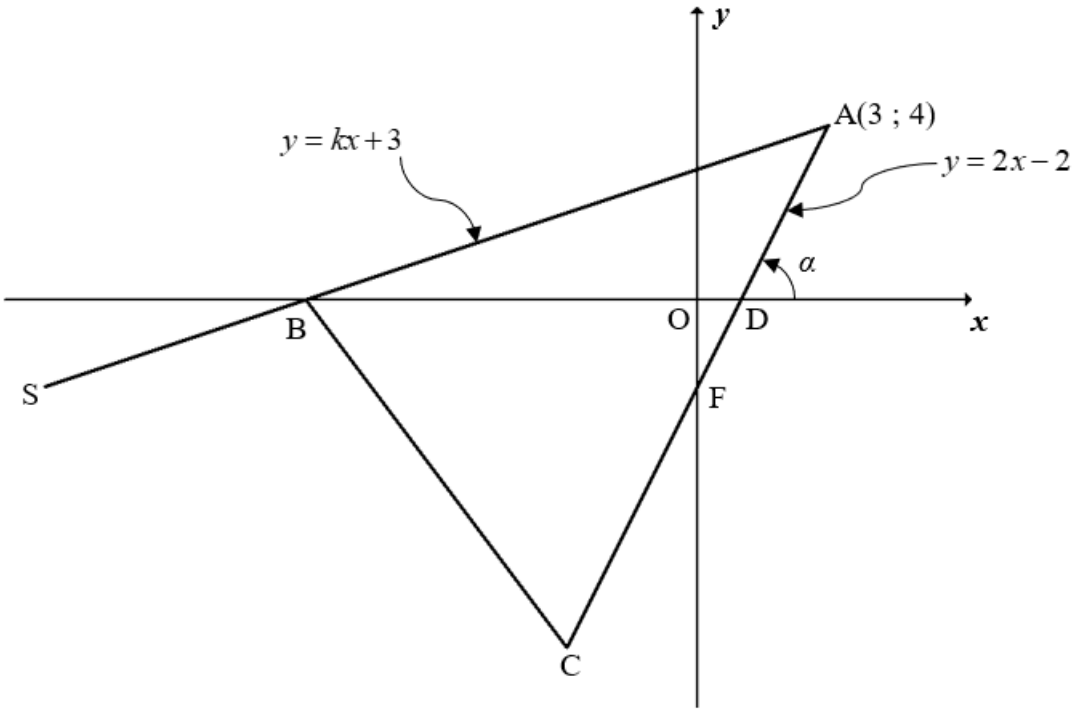
6.6 Calculate the area of ΔLSN.

6.7 Calculate the coordinates of point P, which is equidistant from L, S and N

6.8 Calculate the size of $L\hat{P}S$.

QUESTION 7

In the diagram, A(3 ; 4), B and C are vertices of ΔABC. AB is produced to S. D and F are the x- and y-intercepts of AC respectively. F is the midpoint of AC and the angle of inclination of AC is α. The equation of AB is $y = kx + 3$ and the equation of AC is $y = 2x - 2$.



