SUBJECT: MATHEMATICS

GRADE 12

AUTUMN CLASSES

TEACHER AND LEARNER CONTENT MANUAL

Topic:

TRIGONOMETRY

CONTENTS PAGE

SECTION 1: TRIGONOMETRY FORMULAE

Provided in the Information Sheet

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad a^2=b^2+c^2-2bc.\cos A$ $\quad area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$$\sin 2\alpha=2\sin\alpha.\cos\alpha$$

Not provided in the Information Sheet

$$\underbrace{\sin^2\alpha+\cos^2\alpha=1}_{\begin{array}{l}\sin^2\alpha=1-\cos^2\alpha\\ \cos^2\alpha=1-\sin^2\alpha\end{array}} \qquad \tan\alpha=\frac{\sin\alpha}{\cos\alpha}$$

SECTION 2: SIMPLIFYING TRIGONOMETRIC EXPRESSIONS (APPLICATION OF REDUCTIONS AND SPECIAL ANGLES) AND TRIGONOMETRIC IDENTIES

Reduction Formulae

Reduction formulae are used to reduce the trigonometric ratio of any angle to the trigonometric ratio of an acute angle. The formulae you will use are $180° \pm \theta$, $360° \pm \theta$, $90° \pm \theta$ and $(-\theta)$.

The Formulae

90°

Cofunctions

$\boldsymbol{sin}(90° + A) = \boldsymbol{cos}A$

$\boldsymbol{cos}(90° + A)$

2nd Quadrant

$\sin(180° - A) = \sin A$

$\cos(180° - A) = -\cos A$

$\tan(180° - A) = -\tan A$

$\sin(90° + A) = \cos A$

$\cos(90° + A) = -\sin A$

1st Quadrant

$\sin(90° - A) = \cos A$

$\cos(90° - A) = \sin A$

$\sin(360° + A) = \sin A$

$\cos(360° + A) = \cos A$

$\tan(360° + A) = \tan A$

Cofunctions

$\boldsymbol{sin}(90° - A) = \boldsymbol{cos}A$

$\boldsymbol{cos}(90° - A) = \boldsymbol{sin}A$

180° 360°

3rd Quadrant

$\sin(180° + A) = -\sin A$

$\cos(180° + A) = -\cos A$

$\tan(180° + A) = \tan A$

4th Quadrant

$\sin(360° - A) = -\sin A$

$\cos(360° - A) = \cos A$

$\tan(360° - A) = -\tan A$

$$\sin(-A) = -sinA$$
$$\cos(-A) = cosA$$
$$\tan(-A) = -tanA$$

270°

N.B

1. For angles greater than 360° you can subtract multiples of 360° until you get an angle between 0° and 360°.

   e.g Simplify $sin750°$ **30° = 750° – 360° – 360°**

   $= sin30°$

   $= \frac{1}{2}$

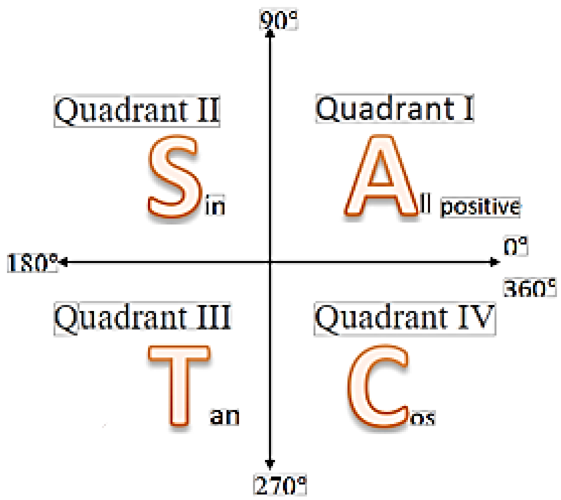
2. For angles less than 0° you can add multiples of 360° until you get an angle between 0° and 360°.

   e.g Simplify $sin - 315°$ **45° = -315° + 360°**

   $= sin45°$

   $= \frac{\sqrt{2}}{2}$

Use **ASTC Rule** to get the correct signs of different ratios in each quadrant.



Quadrant 1	**Quadrant 2**	**Quadrant 3**	**Quadrant 4**
Sine +	Sine +	Sine –	Sine –
Cosine +	Cosine –	Cosine –	Cosine +
Tangent +	Tangent –	Tangent +	Tangent –
All	**Students (sin)**	**Take (tan)**	**Caps (cos)**

Worked Examples

Example 1

Simplify fully: $\sin(90^\circ - x).\cos(180^\circ + x) + \tan x.\cos x.\sin(x - 180^\circ)$

Solution

$$\sin(90^\circ - x).\cos(180^\circ + x) + \tan x.\cos x.\sin(x - 180^\circ)$$
$$= \cos x.(-\cos x) + \frac{\sin x}{\cos x}.\cos x.(-\sin x)$$
$$= -\cos^2 x - \sin^2 x$$
$$= -(\cos^2 x + \sin^2 x)$$
$$= -1$$

$$\sin(x - 180^\circ) = \sin(-(180^\circ - x))$$
$$= -\sin(180^\circ - x)$$
$$= -sinx$$

OR

$$\sin(x - 180^\circ)$$
$$= \sin(x - 180^\circ + 360^\circ)$$
$$= \sin(180^\circ + x)$$
$$= -sinx$$

Example 2

Simplify fully:

$$\frac{\sin(180^\circ - x).\cos(360^\circ + x).\tan(-x)}{\cos(90^\circ - x).\cos(180^\circ + x).\tan(180^\circ + x)}$$

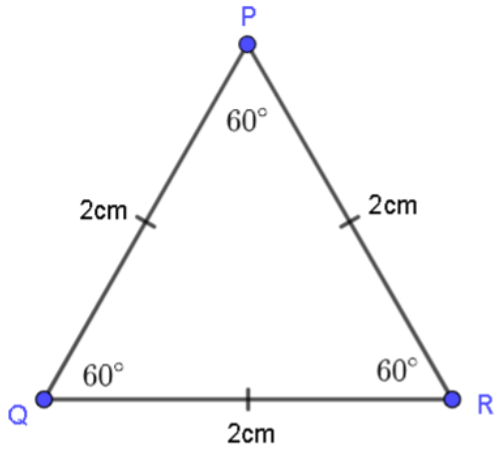
Solution

$$\frac{\sin(180^\circ - x).\cos(360^\circ + x).\tan(-x)}{\cos(90^\circ - x).\cos(180^\circ + x).\tan(180^\circ + x)}$$
$$= \frac{(+\sin x).(+\cos x).(-\tan x)}{(+\sin x).(-\cos x).(+\tan x)}$$
$$= +1$$

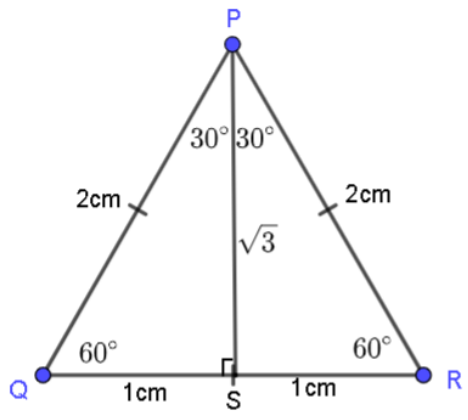
Application of Special Angles

The following are ratios of the special angles (30°, 45° and 60°)

From the equilateral triangle:



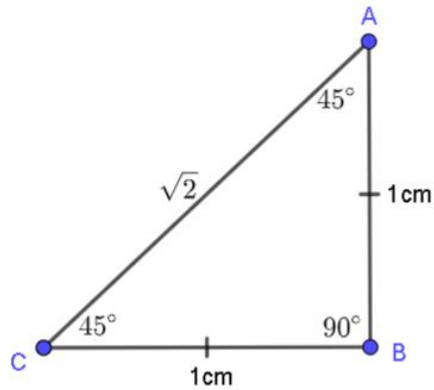
then drop the perpendicular PS



$PS \perp QR$

$PS = \sqrt{3}(from Pythagoras theorem)$

From the isosceles triangle:

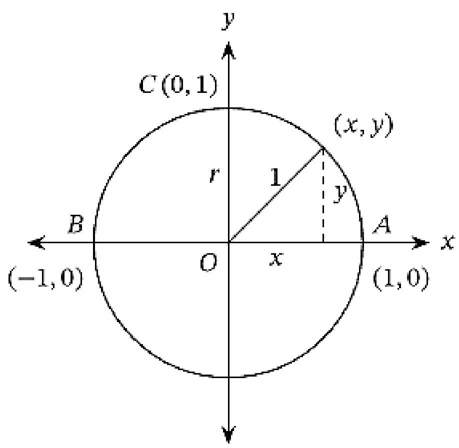


$AC = \sqrt{2}(from Pythagoras theorem)$

If you find it difficult to remember the diagrams, then learn this summary of the special angles.

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

The following are ratios of the special angles (0°, 90°, 180°, and 360°)



Angle A is at position 0°/360°

Angle C is at position 90°

Angle B is at position 180°

N.B: Remember that on the cartesian plane $sin\alpha = \frac{y}{r}$, $cos\alpha = \frac{x}{r}$ $\quad and \quad$ $tan\alpha = \frac{y}{x}$

α	0° / 360°	90°	180°
$sin\alpha$	0	1	0
$cos\alpha$	1	0	-1
$tan\alpha$	0	UNDEFINED	0

Worked Examples

Example 1

Prove, WITHOUT using a calculator, that

$$\frac{\sin 315^\circ . \tan 210^\circ . \sin 190^\circ}{\cos 100^\circ . \sin 120^\circ} = \frac{-\sqrt{2}}{3}$$

Solution

$$\text{LHS} = \frac{\sin 315^\circ . \tan 210^\circ . \sin 190^\circ}{\cos 100^\circ . \sin 120^\circ}$$

$$= \frac{(-\sin 45^\circ).(\tan 30^\circ)(-\sin 10^\circ)}{(-\sin 10^\circ).(\sin 60^\circ)}$$

$$= \frac{-\frac{1}{\sqrt{2}} . \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}}$$

$$= -\frac{\sqrt{2}}{3}$$

Use the reduction formula: (360°-A)

$$\boldsymbol{sin}\mathbf{315^\circ} = \mathbf{\sin(360^\circ - 45^\circ)}$$
$$= (-\boldsymbol{sin}\mathbf{45^\circ})$$

Use the reduction formula : (90+-A)

$$\boldsymbol{cos}\mathbf{100^\circ} = \mathbf{\cos(90^\circ + 10^\circ)}$$
$$= (-\boldsymbol{sin}\mathbf{10^\circ})$$

Example 2

Simplify WITHOUT using a calculator:

$$\frac{\sin 120^\circ . \cos 210^\circ . \tan 315^\circ . \cos 27^\circ}{\sin 63^\circ . \cos 540^\circ}$$

Solution

$$\text{LHS} = \frac{\sin 120°.\cos 210°.\tan 315°.\cos 27°}{\cos 540°.\sin 63°}$$

$$= \frac{\sin 60°.(-\cos 30°)(-\tan 45)°.\sin 63°}{\cos 180°.\sin 63°}$$

$$= \frac{\frac{\sqrt{3}}{2}.\frac{-\sqrt{3}}{2}(-1)}{-1}$$

$$= -\frac{3}{4}$$

Example 3

Simplify fully, WITHOUT the use of a calculator:

$$\frac{\cos(-225°).\sin 135° + \sin 330°}{\tan 225°}$$

$\cos(-225°) = \cos 225°$
$= \cos(180° + 45°)$

Or

Add 360° to -225°

$\cos(-225°) = \cos 135°$
$= \cos(180° - 45°)$
$= -\cos 45°$

Solution

$$\frac{\cos(180° + 45°)\sin(180° - 45°) + \sin(360° - 30°)}{\tan(180° + 45°)}$$

$$= \frac{(-\cos 45°).(\sin 45°) - \sin 30°}{\tan 45°}$$

$$= \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1}$$

$$= -1$$

Trigonometric Identities and Simplification with Compound and Double Angle Identities

We will revise trigonometric identities learnt in Grade 11. We will further have a look at two more identities in Grade 12. The examples that will follow will require you to use the information of the topics already covered above (special angles and reduction formulae)

Grade 11 revision

The square identity formula	The quotient identity formula
$\boldsymbol{\sin^2\alpha + \cos^2\alpha = 1}$ $\boldsymbol{\sin^2\alpha = 1 - \cos^2\alpha}$ $\boldsymbol{\cos^2\alpha = 1 - \sin^2\alpha}$	$\boldsymbol{\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}}$

N.B: The above formulae will not be given on the information sheet; you must learn them by heart.

