JENN
Training and Consultancy
The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT AND ACTIVITY MANUAL

GRADE 12

2024

EUCLIDEAN GEOMETRY

CONTENTS **PAGE**

Outcomes:

Grade 8 -9

Angle relationships
Recognize and describe pairs of angles formed by:
1. Perpendicular lines
2. Intersecting lines
3. Parallel lines cut by a transversal

Solving problems
Solve geometric problems using the relationships between pairs of angles described above

Classifying 2D shapes

• Identify and write clear definitions of triangles in terms of their sides and angles, distinguishing between:
1. Equilateral triangles
2. Isosceles triangles
3. Right-angled triangles

Similar and congruent 2D shapes

• Identify and describe the properties of congruent shapes
• Identify and describe the properties of similar shapes

Solving problems
• Solve geometric problems involving unknown sides and angles in triangles, using known properties and definitions.

Grade 10
(a) Investigate line segments joining the midpoints of two sides of a triangle.
(b) Properties of special quadrilaterals.

Grade 11

(a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles.
(b) Solve circle geometry problems, providing reasons for statements when required.
(c) Prove riders

Grade 12

Prove (accepting results established in earlier grades):
- that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Midpoint Theorem as a special case of this theorem);
- that equiangular triangles are similar;
- that triangles with sides in proportion are similar;
- the Pythagorean Theorem by similar triangles; and
- riders.

(SOURCE: 1. CURRICULUM AND ASSESSMENT POLICY STATEMENT(CAPS)SENIOR PHASE GRADES (7 – 9) MATHEMATICS
2. CURRICULUM AND ASSESSMENT POLICY STATEMENT(CAPS)FET PHASE GRADES (10 – 12) MATHEMATICS)

SECTION1:ALGEBRA NEEDED (NOT LIMITED TO)

Order of operations

B – Brackets

O – Orders (exponents, powers, roots)

D
M } Division or Multiplication (from left to right)

A
S } Addition or Subtraction (from left to right)

Examples

1. $5+3-1\times 2$
$=5+3-2$
$=8-2$
$=6$

2. $10\div 5+3-1\times 2$
$=2+3-2$
$=5-2$
$=3$

3. $10\div 5+(3-1)\times 2$
$=10\div 5+2\times 2$
$=2+4$
$=6$

4. $20\div 2^2+(3-1)\times 2$
$=20\div 2^2+2\times 2$
$=20\div 4+2\times 2$
$=5+4$
$=9$

Addition and subtraction of Like Terms

Examples

1. $3+2x-5-7x$
$=-5x-2$

2. $a+a-b+3+b+c-3c$
$=2a-2c+3$

1. $3+2x-5-7x$
$=-5x-2$

Products: Monomial by Monomial

1. $a\times b=ab$ 2. $a\times bc=abc$

Products: Monomial by Binomial

$b(c+d)=bc+bd$

Products: Binomial by Binomial

$$(a+b)(c+d) = a(c+d) + b(c+d)$$
$$= ac + ad + bc + bd$$

Products: Binomial by Trinomial

$$(a+b)(c+d+e) = a(c+d+e) + b(c+d+e)$$
$$= ac + ad + ae + bc + bd + be$$

Products: Trinomial by Binomial

$$(a+b+c)(d+e) = a(d+e) + b(d+e) + c(d+e)$$
$$= ad + ae + bd + be + cd + ce$$

Fractions

Examples

1. $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$

2. $\frac{1}{2}\div\frac{2}{3}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$

3. $\frac{1}{2}+\frac{2}{3}=\frac{3}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{2}{2}=\frac{3}{6}+\frac{4}{6}=\frac{3+4}{6}=\frac{7}{6}=1\frac{1}{6}$

4. $\frac{1}{2}-\frac{2}{3}=\frac{3}{3}\times\frac{1}{2}-\frac{2}{3}\times\frac{2}{2}=\frac{3}{6}-\frac{4}{6}=\frac{3-4}{6}=-\frac{1}{6}$

Linear equations

Examples

Solve for x:

1. $x+3=5$
   $x=5-3$
   $x=2$

2. $3+x=2x-2$
   $x-2x=-2-3$
   $-x=-5$
   $x=5$

Literal equations (Changing subject of the formula)

Examples

1. Solve for a:

$F = \text{ma}$

$\text{ma} = F$

$\text{a} = \frac{F}{m}$

2. Solve for l:

$P = 2b + 2l$

$-2l = 2b - P$

$l = -b + \frac{P}{2}$

3. Solve for c:

$a = b + \frac{c}{2}$

$2a = 2b + c$

$2a - 2b = c$

$c = 2a - 2b$

Equations with fractions

Examples

Solve for x:

1. $\frac{x}{2} = \frac{3}{2}$

$2x = 6$

$x = 3$

2. $\frac{6}{x} = -5 - x$

restriction $(x \neq 0)$

$6 = -5x - x^2$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x + 3 = 0 \; or \; x + 2 = 0$

$x = -3 \; or \; x = -2$

ACTIVITIES:

1. Calculate:

1.1 $4+2-3\times 3$

1.2 $8\div 2+2-2\times 2$

1.3 $8\div 2+(2-3)\times 3$

1.4 $100\div 5^2+(4-1)\times 3$

2. Simplify:

2.1 $5x-3y+x-8x+9y$

2.2 $\frac{5}{2}x-\frac{3}{4}y+x-8x+\frac{5}{3}y$

2.3 $2(x+3)-4(5x-54)$

2.4 $2x(x+3)-4x(5x-54)$

2.5 $(x+3)(2x-5)$

2.6 $(x+3)(x^2+6x+6-x)$

2.7 $(x^2-x+2)(2-x)$

3. Solve for x:

3.1 $5x=x-8$

3.2 $\frac{5}{2}x-8x=3$

3.3 $2(x+3)-4(5x-54)=1$

3.4 $x+4=-\frac{3}{x}$

4. 4.1 Solve for m: $F=\text{ma}$

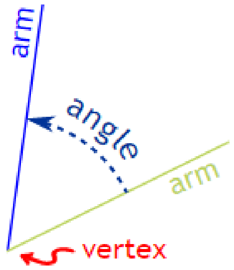
4.2 Solve for b: $P=2b+2l$

4.3 Solve for b: $a=b+\frac{c}{2}$

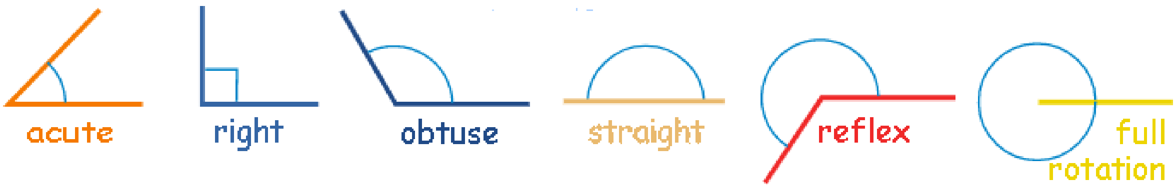
SECTION2: LINES, ANGLES, TRIANGLES, AND QUADRILATERALS