7.1 Show that $k = \frac{1}{3}$. (1)

7.2 Calculate the coordinates of B, the x-intercept of line AS. (2)

7.3 Calculate the coordinates of C. (4)

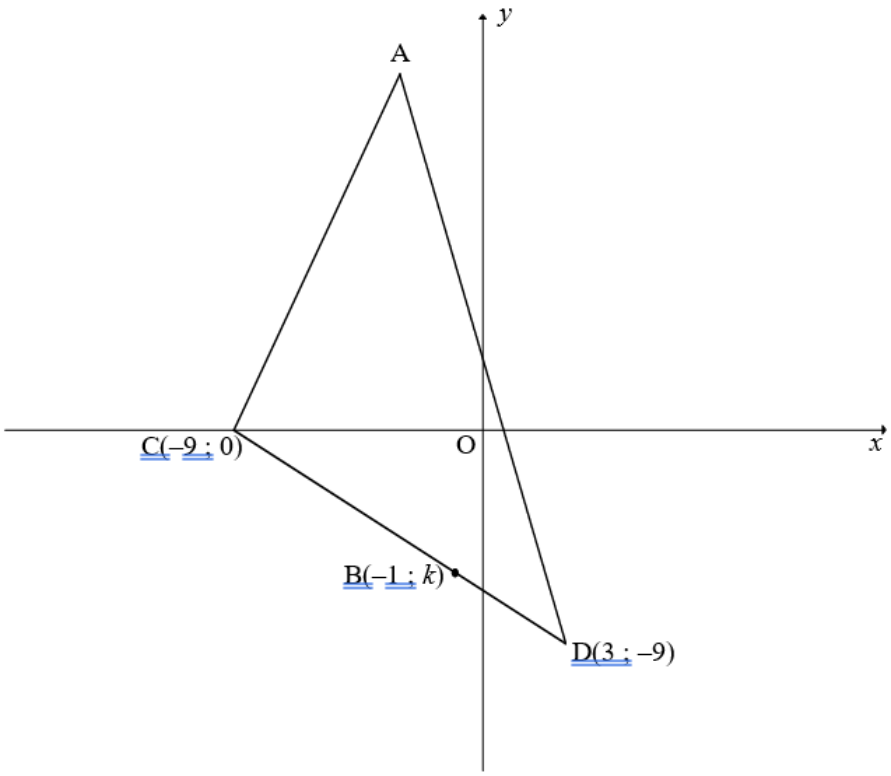
7.4 Determine the equation of the line parallel to BC and passing through S(–15 ; –2). Write your answer in the form $y = mx + c$. (5)

7.5 Calculate the size of $B\hat{A}C$. (5)

7.6 If it is further given that the length of AC is $6\sqrt{5}$ units, calculate the value of $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC}$. (5)

QUESTION 8

In the diagram below, ΔACD has vertices A, D(3 ; –9) and C(–9 ; 0), where A is a point in the second quadrant. B(–1 ; k) lies on side DC.



8.1 Calculate the gradient of DC. (2)

8.2 Determine the equation of DC in the form $y = mx + c$. (2)

8.3 Show that $k = -6$. (1)

8.4 Calculate the length of DC. (2)

8.5 Calculate the ratio of $\frac{DB}{DC}$. (2)

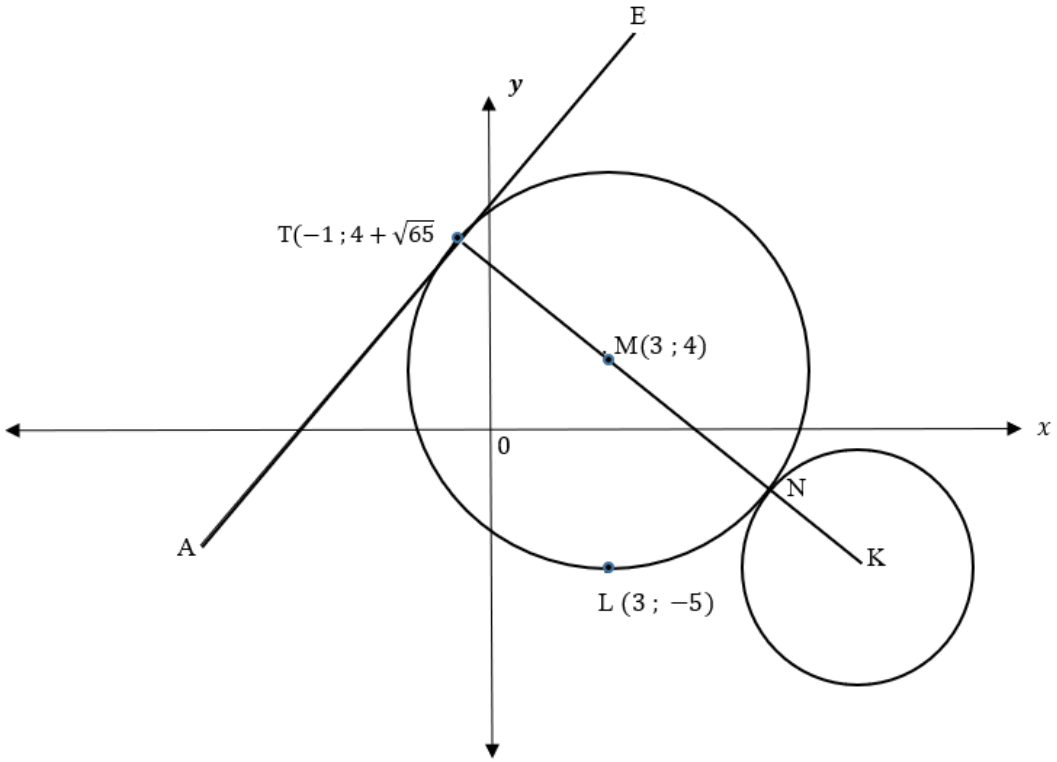
8.6 If M is a point on AD such that AC || MB, calculate the ratio of $\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD}$. (4)