Grade 12 Identities (provided on the information sheet)

Compound Angle Formulas	$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$	$\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$
Double Angle Formulas	$\cos 2\alpha = \begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$	$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

N.B: if you come across questions with double or compound angles with the tangent ratio, use the quotien identity formula:

$$1.\ \tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$2.\ \tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$$

Worked Examples

Example 1

Without using a calculator, evaluate

$\cos 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$

Solution

$$\cos 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$$
$$= \cos 79^\circ \cos 49^\circ + \sin 79^\circ \sin 49^\circ$$
$$= \cos(79^\circ - 49^\circ)$$
$$= \cos 30^\circ$$
$$= \frac{\sqrt{3}}{2}$$

$\cos\ 311^\circ = \cos(360^\circ - 49^\circ)$
$= cos49^\circ$

$sin101^\circ = \sin(180^\circ - 79^\circ)$
$= sin79^\circ$

Example 2

Given: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$

Prove that $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$

Solution

LHS: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x}$

$$= \frac{\cos x}{2\sin x \cos x} - \frac{1 - 2sin^2 x}{2\sin x}$$
$$= \frac{1}{2\sin x} - \frac{(1 - 2sin^2 x)}{2\sin x}$$
$$= \frac{1 - 1 + 2sin^2 x}{2\sin x}$$
$$= \frac{2sin^2 x}{2\sin x}$$
$$= \sin x$$

=RHS

Double angel($\sin 2x$) – immediately apply the formula as there is a single angle ($\sin x$) in the denominator

Double angel ($cos2x$) – immediately apply the formula as there is a single angle ($cosx$) in the numerator

Example 3

Simplify the following expression.

$$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$

Solution

$$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$

$$=\frac{(-\sin x)(-\sin x)-\cos^2 x}{\cos 2x}$$

$\cos^2(180°-x)$
$=[\cos(180°-x)]^2$
$=[-\cos x]^2$
$=\cos^2 x$

$$=\frac{\sin^2 x-\cos^2 x}{\cos^2 x-\sin^2 x} \quad \textbf{OR} \quad \frac{-\cos 2x}{\cos 2x}$$

$$=\frac{-(\cos^2 x-\sin^2 x)}{\cos^2 x-\sin^2 x} \qquad\qquad =-1$$

$$=-1$$

Example 4

Prove that $\dfrac{\sin 2x+\sin x}{\cos 2x+\cos x+1}=\tan x$

Solution

$$LHS=\frac{\sin 2x+\sin x}{\cos 2x+\cos x+1}$$

Double angel($sin2x$) – immediately apply the formula as there is a single angle ($sinx$) in the numerator

Double angel($cos2x$) – immediately apply the formula as there is a single angle ($cosx$). N.B apply the formula that has (-1) to eliminate (+1) in the denominator

$$=\frac{2\sin x\cos x+\sin x}{2\cos^2 x-1+\cos x+1}$$

$$=\frac{\sin x(2\cos x+1)}{2\cos^2 x+\cos x}$$

$$=\frac{\sin x(2\cos x+1)}{\cos x(2\cos x+1)}$$

$=\tan x$

= RHS

Example 5

Simplify

$$\frac{\sin^3 x + \sin x \cos^2 x}{\cos(\,360° - x)}$$

Solution

Use the square identity formula:

$$\boldsymbol{\sin^2 x + \cos^2 x = 1}$$

$$\frac{\sin^3 x+\sin x \cos^2 x}{\cos(360°-x)}$$

$$= \frac{\sin x\,(\sin^2 x + \cos^2 x)}{\cos x}$$

$$= \frac{(\sin x)(1)}{\cos x}$$

$$= \tan x$$

Activities:

Activity 1

FS 2025 Trial Exam Paper 2

5.2 Simplify:

$$\frac{\cos(90° + x).\cos(x - 180°).\tan(360° + x)}{\cos 240°.\tan 225°}$$

5.3 Prove the identity:

$$\frac{1 + \cos 2A}{\cos 2A} = \frac{\tan 2A}{\tan A}$$

5.5 Given: $\cos(x + y) - \cos(x - y) = -2\sin x \sin y$

5.5.1 Prove the above identity

5.5.2 Hence deduce that $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

Activity 2

DBE November 2025 paper 2

5.2 Given: $\dfrac{\sin(540^0 + x).\cos(90^0 + x)}{\sin(-x)}$

5.2.1 Simplify the expression above fully to a single trigonometric ratio.

5.2.2 Hence, determine the value of x in the interval $x \in [0^0; 360^0]$ for which $\sqrt{\dfrac{\sin(540^0 + x).\cos(90^0 + x)}{\sin(-x)}}$ will be real

6.1 Prove that: $\left[\tan(180^0 - x)\right](1 - \cos^2 x) + \cos^2 x = \dfrac{(\sin x - \cos x)(1 + \sin x.\cos x)}{-\cos x}$

Activity 3

LP 2025 Trial Exam Paper 2

6.1 Simplify the following without using a calculator:

$$\frac{2\cos 105^\circ \cos 15^\circ}{\cos(45^\circ - x)\cos x - \sin(45^\circ - x)\sin x}$$

GP 2025 Trial Exam Paper 2

6.1 Simplify the following to a single trigonometric term, **without the use of a calculator**:

$$\frac{\tan(180^\circ - x)\cos(180^\circ - x)}{\cos 240^\circ \left(\tan^2 y - \frac{1}{\cos^2 y}\right)}$$

6.2 Prove the identity: $\frac{\sin 3x}{\sin x \cos x} = \frac{4\cos^2 x - 1}{\cos x}$

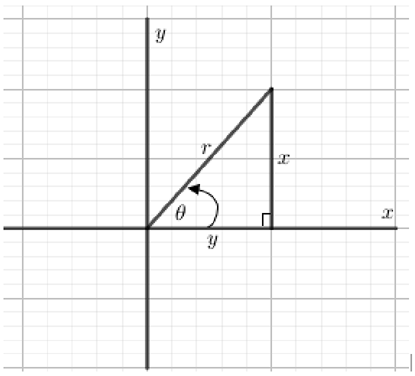
SECTION 3: EQUATIONS (AND THE USE OF A DIAGRAM)

<u>Cartesian Plane (With the Aid of a Diagram)</u>

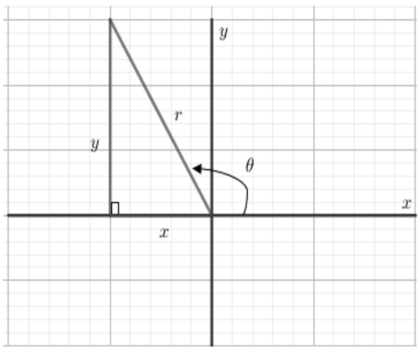
In the exam or test you will need to use the aid of the diagram to answer some questions. The following diagrams will show us how to draw our right-angled triangles in different quadrants.

N.B $\boldsymbol{\theta}$ *is an angle measured from the positive* $\boldsymbol{x}$ *– axis to the termila arm (radius)*

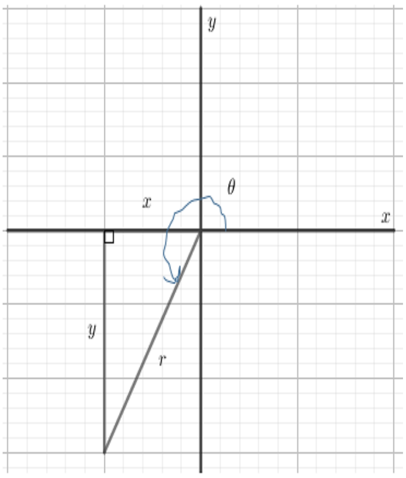
Quadrant 1



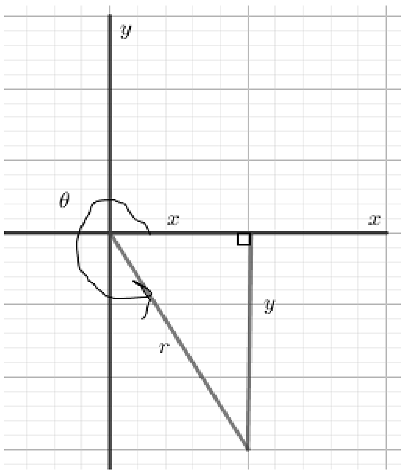
Quadrant 2



Quadrant 3



Quadrant 4



In this section you will represent the trigonometric ratios in terms of x, y *and* r, since you will be working in a cartesian plane.

$$sin\theta = \frac{y}{r} \quad , \quad \cos\theta = \frac{x}{r} \quad , \quad \tan\theta = \frac{y}{x}$$

N.B: Pythagoras Theorem

$x^2 + y^2 = r^2$

Example 1

Draw the right-angled triangle under the conditions below:

a. $\sin\theta < 0$ and $90 < \theta < 270$.
b. $\cos\theta > 0$ and $\tan\theta < 0$.

Draw the right-angle triangle in the correct quadrant and find the values of $x, y, and\ r$:
c. $7\tan\theta + 3 = 0$ and $\theta \in [90°: 270°,]$

Solutions

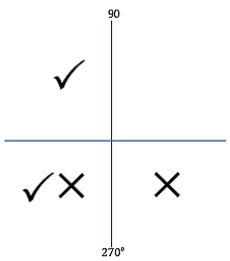
a. $\sin\theta < 0$ and $90 < \theta < 270$.

Step 1:
→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

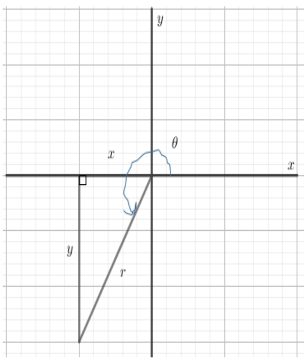
Sine is negative in the 3rd and 4th quadrants.

This interval represents 2nd and 3rd quadrants

Given that $\sin\theta < 0$ and $90 < \theta < 270$.



Where **X** represents the quadrant we obtained from sine ratio and √ is the quadrants obtained from the interval. We can notice the 3rd quadrant satisfies both conditions , hence the terminal arm will be in the 3rd quadrant.
→ Step 2: Complete the right-angled triangle.



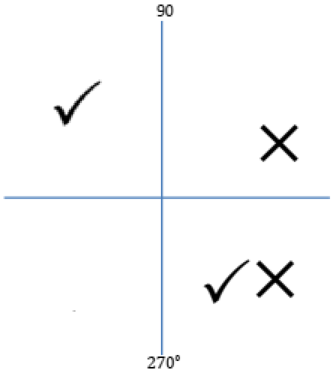
b. $\cos\theta > 0$ and $\tan\theta < 0$

Step 1:
→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

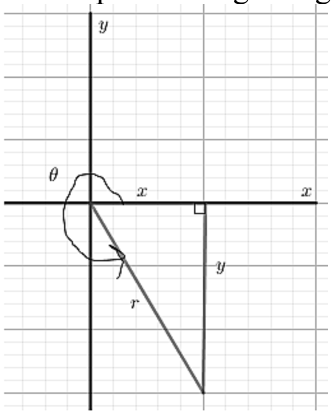
cosine is positive in the 1st and 4th quadrants.

tangent is negative in the 2nd and 4th quadrants

Given that $\cos\theta > 0$ and $\tan\theta < 0$.



Where X represents the quadrant we obtained from cos and √ is the quadrants obtained from the tan . We can notice the 4th quadrant it satisfy both conditions , hence the terminal arm will be in the 4th quadrant.
→ Complete the right-angled triangle.



c. $7\tan\theta + 3 = 0$ and $\theta \in [90°: 270°]$

Step1:
Simplify the first trig equation in such a way that we only have trig ratio(tan θ) on the left hand side.

$7\tan\theta + 3 = 0$

$7\tan\theta = -3$

$\tan\theta = -\frac{3}{7}$

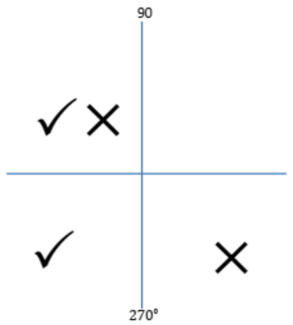
Step 2:

→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

tangent is negative in the 2^{nd} and 4^{th} quadrants.

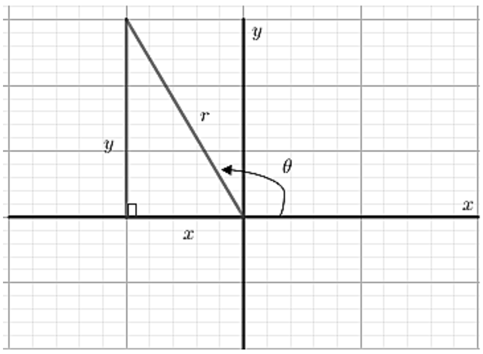
This interval represents 2^{nd} and 3^{rd} quadrants

After simplifying $\tan\theta = -\frac{3}{7} = \frac{y}{x}$ and $\theta \in [90°: 270°]$



Where X represents the quadrant we obtained from tan and tick is the quadrants obtained from the interval. We can notice the 2^{nd} quadrant satisfies both conditions , hence the terminal arm will be in the 2^{nd} quadrant.

→ Complete the right-angled triangle.



Now determine the values of $x, y, and\ r$:

$x^2 + y^2 = r^2 \qquad (Pythagoras)$

$(-7)^2 + (3)^2 = r^2$

$r^2 = 58$

$r = \sqrt{58}$

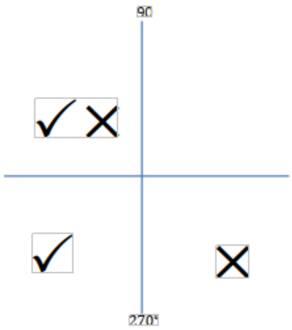
N.B: Remember hypotenuse (r) is ALWAYS positive, and for the values of x and y, they can be positive or negative depending on the quadrant you are working in.