Definitions

Angles: an angle is "the amount of turn between two lines around their common point (the vertex)" – (www.mathisfun.com)



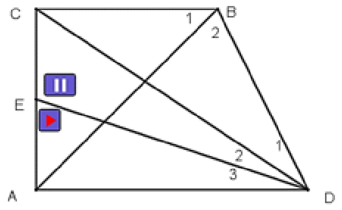
Types of angles



(www.mathisfun.com)

- Acute angle → an angle between 0° *and* 90°
- Right angle → an angle equal to 90°
- Obtuse angle → an angle between 90° *and* 180°
- Straight angle → an angle equal to 180°
- Reflex angle → an angle between 180° *and* 360°
- Full rotation (Revolution) → an angle equal to 360°

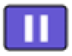
Naming angles

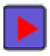


$\hat{B}_1 = C\hat{B}A$

$\hat{B}_2 = A\hat{B}D$

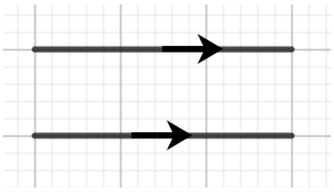
$\hat{B}_1 + \hat{B}_2 = C\hat{B}D$

 is equal to $C\hat{E}D$

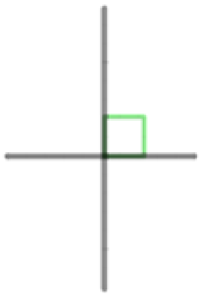
 is equal to $A\hat{E}D$

Lines

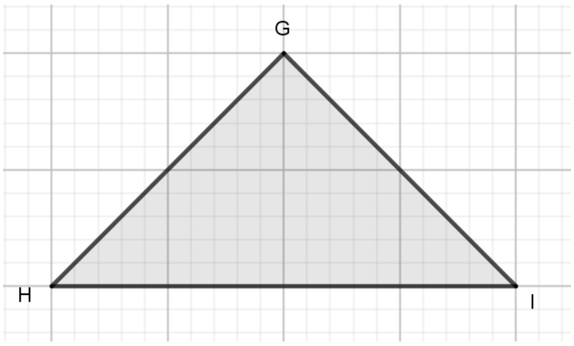
Parallel lines → lines on the same plane and that can never intersect as they are extended.



Perpendicular lines → lines that intersect at right angles (90°)

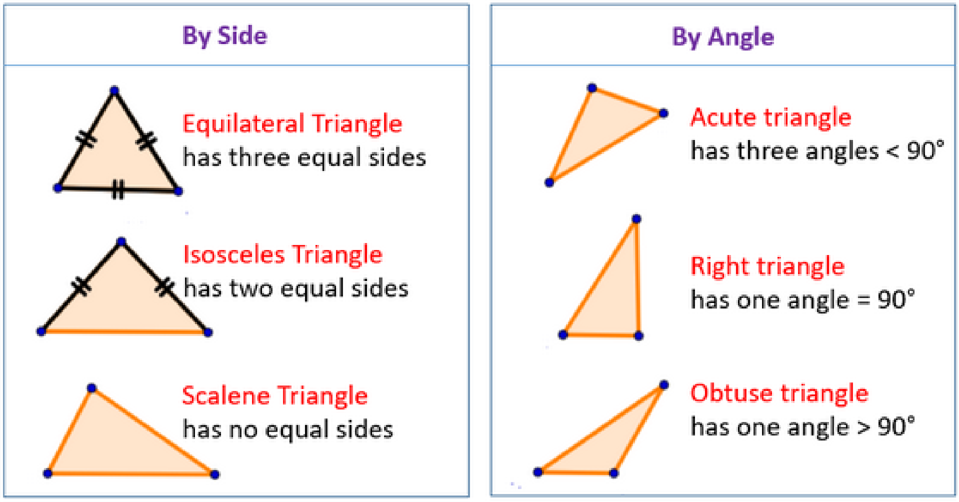


Triangles: a triangle is shape made of 3 straight lines, the shape is closed. It has 3 sides and 3 angles.



ΔGHI

Types of triangles

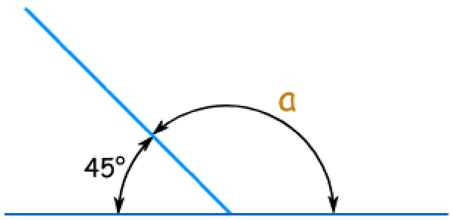


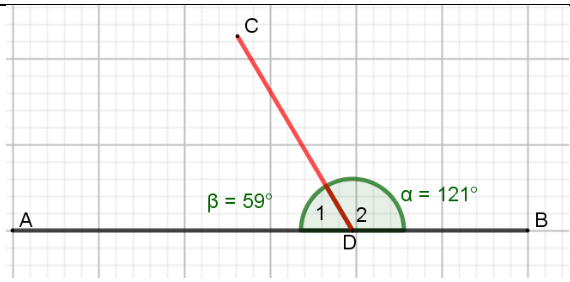
(www.curemath.com)

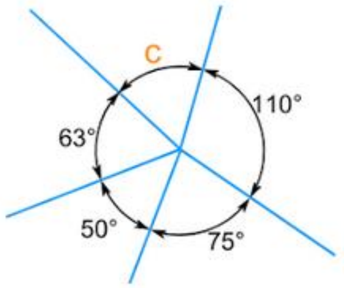
Theorem Statements

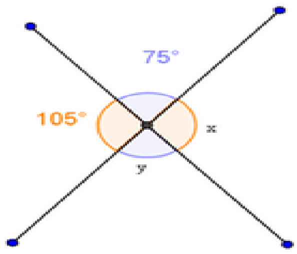
N.B All the theorem statements were taken from 2021 grade 12 mathematics examination guidelines.