8.7 If it is further given that the gradient of AD is -4 and the length of AD is $\sqrt{612}$ units, calculate the coordinates of A. (6)

CIRCLES

QUESTION 1

Drawn below is the BIGGER circle centered at M and SMALLER circle centered at K. ATE is a tangent to the bigger circle at T. TN is a diameter of the bigger circle and NK is a radius of the smaller circle. The coordinates of T (– 1; 8), M (3 ; 4) and L (3 ; – 5)



1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (2)

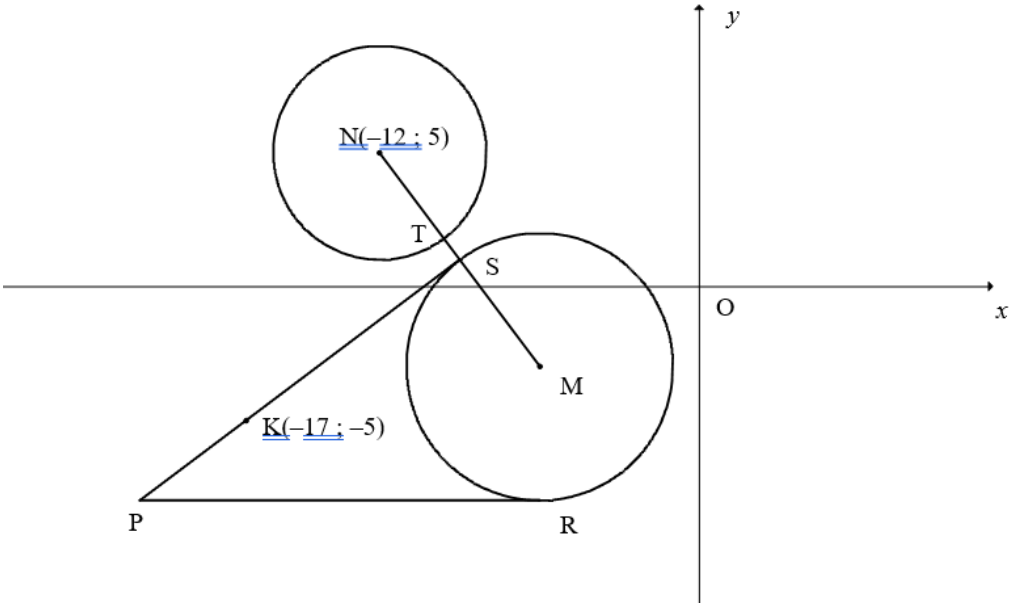
1.2 Determine the equation of the tangent through point T. (5)

1.3 Does point P(7; 3) lie inside, outside or on the circle. Show all calculations. (4)

1.4 If it is further given that KL is a tangent at L, to the circle centered at M. Determine the coordinates of K, the center of the smaller circle. (5)

QUESTION 2

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the *x*-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