Example 2

Given that $\sin\theta = \frac{3}{5}$, and $\theta \in [90°: 360°,]$ find the value of the following **without the use of a calculator:**
Now Evaluate:

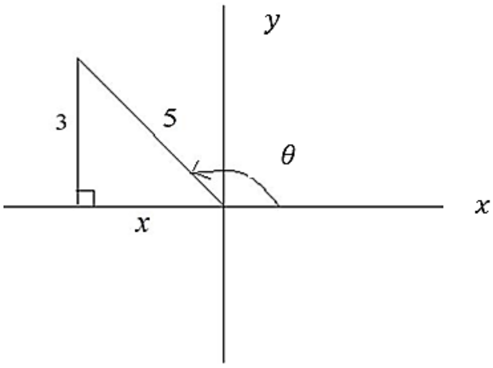
$a.\quad \cos\theta$
$b.\quad \tan(180° - \theta)$
$c.\quad \sin(-\theta)$
$d.\quad \sin(\theta - 180°)$
$e.\quad \cos(450° + 2\theta)$
$f.\quad \sin(45° - \theta)$

Solution:



We are going to draw the triangle in a 2nd quadrant.

$\sin\theta = \frac{3}{5} = \frac{y}{r}$,



In the triangle above we do not have the value of x, then we will calculate it using **theorem of Pythagoras**:

$x^2 + y^2 = r^2 \quad (Pythagoras)$
$x^2 + (3)^2 = (5)^2$
$x^2 + 9 = 25$
$x^2 = 25 - 9$
$x^2 = 16$
$x = \sqrt{16}$
$x = \pm 4$
$x = -4$, because the x value in the 2nd quadrant where x is negative.

a. $\cos\theta = \frac{x}{r}$

$= \frac{-4}{5}$

$= -\frac{4}{5}$

b. $\tan(180 - \theta)$

$= -\tan\theta$

And $\tan\theta = \frac{y}{x}$

$= -(\frac{3}{-4})$

$= \frac{3}{4}$

c. $\sin(-\theta)$

$= -\sin\theta$

$= -\frac{3}{5}$

d. $\sin(\theta - 180°)$

$= \sin(-180° + \theta)$

$= \sin[-(180° - \theta)]$

$= -\sin(180° - \theta)$

$= -\sin\theta$

$= -\frac{3}{5}$

e. $\cos(450 + 2\theta)$

$= \cos(450°-360° + 2\theta)$

$= \cos(90° + 2\theta)$

$= -\sin 2\theta$

$= -2\sin\theta.\cos\theta$

$= -2(\frac{3}{5}).(-\frac{4}{5})$

$= \frac{24}{25}$

f. $\sin(45° - \theta)$

$= \sin(45° - \theta°)$

$= \sin 45°.\cos\theta° - \cos 45°.\sin\theta°$

$= \frac{1}{\sqrt{2}}.\left(-\frac{4}{5}\right) - \frac{1}{\sqrt{2}}.\frac{3}{5}$

$= -\frac{4}{5\sqrt{2}} - \frac{3}{5\sqrt{2}}$

$= -\frac{7}{5\sqrt{2}}$

$= -\frac{7}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= -\frac{7\sqrt{2}}{10}$

Example 3

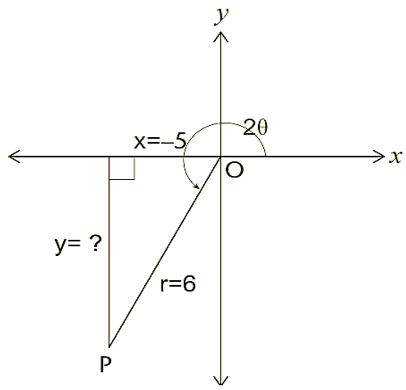
If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator,** the values in simplest form of:

a. $\boldsymbol{\sin 2\theta}$

b. $\boldsymbol{\sin^2\theta}$

c. $\boldsymbol{\tan 2\theta}$

Solutions



Before we can answer the questions, we can note that in the triangle we do not have y value.

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = (6)^2$

$y^2 = 36 - 25$

$y^2 = 11$

$y = \sqrt{11}$

$y = \pm\sqrt{11}$

$y = -\sqrt{11}$ **, because the y value in the 3rd quadrant is negative.**

a. Sin 2θ

$= \frac{-\sqrt{11}}{6}$

b. $\sin^2\theta$

$\cos 2\theta = -\frac{5}{6}$

$1 - 2\sin^2\theta = -\frac{5}{6}$

$2\sin^2\theta = 1 + \frac{5}{6}$

$2\sin^2\theta = \frac{11}{6}$

$\sin^2\theta = \frac{11}{6} \div 2$

$\sin^2\theta = \frac{11}{6} \times \frac{1}{2}$

$\sin^2\theta = \frac{11}{12}$

Use the double angle identity formula:

$\cos 2\theta = 1 - 2\sin^2\theta$

c. $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$= -\frac{\sqrt{11}}{6} \Big/ -\frac{5}{6}$

$= -\frac{\sqrt{11}}{6} \times \left(-\frac{6}{5}\right)$

$= \frac{\sqrt{11}}{5}$

Example 4

It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°\,;270°]$ and $\beta \in [90°\,;270°]$. Determine, without using a calculator, the values of the following:

a) $\cos\alpha$
b) $\cos(\alpha + \beta)$

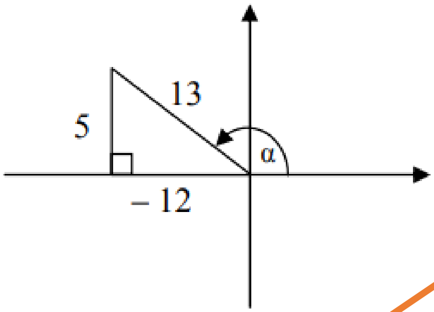
Solution
a)

$\sin\alpha = \frac{5}{13}$

$y_\alpha = 5 \quad r_\alpha = 13$

$x_\alpha = -12$

$\cos\alpha = -\frac{12}{13}$



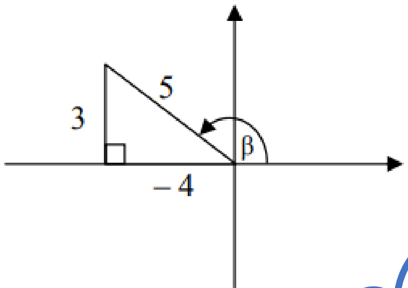
$x^2 + y^2 = r^2$
$x^2 + 5^2 = 13^2$
$x^2 = 144$
$x = \pm 12$
$\therefore x = -12$

b)

$\tan\beta = -\frac{3}{4}$

$y_\beta = 3 \quad x_\beta = -4$

$r = 5$



$x^2 + y^2 = r^2$
$(-4)^2 + 3^2 = r^2$
$r^2 = 25$
$\therefore r = 5$

$\cos(\alpha + \beta)$

$= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{48 - 15}{65}$

$= \frac{33}{65}$

Ratios With Known Angle and Unknow Value

We now focus on questions where the angle of a ratio is given, and we have to simplify in terms of the given letter (unkown value of the ratio).

Worked Examples

Example 1

If, $sin31° = p$ determine, without using a calculator, the following in terms of p:

a. $\sin 149°$
b. $\cos(-59°)$
c. $cos62°$
d. $sin59°$

Use the reduction formula:
$\boldsymbol{sin(180° - \alpha) = sin\alpha}$
$\boldsymbol{sin149° = sin(180° - 31°)}$
$\boldsymbol{= sin31°}$

Solution

Method 1

a.
$\sin 149° = \sin 31° = p$

b.
$\cos(-59°) = \cos 59° = \sin 31° = p$

c.
$\cos 62° = 1 - 2\sin^2 31° = 1 - 2p^2$

Use the reduction formula
$\boldsymbol{cos(90° - \alpha) = sin\alpha}$

$\therefore$

$\boldsymbol{cos59° = cos(90° - 31°)}$
$\boldsymbol{= sin31°}$

Use the double angle formul
$\boldsymbol{cos2\alpha = 1 - 2\sin^2\alpha}$

$\therefore$

$\boldsymbol{cos62° = cos2(31°)}$
$\boldsymbol{= 1 - 2\sin^2 31°}$

d.
$\boldsymbol{sin^2 59° + cos^2 59° = 1}$
$\boldsymbol{sin^2 59 = 1 - cos^2 59}$
$\boldsymbol{sin59° = \sqrt{1 - cos^2 59}}$
$\boldsymbol{sin59° = \sqrt{1 - p^2}}$

We have already shown in b. that, $\boldsymbol{cos59° = sin31° = p}$
$\boldsymbol{\therefore cos^2 59° = p^2}$

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Method 2

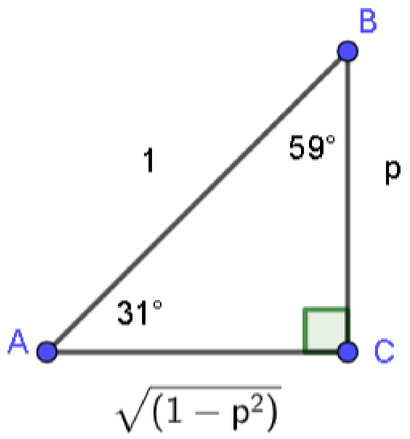
a.
$\sin 149° = \sin 31° = p$

b.
$\cos(-59°) = \cos 59° = \sin 31° = p$

c.
$\cos 62° = 1 - 2\sin^2 31° = 1 - 2p^2$

d. (solve the right triangle)

$\boldsymbol{sn31° = \frac{p}{1}}\ \left(\frac{\textbf{opposite}}{\textbf{hypotenuse}}\right)$



Step 1: fill the right triangle with given information (31°, p, and 1)

Step 2: find the missing side using Pythagoras theorem:

$AC^2 + BC^2 = AB^2 \qquad (Pythagoras)$
$AC^2 + p^2 = 1$
$AC^2 = 1 - p^2$
$AC = \sqrt{1-p^2}$ {no need for $\pm$ *as the length of a triangle is alwasy+*}

Step 3: calculate the size of the missing angle (angle B) using sum of angles in a triangle theorem:
$\hat{B} + 31° + 90° = 180° \qquad (sum\ of\ angles\ in\ a\ triangle)$
$\hat{B} = 59°$

$sin\ 59° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$

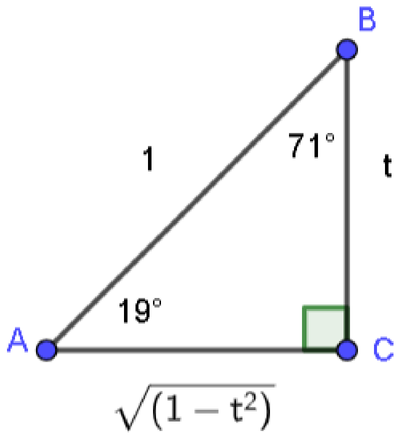
N.B: for examples 2 and 3, we will use only one method, however, you are free to use any method you are comfortable with.

Example 2

$If\ sin161° = t, express\ tan71°\ in\ terms\ of\ t.$

Solution

$sin161° = \sin(180° - 19°)$
$= sin19°$



$AC = \sqrt{1-t^2}$ {see example 1 d. Method 2}

$\hat{B} = 71°$

$\therefore sin19° = \frac{t}{1}\left(\frac{opp}{hyp}\right)$

$tan71° = \frac{\sqrt{1-t^2}}{t}$

SohCahToa

$tan71° = \frac{opp}{adj}$

Example 3

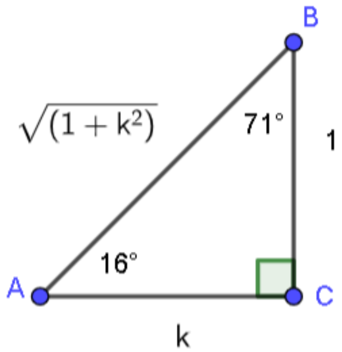
If $\sin 16° = \frac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a**

a. $tan16°$

b. $cos32°$

Solution

a.



$sin16° = \frac{1}{\sqrt{1+k^2}}$ $\left(\frac{opp}{hyp}\right)$

$$AC^2 + BC^2 = AB^2 \qquad (Pythagoras)$$
$$AC^2 + 1 = 1 + k^2$$
$$Ac^2 = k^2$$
$$\therefore AC = k$$

$tan16° = \frac{1}{k}$

b.

$cos32° = cos2(16°)$

$= 1 - 2\sin^2 16°$

$= 1 - 2\left(\frac{1}{\sqrt{1+k^2}}\right)^2$

$= 1 - 2\left(\frac{1}{1+k^2}\right)$

General Solutions

Use the following formulae to determine the general solutions of trigonometric equations:

If $sin\alpha = m$

Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $sin\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$ BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.c).

Formula	$\alpha = \sin^{-1}(m) + k.360°$ or $\alpha = 180° - \sin^{-1}(m) + k.360°$, $k \in Z$
Example 1, determine the genral solution of: $sinx = \frac{1}{2}$	$x = \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ $x = 30° + k.360°$ or $x = 150° + k.360°$, $k \in Z$
Example 2, determine the genral solution of: $sinx = -\frac{1}{2}$	$x = \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ $x = -30° + k.360°$ or $x = 210° + k.360°$, $k \in Z$

If $\cos\alpha = m$

Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $cos\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$. BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.b).