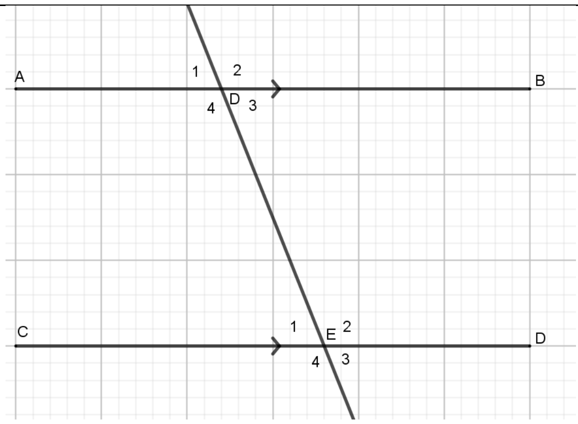
(Angles and lines)

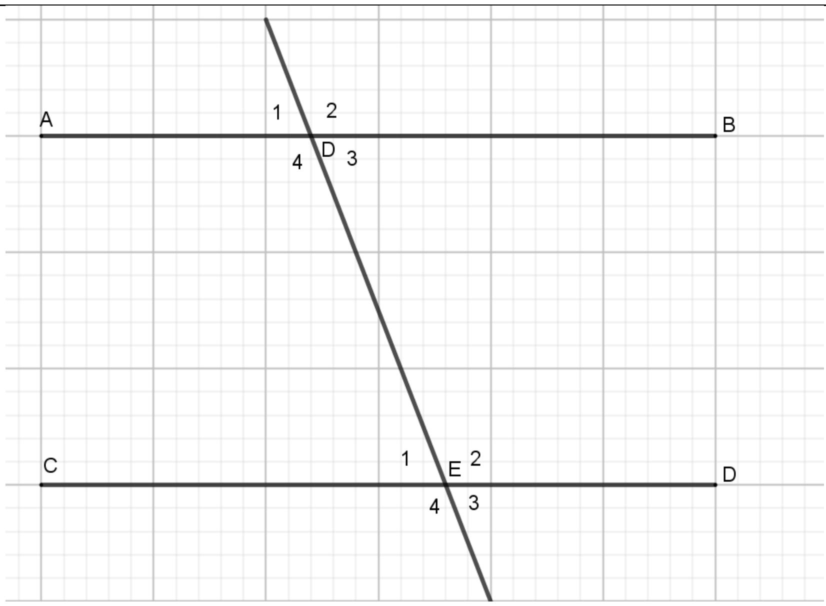
1.	Theorem statement	The adjacent angles on a straight line are supplementary
	Diagram	
	Mathematical statement	$a + 45° = 180°$ $a = 180° - 45°$ $a = 135°$
	Reason	$\angle$s on a str line

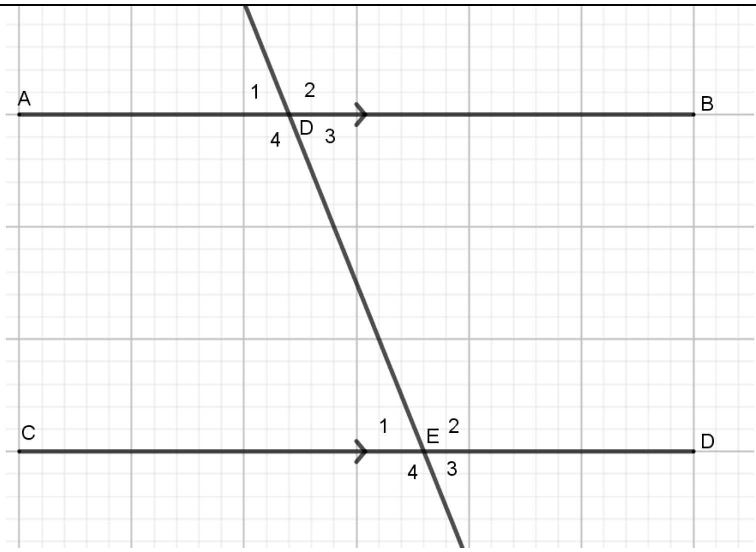
2.	Theorem statement	If the adjacent angles are supplementary, the outer arms of these angles form a straight line
	Diagram	
	Mathematical statement	$\hat{D}_1+\hat{D}_2=59^\circ+121^\circ=180^\circ$
	Reason	adj $\angle$s supp

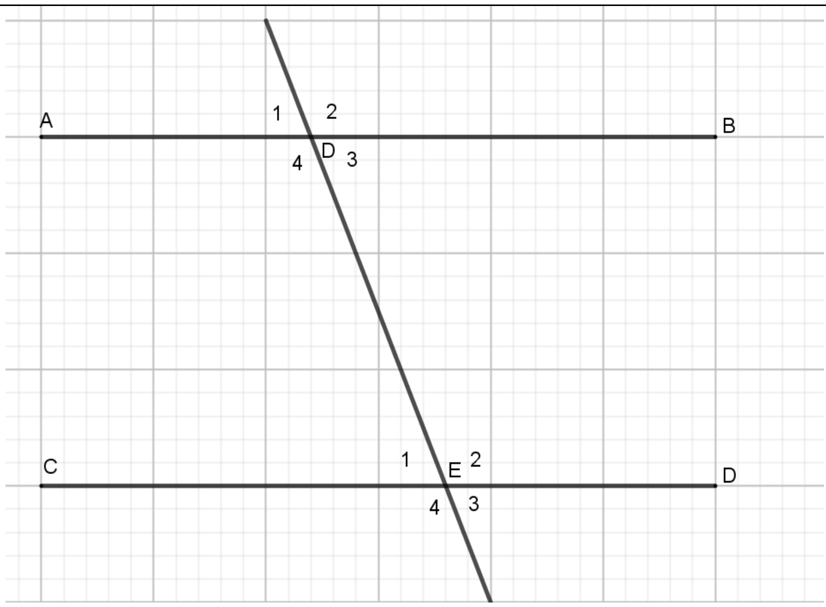
3.	Theorem statement	The adjacent angles in a revolution add up to 360°
	Diagram	
	Mathematical statement	$\hat{C}+110^\circ+75^\circ+50^\circ+63^\circ=360^\circ$ $\hat{C}=62^\circ$
	Reason	$\angle$s around a pt