2.1 Write down the coordinates of M. (2)

2.2 Calculate the:

2.2.1 Length of the radius of the smaller circle (2)

2.2.2 Length of TS (4)

2.3 Determine the equation of the tangent:

2.3.1 PR (2)

2.3.2 PS, in the form $y = mx + c$ (5)

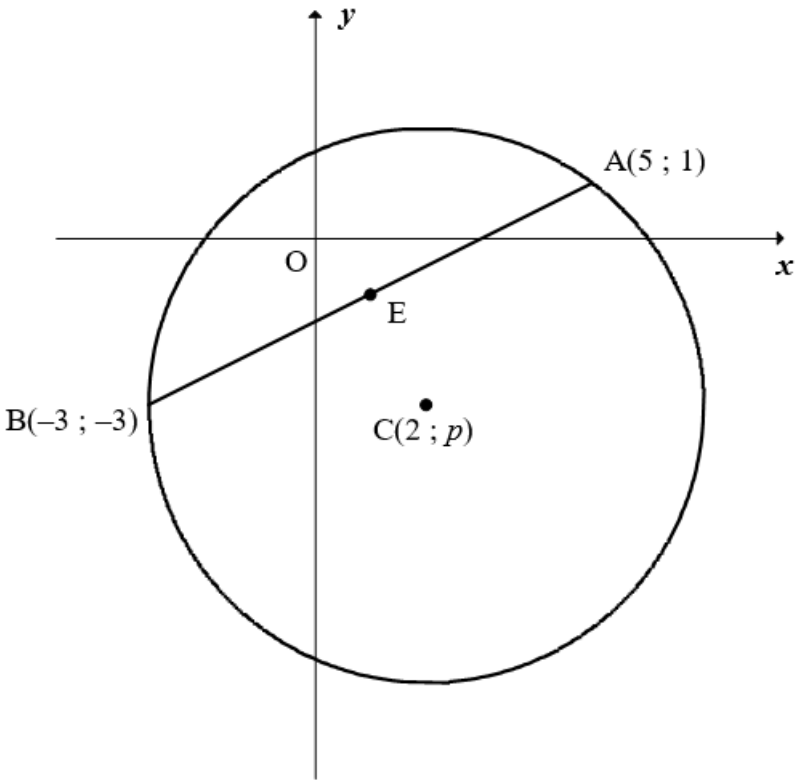
2.4 Quadrilateral PSMR is drawn. Calculate the:

2.4.1 Perimeter of PSMR (5)

2.4.2 Ratio of $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ (2)

QUESTION 3

In the diagram, the circle centred at $C(2\,;p)$ is drawn. $A(5\,;1)$ and $B(-3;-3)$ are points on the circle. E is the midpoint of AB.



3.1 Calculate the coordinates of E, the midpoint of AB. (2)

3.2 Calculate the length of AB. Leave your answer in surd form. (1)

3.3 Determine the equation of the perpendicular bisector of AB in the form $y = mx + c$. (4)

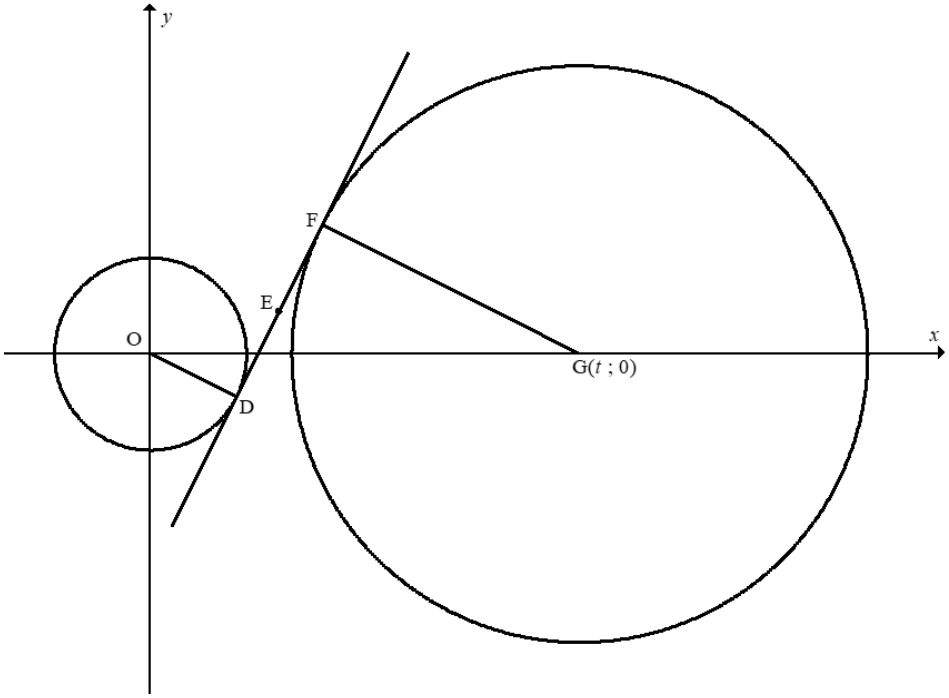
3.4 Show that $p = -3$. (1)

3.5 Show, by calculation, that the equation of the circle is $x^2 + y^2 - 4x + 6y - 12 = 0$. (4)

3.6 Calculate the values of t for which the straight line $y = tx + 8$ will not intersect the circle. (6)

QUESTION 4

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. G$(t\ ;\ 0)$ is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that D$(p\ ;\ -2)$ lies in the 4th quadrant.



