



# JENN

Training and Consultancy

The path to enlightened education

**SUBJECT: MATHEMATICS**

**ACTIVITY MANUAL**

**GRADE 12**

**2024**

**EUCLIDEAN GEOMETRY**

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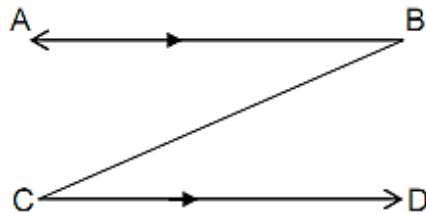
## ACTIVITIES

**N.B GIVE REASONS FOR ALL YOUR STATEMENTS**

### Lines, Angles and Triangles

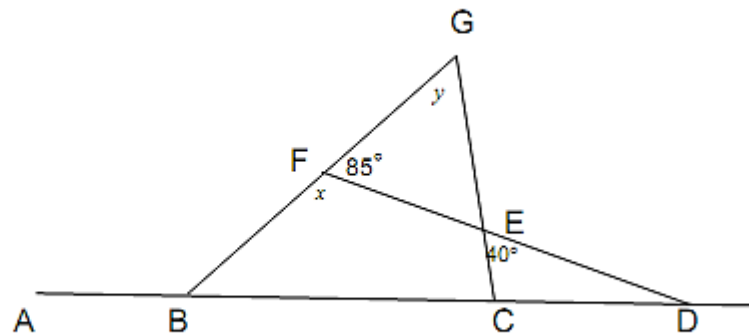
#### Question 1

1. A Z-letter shape is drawn below ( $AB \parallel CD$ ) showing a pair of alternate angles.



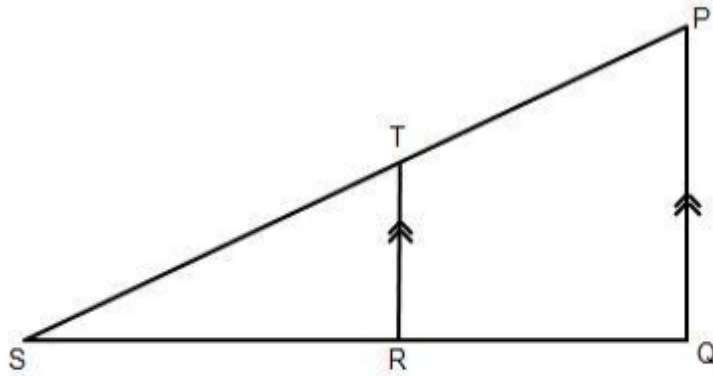
In the shape above identify the:

- 1.1 Angle of elevation
  - 1.2 Angle of depression
2. Study the figure below:



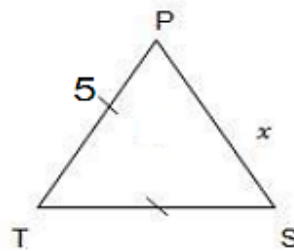
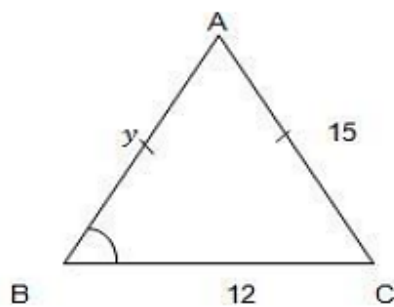
AD; BG; FD and GC are straight lines. Calculate the value of  $y$ .

- 3 In the diagram below,  $TR \parallel PQ$ ,  $\hat{S} = 28^\circ$ ,  $\hat{TRS} = x + 70^\circ$  and  $\hat{P} = x + 10^\circ$



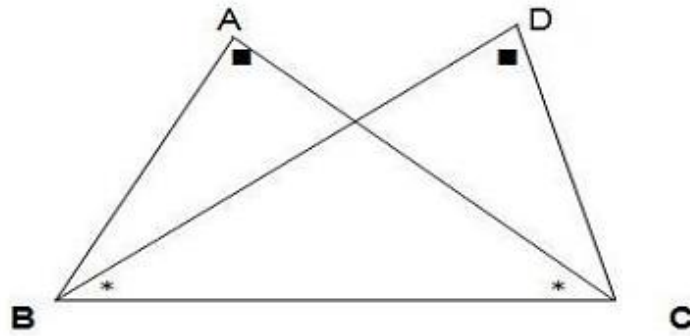
- 3.1 Calculate the value of  $x$ , giving reasons.
- 3.2 Calculate the value of  $\hat{STR}$ , giving reasons.
- 3.3 Is  $\triangle PQS$  a right angled triangle? Justify your answer by means of calculations.

- 4 In  $\triangle ABC$  and  $\triangle PTS$   $B = 70^\circ$  and  $P = 70^\circ$



- 4.1 Prove with reasons that  $\triangle ABC \sim \triangle TSP$
- 4.2 Determine  $y$  and  $x$ .

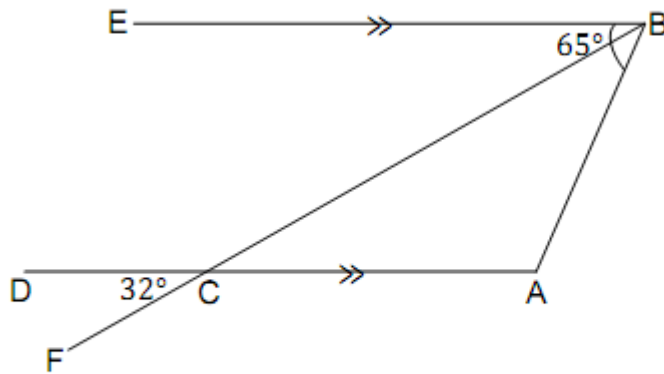
5 Study the figure below and answer the questions that follow.



- 5.1 Prove with reasons that  $\triangle ABC \cong \triangle DCB$
- 5.2 If  $AB = 4$  units, what is the length of  $DC$ ?

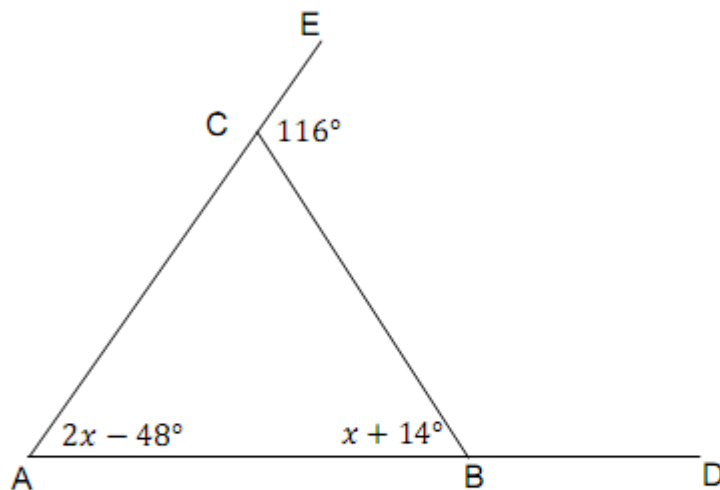
## Question 2

2.1 In the diagram below  $\angle ABE = 65^\circ$  and  $\angle DCF = 32^\circ$ .



- 2.1.1 Calculate the size of  $\angle EBC$ . Give reasons for your answer.
- 2.1.2 Calculate the size of  $\angle CAB$ . Give reasons for your answer.

2.2 In the diagram below,  $\widehat{CAB} = 2x - 48^\circ$ ,  $\widehat{ABC} = x + 14^\circ$  and  $\widehat{BCE} = 116^\circ$ .

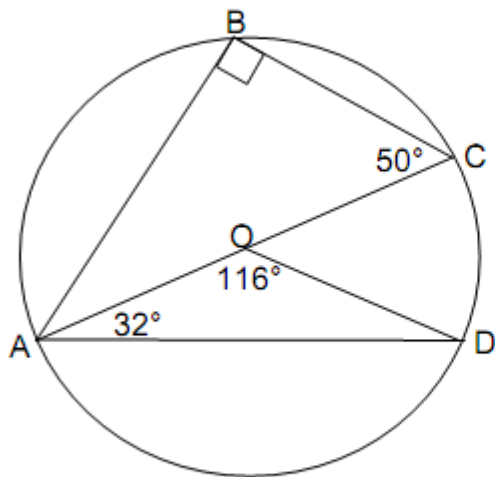


2.2.1 Calculate the value of  $x$ . Give reasons for your answer.

2.2.2 Calculate the actual size of  $\widehat{CAB}$ .

2.2.3 What type of  $\Delta$  is  $\Delta ABC$ ? Give reasons for your answer.

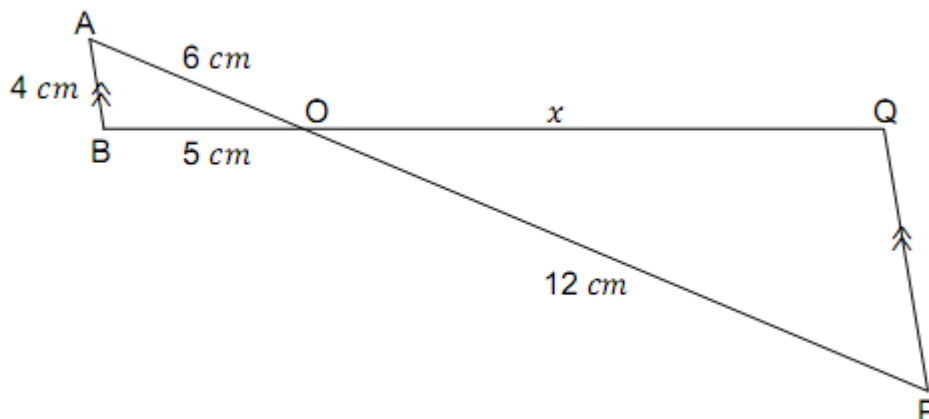
2.3 In the figure below, O is the centre of the circle.



2.3.1 Calculate the size of  $\hat{CAB}$ . Give a reason for your answer.

2.3.2 Calculate the size of  $\hat{ADO}$ . Give a reason for your answer.

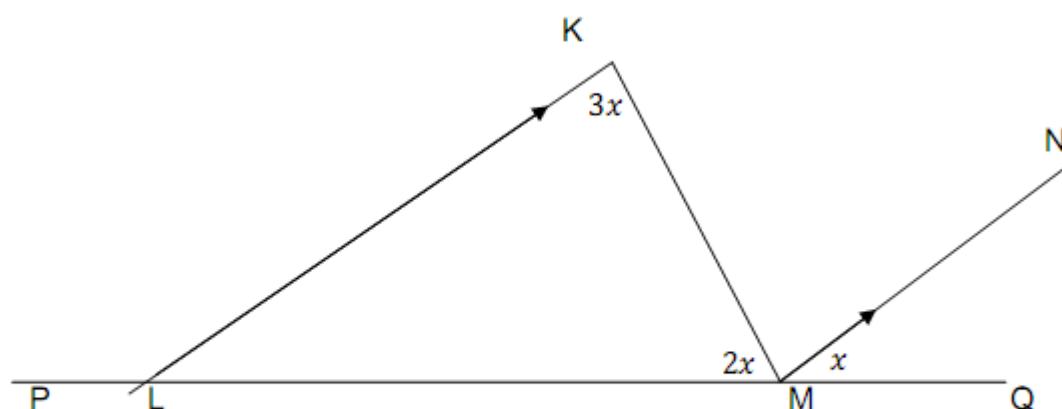
2.4 In the diagram below,  $AB \parallel PQ$ .



2.4.1 Prove that  $\triangle ABO \parallel \triangle PQO$ .

2.4.2 Calculate the value of  $x$ .

- 2.5 The figure below shows that  $LK \parallel MN$ ;  $\hat{LKM} = 3x$ ;  $\hat{KML} = 2x$  and  $\hat{NMQ} = x$ .

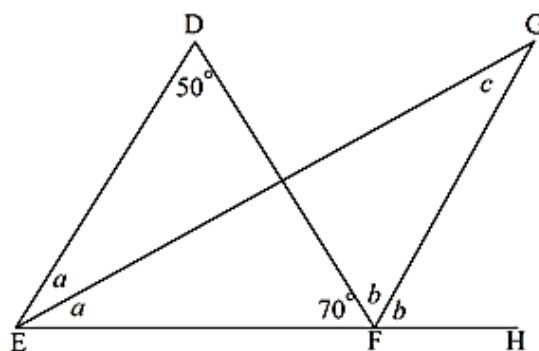


- 2.5.1 Calculate with reasons the value of  $x$ .
- 2.5.2 What is the size of  $\hat{LKM}$ ?
- 2.5.3 Which type of triangle is  $\triangle MKL$ ?