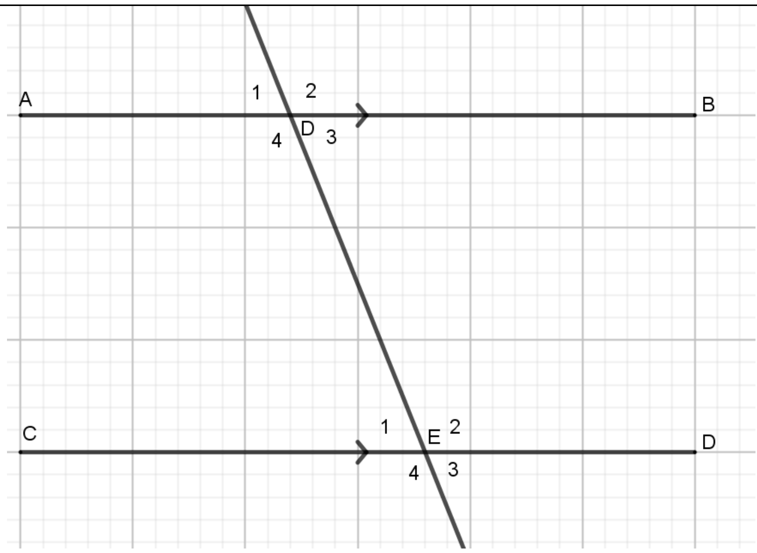
4.	Theorem statement	Vertically opposite angles are equal
	Diagram	
	Mathematical statement	$x=105^\circ$ and $y=75^\circ$
	Reason	vert opp $\angle$s =

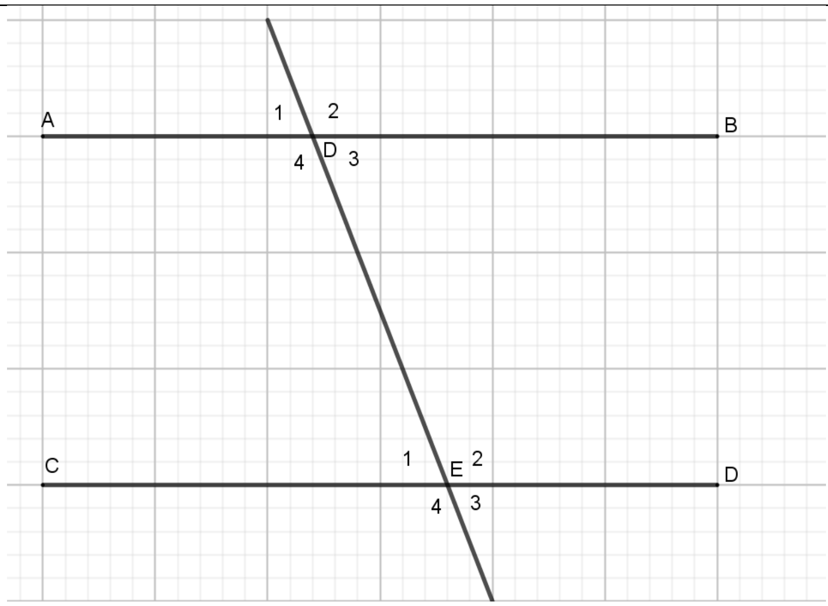
5.	Theorem statement	If AB || CD, then the alternate angles are equal
	Diagram	
	Mathematical statement	$\hat{D}_3 = \hat{E}_1$ and $\hat{D}_4 = \hat{E}_2$
	Reason	alt $\angle$s ; AB || CD

6.	Theorem statement	If the alternate angles between two lines are equal, then the lines are parallel
	Diagram	
	Mathematical statement	If $\widehat{D}_3 = \hat{E}_1$ or $\widehat{D}_4 = \hat{E}_2$ then AB || CD
	Reason	alt $\angle$s =

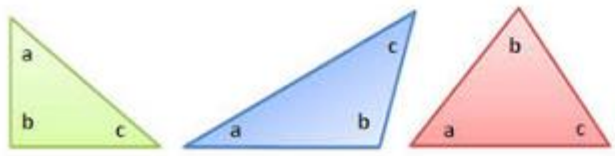
7.	Theorem statement	If AB || CD, then the corresponding angles are equal
	Diagram	
	Mathematical statement	$\hat{D}_3 = \hat{E}_3$, $\hat{D}_4 = \hat{E}_4$, $\hat{D}_1 = \hat{E}_1$ and $\hat{D}_2 = \hat{E}_2$
	Reason	corresp $\angle$s ; AB || CD

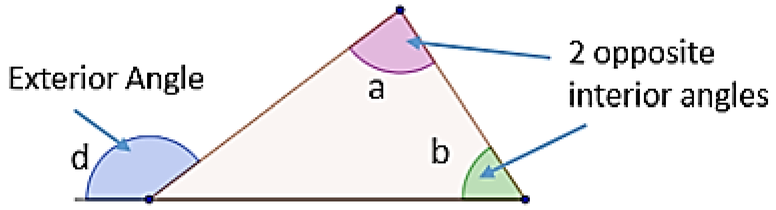
8.	Theorem statement	If the corresponding angles between two lines are equal, then the lines are parallel
	Diagram	
	Mathematical statement	If $\widehat{D}_3 = \widehat{E}_3, \widehat{D}_4 = \widehat{E}_4,\ \widehat{D}_1 = \widehat{E}_1,\ or\ \widehat{D}_2 = \widehat{E}_2,$ then AB || CD
	Reason	corresp $\angle$s =

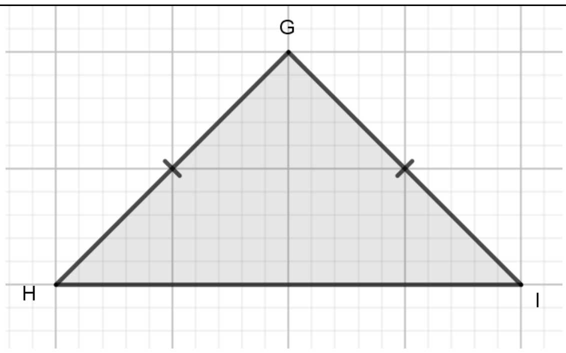
9.	Theorem statement	If AB || CD, then the co-interior angles are supplementary
	Diagram	
	Mathematical statement	$\hat{D}_3 + \hat{E}_2 = 180^0$ and $\hat{D}_4 + \hat{E}_1 = 180^0$
	Reason	co-int $\angle$s ; AB || CD

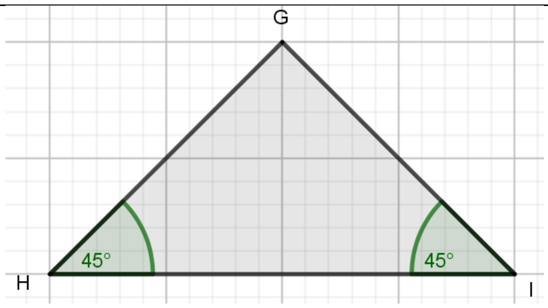
10.	Theorem statement	If the co-interior angles between two lines are supplementary, then the lines are parallel
	Diagram	
	Mathematical statement	If $\hat{D}_3 + \hat{E}_2 = 180^0$ or $\hat{D}_4 + \hat{E}_1 = 180^0$ then AB || CD
	Reason	co-int $\angle$s supp