Formula	$\alpha = cos^{-1}(m) + k.360°$ or $\alpha = -cos^{-1}(m) + k.360°$, $k \in Z$
Example 3, determine the genral solution of: $cosx = 1$	$x = cos^{-1}(1) + k.360°$ or $x = -\cos^{-1}(1) + k.360°$ $x = 0° + k.360°$ or $x = 0° + k.360°$ $\therefore x = 0° + k.360°$, $k \in Z$
Example 4, determine the genral solution of: $cosx = -1$	$x = cos^{-1}(-1) + k.360°$ or $x = -\cos^{-1}(-1) + k.360°$, $k \in Z$ $x = 180° + k.360°$ or $x = -180° + k.360°$, $k \in Z$

If $\tan\alpha = m$

Formula	$\alpha = tan^{-1}(m) + k.180°$, $k \in Z$
Example 5, determine the genral solution of: $tanx = 3$ (Round your answer to TWO decimal places)	$x = tan^{-1}(3) + k.180°$ $x = 71{,}57° + k.180°$, $k \in Z$
Example 6, determine the genral solution of: $tanx = -3$ (Round your answer to TWO decimal places)	$x = tan^{-1}(-3) + k.180°$ $x = -71{,}57° + k.180°$, $k \in Z$

N.B: You may use other methods to determine the general solutions, however, the above method is highly recommeded

Example 7

Determine the general solutions of the following (round off answers to TWO decimal places where necessary:

$\boldsymbol{a}.\ \ \sin^2 x - sinxcosx = 0$
$\boldsymbol{b}.\ \ \cos^2 x - 2cosx - 3 = 0$
$\boldsymbol{c}.\ \ 2\cos^2 x + 7sinx = 5$
$\boldsymbol{d}.\ \ \cos(x - 45°) = \sin 15°$
$\boldsymbol{e}.\ \ sin3x = cosx$

Solution

Take out the common factor ($sinx$)

$\boldsymbol{a}.\ \ \sin^2 x - sinxcosx = 0$
$sinx(sinx - cosx) = 0$
$sinx = 0 \quad or \quad sinx - cosx = 0$
$sinx = cosx$
$\frac{sinx}{cosx} = 1$
$tanx = 1$
$Now\ determine\ the\ general\ solutions$

$\boldsymbol{sinx = 0}$
$\boldsymbol{x = \sin^{-1}(0) + k.360° \quad or \quad x = 180° - sin^{-1}(0) + k.360°}$
$\boldsymbol{x = 0° + k.360° \quad or \quad x = 180° + k.360° \quad , k \in Z}$

OR

$\boldsymbol{tanx = 1}$
$\boldsymbol{x = \tan^{-1}(1) + k.180°}$
$\boldsymbol{x = 45° + k.180° \quad , k \in Z}$

b.

Quadratic equation

$\cos^2 x - 2cosx - 3 = 0$
$\cos^2 x - 2cox - 3 = 0$
$(cosx + 1)(cosx - 3) = 0$
$cosx = -1 \quad or \quad cosx = 3$

$Now\ determine\ the\ general\ solutions$

$\boldsymbol{cosx = -1}$
$\boldsymbol{x = \cos^{-1}(-1) + k.360° \quad or \quad x = -cos^{-1}(-1) + k.360°}$
$\boldsymbol{x = 180° + k.360° \quad or \quad x = -180° + k.360° \quad , k \in Z}$

OR

$\boldsymbol{cosx = 3}$
$\boldsymbol{No\ Solution\ (-1 \leq cosx \leq 1)}$

c.

$2\cos^2 x + 7sinx = 5$
$2\cos^2 x + 7sinx - 5 = 0$
$2(1 - \sin^2 x) + 7sinx - 5 = 0$
$-2\sin^2 x + 7sinx - 3 = 0$
$2\sin^2 x - 7sinx + 3 = 0$
$(2sinx - 1)(sinx - 3) = 0$
$sinx = \frac{1}{2} \quad or \quad sinx = 3$

Quadratic equation

Now determine the general solutions

$$\boldsymbol{sinx = \frac{1}{2}}$$
$$\boldsymbol{x = \sin^{-1}\left(\frac{1}{2}\right) + k.360° \quad or \quad x = 180° - sin^{-1}\left(\frac{1}{2}\right) + k.360°}$$
$$\boldsymbol{x = 30° + k.360° \quad or \quad x = 150° + k.360° \quad , k \in Z}$$

OR

$$\boldsymbol{sinx = 3}$$
$$\boldsymbol{No\ Solution\ (-1 \leq sinx \leq 1)}$$

d.

$\cos(x - 45°) = \sin 15°$
$\cos(x - 45°) = \sin(90° - 75°)$
$\cos(x - 45°) = cos75°$

Cofunctions

Now determine the general solutions

$x - 45° = \cos^{-1}(cos75°) + k.360° \quad or \quad x - 45° = -\cos^{-1}(cos75°) + k.360°$
$x - 45° = 75° + k.360° \quad or \quad x - 45° = -75° + k.360°$
$x = 120° + k.360° \quad or \quad x = -30° + k.360° \quad , k \in Z$

e.

$sin3x = cosx$
$sin3x = \sin(90° - x)$

Co-functions

Now determine the general solutions

$3x = \sin^{-1}[\sin(90° - x)] \quad or \quad 3x = 180° - \sin\text{\textasciicircum}(-1)\,[\sin(90° - x)]$
$3x = 90° - x + k.360° \quad or \quad 3x = 180 - (90° - x) + k.360°$
$4x = 90° + k.360° \quad or \quad 3x = 90° + x + k.360°$
$4x = 90° + k.360° \quad or \quad 2x = 90° + k.360$
$x = 22{,}5° + k.90° \quad or \quad x = 45 + k.180° \quad , k \in Z$

Example 8 (restrictions on identities)

- **An identity with the function** $\boldsymbol{tanx}$ is undefined for $\boldsymbol{x = 90° + k.180° \quad , k \in Z}$.
- **An identity is undefined when any denominator is zero.**

a. For which values of B is the identity $\frac{cosB}{1+sinB} = \frac{1-sinB}{cosB}$ undefined?

b. For which values of A is the identity $\frac{1}{cosA} + tanA = \frac{cosA}{1-sinA}$ undefined?

Solution

a.
The indentity will be undefined when:
$1 + sinB = 0 \quad or \quad cosB = 0$

$\boldsymbol{1 + sinB = 0}$
$\boldsymbol{sinB = -1}$
$\boldsymbol{B = -90° + k.360° \quad or \quad B = 270° + k.360° , k \in Z}$

OR

$\boldsymbol{cosB = 0}$
$\boldsymbol{B = 90° + k.360° \quad or \quad B = -90° + k.360° , k \in Z}$

b.
The indentity will be undefined when:
$(\boldsymbol{we\ have\ tanA})$
$\boldsymbol{\therefore A = 90° + k.180° \quad , k \in Z}$

OR

$\boldsymbol{cosA = 0}$
$\boldsymbol{A = 90° + k.360° \quad or \quad A = -90° + k.360° , k \in Z}$

OR

$\boldsymbol{1 - sinA = 0}$
$\boldsymbol{sinA = 1}$
$\boldsymbol{A = 90° + k.360° \quad or \quad A = 90° + k.360° , k \in Z}$

Example 9 (finding the size of the angle): Round off answers to Two decimals places where needed

Solve for x:

a. $3\tan(x-20°) = -4{,}456 \quad , \quad x \in [-90°; 270°]$
b. $\sin^2 x = sinx \quad , \quad -180° \leq x \leq 270°$
c. $tan2x + 3 = 7{,}5$

Solution

a.

$3\tan(x-20°) = -4{,}456$

$\tan(x-20°) = -\frac{557}{375}$

first determine the general solution

$x - 20° = -56{,}05° + k.180°$

$x = -36{,}05° + k.180° \ , k \in Z$

Then solve for x

$x = -36{,}05° + k.180° \ , k \in Z \qquad\qquad x \in [-90°; 270°]$

k	-3	-2	-1	0	1	2
x			~~-216,05°~~	-36,05°	143,95°	~~323,95°~~

$$\therefore \boldsymbol{x = -36{,}05° ; 143{,}95°}$$

b.

$sin^2x = sinx$

$sin^2x - sinx = 0$

$sinx(sinx - 1) = 0$

$sinx = 0 \quad or\ sinx = 1$

first determine the general solutions

$sinx = 0$

$\boldsymbol{x = 0° + k.360° \quad or \quad x = 180° + k.360° \ , k \in Z}$

OR

$sinx = 1$

$\boldsymbol{x = 90° + k.360° \ , k \in Z}$ **{no need to repeat the equation (as 180°-90° gives the same equation)}**

Then solve for $\boldsymbol{x}$ ***(for each general solution)***

$x = 0° + k.360° \quad ,k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x			~~-360°~~	0°	~~360°~~	

$$\therefore \boldsymbol{x = 0°}$$

OR

$x = 180° + k.360° \quad ,k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x		~~-540°~~	-180°	180°	~~540°~~	

$$\therefore \boldsymbol{x = -180°; 180°}$$

OR

$x = 90° + k.360° \quad ,k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x			~~-270°~~	90°	~~450°~~	

$$\therefore \boldsymbol{x = 90°}$$

c.

$tan2x + 3 = 7{,}5$
$tan2x = 4{,}5$
$2x = 77{,}47° + k.180°$
$x = 38{,}74° + k.90° \quad ,k \in Z$

{Note that we were not given the interval, therefore we will stop ater finding the general solution}

Activities:

Activity 1

DBE November 2025 Paper 2

QUESTION 5

5.1 It is given that $\tan 50^0 = k$. Express EACH of the following in terms of k:

5.1.1 $\cos 40^0$

5.1.2 $\dfrac{2\sin 25^0.\cos 25^0}{-2+4\sin^2 25^0}$

5.1.3 $\sin 10^0$

FS 2025 Trial Exam Paper 2

5.4 If $\sin 32° = t$ determine $\sin 16°$ in terms of t.

Activity 2

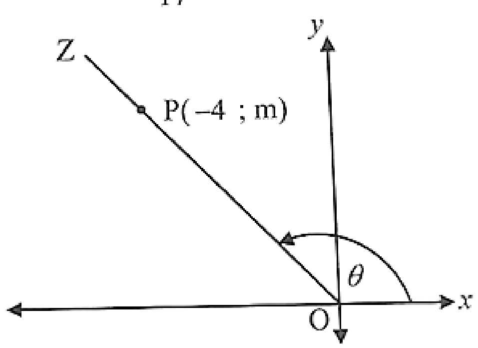
GP 2025 Trial Exam Paper 2

5.1 Given: $\sin\beta = \frac{12}{13}$, where $\tan\beta < 0$.

With the aid of a diagram, and **without the use of a calculator**, determine the value of $\sin 2\beta$.

LP 2025 Trial Exam Paper 2

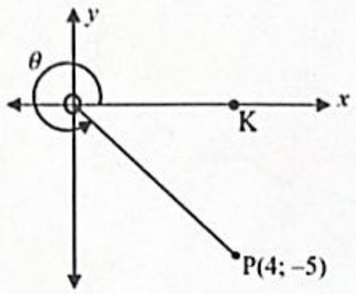
5.2 In the diagram, P(−4 ; m) is a point in the Cartesian plane. It is also given that $Z\hat{O}X = \theta$ and $\sin\theta = \dfrac{15}{17}$.



Determine the value of m, without using a calculator.