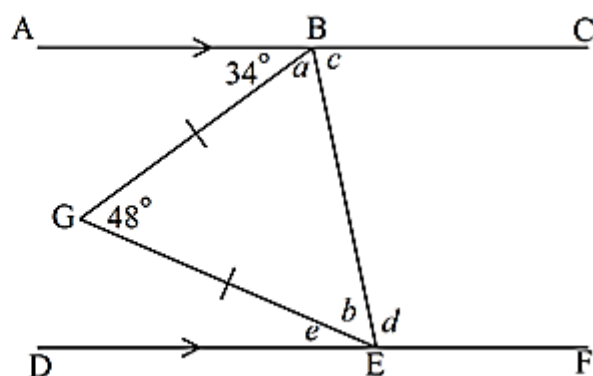
### Question 3

- 3.1 In the given sketches angles that are marked with the same letter are equal to each other. Find the size of each of the following angles:

- 3.1.1  $a$ ,  $b$ , and  $c$

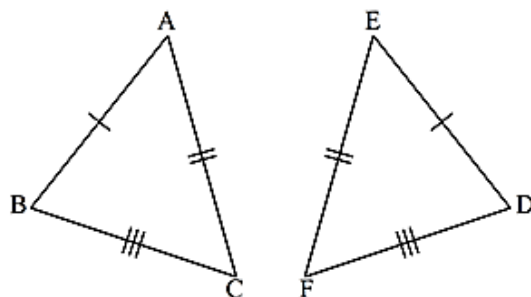


3.1.2  $a, b, c, d$  and  $e$

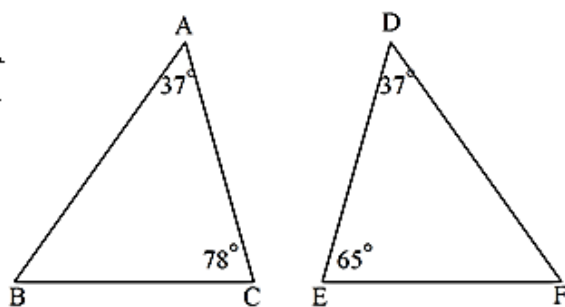


3.3 In each of the following, circle the correct answer from the options given that matches the statement to the given sketch:

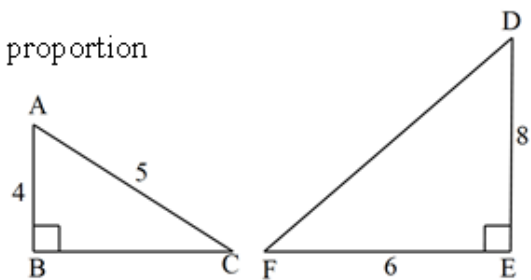
- 3.3.1 A)  $\triangle ABC \equiv \triangle DEF$  S, S, S  
 B)  $\triangle ABC \equiv \triangle EDF$  S, S, S  
 C)  $\triangle ABC \equiv \triangle FED$  S, S, S  
 D) None of the above



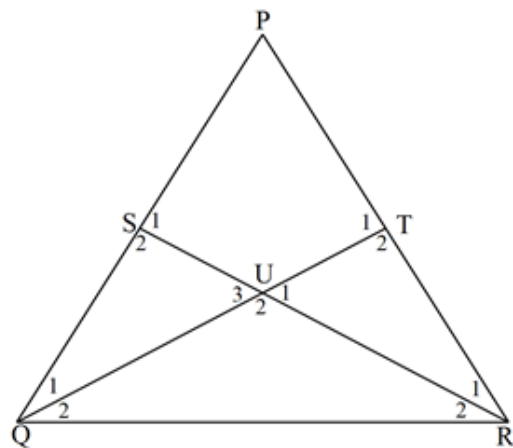
- 3.3.2 A)  $\triangle ABC \equiv \triangle DEF$  A, A, A  
 B)  $\triangle ABC \equiv \triangle DEF$  A, A, A  
 C)  $\triangle ABC \equiv \triangle DEF$  A, S, A  
 D) None of the above



- 3.3.3 A)  $\triangle ABC \parallel \triangle DEF$  sides are in proportion  
 B)  $\triangle ABC \parallel \triangle DEF$  S, S, S  
 C)  $\triangle ABC \equiv \triangle DEF$  R, H, S  
 D) None of the above



- 3.4 In the given sketch,  $\triangle PQR$  is isosceles with  $PQ = PR$  and  $\hat{Q}_2 = \hat{R}_2$   
 Prove  $\triangle QTP \equiv \triangle RSP$



## Circle Geometry

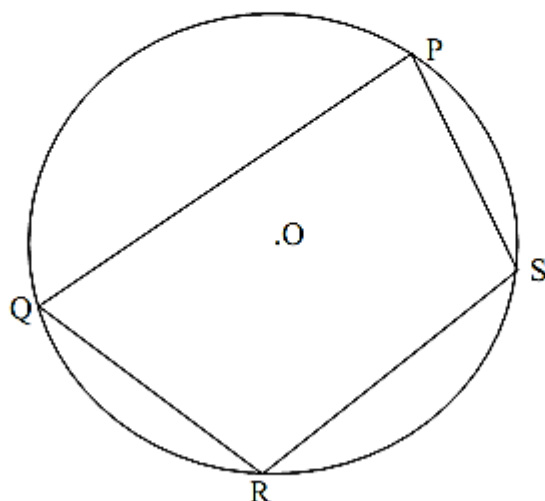
### QUESTION 1

1.1 Complete the statements below by filling in the missing word(s) to make the statements correct.

1.1.1 The angle between a tangent and a chord is ...

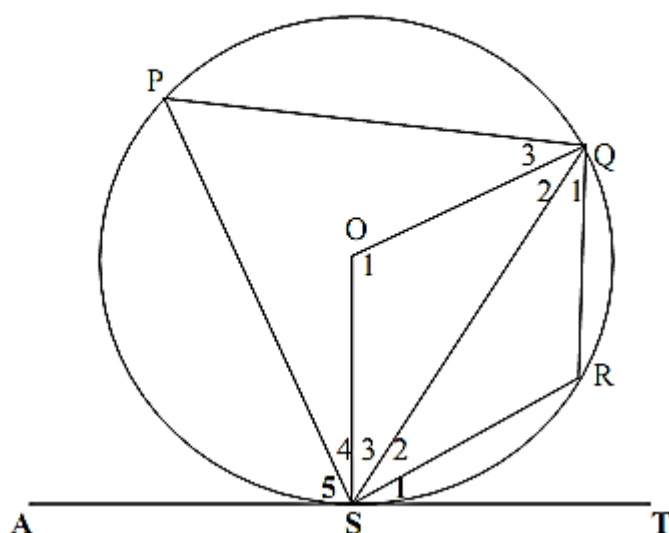
1.1.2 The exterior angle of a cyclic quadrilateral is equal to ...

1.2 In the diagram below O is the centre of the circle. PQRS is cyclic quadrilateral.



Redraw the diagram or use the diagram on DIAGRAM SHEET 2 to prove the theorem which states that  $\hat{P} + \hat{R} = 180^\circ$ .

1.3 In the diagram below, AST is a tangent to a circle O at S.  $\hat{RST} = \hat{S}_1 = 23^\circ$  and  $QR = RS$ .



Calculate, with reasons, the sizes of:

1.3.1  $\widehat{QSR}$

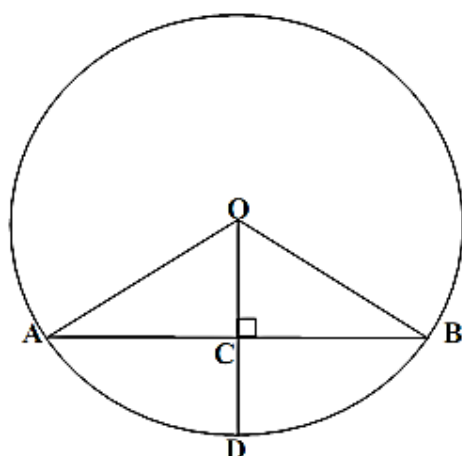
1.3.2  $\widehat{R}$

1.3.3  $\widehat{P}$

1.3.4  $\widehat{O}_1$

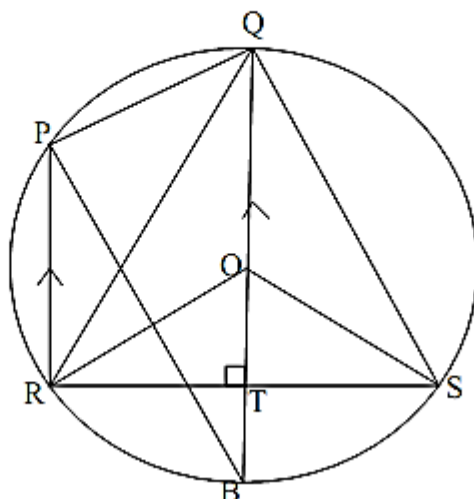
## QUESTION 2

- 2.1 In the diagram below, AB is a chord of a circle with centre O. D is a point on the circle. OD is perpendicular to AB. OA = 25 cm and CD = 18 cm.



Calculate, with reasons, the length of AB.

- 2.2 In the diagram below, QOB is the diameter of the circle with centre O.  $PR \parallel QB$ ,  $QB \perp RS$  and  $\widehat{PBQ} = 25^\circ$ . P, R and S are points on the circle.



2.2.1 Determine, with reasons, three other angles each equal to  $25^\circ$ .

2.2.2 Determine, with reasons:

(a)  $\widehat{ROB}$

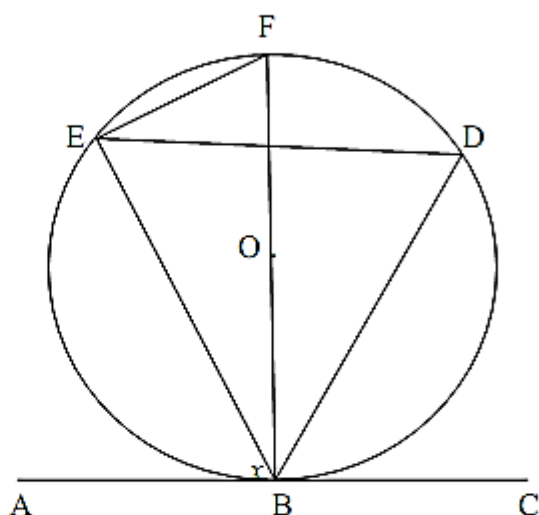
(b)  $\widehat{ORT}$

(c)  $\widehat{ROS}$

(d)  $\widehat{RPQ}$

### QUESTION 3

3.1 In the diagram below, ABC is a tangent at B to the circle with centre O. D and E are points on this circle.  $\widehat{ABE} = x$ .



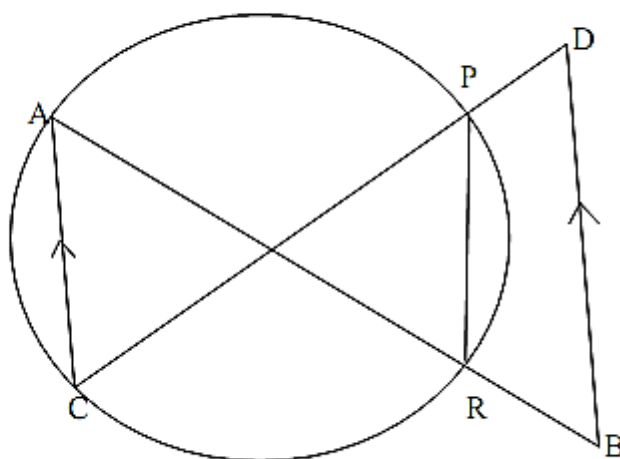
3.1.1 Express in terms of  $x$ :

(a)  $\widehat{FBE}$

(b)  $\widehat{F}$

3.1.2 Prove that AB is NOT a tangent to circle OEB.

- 3.2 In the diagram, chords AR and CP intersect inside the circle.  
AR and CP are respectively produced to B and D such that  $AC \parallel DB$ .



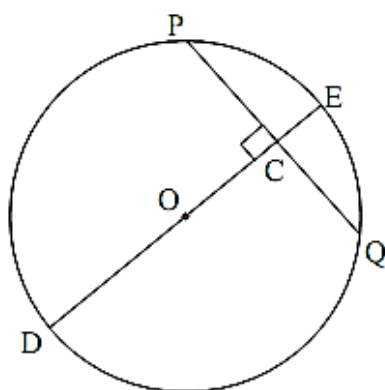
Prove that PDBR is a cyclic quadrilateral.

#### QUESTION 4

- 4.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord ...

- 4.2 In the diagram, O is the centre of the circle. The diameter DE is perpendicular to the chord PQ at C.  $DE = 20$  cm and  $CE = 2$  cm.



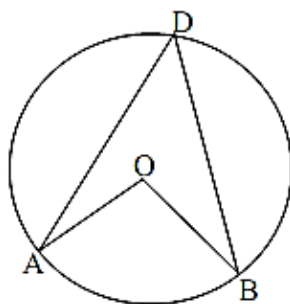
Calculate the length of the following with reasons:

4.2.1 OC

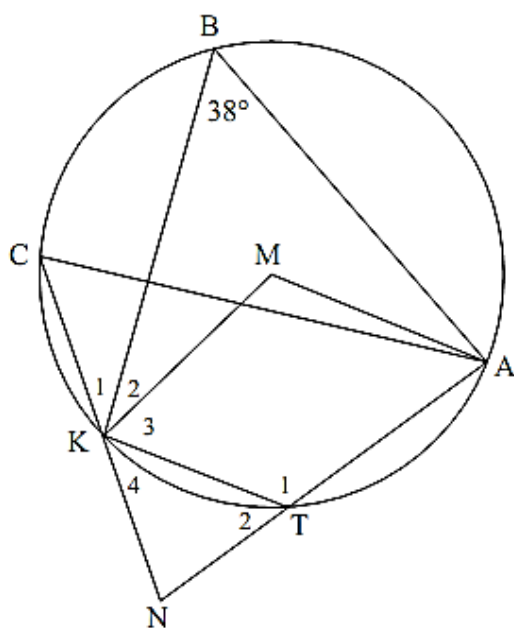
4.2.2 PQ

### QUESTION 5

- 5.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that  $\hat{AOB} = 2\hat{ADB}$ .



- 5.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also  $NA = NC$  and  $\hat{B} = 38^\circ$ .
- 5.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also  $NA = NC$  and  $\hat{B} = 38^\circ$ .