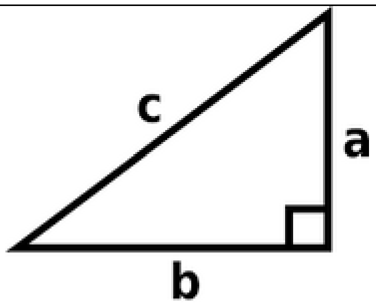
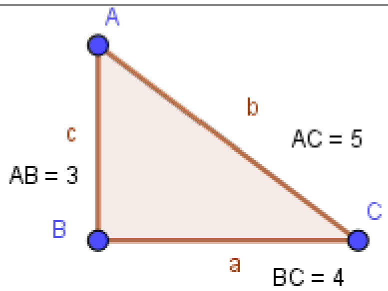
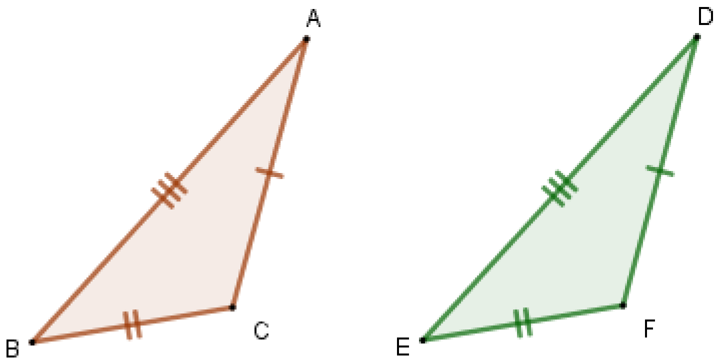
(TRIANGLES)

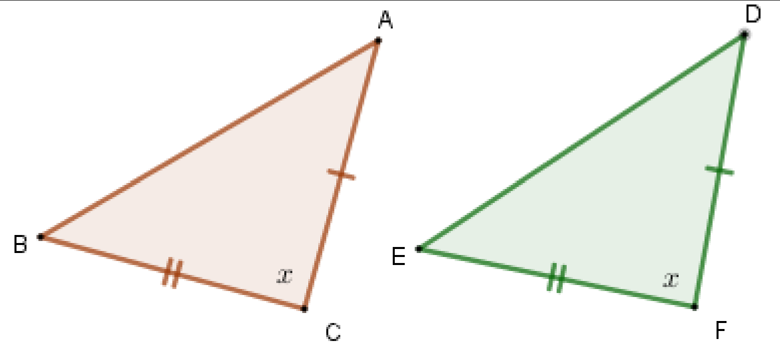
11.	Theorem statement	The interior angles of a triangle are supplementary
	Diagram	
	Mathematical statement	$a+b+c=180^\circ$
	Reason	sum of $\angle$s in Δ

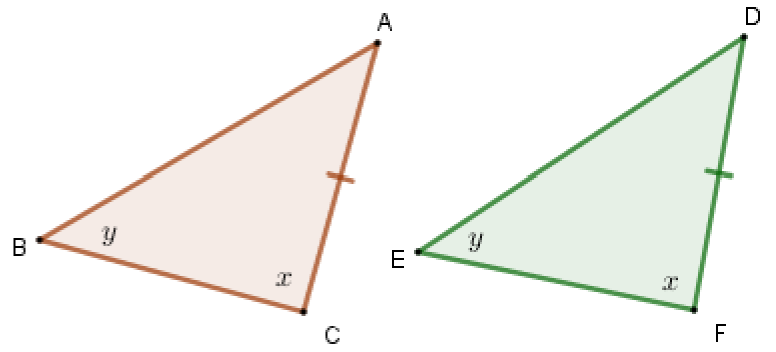
12.	Theorem statement	The exterior angle of a triangle is equal to the sum of the interior opposite angles
	Diagram	 (https://www.google.co.za/search?q=exterior+angle+of+a+triangle&sxsrf=ALeKk03ixLhT157jKazMF3jbVcdYdRskFw:1610695669220&source=lnms&tbm=isch&sa=X&ved=2ahUKEwioz9H5tJ3uAhXLTMAKHTH4CcYQ_AUoAXoECBAQAw&=1362&bih=636#imgrc=L8L9eu-YE6gviM)
	Mathematical statement	d = a + b
	Reason	ext $\angle$ of Δ

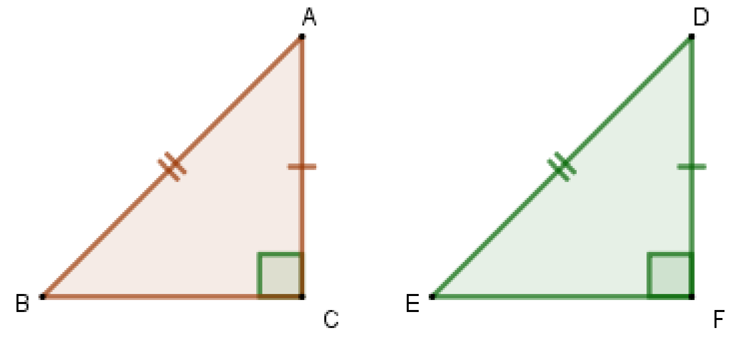
13.	Theorem statement	The angles opposite the equal sides in an isosceles triangle are equal
	Diagram	
	Mathematical statement	If GH = GI, then $\hat{H} = \hat{I}$
	Reason	$\angle s$ opp equal sides

14.	Theorem statement	The sides opposite the equal angles in an isosceles triangle are equal
	Diagram	
	Mathematical statement	If $\hat{H} = \hat{I}$, then HG = GI
	Reason	sides opp equal $\angle s$

15.	Theorem statement	In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides
	Diagram	
	Mathematical statement	$c^2 = a^2 + b^2$
	Reason	Pythagoras
16.	Theorem statement	If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides, then the triangle is right-angled.
	Diagram	
	Mathematical statement	If $b^2 = a^2 + c^2$, the ΔABC is right-angled
	Reason	Converse Pythagoras
17.	Theorem statement	If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent
	Diagram	
	Mathematical statement	If AB=DE, AC=DF and BC=EF, then $\Delta ABC \equiv \Delta DEF$
	Reason	SSS

18.	Theorem statement	If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent
	Diagram	
	Mathematical statement	If $\hat{C}=\hat{F}$, AC=DF and BC=EF, then $\Delta ABC \equiv \Delta DEF$
	Reason	SAS