NW 2025 Trial Exam Paper 2

5.1 In the diagram below, line OP is given with $P(4;-5)$. $K\hat{O}P = \theta$



Calculate, **without using a calculator**, the numerical value of the following:

5.1.1 $\sin^2(180° + \theta)$

5.1.2 $\tan(-\theta)$

Activity 3

WC 2025 Trial Exam Paper 2

5.4 Determine the general solution for:

$$2\sin x.\cos x - 0{,}8 = 0$$

LP 2025 Trial Exam Paper 2

6.2 Determine the general solution of $1 - \tan x = \cos 2x$ if $\sin x \neq \cos x$.

GP 2025 Trial Exam Paper 2

6.3 Determine the general solution of $\cos x + 1 = \sin x$.

QUESTION 5

5.1 Solve:

5.1.1 If $\tan\theta = \frac{8}{6}$ and $0° < \theta < 90°$ show that

$10\sin(\theta + x) = 6\sin x + 8\cos x$

5.1.2 Hence, solve the following equation:

$6\sin x + 8\cos x = 9$ for $x\ \varepsilon\ [0°; 360°]$

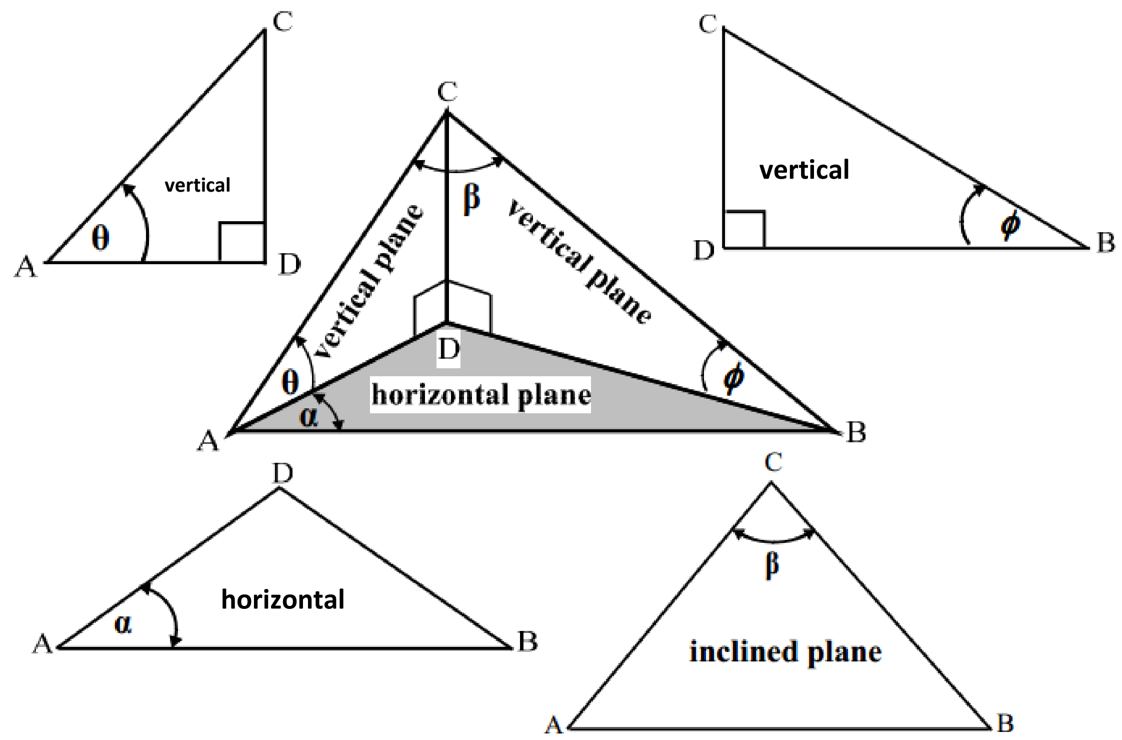
SECTION 4: SOLUTION OF TRIANGLES (2-D & 3-D)

3-dimensional space takes up 3 planes (horizontal, vertical and inclined/slanted).

The 3-dimensional diagram below is split such that you can work separately on each 2-D plane.

The 3-D diagram below has 4 planes:

- Vertical plane 1 → ΔADC
- Vertical plane 2 → ΔBDC
- Horizontal plane → ΔADB
- Inclined plane → ΔABC

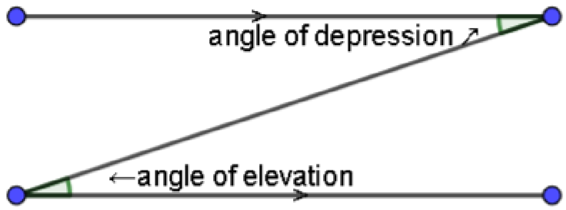


Take note that:

- You cannot add two angles from different planes to get the sum, from the 3-D diagram, $\boldsymbol{\theta + \alpha \neq C\hat{A}B}$

Angle of elevation vs Angle of depression

Angle of **depression** (measured from horizontal going down)



Angle of **elevation** (measured from the horizontal going up)

For right-angled triangles

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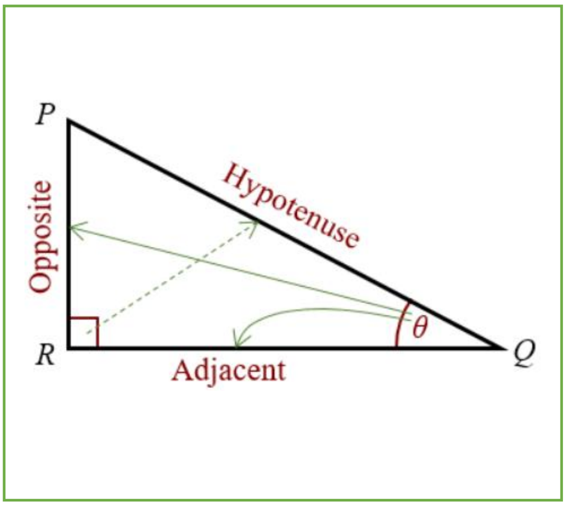


1.

$$\text{sine of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2.

$$\text{cosine of an angle } \theta = \frac{\text{length of the side adjacent to angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3

$$\text{tangent of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the side adjacent to angle } \theta}$$

$$\therefore \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

To calculate the area of a right-angled triangle PQR above, use the formula $Area\ \Delta = \frac{1}{2} b \times h$

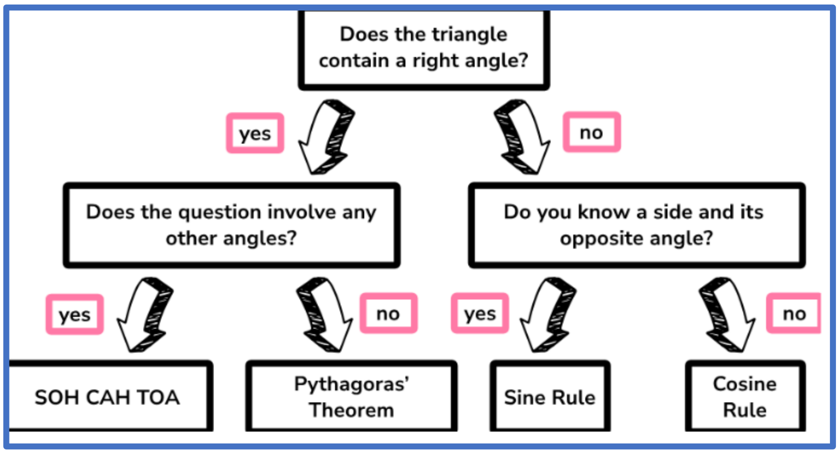
$$\therefore Area\ \Delta PQR \ = \frac{1}{2} RQ \times PR$$

For triangles that are not right-angled triangles

Rule	Formula	When to use
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Given two sides and the angle opposite one of those sides.
		One side and any two angles.
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$	Given two sides and the included angle.
		Three sides.
Area rule	$\frac{1}{2} ab\sin C$	Area is required. In order to use the formula for Area, two sides and the included angle are required.

N.B Only use area formula when you are asked to calculate the area or when you are given the area.

2-D & 3-D Approach



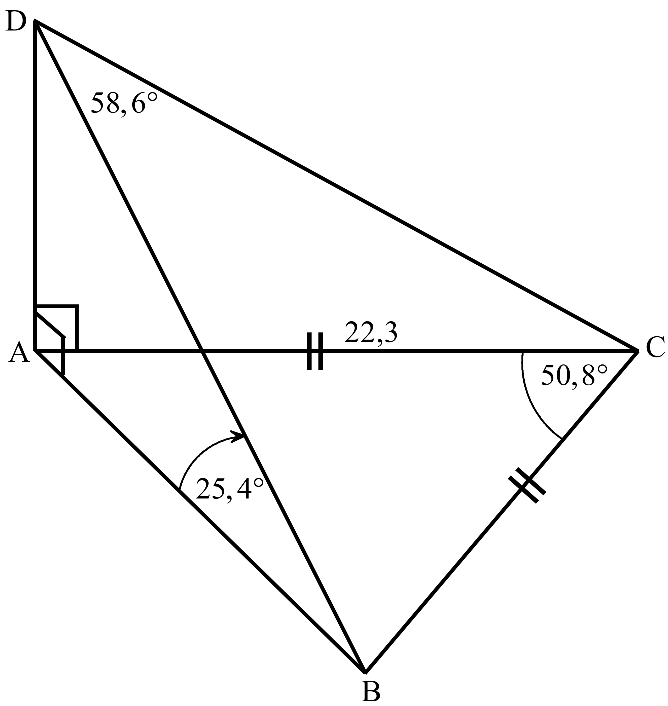
Worked Examples

Example 1

In the diagram below, A, B and C are points in the same horizontal plane with

AC = BC = 22,3 metres. AD is a vertical tower which is anchored at B and C.

The angle of elevation of D, from B is 25,4°. $B\hat{D}C = 58{,}6°$ and $A\hat{C}B = 50{,}8°$.



Calculate, correct to ONE decimal place:

1.1 the area of ΔABC

1.2 the length of AB

1.3 the height of the tower, AD

1.4 the length of DC if $D\hat{B}C = 67{,}3°$

Solution

1.1

$$\text{Area } \Delta ABC = \frac{1}{2}(22{,}3)(22{,}3(\sin 50{,}8°)$$

$$= 192{,}7 \text{ units}^2$$

Area Rule

1.2

$$AB^2 = (22{,}3)^2 + (22{,}3)^2 - 2(22{,}3)(22{,}3)\cos 50{,}8°$$

$$= 365{,}9762962$$

$$\therefore AB = 19{,}1 \text{ units}$$

Cosine Rule

1.3

$$\frac{AD}{19{,}1} = \tan 25{,}4°$$

$$\therefore AD = 19{,}1 \tan 25{,}4°$$

$$\therefore AD = 9{,}1 \text{ units}$$

SohCahToa

$$\boldsymbol{tan\theta = \frac{opp}{adj}}$$

1.4

$$\frac{DC}{\sin 67{,}3°} = \frac{22{,}3}{\sin 58{,}6°}$$

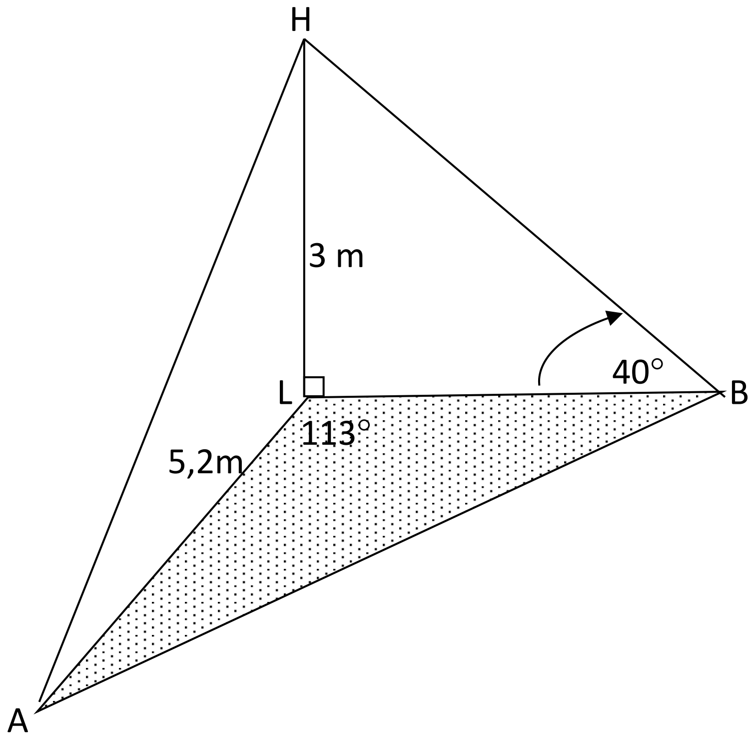
$$\therefore DC = \frac{22{,}3 \sin 67{,}3°}{\sin 58{,}6°}$$

$$\therefore DC = 24{,}1 \text{ units}$$

Sine Rule

Example 2

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.



2.1 Calculate the length of LB.

2.2 Hence, or otherwise, calculate the length of AB.

2.3 Determine the area of ΔABL.

Solution

2.1

$$\frac{3}{\text{LB}} = \tan 40° \qquad \text{or} \qquad \frac{\text{LB}}{\sin 50°} = \frac{3}{\sin 40°}$$

$$\therefore \text{LB} = \frac{3}{\tan 40°} \qquad \qquad \therefore \text{LB} = \frac{3\sin 50°}{\sin 40°}$$

$$\therefore \text{LB} = 3,58 \text{ m} \qquad \qquad \therefore \text{LB} = 3,58 \text{ m}$$

2.2 $$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113^\circ$$

$$\therefore AB^2 = (5,2)^2 + (3,58)^2 - 2(5,2)(3,58)\cos 113^\circ$$

$$\therefore AB^2 = 54,40410138 \text{ m}^2$$

$$\therefore AB = 7,38 \text{ m}$$

2.3 $$\text{Area of } \Delta ABL = \frac{1}{2} AL.BL.\sin A\hat{L}B$$

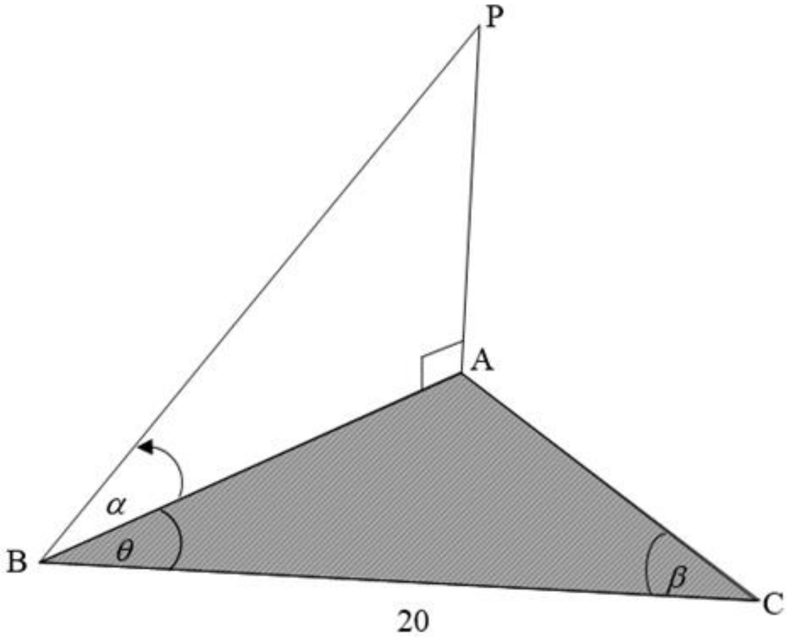
$$= \frac{1}{2}(5,2)(3,58)\sin 113^\circ$$

$$= 8.568059176$$

$$= 8,57 \text{ m}$$

Example 3

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is α. $A\hat{C}B = \beta$, $A\hat{B}C = \theta$ and BC = 20 units.