5.2.1 Calculate, with reasons, the size of the following angles:

(a)  $\hat{KMA}$

(b)  $\hat{T}_2$

(c)  $\hat{C}$

(d)  $\hat{K}_4$

5.2.2 Show that  $NK = NT$ .

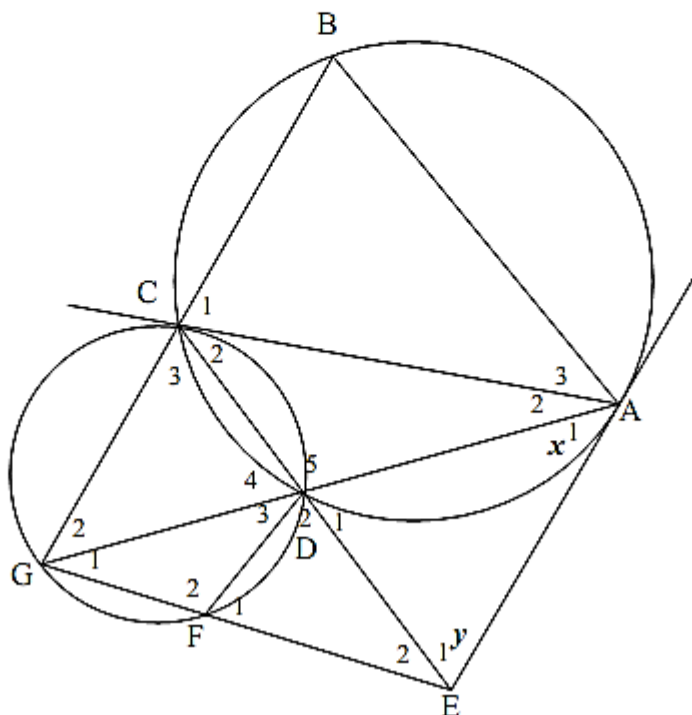
5.2.3 Prove that  $AMKN$  is a cyclic quadrilateral.

## QUESTION 6

6.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is ...

6.2 In the diagram,  $EA$  is a tangent to circle  $ABCD$  at  $A$ .  
 $AC$  is a tangent to circle  $CDFG$  at  $C$ .  
 $CE$  and  $AG$  intersect in  $D$ .



If  $\hat{A}_1 = x$  and  $\hat{E}_1 = y$ , prove the following with reasons:

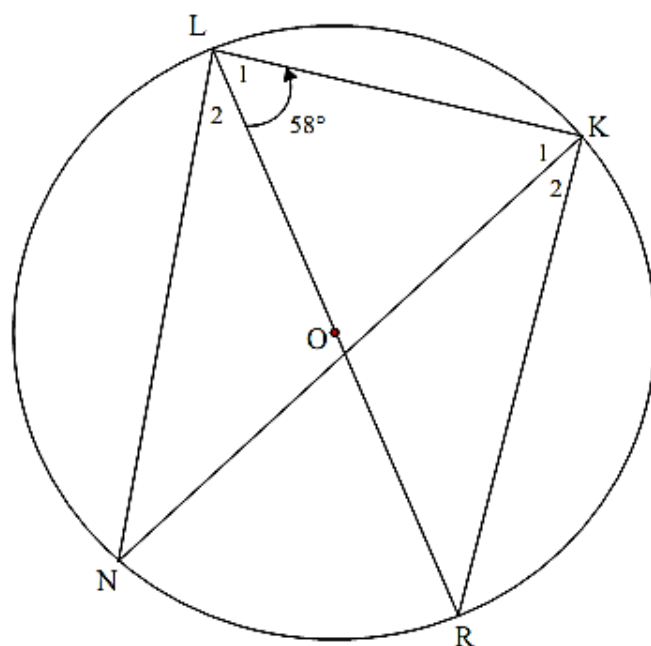
6.2.1  $BCG \parallel AE$

6.2.2  $AE$  is a tangent to circle  $FED$

6.2.3  $AB = AC$

### QUESTION 7

In the diagram,  $O$  is the centre of the circle. Diameter  $LR$  subtends  $\hat{LKR}$  at the circumference of the circle.  $N$  is another point on the circumference and chords  $LN$  and  $KN$  are drawn.  $\hat{L}_1 = 58^\circ$ .



Calculate, giving reasons, the size of:

7.1  $\hat{LKR}$

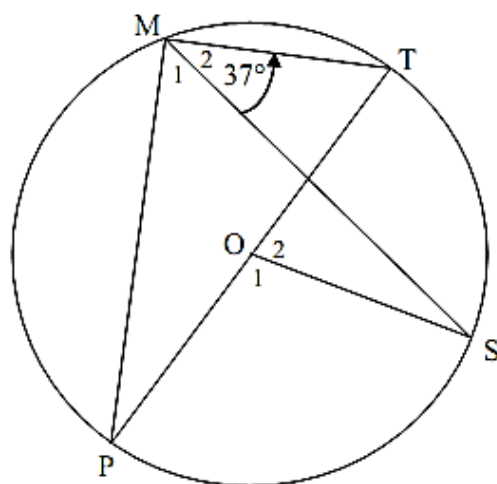
7.2  $\hat{R}$

7.3  $\hat{N}$

**QUESTION 8**

- 8.1 In the diagram below,  $PT$  is a diameter of the circle with centre  $O$ .  $M$  and  $S$  are points on the circle on either side of  $PT$ .  $MP$ ,  $MT$ ,  $MS$  and  $OS$  are drawn.

$$\hat{M}_2 = 37^\circ$$

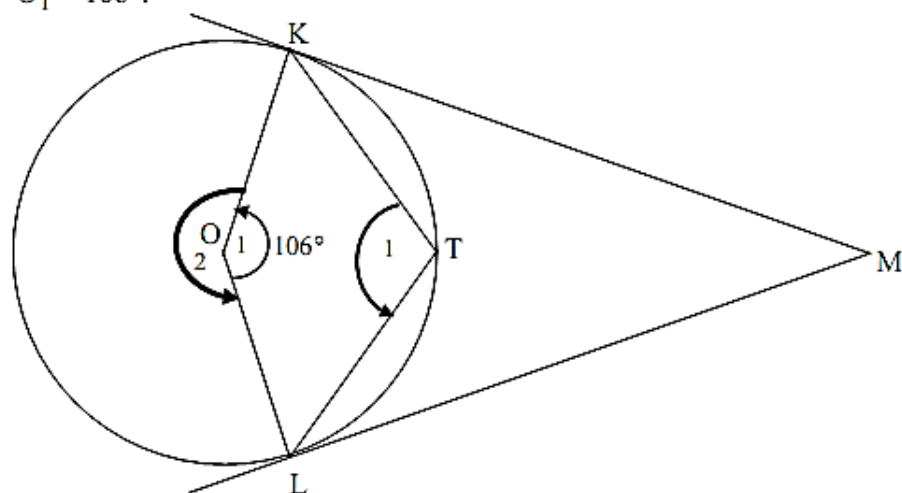


Calculate, with reasons, the size of:

8.1.1  $\hat{M}_1$

8.1.2  $\hat{O}_1$

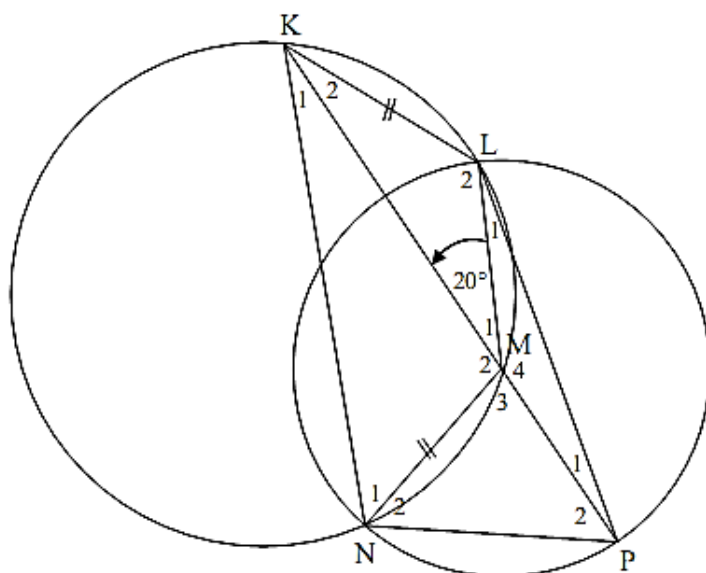
- 8.2 In the diagram  $O$  is the centre of the circle.  $KM$  and  $LM$  are tangents to the circle at  $K$  and  $L$  respectively.  $T$  is a point on the circumference of the circle.  $KT$  and  $LT$  are joined.  $\hat{O}_1 = 106^\circ$ .



- 8.2.1 Calculate, with reasons, the size of  $\hat{T}_1$ .
- 8.2.2 Prove that quadrilateral  $OKML$  is a kite.
- 8.2.3 Prove that quadrilateral  $OKML$  is a cyclic quadrilateral.
- 8.2.4 Calculate, with reasons, the size of  $\hat{M}$ .

### QUESTION 9

In the diagram  $M$  is the centre of the circle passing through points  $L$ ,  $N$  and  $P$ .  $PM$  is produced to  $K$ .  $KLMN$  is a cyclic quadrilateral in the larger circle having  $KL = MN$ .  $LP$  is joined.  $\hat{KML} = 20^\circ$ .



9.1 Write down, with a reason, the size of  $\hat{NKM}$ .

9.2 Give a reason why  $KN \parallel LM$ .

9.3 Prove that  $KL = LM$ .

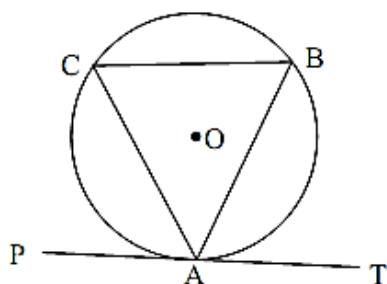
9.4 Calculate, with reasons, the size of:

9.4.1  $\hat{KNM}$

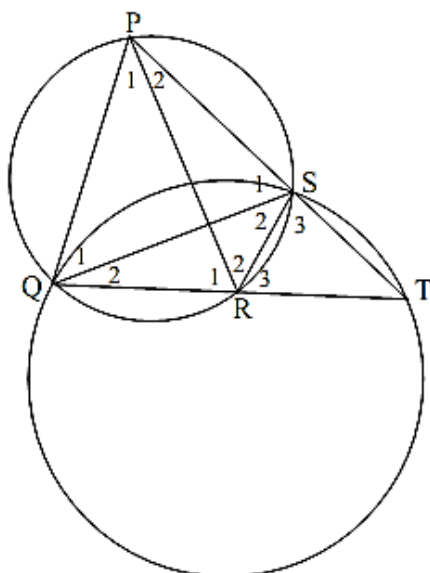
9.4.2  $\hat{LPN}$

# QUESTION 10

- 10.1 Use the sketch in the SPECIAL ANSWER BOOK to prove the theorem which states that  $\hat{BAT} = \hat{C}$ .



- 10.2 In the diagram PQ is a tangent to the circle QST at Q such that QT is a chord of the circle and TS produced meets the tangent at P. R is a point on QT such that PQRS is a cyclic quadrilateral in another circle. PR, QS and RS are joined.



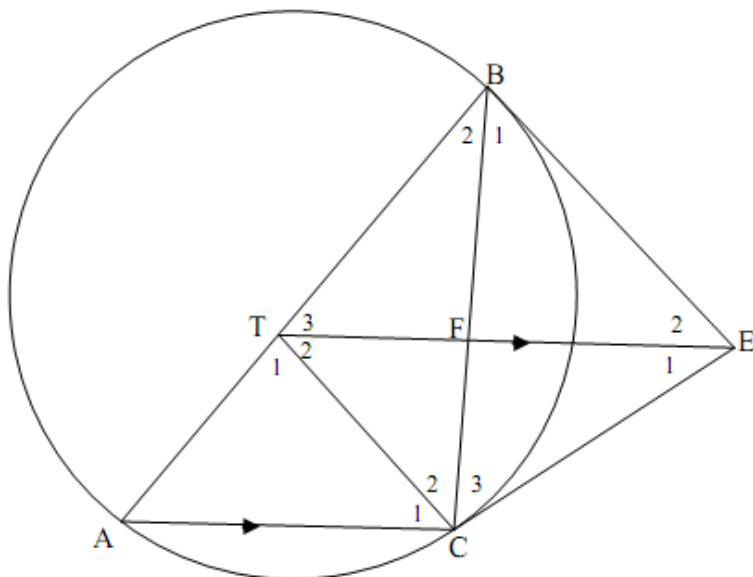
- 10.2.1 Give a reason for each statement. Write down only the reason next to the question number in the SPECIAL ANSWER BOOK.

Statement	Reason
$\hat{Q}_1 = \hat{T}$	10.2.1 (a)
$\hat{Q}_2 = \hat{P}_2$	10.2.1 (b)

- 10.2.2 Prove that PQR is an isosceles triangle.
- 10.2.3 Prove that PR is a tangent to the circle RST at point R.

**QUESTION 11**

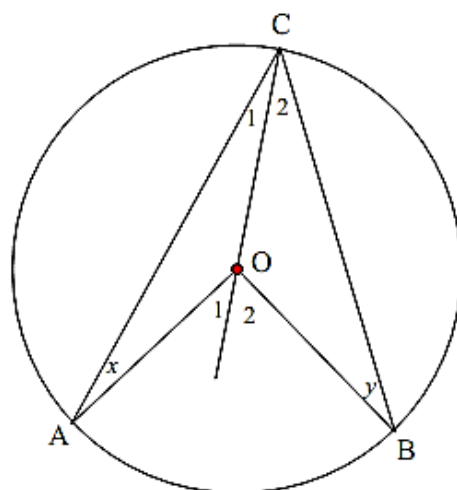
In the diagram, the vertices  $A$ ,  $B$  and  $C$  of  $\triangle ABC$  are concyclic.  $EB$  and  $EC$  are tangents to the circle at  $B$  and  $C$  respectively.  $T$  is a point on  $AB$  such that  $TE \parallel AC$ .  $BC$  cuts  $TE$  in  $F$ .