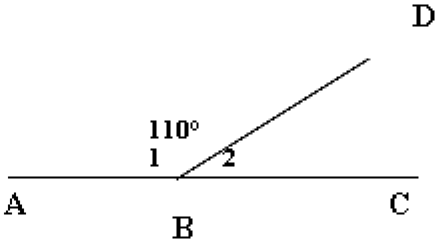
19.	Theorem statement	If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent
	Diagram	
	Mathematical statement	If $\hat{C}=\hat{F}$, $\hat{B}=\hat{E}$ and AC=DF, then $\Delta ABC \equiv \Delta DEF$
	Reason	AAS

20.	Theorem statement	If in two right-angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent
	Diagram	
	Mathematical statement	If AC=DF and AB=DE, then $\Delta ABC \equiv \Delta DEF$
	Reason	RHS

WORKED EXAMPLES

Example 1

Sum of adjacent angles on a straight line equals 180° ($\angle^s$ on straight line = 180°)



Calculate the size of $\hat{B}_2$

$\hat{B}_1 + \hat{B}_2 = 180°$ ($\angle^s$ on straight line =180°)

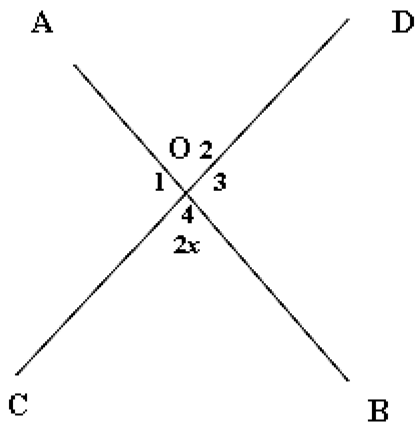
$\hat{B}_1 = 110°$ (given)

$\therefore\ 110° + \hat{B}_2 = 180°$

$\therefore\ \hat{B}_2 = 180° - 110°$

$\hat{B}_2 = 70°$

Example 2



Vertically opposite $\angle^s$

Given that $O_2 = 100°$

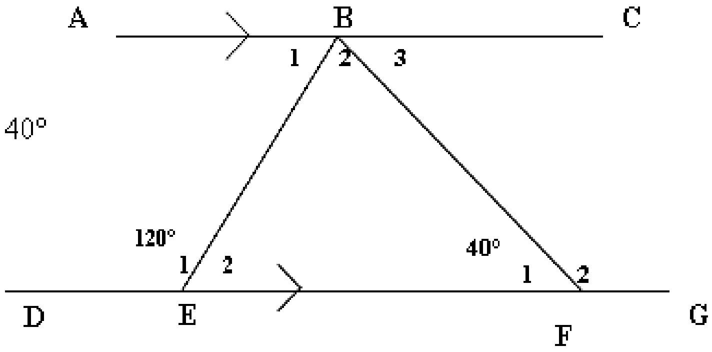
Calculate the value of x

$\hat{O}_4 = \hat{O}_2$ (vertically opposite $\angle^s$)

$2x = 100°$

$x = 50°$

Example 3



Given: $AC \parallel DG$, $\hat{E}_1 = 120°$ and $\hat{F}_1 = 40°$

Calculate the sizes of $\hat{B}_1$ and $\hat{B}_3$

Solution:

$\hat{B}_1 + \hat{E}_1 = 180°$ (co-interior $\angle^s$, $AC \parallel DG$)

$\hat{E}_1 = 120°$ (given)

$\therefore \hat{B}_1 + 120° = 180°$

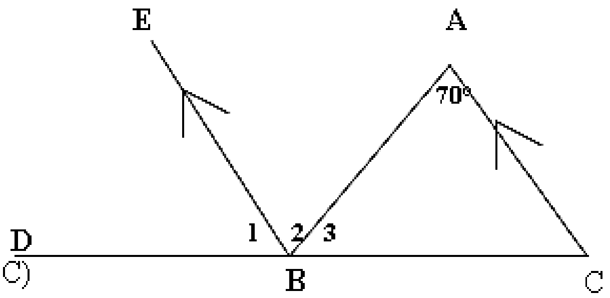
$\hat{B}_1 = 60°$

$\hat{B}_3 = \hat{F}_1$ (alternating $\angle^s$, $AC \parallel DG$)

$\hat{F}_1 = 40°$ (given)

$\therefore \hat{B}_3 = 40°$

Example 4



Given: $EB \parallel AC$ and $\hat{B}_1 = \hat{B}_2$

Calculate the sizes of $\hat{B}_1$ and $\hat{C}$

Solution:

$\hat{A} = \hat{B}_2$ (alternating $\angle^s$, $EB \parallel AC$)

$\hat{A} = 70°$ (given)

$\therefore \hat{B}_2 = 70°$

$\hat{B}_1 = \hat{B}_2$ (given)

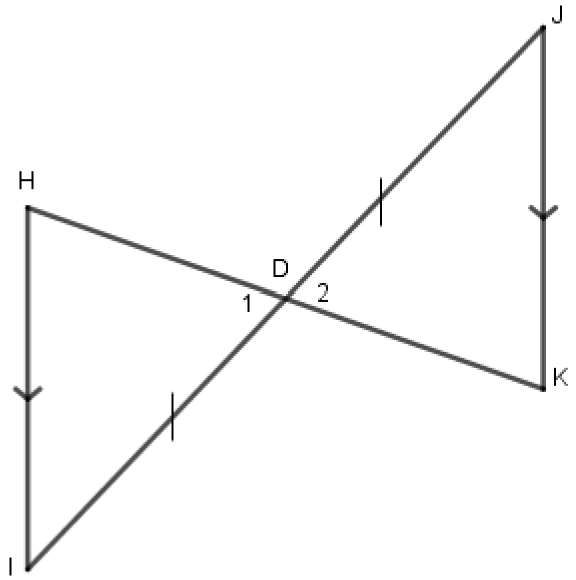
$\therefore \hat{B}_1 = 70°$

$\hat{C} = \hat{B}_1$ (corresponding $\angle^s$, $EB \parallel AC$)