3.1 Write AP in terms of AB and α.

3.2 Prove that $AP = \frac{20 \sin\beta \tan\alpha}{\sin(\theta+\beta)}$

3.3 Given that AB = AC, determine AP in terms of α and β in its simplest form.

Solution

3.1

$$\frac{AP}{AB} = \tan\alpha$$

$$\therefore AP = AB\tan\alpha$$

3.2

$$\frac{AB}{\sin\beta} = \frac{20}{\sin[180° - (\theta + \beta)]}$$

$$\therefore \frac{AB}{\sin\beta} = \frac{20}{\sin(\theta + \beta)}$$

$$\therefore AB = \frac{20\sin\beta}{\sin(\theta + \beta)}$$

$$\therefore AP = \frac{20\sin\beta}{\sin(\theta + \beta)}\tan\alpha$$

$$\therefore AP = \frac{20\sin\beta\tan\alpha}{\sin(\theta + \beta)}$$

3.3

$$AP = \frac{20\sin\beta\tan\alpha}{\sin(\theta + \beta)}$$

$$= \frac{20\sin\beta\tan\alpha}{\sin(\beta + \beta)}$$

$$= \frac{20\sin\beta\tan\alpha}{\sin 2\beta}$$

$$= \frac{20\sin\beta\tan\alpha}{2\sin\beta\cos\beta}$$

$$= \frac{10\tan\alpha}{\cos\beta}$$

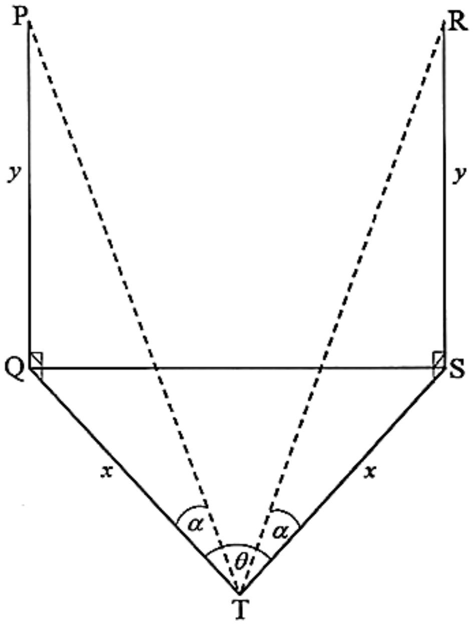
Activities:

Activity 1

LP 2025 Trial Exam Paper 2

QUESTION 8

Two poles, PQ and RS with equal lengths of y meters, stand with their lower points Q and S in the same horizontal plane as the point T. They are also equidistant from T, distance equal to x. The angles of elevation of the top of the poles are both α. QS subtend an angle θ at T.



Prove that: $QS^2 = \dfrac{2y^2(1-\cos\theta)}{\tan^2\alpha}$

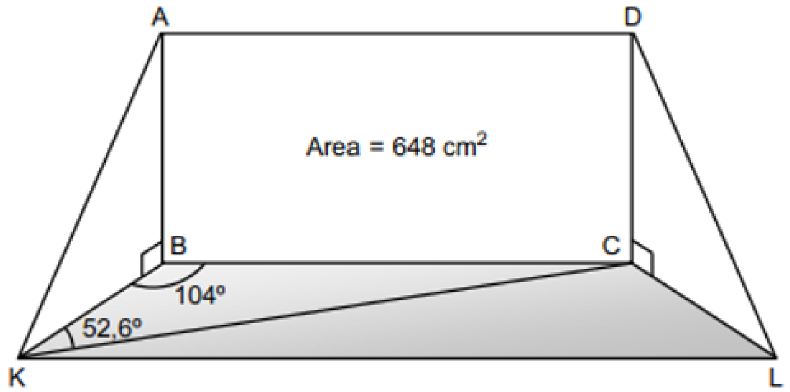
Activity 2

DBE November 2025 Paper 2

QUESTION 8

As part of the school project, learners are required to design a portable stage for a puppet show, as shown in the diagram below. The design must fulfil the following requirements:

- BKLC is a horizontal base having $K\hat{B}C = 104^0$ and $B\hat{K}C = 52,6^0$.
- The rectangular backdrop, ABCD, is vertical to the horizontal base and must have an area of 648 cm^2.
- The sides of ABCD must be in the ratio AB : BC = 1: 2.
- The stage must be partly enclosed with triangular sides ABK and DCL.



8.1 Show that AB = 18 cm.

8.2 Calculate the length of AC.

8.3 Calculate the length of diagonal KC.

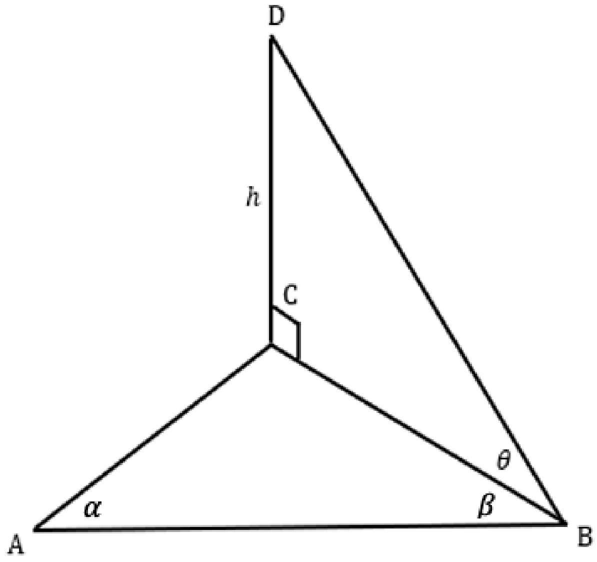
8.4 If AB = BK, calculate the size of $K\hat{A}C$.

Activity 3

WC 2025 Trial Exam Paper 2

QUESTION 7

In the diagram below, DC is a vertical corner-post on a horizontal plane ABC. The angle of elevation of D (from B), is θ. $C\hat{A}B = \alpha$, $C\hat{B}A = \beta$ and the length of DC is h.



7.1 Determine the length of CB in terms of h and θ.

7.2 Prove that $AB = \frac{h\sin(\alpha+\beta)}{\tan\theta . \sin\alpha}$

7.3 Calculate the value of AB if, $h = 31{,}2\text{m}$, $\alpha = 52{,}2°$, $\beta = 23{,}5°$ and $\theta = 42{,}3°$.

SECTION 5: TRIGONOMETRIC FUNCTIONS

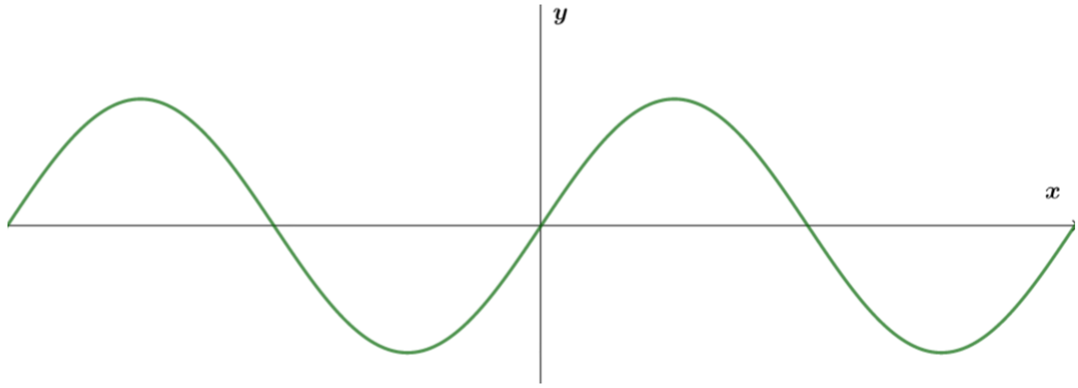
Summary of The sine function $y = asink(x + p) + q$

Vertical translation(q)

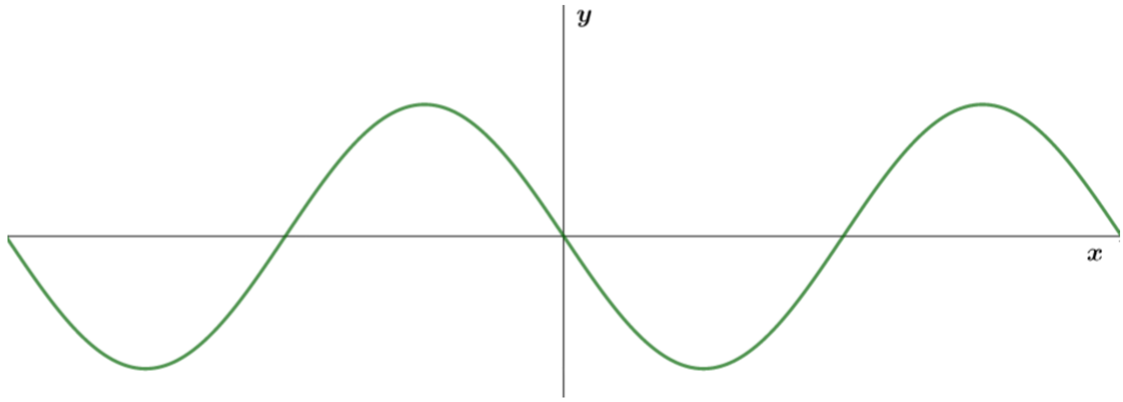
helps to find amplitude(a) helps with period(k) horizontal translation(p)

- **Shape**

$a > 0$



$a < 0$



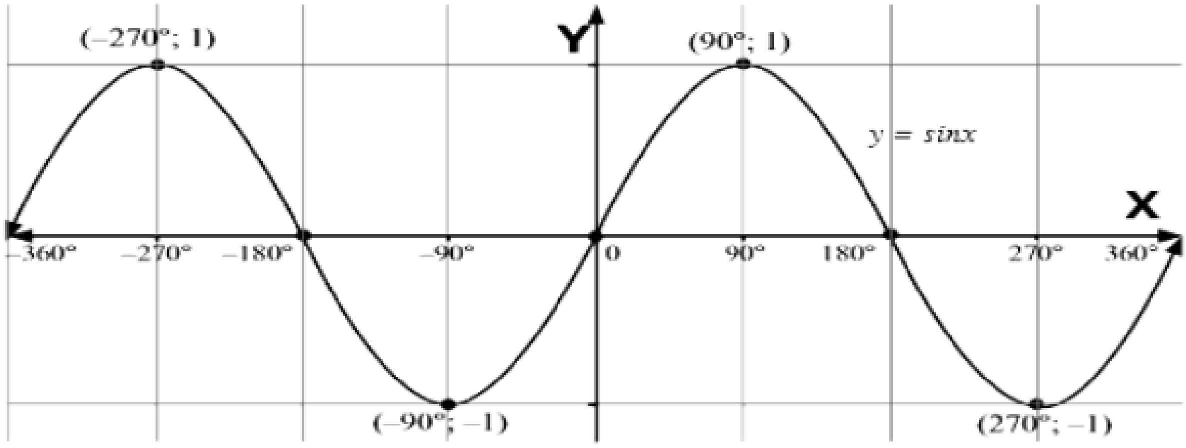
- **Amplitude** – halfway between the maximum and the minimum → $\frac{max - min}{2}$.
  - If $y = 2sinx, then\ the\ amplitude\ is\ 2$
  - If $y = -3sinx, then\ the\ amplitude\ is\ 3$
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow the\ horizontal\ shift$
  - If $y = \sin(x + 45°)$ $\rightarrow shifts\ 45°\ to\ the\ left$
  - If $y = \sin(x - 30°)$ $\rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow the\ vertical\ shift$
  - If $y = \sin\ x + 3$ $\rightarrow shifts\ 3\ units\ up$
  - If $y = \sin\ x - 2 \rightarrow shifts\ 2\ units\ down$

Example 1

sketch the graph $y = sinx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0



Take note of the following key aspects of the graph of $y = sinx \quad for\ x \in [-360°; 360°]$