- 11.1 Prove that  $\hat{B}_1 = \hat{T}_3$ .
- 11.2 Prove that  $TBEC$  is a cyclic quadrilateral.
- 11.3 Prove that  $ET$  bisects  $\hat{BTC}$ .
- 11.4 If it is given that  $TB$  is a tangent to the circle through  $B$ ,  $F$  and  $E$ , prove that  $TB = TC$ .
- 11.5 Hence, prove that  $T$  is the centre of the circle through  $A$ ,  $B$  and  $C$ .

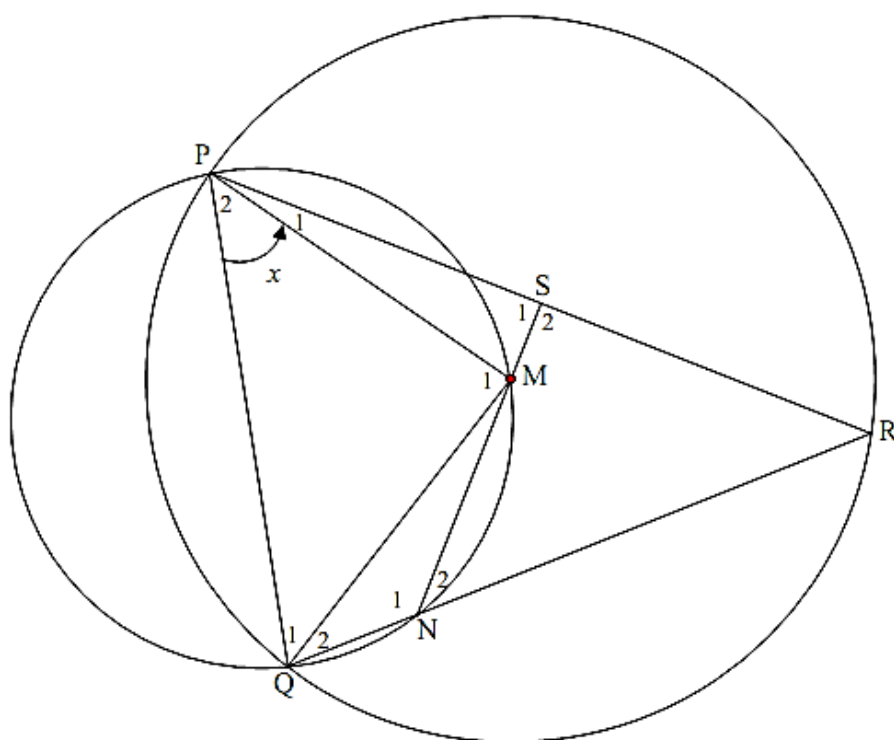
## QUESTION 12

- 12.1 In the diagram,  $O$  is the centre of the circle.  $A$ ,  $B$  and  $C$  are points on the circumference of the circle. Chords  $AC$  and  $BC$  and radii  $AO$ ,  $BO$  and  $CO$  are drawn.  $\hat{A} = x$  and  $\hat{B} = y$ .



- 12.1.1 Determine the size of  $\hat{O}_1$  in terms of  $x$ .
- 12.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is  $\hat{AOB} = 2\hat{ACB}$ .

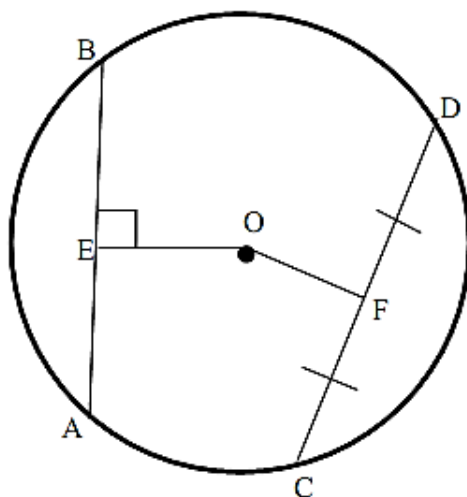
- 12.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S.  $\hat{P}_2 = x$ .



- 12.2.1 Give a reason why  $\hat{N}_2 = x$ .
- 12.2.2 Write down another angle equal in size to  $x$ . Give a reason.
- 12.2.3 Determine the size of  $\hat{R}$  in terms of  $x$ .
- 12.2.4 Prove that  $PS = SR$ .

### QUESTION 13

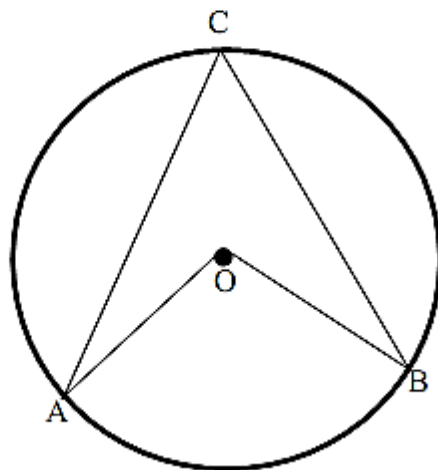
- 13.1 Complete: The line drawn from the centre of the circle perpendicular to the chord ...
- 13.2 In the figure below, AB and CD are chords of the circle with centre O.  $OE \perp AB$ .  $CF = FD$ .  $OE = 4$  cm,  $OF = 3$  cm and  $CD = 8$  cm.



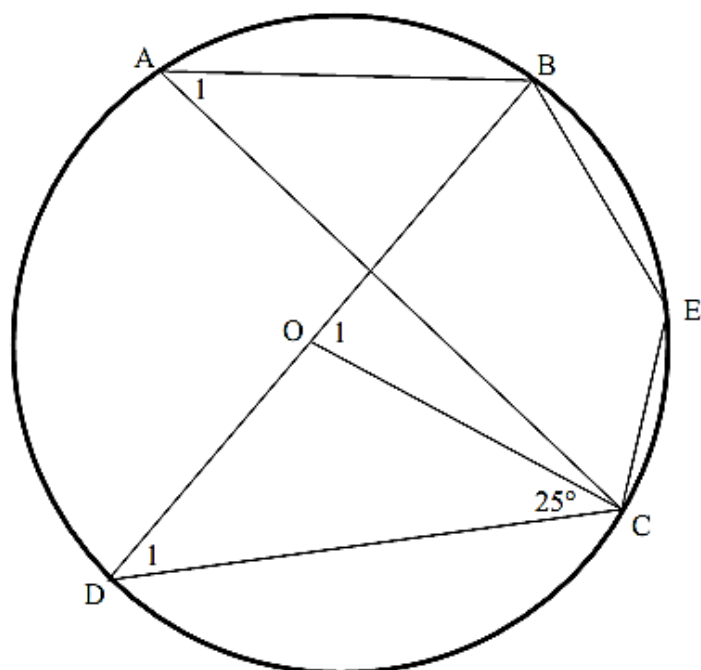
- 13.2.1 Calculate the length of OD.
- 13.2.2 Hence calculate the length of AB.

### QUESTION 14

- 14.1 In the diagram O is the centre of the circle and ABC are points on the circle. Use the diagram in your SPECIAL ANSWER BOOK to prove that:  $\angle AOB = 2\angle ACB$ .



- 14.2 In the figure below,  $\angle DCO = 25^\circ$  and O is the centre of the circle. A, B, E, C and D are points on the circumference. Calculate, giving reasons, the sizes of:



14.2.1  $\hat{D}_1$

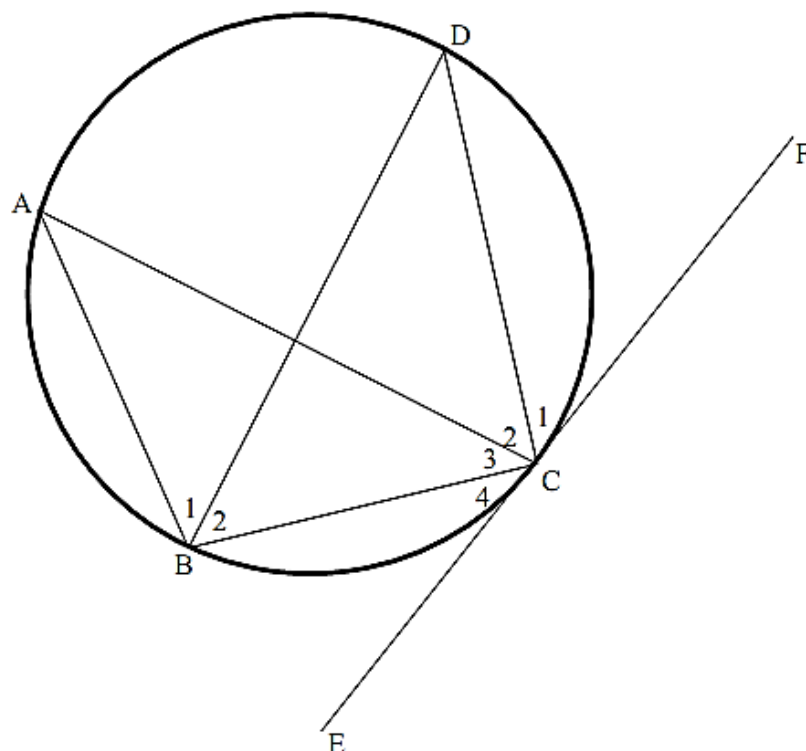
14.2.2  $\hat{O}_1$

14.2.3  $\hat{A}_1$

14.2.4  $\hat{E}$

### QUESTION 15

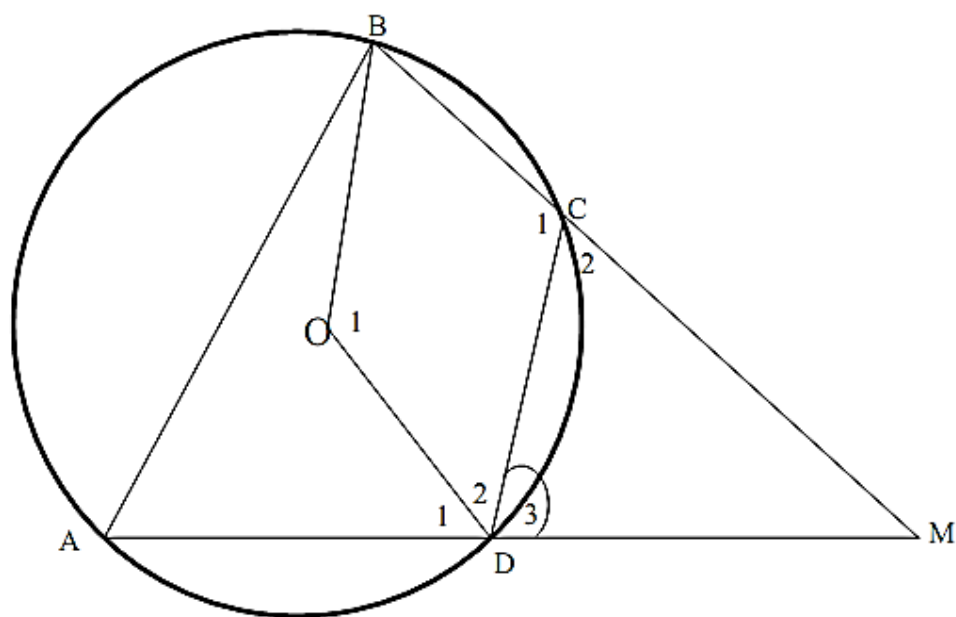
A, B, C and D are points on the circumference of the circle in the diagram below. ECF is a tangent at C,  $B_1 = B_2$ .



- 15.1 If  $B_1 = x$ , find, with reasons, TWO other angles equal to  $x$ .
- 15.2 Hence, show that DC bisects  $\widehat{ACF}$ .

### QUESTION 16

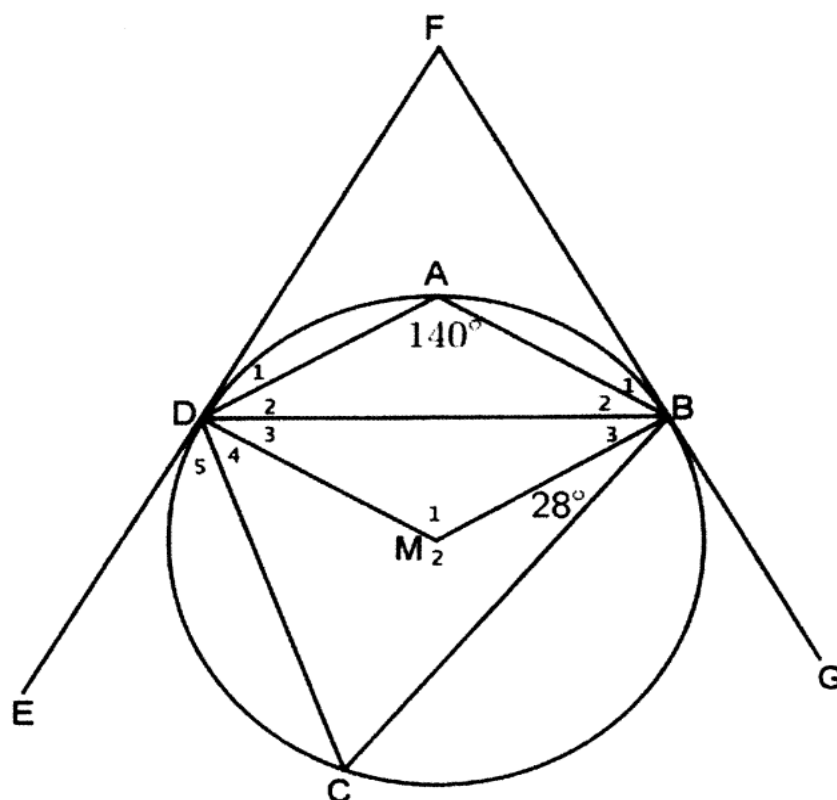
- 16.1 Complete: Opposite angles of a cyclic quadrilateral ...
- 16.2 In the figure, ABCD is a cyclic quadrilateral.  $AB \parallel DC$  in circle with centre O. BC and AD produced meet at M.  $D_3 = x$



- 16.2.1 Show that  $MC = MD$ .
- 16.2.2 If  $\angle D_3 = x$ , determine the value of  $\hat{M}$ , in terms of  $x$ .
- 16.2.3 Hence, show that BODM is a cyclic quadrilateral.