$\therefore \hat{C} = 70°$

Example 5

Prove that $\Delta HDI \equiv \Delta KDJ$



Solution

In ΔHDI and ΔKDJ

$\hat{H} = \hat{K}$ $(alt\angle s; HI \parallel JK)$

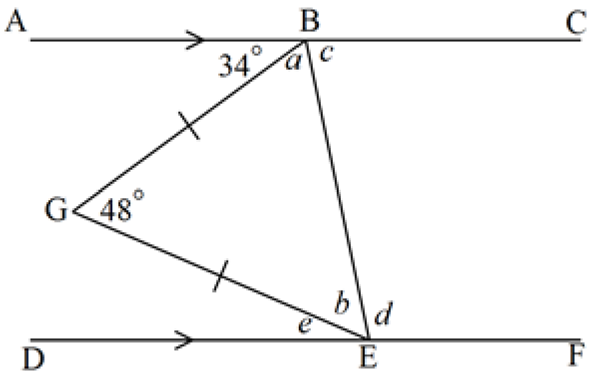
$\hat{D}_1 = D_2$ (vert opp $\angle$s =)

ID = DJ (given)

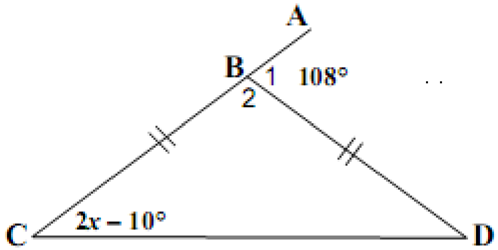
$\therefore \Delta HDI \equiv \Delta KDJ$ (AAS)

ACTIVITIES: GIVE REASONS FOR YOUR STATEMENTS

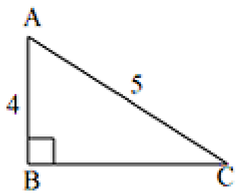
1. Find the size of the angles a, b, c, d and e



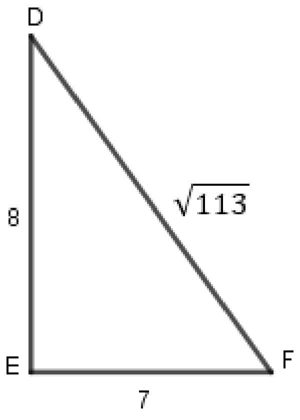
2. Calculate x, if $\hat{B}_1 = 108^\circ$ and $\hat{B}_2 = 2x - 10^\circ$



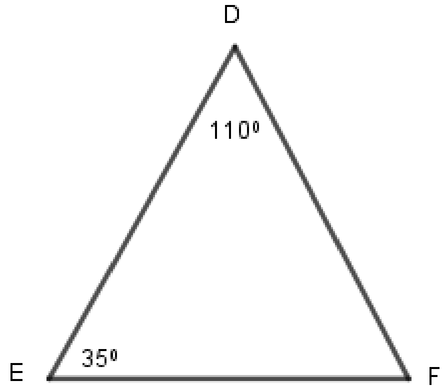
3. Calculate the length of BC



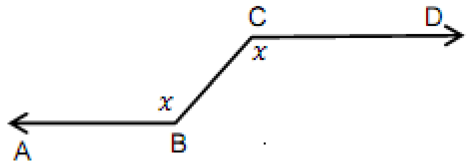
4. Prove that triangle ΔDEF is a right-angled triangle. Show which angle is = 90^o



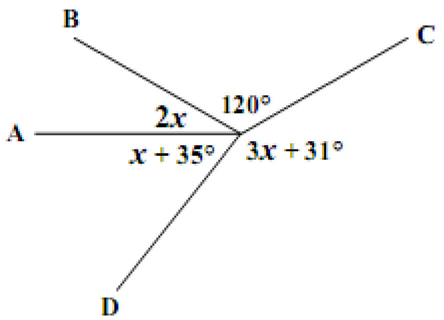
5. Prove that DE = DF



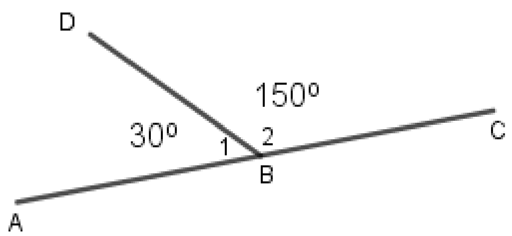
6. B and C are alternating angles equal to x. Is AB || CD? Give a reason for your answer.



7. Calculate x

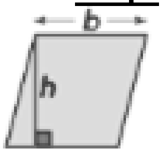
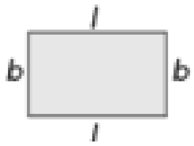
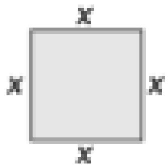
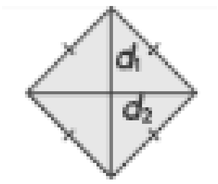
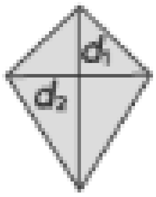
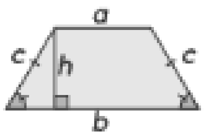


8. Given that: $A\hat{B}D = 30^o$ and $D\hat{B}C = 150^o$.Prove, giving reasons, that ABC is a straight line



9. 9.1 If the corresponding angles between two lines are equal, then the lines are ………………….
   9.2 If the ………………..angles between two lines are supplementary, then the lines are parallel

PROPERTIES OF QUADRILATERALS

Quadrilateral	**Shape**	**Properties**	**Area**
Parallelogram		• Opposite sides parallel • Opposite sides equal • Opposite angles equal • Diagonals bisect each other	$b \times h$
Rectangle		• All properties of parallelogram • All angles are right angles • Diagonals are equal in length	$l \times b$
Square		• All properties of rectangle • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles	x^2
Rhombus		• All properties of a parallelogram • All sides are equal • Diagonals bisect at right angle • Diagonals bisect corner angles	$\frac{1}{2} \times d_1 \times d_2$
Kite		• Two pairs of adjacent sides are equal • One pair of opposite angles are equal • One diagonal bisect the other at right angle • One diagonal bisects corner angles	$\frac{1}{2} \times d_1 \times d_2$
Trapezium		One pair of opposite sides parallel	$\frac{1}{2} \times (a + b) \times h$