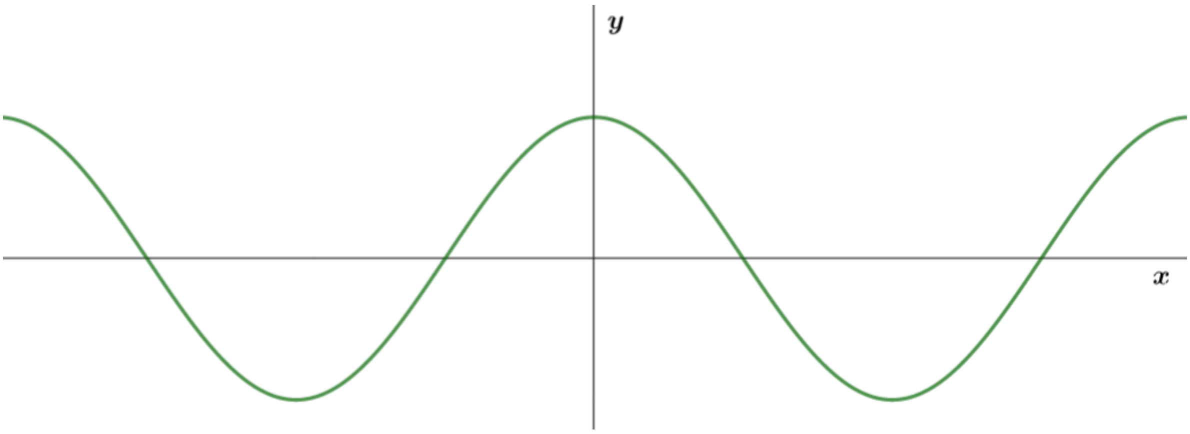
1	**Maximum Value**	**1** (at $x = -270°$ *and* $x = 90°$)
	Minimum Value	-1 (at $x = -90°$ *and* $x = 270°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-360°, -180°, 0°, 180°, 360°$
	y-intercept	**0**
4	**Amplitude**	**1** $\left\{\frac{max-min}{2} \to \frac{1-(-1)}{2} = 1\right\}$
5	**Period**	$360°$ $\left\{period = \frac{360°}{k} \to \frac{360°}{1} = 360°\right\}$

Summary of The cosine function $y = acosk(x+p)+q$

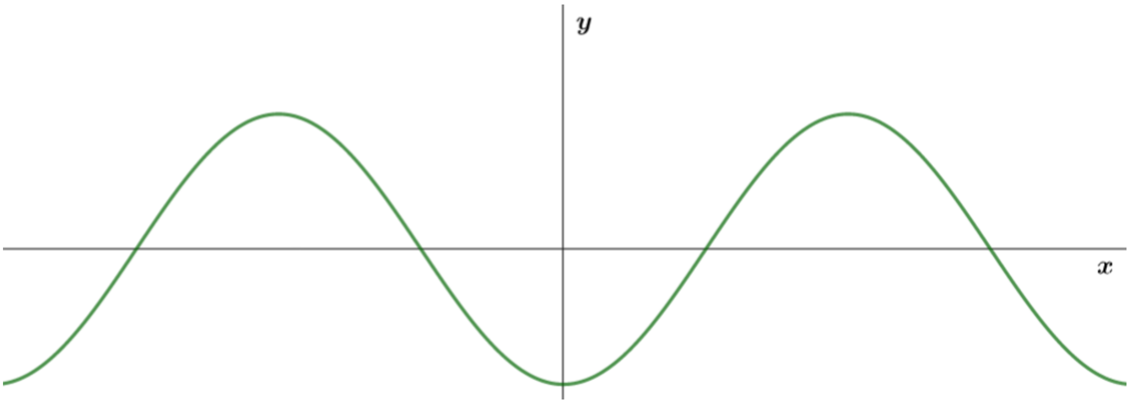
helps to find amplitude(a) **helps with period(k)** **horizontal translation(p)** **vertical translation(q)**

- **Shape**

$a > 0$



$a < 0$



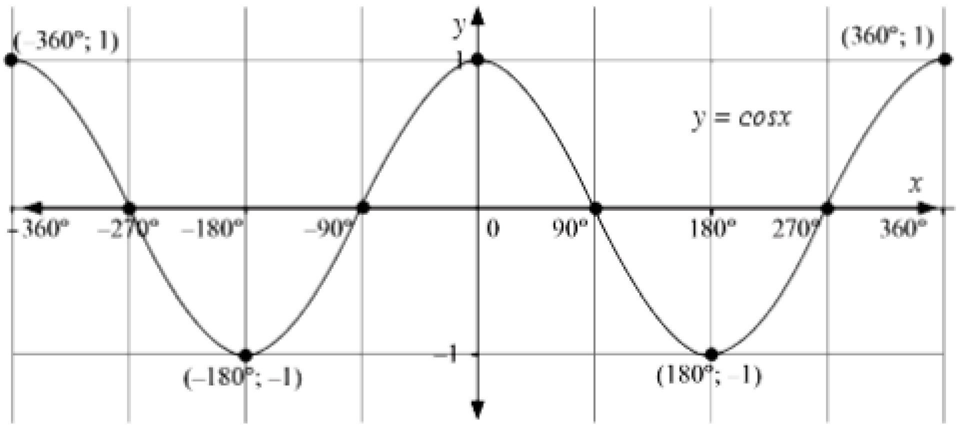
- **Amplitude** – halfway between the maximum and the minimum $\rightarrow \frac{max-min}{2}$.
    - If $y = 2cosx$, then the amplitude is 2
    - If $y = -3cosx$, then the amplitude is 3
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow$ *the horizontal shift*
    - $y = \cos(x+45°) \rightarrow$ *shifts* $45°$ *to the left*
    - $y = \cos(x-30°) \rightarrow$ *shifts* $30°$ *to the right*
- $\boldsymbol{q} \rightarrow$ *the vertical shift*
    - $y = cos\ x + 3 \rightarrow$ *shifts* 3 *units up*
    - $y = \cos\ x - 2 \rightarrow$ *shifts* 2 *units down*

Example 2

$sketch\ the\ graph\ y = cosx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1



Take note of the following key aspects of the graph of $y = cosx \quad for\ x \in [-360°; 360°]$

1	**Maximum Value**	**1** (at $x = -360°, x = 0°\ and\ x = 90°$)
	Minimum Value	-1 (at $x = -180°\ and\ x = 180°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-270°, -90°, 90°, 270°$
	y-intercept	**1**
4	**Amplitude**	**1** $\{\frac{max-min}{2} \rightarrow \frac{1-(-1)}{2} = 1\}$
5	**Period**	$360°$ $\{period = \frac{360°}{k} \rightarrow \frac{360°}{1} = 360°\}$

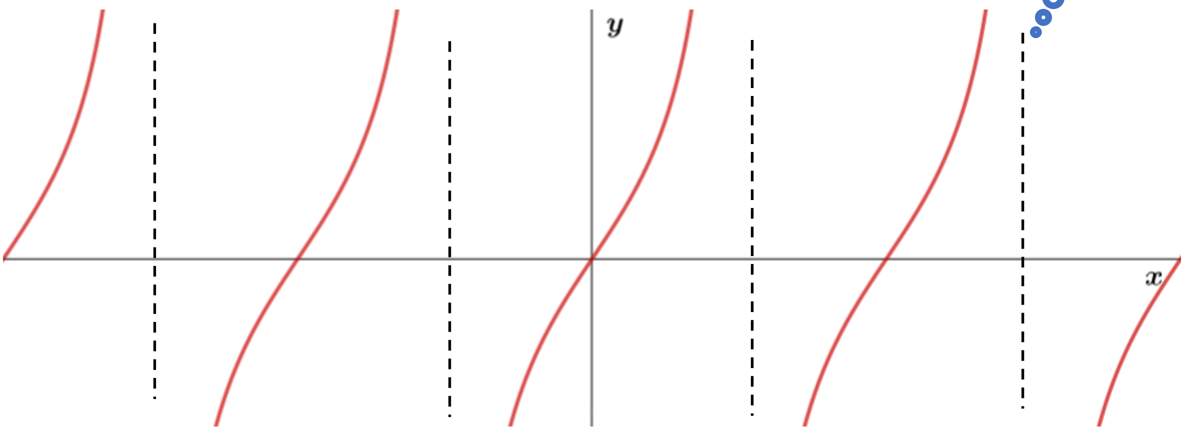
Summary of The cosine function $y = atank(x + p) + q$

helps with slope point(a) **helps with period(k)** **horizontal translation(p)** **vertical translation(q)**

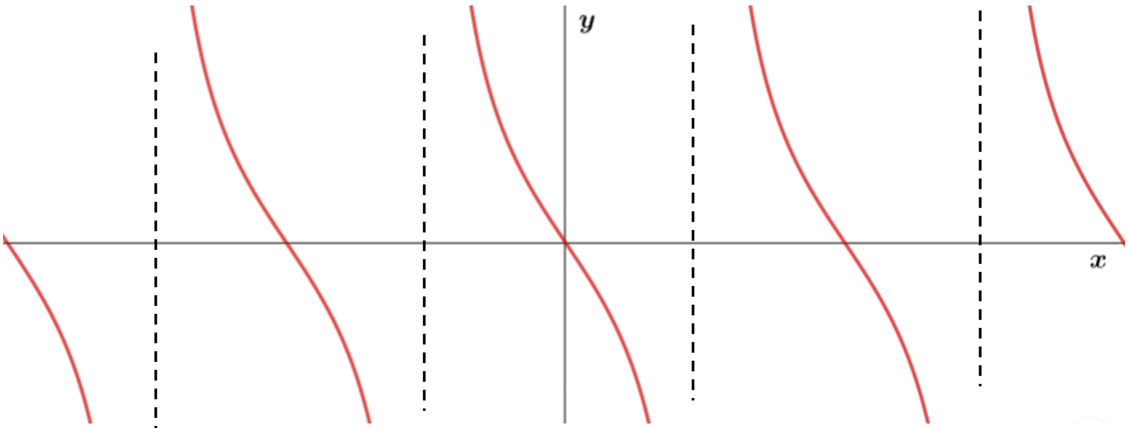
- **Shape**

Asymptote line

$\boldsymbol{a > 0}$



$\boldsymbol{a < 0}$



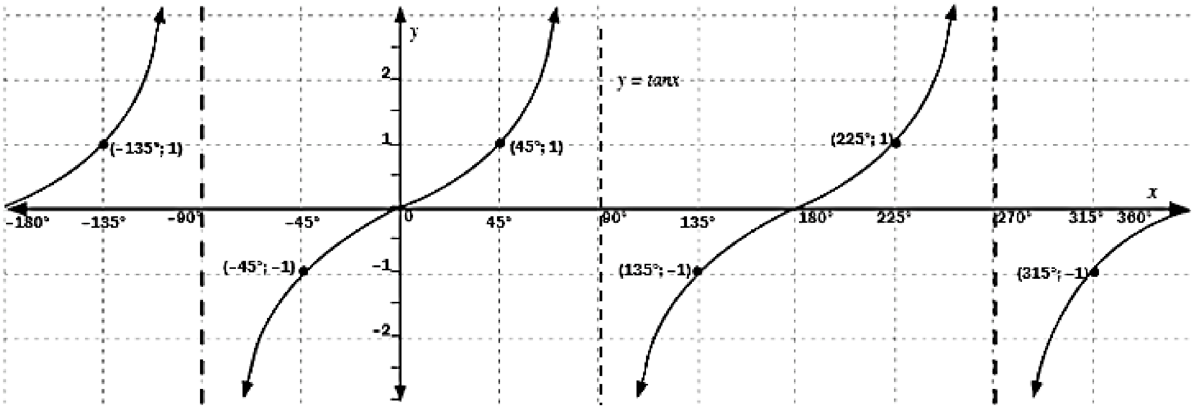
- **Amplitude** – tangent graph does not have maximum or the minimun VALUE, thus **THERE IS NO AMPLITUDE** for the tangent function.
- **Asymptotes**
  - Positions of the first asymptotes are at $\mathbf{0° \pm \frac{period}{2}}$
  - Then, other asymptotes are found every period.
- $\boldsymbol{Period} = \frac{180°}{k}$
- $\boldsymbol{p} \rightarrow the\ horizontal\ shift$
  - $y = \tan\,(x + 45°)\ \rightarrow shifts\ 45°\ to\ the\ left$
  - $y = \tan\,(x - 30°)\ \rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow the\ vertical\ shift$
  - $y = tan\ x + 3\ \rightarrow shifts\ 3\ units\ up$
  - $y = \tan\ x - 2 \rightarrow shifts\ 2\ units\ down$

Example 3

sketch the graph $y = tanx \quad for\ x \in [-180°; 360°]$

Solution

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
y	0	1	unde-fined	-1	0	1	unde-fined	-1	0	1	unde-fined	-1	0