## QUESTION 17

In the diagram below, M is the centre of the circle DABC. EDF is a tangent to the circle at D and FBG is another tangent to the circle at B.



Calculate the following angles, with reasons:

17.1  $\hat{C}$

17.2  $\hat{M}_1$

17.3  $\hat{B}_3$

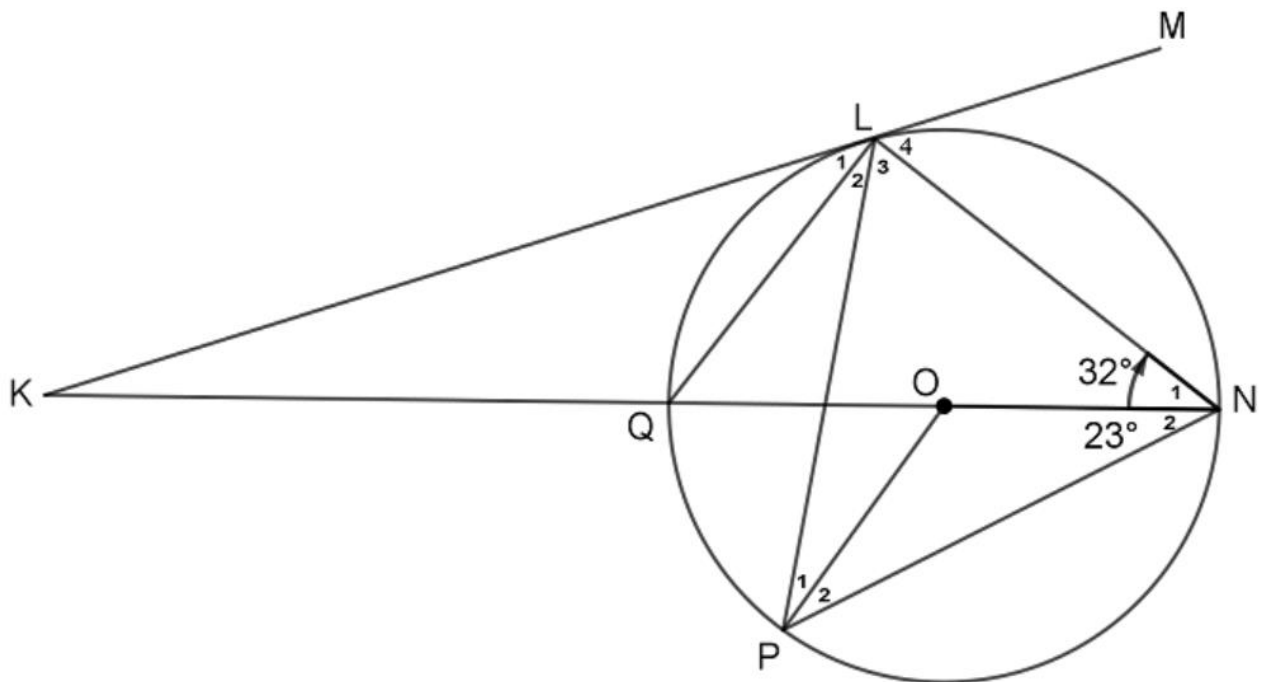
17.4  $\hat{D}_5$

## QUESTION 18

18.1 The angle at the point of contact between a tangent to a circle and a chord is ----- .

18.2 In the sketch below, circle centre O has a tangent KLM.  
Diameter NQ produced meet the tangent in K.

$$\hat{N}_1 = 32^\circ \text{ and } \hat{N}_2 = 23^\circ.$$



Calculate, with reasons, the size of:

18.2.1  $\hat{P}_2$

18.2.2  $\hat{POQ}$

18.2.3  $\hat{L}_2$

18.2.4  $\hat{NLQ}$

18.2.5  $\hat{L}_3$

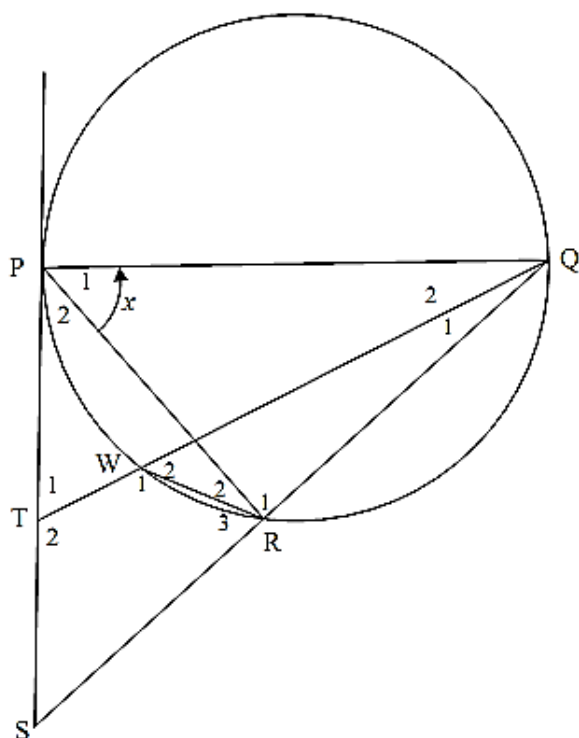
18.2.6  $\hat{PLK}$

## Circle Geometry, and Similarity and Proportionality

### QUESTION 1

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

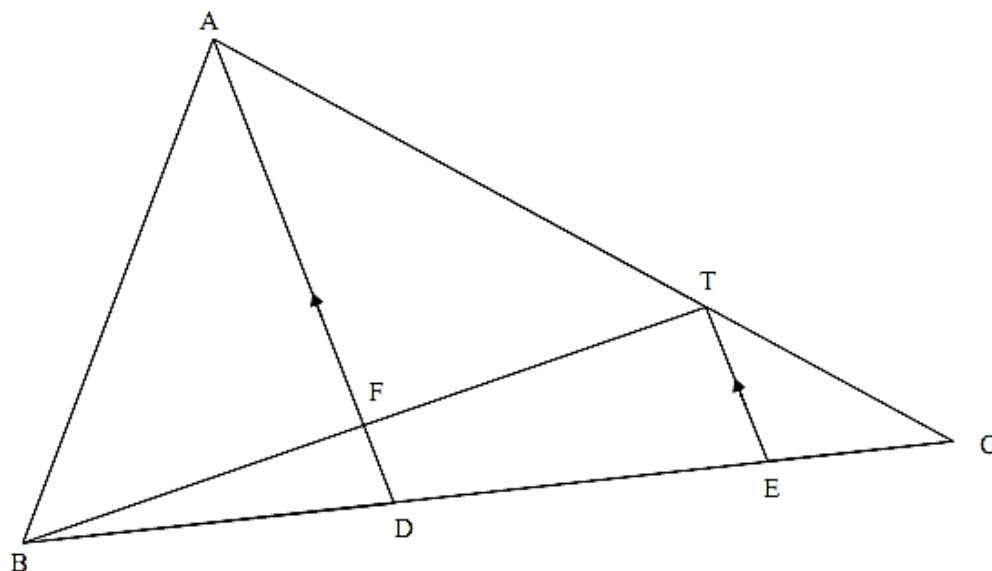
Let  $\hat{P}_1 = x$



- 1.1 Why is  $\hat{P}RQ = 90^\circ$ ?
- 1.2 Prove that  $\hat{P}_1 = \hat{S}$ .
- 1.3 Prove that SRWT is a cyclic quadrilateral.
- 1.4 Prove that  $\triangle QWR \sim \triangle QST$ .
- 1.5 If  $QW = 5$  cm,  $TW = 3$  cm,  $QR = 4$  cm and  $WR = 2$  cm, calculate the length of:
  - 1.5.1 TS
  - 1.5.2 SR

**QUESTION 2**

In the figure below,  $\triangle ABC$  has D and E on BC.  $BD = 6$  cm and  $DC = 9$  cm.  
 $AT : TC = 2 : 1$  and  $AD \parallel TE$ .



2.1 Write down the numerical value of  $\frac{CE}{ED}$

2.2 Show that D is the midpoint of BE.

2.3 If  $FD = 2$  cm, calculate the length of TE.

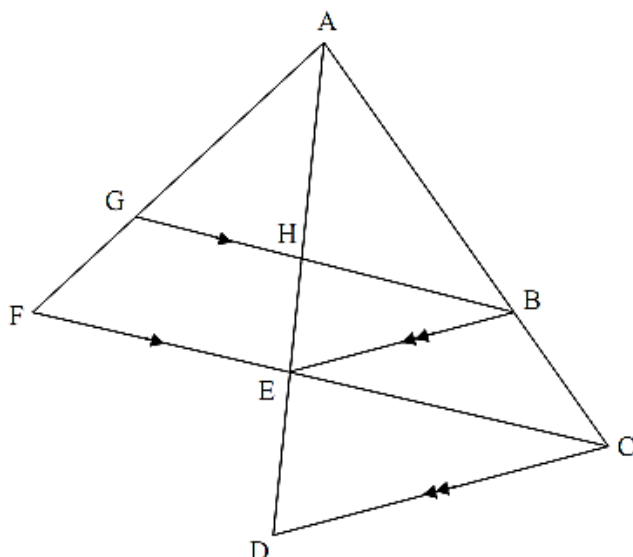
2.4 Calculate the numerical value of:

2.4.1  $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$

2.4.2  $\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC}$

### QUESTION 3

In the figure below,  $GB \parallel FC$  and  $BE \parallel CD$ .  $AC = 6$  cm and  $\frac{AB}{BC} = 2$ .



3.1 Calculate with reasons:

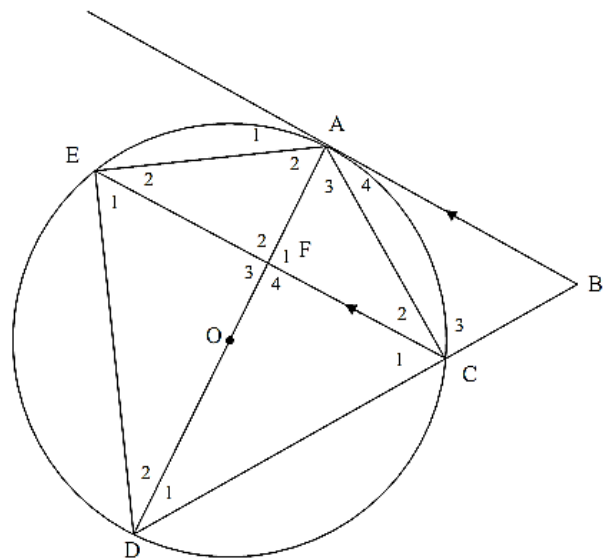
3.1.1  $AH : ED$

3.1.2  $\frac{BE}{CD}$

3.2 If  $HE = 2$  cm, calculate the value of  $AD \times HE$ .

### QUESTION 4

In the figure below,  $AB$  is a tangent to the circle with centre  $O$ .  $AC = AO$  and  $BA \parallel CE$ .  $DC$  produced, cuts tangent  $BA$  at  $B$ .



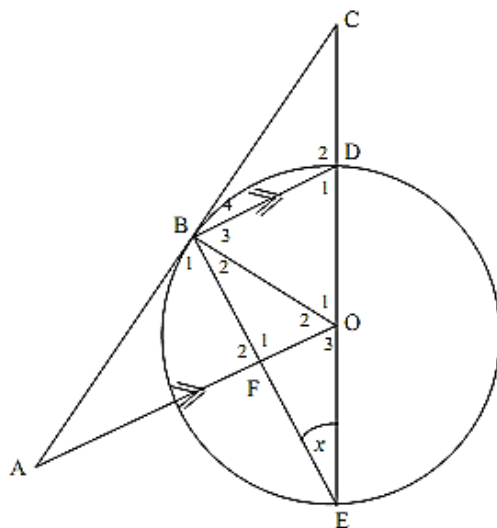
4.1 Show  $\hat{C}_2 = \hat{D}_1$ .