4.1 Given that D$(p\ ;\ -2)$ lies on the smaller circle, show that $p = 4$. (2)

4.2 E$(6\ ;\ 2)$ is the midpoint of DF. Determine the coordinates of F. (3)

4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

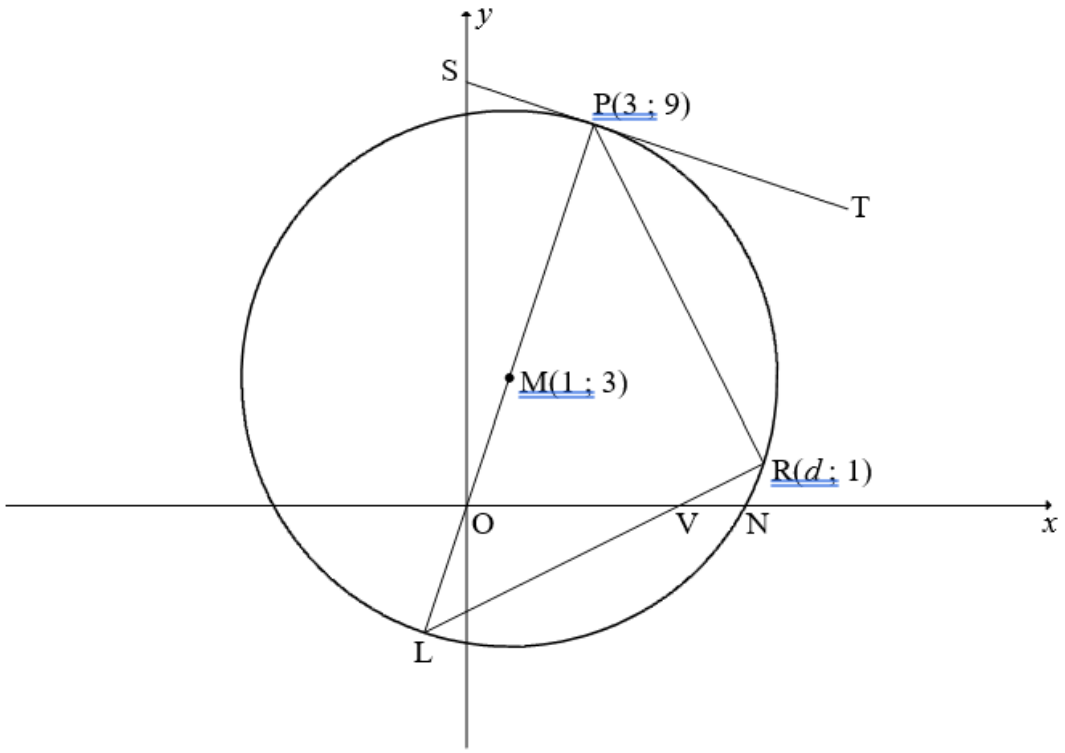
4.4 Calculate the value of t. Show ALL working. (3)

4.5 Determine the equation of the larger circle in the form $ax^2 + by^2 + cx + dy + e = 0$. (4)

4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)

QUESTION 5

In the diagram, M(1 ; 3) is the centre of the circle. The circle cuts the x-axis at N. ST is a tangent to the circle at P(3 ; 9). R(d ; 1), with $d > 0$, and L lie on the circle. O and V are the x-intercepts of PL and RL respectively.



5.1 Write down the coordinates of L. (2)

5.2 Determine the equation of tangent ST to the circle at P. (4)

5.3 Show that the equation of the circle with centre M is $x^2 + y^2 - 2x - 6y - 30 = 0$. (4)

5.4 Show that $d = 7$. (2)

5.5 Calculate the size of $\hat{L}$. (5)

5.6 TR is a tangent to the circle at R. Prove that PT $\perp$ RT. (3)

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

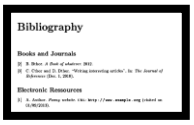
$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$



BIBLIOGRAPHY

1	Mathematics CAPS Document (Grade 10 – 12)
2	Past Exam Papers and Memos
3	Diagnostic reports