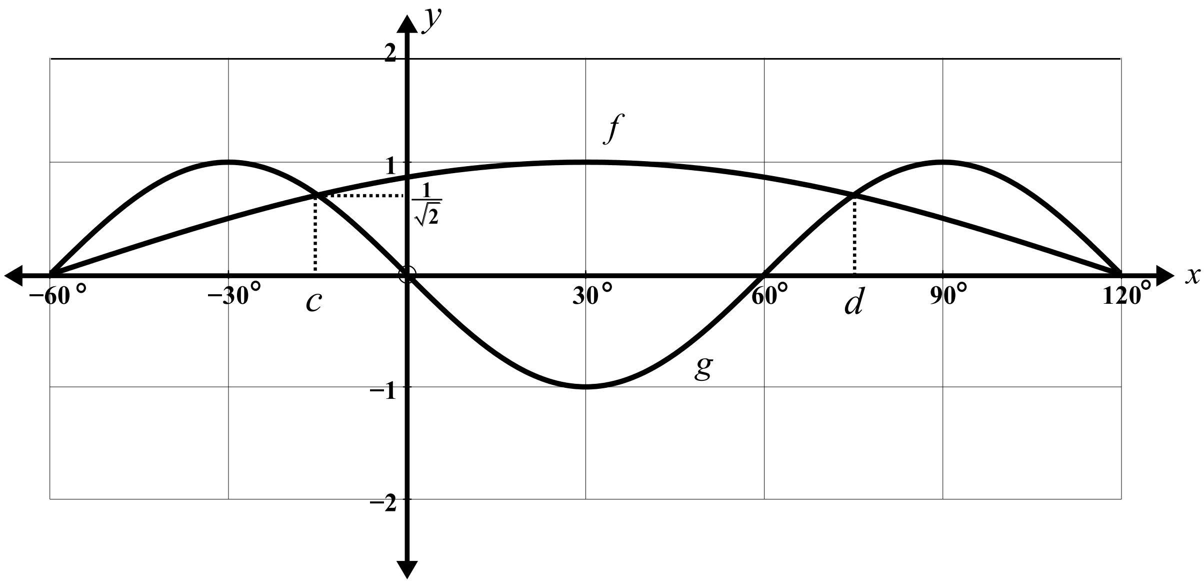


Take note of the following key aspects of the graph of $y = tanx \quad for\ x \in [-180°; 360°]$

1	**Maximum Value**	**N/A**
	Minimum Value	**N/A**
2	**Domain**	$x \in [-180°; 360°], \quad but\ x \neq -90°, 90°, 270°$
	Range	$y \in (-\infty; \infty), y \in R$
3	**x-intercepts**	$-180°, 0°, 180°, 360°$
	y-intercept	**0**
4	**Amplitude**	**N/A**
5	**Period**	$180° \quad \{period = \frac{180°}{k} \rightarrow \frac{180°}{1} = 180°\}$
6	**Equations of asymptotes**	$x = -90°, x = 90°, \quad and\ x = 270$
7	**Slope points**	$(-135°; 1), (-45°; -1), (45°; 1), (135°; -1), (225°; 1), (315°; -1)$

Example 4

In the diagram below, the graphs of $f(x) = cos(\, x + p)$ and $g(x) = a\, sin\, b\, x$ are shown in the interval $-60° \leq x \leq 120°$. The y-value at c is $\frac{1}{\sqrt{2}}$. The graphs intersect at c and d.



4.1 Determine the values of a, b and p.

4.2 Calculate the values of c and d.

4.3 Determine graphically the value(s) of x in the interval $-60° \leq x \leq 120°$ for which:

4.3.1 $f(x) - g(x) \geq 0$

Solutions

4.1

$f(x) = \cos(x + p)$

$p = -30°$

$g(x) = a \sin bx$:

Period:

$\frac{360°}{b} = 120°$

$\therefore b = 3$

Also $a = -1$

4.2 $\frac{1}{\sqrt{2}} = -sin\ 3\,x$

$\therefore sin\ 3\,x = -\frac{1}{\sqrt{2}}$

$\therefore 3x = \ -45° + k.360 \quad or \quad 3x = \ 225° + k.360°, \qquad k \in Z$

$x = -15° \qquad\qquad or \quad x = 75°$

$\therefore c = -15°$

$\therefore d = 75°$

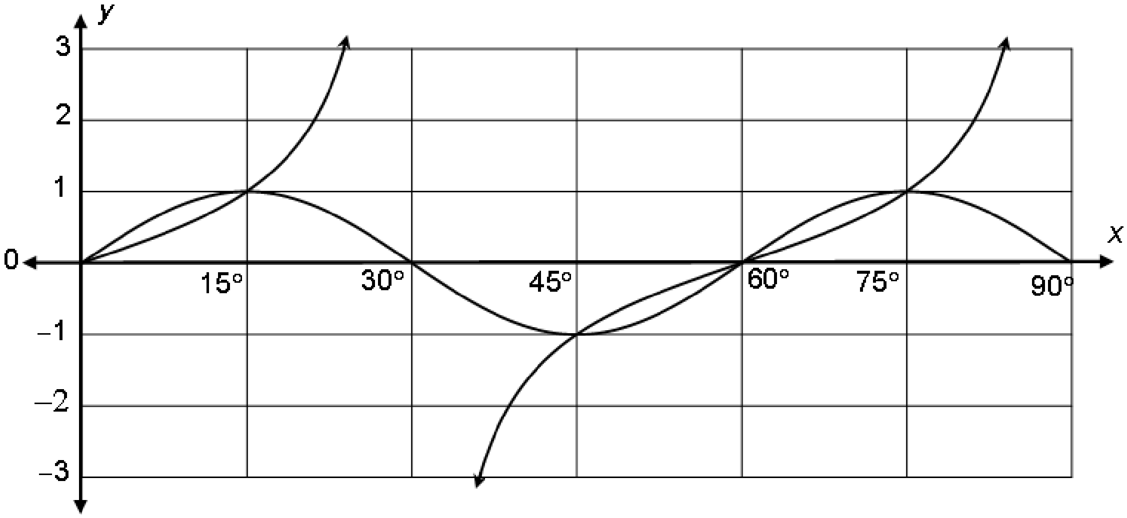
4.3.1 $f(x) - g(x) \geq 0$

$f(x) \geq g(x)$

$\therefore -15° \leq x \leq 75° \ \ or \ \ x = -60°" \ \ or \ \ x = 120°$

Example 5

On the set of axes, the graphs of $f(x) = tan\,3\,x$ and $g(x) = sin\,6\,x$ are shown for the interval $x \in [0°\,;\ 90°]$.



5.1 Write down the period of *f*.

5.2 Determine graphically the values of *x* for which $f(x) \leq g(x)$.

5.3 If the graph of *g* is shifted 2 units vertically up, write down the range of the resulting graph.

Solution

5.1 $60°$

5.2 $l0° \leq x \leq 15°$ or $30° < x \leq 45°$ or $60° \leq x \leq 75°$

5.3 $y \in [1\ ;\ 3]$

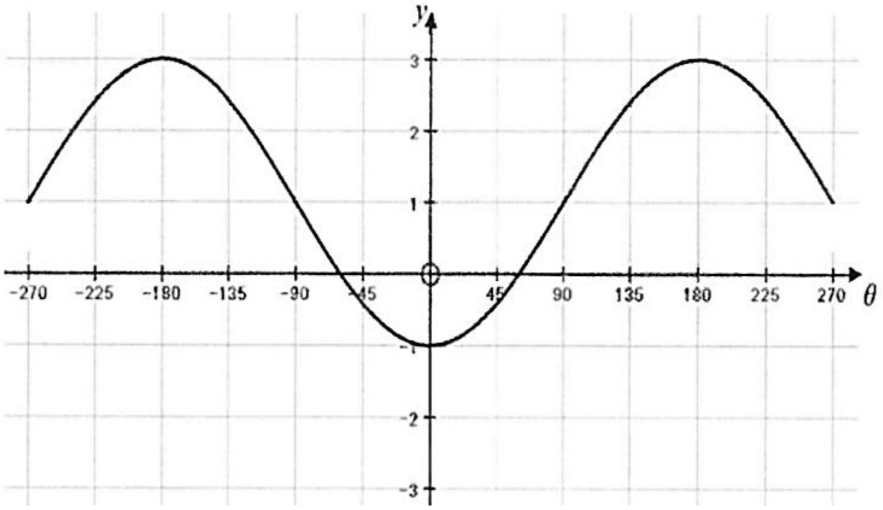
Activities:

Activity 1

FS 2025 Trail Exam Paper 2

Question 6

In the diagram below, the graph of $f(\theta) = pcos\theta + q$ is sketched for $-270° \leq \theta \leq 270°$



6.1 Write down the amplitude of *f*.

6.2 Write down the range of *f*.

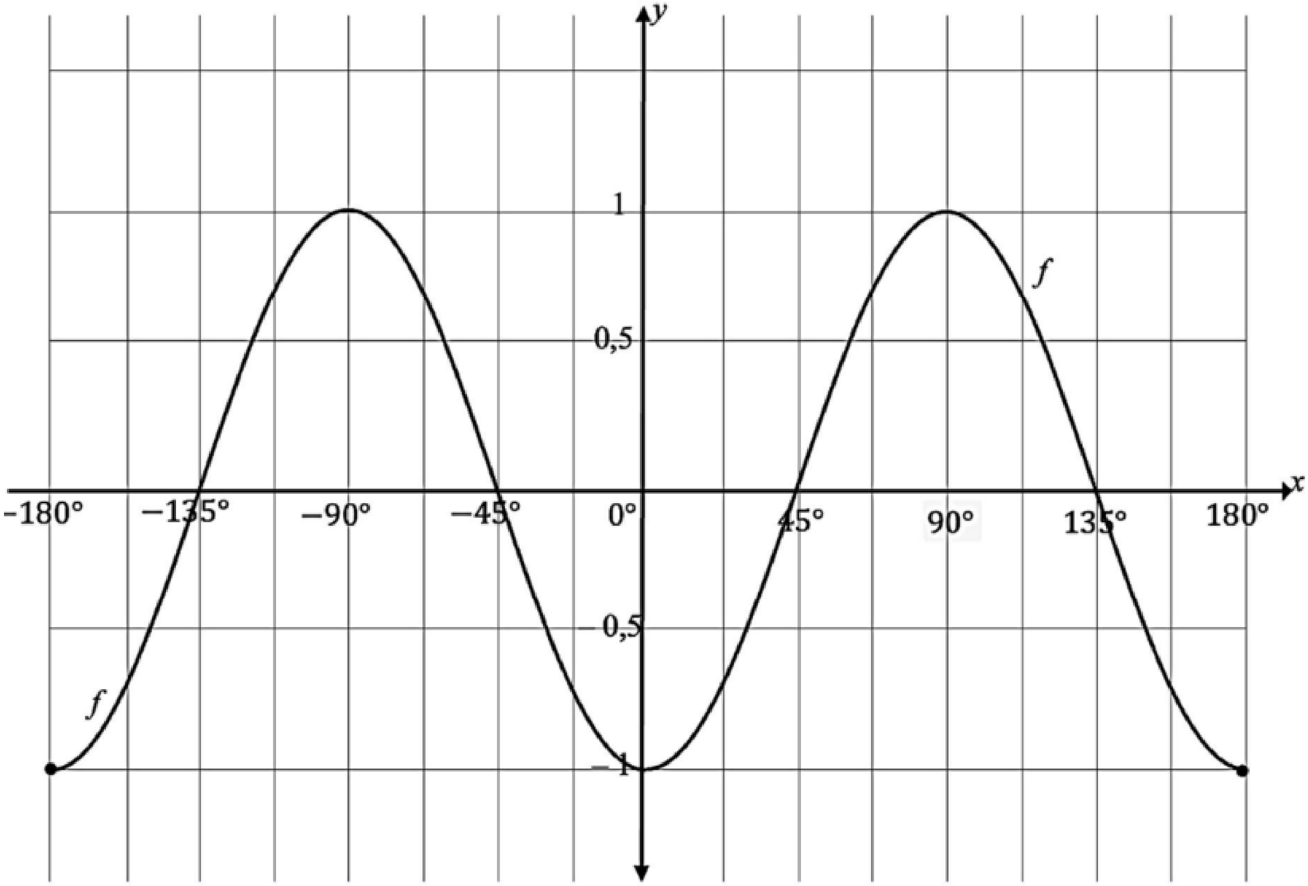
6.3 Write down the values of p and q.

Activity 2

GP 2025 Trial Exam Paper 2

Question 7

In the diagram below, the graph of $f(x) = -\cos 2x$ is drawn for the interval $x \in [-180°\,;\,180°]$.



7.1 Write down the period of *f*.

7.2 Write down the range of *f*.

7.3 On the grid provided above, draw the graph of $g(x) = \tan(x - 45°)$ for the interval $x \in [-180°\,;\,180°]$. Clearly show the asymptotes and intercepts with the axes.

7.4 For which value(s) of *x* is $f(x) \leq g(x)$, for the interval $x \in [-180°;\,0°]$.

7.5 What is the maximum value of $h(x) = 4^{2\sin^2 x - 1}$ for $x \in \mathbb{R}$.

SECTION 6: EXAMINATION GUIDELINES (TRIGONOMETRY)

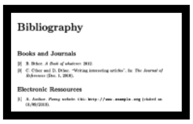
Source: Mathematics Grade 12 Examination Guidelines 2021

ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as learners should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.



BIBLIOGRAPHY

1 FS 2025 Trial Exam Paper 2

2 GP 2025 Trial Exam Paper 2

3 WC 2025 Trial Exam Paper 2

4 LP 2025 Trial Exam Paper 2

5 DBE November 2025 Paper 2

6 Grade 12 Mathematics Examination Guidelines 2021

7 2026 Grade 12 Mathematics Annual Teaching Plan

8 FET Grade 10 – 12 Mathematics CAPS Document

9 JENN Term 1 Trigonometry Teacher and Learner Content Manual

10 NW 2025 Trial Exam Paper 2