4.2 Prove that  $\triangle ACF \parallel \triangle ADC$ .

4.3 Prove that  $AD = 4AF$ .

#### QUESTION 5

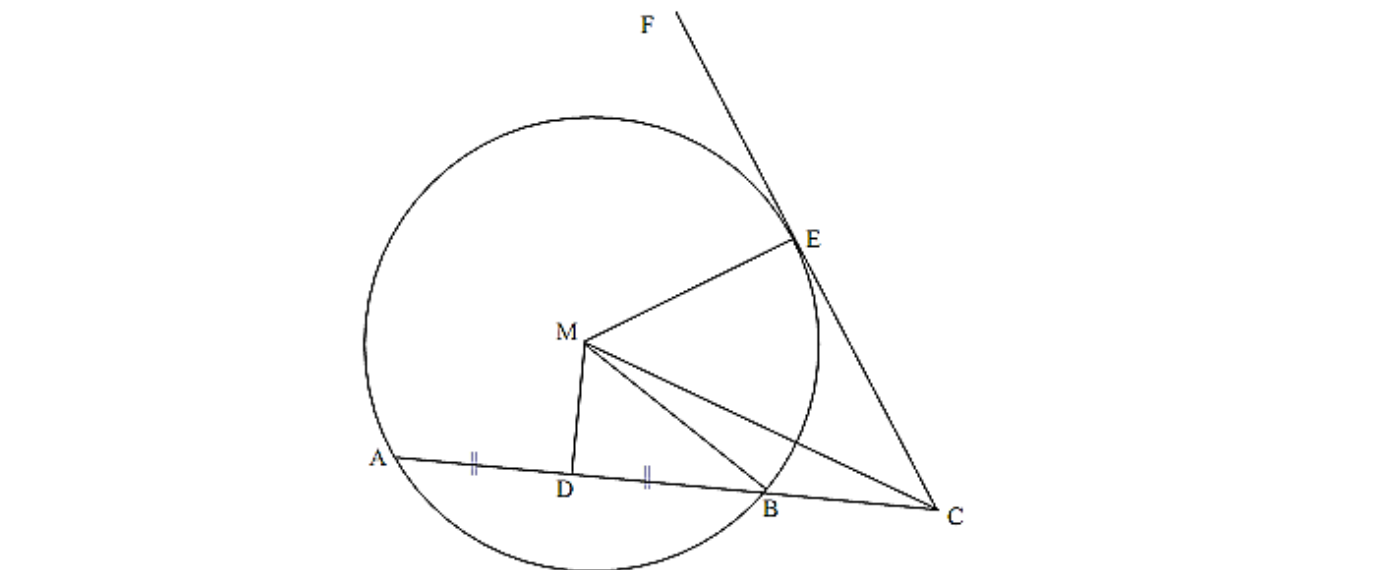
5.1 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F.  $BD \parallel AO$ .  $\hat{E} = x$ .



- 5.1.1 Write down, with reasons, THREE other angles equal to  $x$ .
- 5.1.2 Determine, with reasons,  $\hat{CBE}$  in terms of  $x$ .
- 5.1.3 Prove that  $F$  is the midpoint of  $BE$ .
- 5.1.4 Prove that  $\triangle CBD \parallel \triangle CEB$ .
- 5.1.5 Prove that  $2EF \cdot CB = CE \cdot BD$ .

**QUESTION 6**

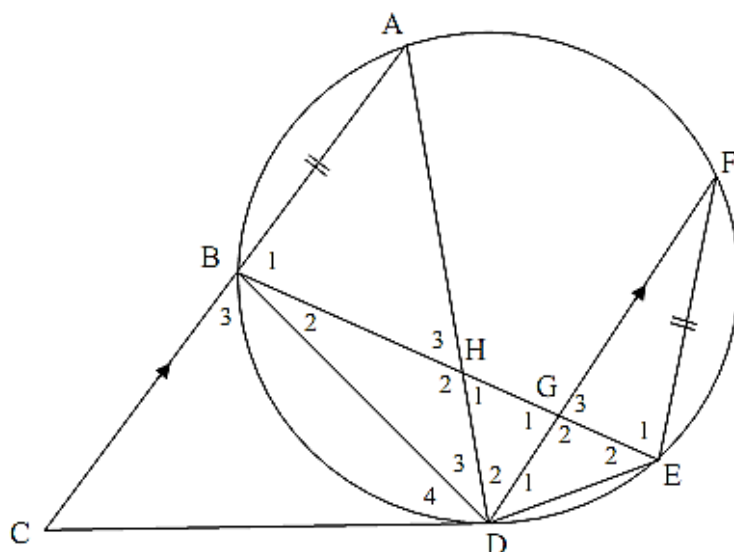
In the diagram below  $M$  is the centre of the circle.  $FEC$  is a tangent to the circle at  $E$ .  $D$  is the midpoint of  $AB$ .



- 6.1 Prove MDCE is a cyclic quadrilateral.
- 6.2 Prove that  $MC^2 = MB^2 + DC^2 - DB^2$ .
- 6.3 Calculate CE if AB = 60 mm, ME = 40 mm and BC = 20 mm.

### QUESTION 7

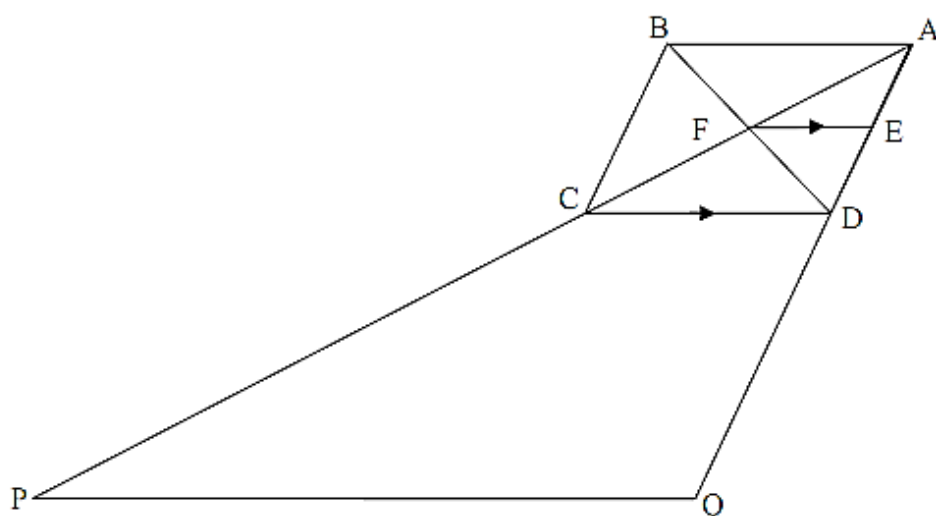
CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G.  $AC \parallel FD$  and  $FE = AB$ . Let  $\hat{D}_4 = x$  and  $\hat{D}_1 = y$ .



- 7.1 Determine THREE other angles that are each equal to  $x$ .
- 7.2 Prove that  $\triangle BHD \parallel \triangle FED$ .
- 7.3 Hence, or otherwise, prove that  $AB \cdot BD = FD \cdot BH$ .

**QUESTION 8**

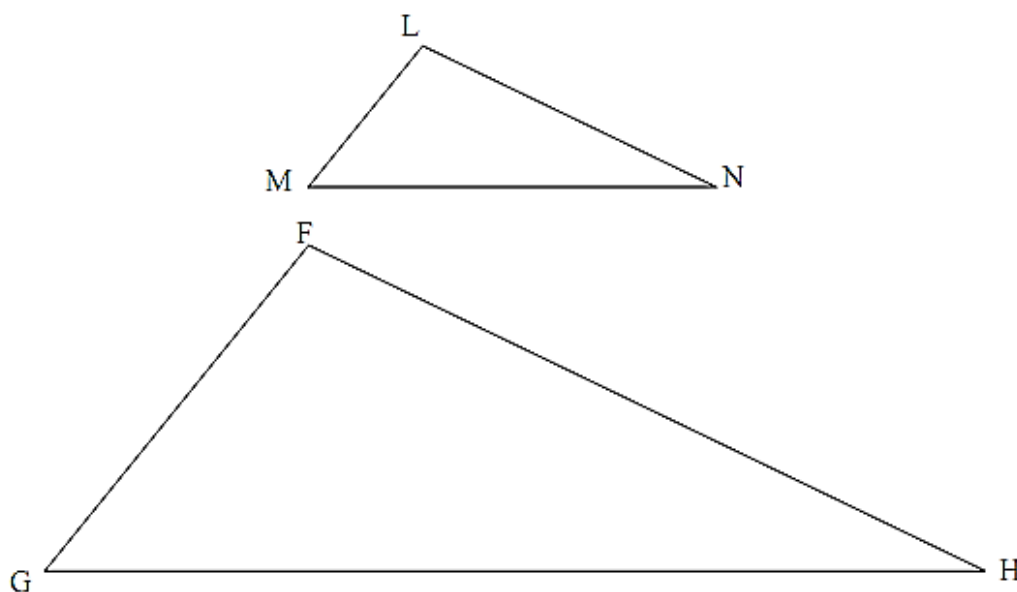
ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that  $PC = 2AC$  and AD is produced to Q such that  $DQ = 2AD$ .



- 8.1 Show that E is the midpoint of AD.
- 8.2 Prove  $PQ \parallel FE$ .
- 8.3 If PQ is 60 cm, calculate the length of FE.

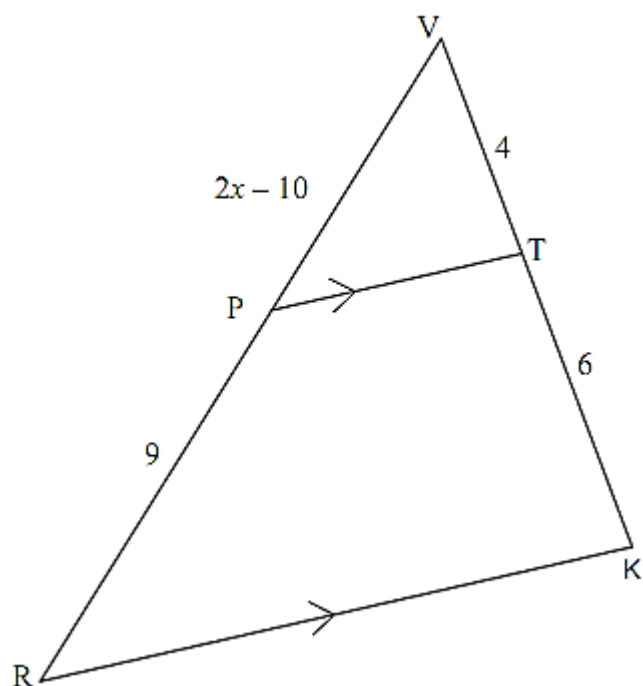
**QUESTION 9**

- 9.1 If in  $\triangle LMN$  and  $\triangle FGH$  it is given that  $\hat{L} = \hat{F}$  and  $\hat{M} = \hat{G}$ , prove the theorem that states  $\frac{LM}{FG} = \frac{LN}{FH}$ .



- 9.2 In the diagram below,  $\triangle VRK$  has P on VR and T on VK such that  $PT \parallel RK$ .  
 $VT = 4$  units,  $PR = 9$  units,  $TK = 6$  units and  $VP = 2x - 10$  units.

Calculate the value of  $x$ .

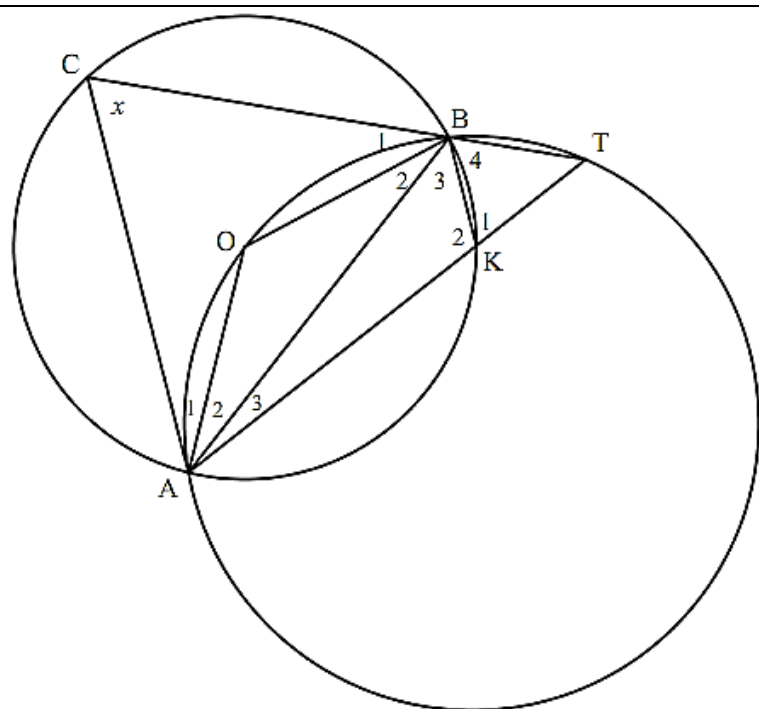


#### QUESTION 10

O is the centre of the circle CAKB.

AK produced intersects circle AOBT at T.

$\angle ACB = x$



10.1 Prove that  $\hat{T} = 180^\circ - 2x$ .

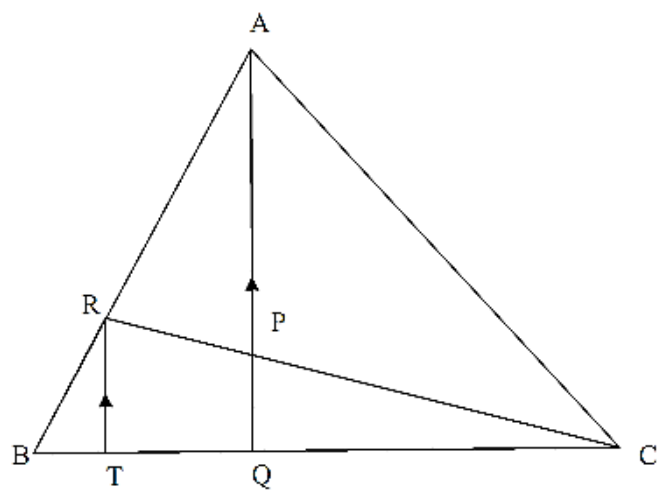
10.2 Prove  $AC \parallel KB$ .

10.3 Prove  $\triangle BKT \parallel \triangle CAT$

10.4 If  $AK : KT = 5 : 2$ , determine the value of  $\frac{AC}{KB}$

### QUESTION 11

In the figure  $AQ \parallel RT$ ,  $\frac{BQ}{QC} = \frac{3}{5}$  and  $\frac{BR}{RA} = \frac{1}{2}$ .



11.1 If  $BT = k$ , calculate  $TQ$  in terms of  $k$ .

11.2 Hence, or otherwise, calculate the numerical value of:

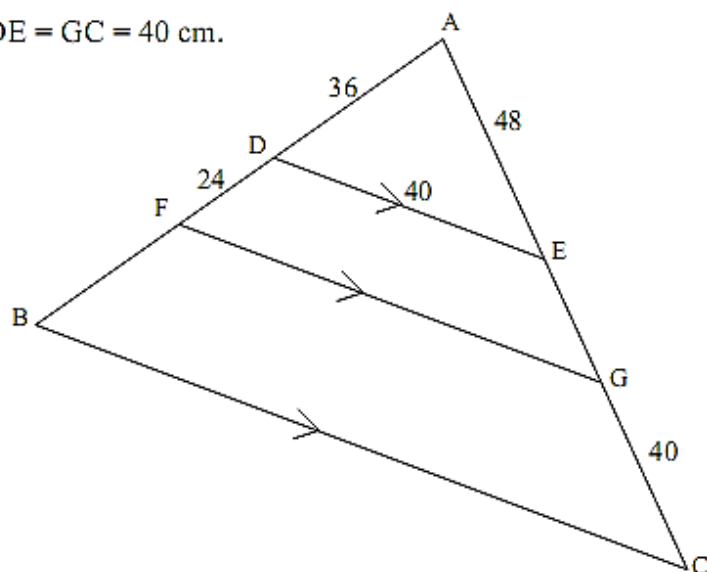
11.2.1  $\frac{CP}{PR}$

11.2.2  $\frac{\text{Area } \triangle RCT}{\text{Area } \triangle ABC}$

### QUESTION 12

In the figure below  $DE \parallel FG \parallel BC$ .

$AD = 36$  cm,  $DF = 24$  cm,  $AE = 48$  cm and  $DE = GC = 40$  cm.



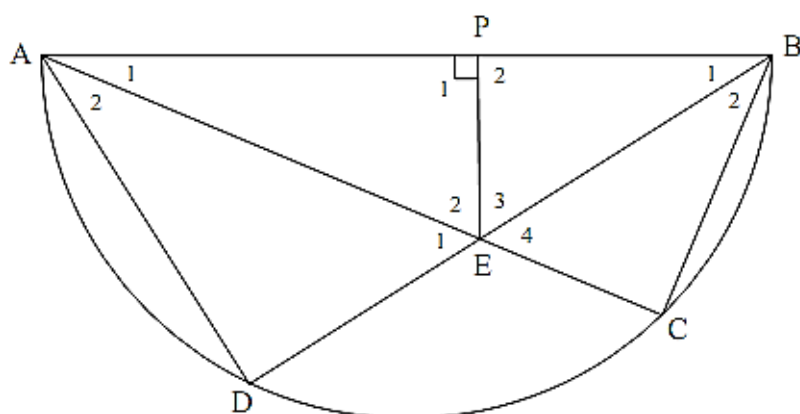
Determine, with reasons, the lengths of:

12.1  $EG$

12.2  $BC$

### QUESTION 13

In the accompanying figure,  $AB$  is the diameter of circle  $ADCB$ . Chords  $AC$  and  $BD$  intersect at  $E$ .  $EP$  is perpendicular to  $AB$ .



- 13.1 Prove that  $\triangle BPE \parallel \triangle BDA$ .
- 13.2 Hence show that  $\frac{BP}{BD} = \frac{PE}{AD}$ .
- 13.3 Prove that  $AB^2 = BD^2 + \frac{BD^2 \cdot PE^2}{BP^2}$ .

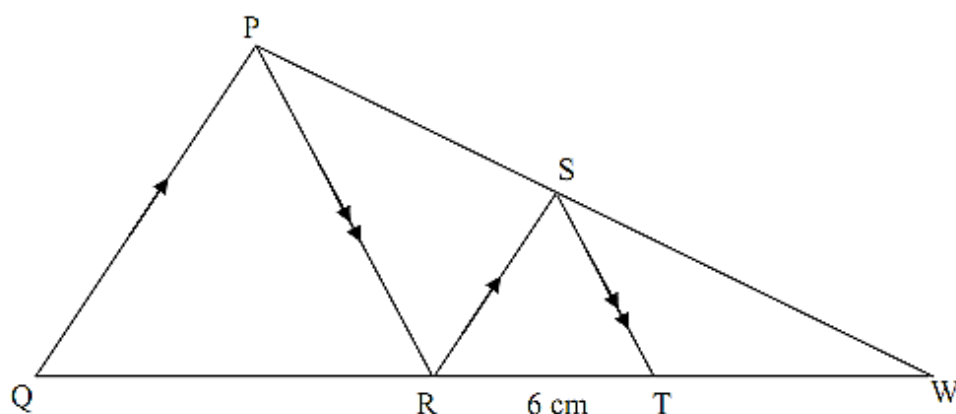
#### QUESTION 14

In  $\triangle PQW$ , S is a point on PW and R is a point on QW such that  $SR \parallel PQ$ .

T is a point on QW such that  $ST \parallel PR$ .

$RT = 6$  cm

$WS : SP = 3 : 2$



Calculate:

- 14.1 WT
- 14.2 WQ

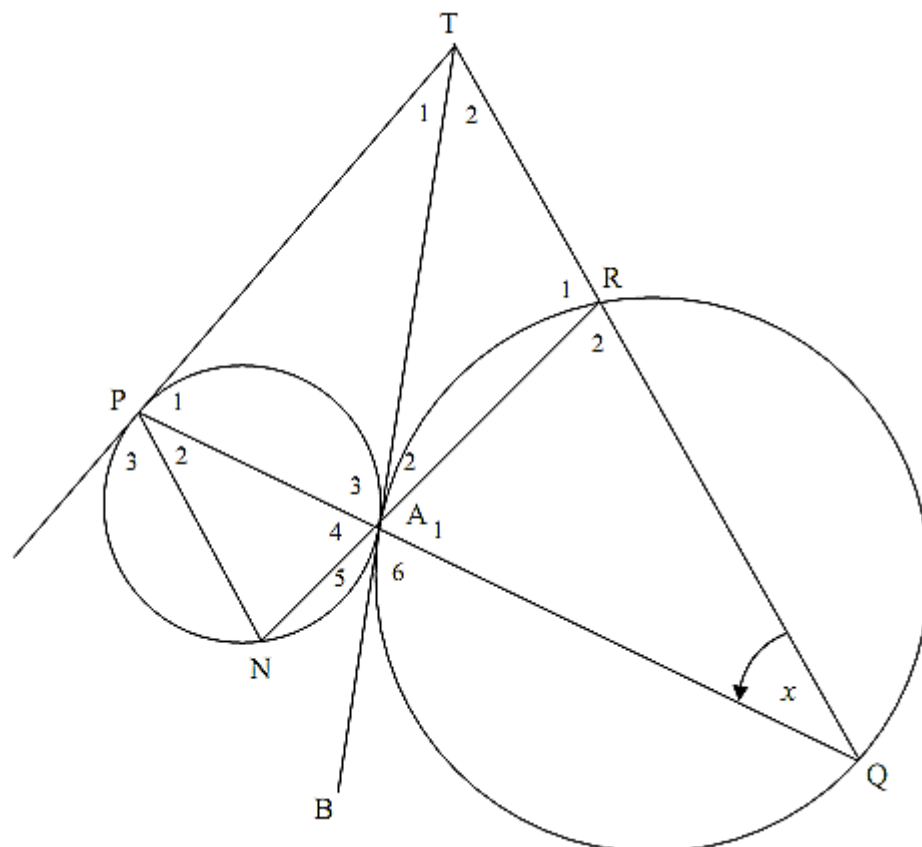
# QUESTION 15

15.1 Complete the following statement:

The angle between the tangent and the chord ...

15.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines. PAQ, QRT and NAR are straight lines.

Let  $\hat{Q} = x$ .

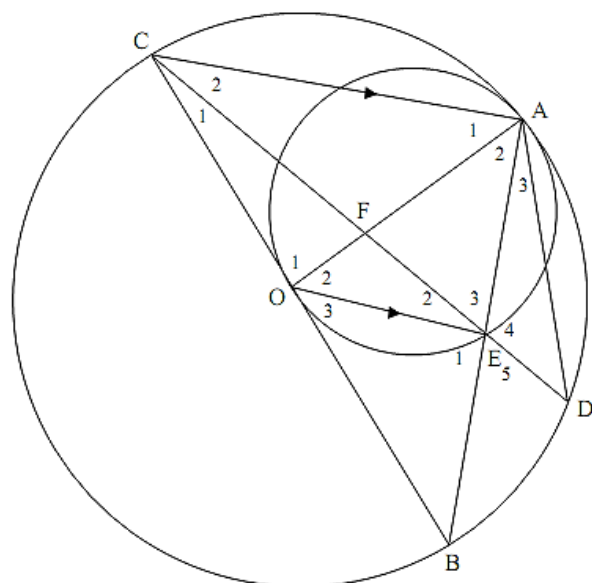


15.2.1 Name, with reasons, THREE other angles equal to  $x$ .

15.2.2 Prove that APTR is a cyclic quadrilateral.

# QUESTION 16

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle. OE  $\parallel$  CA.



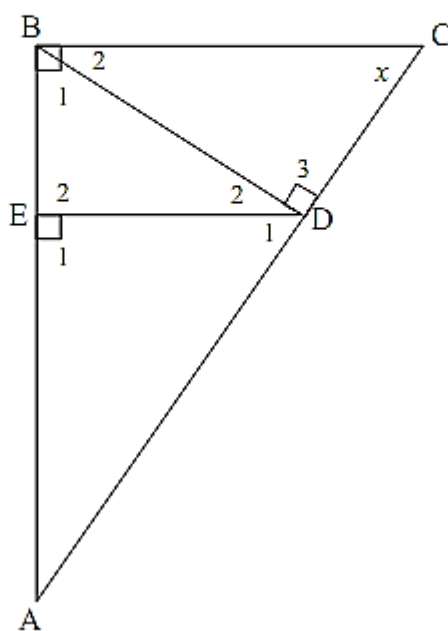
- 16.1 Prove, with reasons, that  $AE = BE$ .
- 16.2 Prove that  $\triangle AED \sim \triangle CEB$ .
- 16.3 Hence, or otherwise, show that  $AE^2 = DE \cdot CE$ .
- 16.4 If  $AE \cdot EB = EF \cdot EC$ , show that E is the midpoint of DF.

**QUESTION 17**

$\triangle ABC$  is a right-angled triangle with  $\hat{B} = 90^\circ$ . D is a point on AC such that  $BD \perp AC$  and E is a point on AB such that  $DE \perp AB$ . E and D are joined.

$AD : DC = 3 : 2$ .

$AD = 15$  cm.

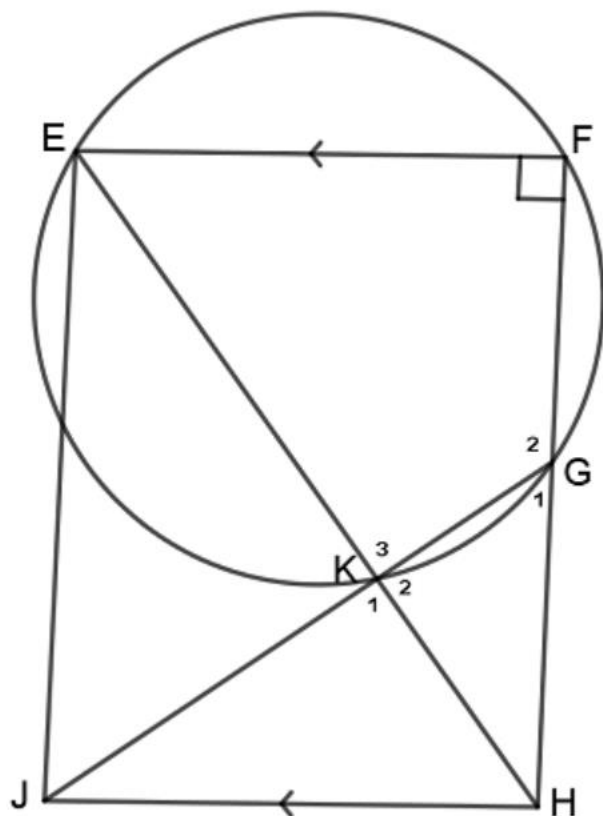


- 17.1 Prove that  $\triangle BDA \parallel \triangle CDB$ .
- 17.2 Calculate BD (Leave your answer in surd form).
- 17.3 Calculate AE (Leave your answer in surd form).

## QUESTION 18

In the diagram below, EFGK is a cyclic quadrilateral with  $\hat{F} = 90^\circ$ .

EK and FG are produced to meet at H. HJ is drawn parallel to FE. GK produced meets HJ at J.



18.1 Prove that:

18.1.1  $\hat{JHF} = 90^\circ$

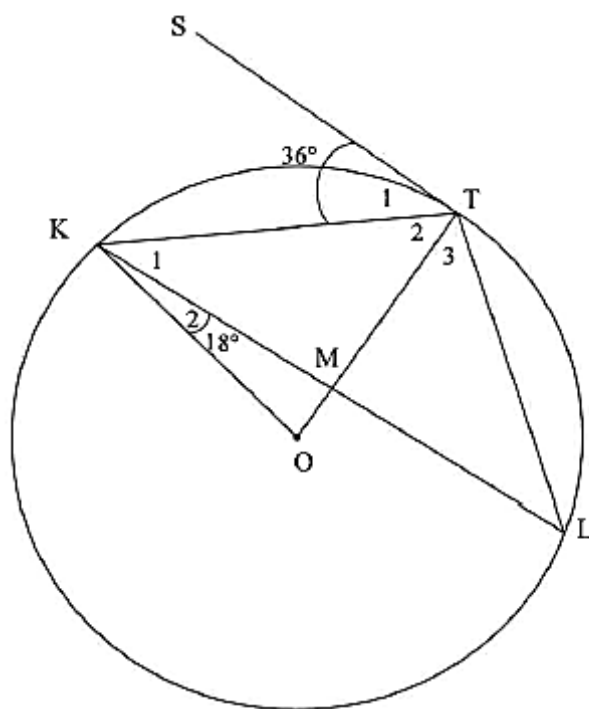
18.1.2  $\hat{K}_2 = 90^\circ$

18.1.3  $\triangle HKG \parallel \triangle JHG$

18.2 Calculate JG and KG if  $HG = 5\text{cm}$  and  $JH = 10\text{cm}$ .

## QUESTION 8

- 8.1 In the diagram,  $O$  is the centre of the circle.  $K$ ,  $T$  and  $L$  are points on the circle.  $KT$ ,  $TL$ ,  $KL$ ,  $OK$  and  $OT$  are drawn.  $OT$  intersects  $KL$  at  $M$ .  $ST$  is a tangent to the circle at  $T$ .  $\hat{S}TK = 36^\circ$  and  $\hat{OKL} = 18^\circ$ .

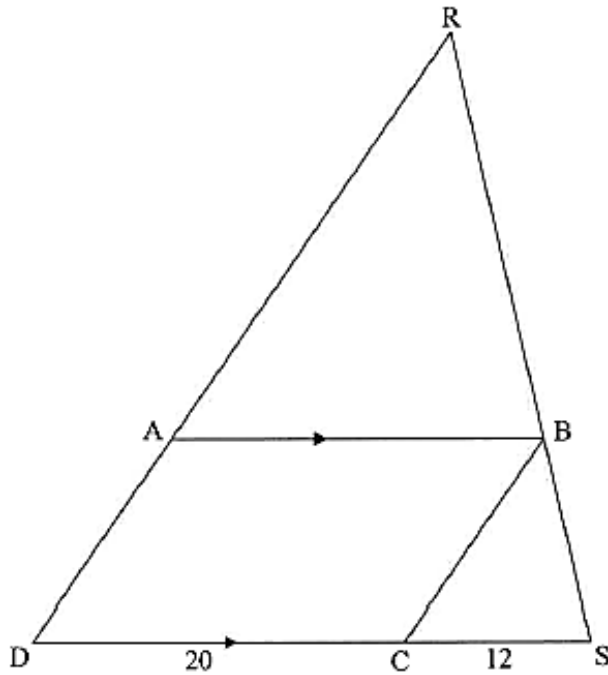


- 8.1.1 Determine, giving reasons, the size of:

- (a)  $\hat{T}_2$  (2)  
 (b)  $\hat{L}$  (2)  
 (c)  $\hat{KOT}$  (2)

- 8.1.2 Prove, giving reasons, that  $KM = ML$ . (3)

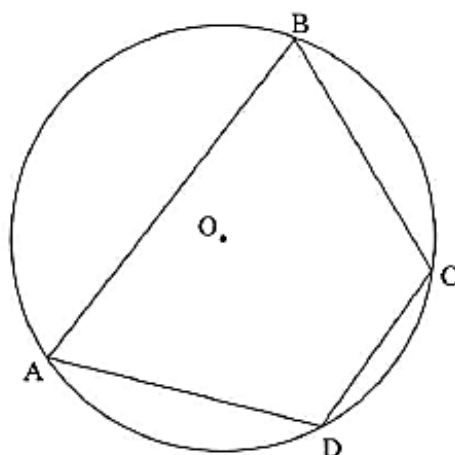
- 8.2 In the diagram,  $\triangle RDS$  is drawn. A, B and C are points on RD, RS and DS respectively such that  $AB \parallel DS$  and  $RB : BS = 5 : 3$ .  $DC = 20$  units and  $CS = 12$  units.



- 8.2.1 Prove, giving reasons, that  $BC \parallel AD$ . (3)
- 8.2.2 If it is further given that  $RD = 48$  units, calculate, giving reasons, the value of the ratio  $AD : AB$ . (3)  
[15]

# QUESTION 9

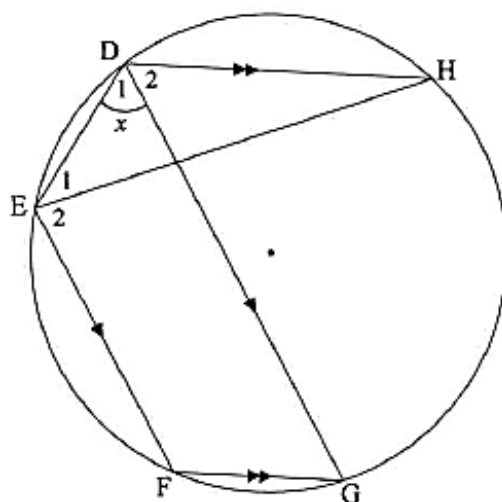
- 9.1 In the diagram,  $O$  is the centre of the circle.  $ABCD$  is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that  $\hat{B} + \hat{D} = 180^\circ$ .

(5)

- 9.2 In the diagram,  $DEFG$  is a cyclic quadrilateral such that  $EF \parallel DG$ .  $H$  is another point on the circle such that  $DH \parallel FG$ . Chord  $EH$  is drawn. Let  $\hat{D}_1 = x$ .



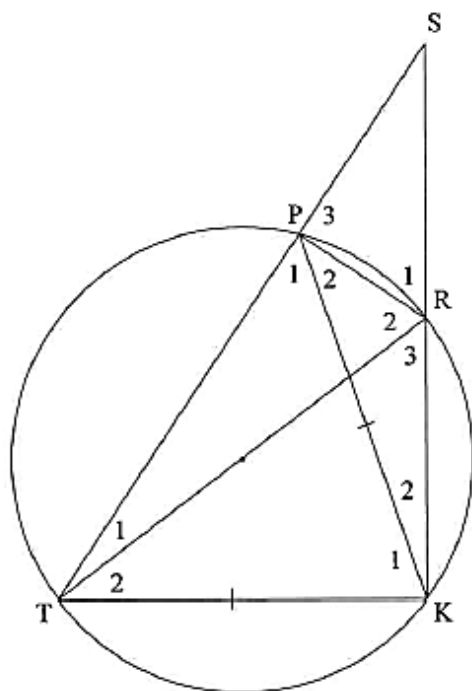
Prove, giving reasons, that  $\hat{D}_1 = \hat{D}_2$ .

(4)

[9]

### QUESTION 10

In the diagram,  $TR$  is a diameter of the circle.  $PRKT$  is a cyclic quadrilateral. Chords  $TP$  and  $KR$  are produced to intersect at  $S$ . Chord  $PK$  is drawn such that  $PK = TK$ .



10.1 Prove, giving reasons, that:

10.1.1  $SR$  is a diameter of a circle passing through points  $S$ ,  $P$  and  $R$  (4)

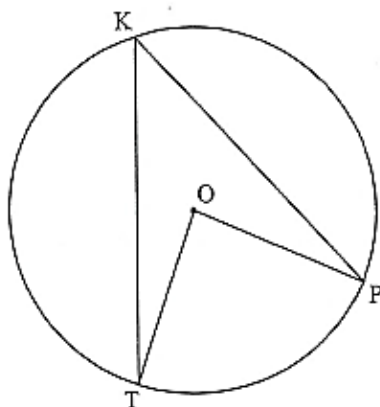
10.1.2  $\hat{S} = \hat{P}_2$  (5)

10.1.3  $\triangle SPK \parallel \triangle PRK$  (3)

10.2 If it is further given that  $SR = RK$ , prove that  $ST = \sqrt{6}RK$ . (5)  
[17]

**QUESTION 8**

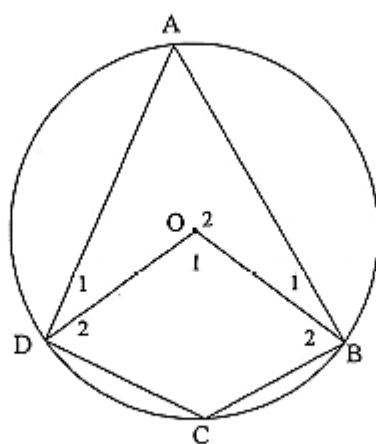
8.1 In the diagram, O is the centre of the circle.



Use the diagram above to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that  $\hat{TOP} = 2\hat{TKP}$ .

(5)

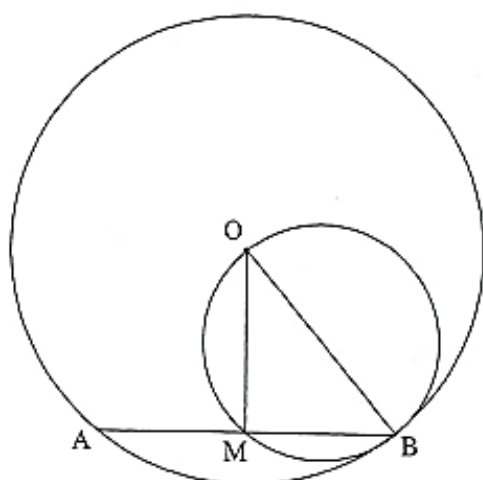
8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.



If  $\hat{O}_1 = 4x + 100^\circ$  and  $\hat{C} = x + 34^\circ$ , calculate, giving reasons, the size of  $x$ .

(5)

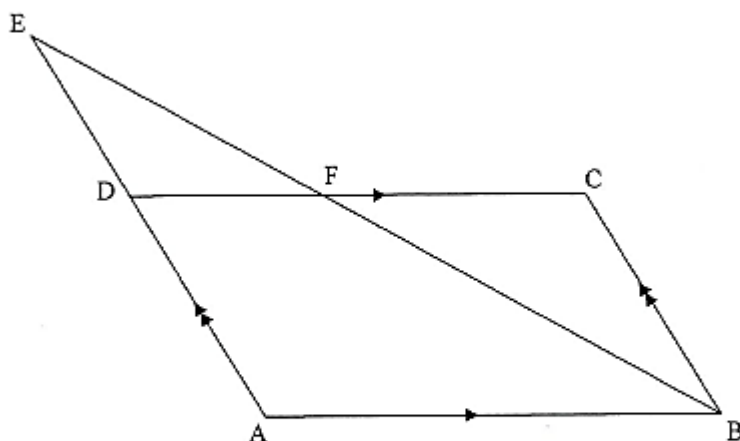
- 8.3 In the diagram,  $O$  is the centre of the larger circle.  $OB$  is a diameter of the smaller circle. Chord  $AB$  of the larger circle intersects the smaller circle at  $M$  and  $B$ .



- 8.3.1 Write down the size of  $\angle OMB$ . Provide a reason. (2)
- 8.3.2 If  $AB = \sqrt{300}$  units and  $OM = 5$  units, calculate, giving reasons, the length of  $OB$ . (4)
- [16]

#### QUESTION 9

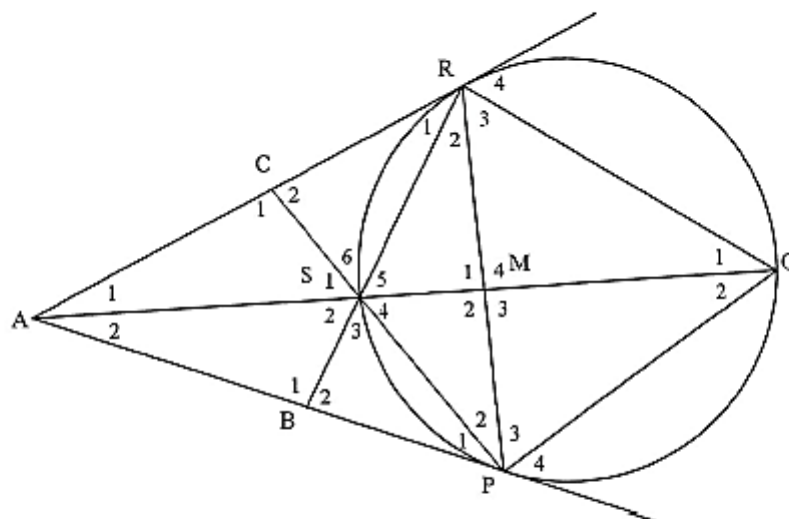
In the diagram,  $ABCD$  is a parallelogram with  $AB = 14$  units.  $AD$  is produced to  $E$  such that  $AD : DE = 4 : 3$ .  $EB$  intersects  $DC$  in  $F$ .  $EB = 21$  units.



- 9.1 Calculate, with reasons, the length of  $FB$ . (3)
- 9.2 Prove, with reasons, that  $\triangle EDF \parallel \triangle EAB$ . (3)
- 9.3 Calculate, with reasons, the length of  $FC$ . (3)
- [9]

### QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that  $PQ = PR$ . The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

- 10.1  $\hat{S}_3 = \hat{S}_4$  (5)
  - 10.2 SMRC is a cyclic quadrilateral (4)
  - 10.3 RP is a tangent to the circle passing through P, S and A at P (6)
- [15]

**Appendix A: Information Sheet**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