SECTION3: CIRCLE GEOMETRY

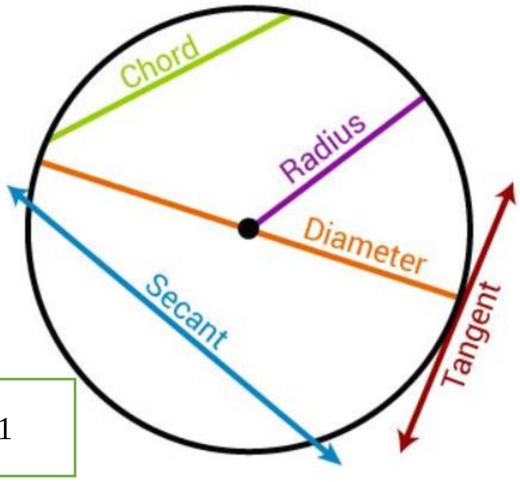


Figure 1

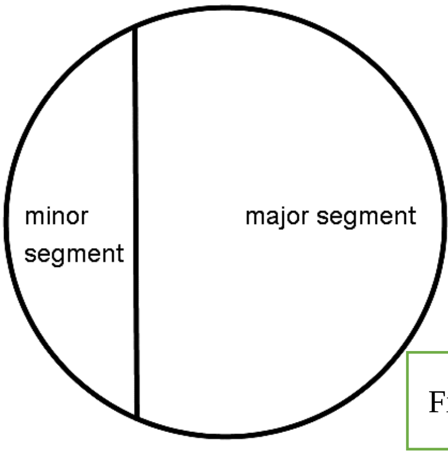


Figure 2

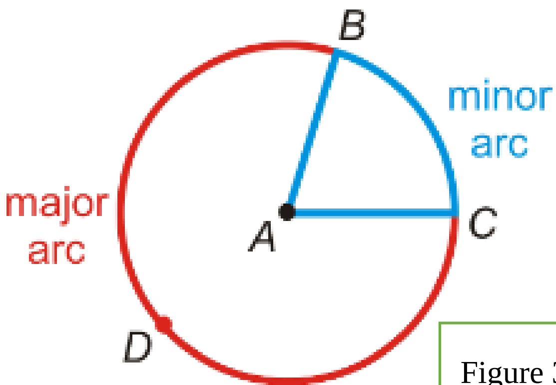


Figure 3

Some definitions related to the circle:

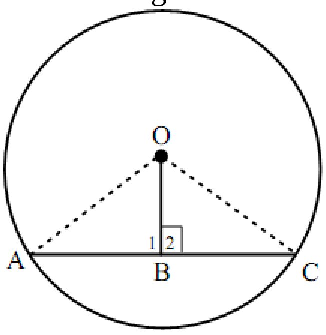
Chord	A line segment connecting two points on a circle.
Diameter	The distance from one point on a circle through the center to another point on the circle.
Radius	The distance from the center to the circumference of a circle.
Secant	A line that intersects two points on a circle.
Tangent	A line that just touches a circle at a point, and when extended it will not cut the circle.
Circumference	The distance around the edge of a circle.
Arc	Part of the circumference of a circle.
Segment	A segment of a circle is the area enclosed by an arc of a circle and a chord

(mathsisfun.com)

Theorems focused on the Centre.

Theorem statement: The line drawn from the Center of the circle perpendicular to the chord bisects the chord.

Diagram

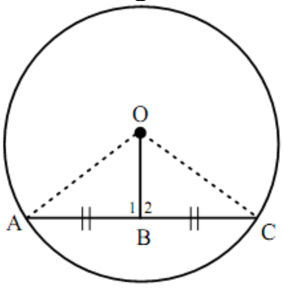


Mathematical Statement: if $OB \perp AC$ then $AB = BC$

Acceptable Reason: Line from center $\perp$ to chord

Theorem statement: The line segment joining the center of the circle to the midpoint of the chord is perpendicular to the chord.

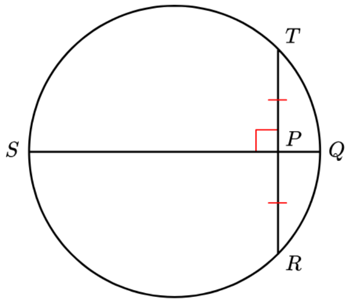
Diagram



Mathematical Statement: if $AB = BC$ then $OB \perp AC$

Acceptable Reason: Line from center to the midpoint of chord

Theorem statement: The perpendicular bisector of a chord passes through the center of the circle.



Mathematical Statement: $SQ \perp TR \text{ and } PR = PT$ (Given)

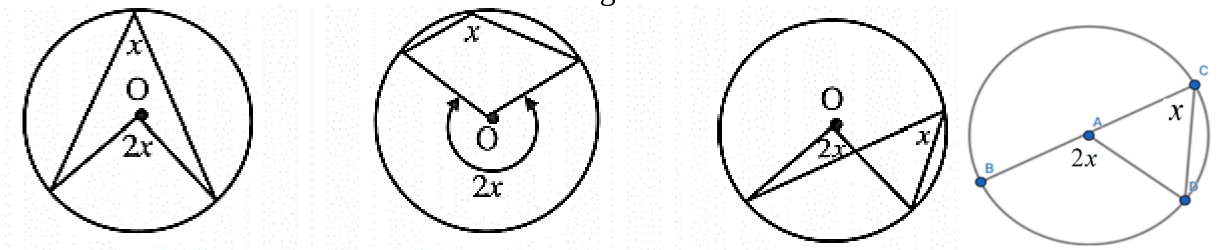
$\therefore SQ \text{ passes throughthe center}$

$\therefore SQ \text{ is a diameter}$

Acceptable Reason: perp bisector of chord

Theorem statement: The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.

Diagram

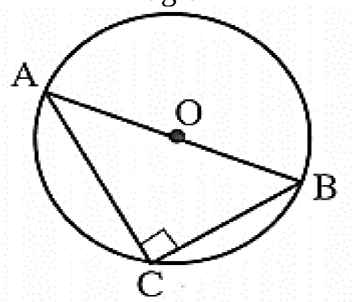


Mathematical Statement:

Acceptable Reason: $\angle\ at\ Center = 2 \times \angle\ at\ Circumference$

Theorem statement: The angle subtended by the diameter at the circumference of the circle is 90^0.

Diagram

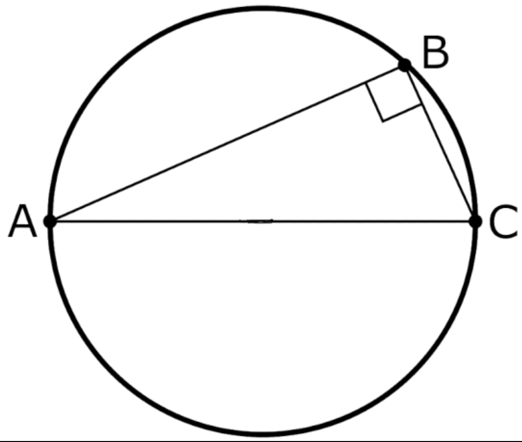


Mathematical Statement: $\hat{C} = 90^0$

Acceptable Reason: $\angle s\ in\ semi - circle$

Theorem statement: If the angle subtended by a chord at the circumference of the circle is 90^0, then the chord is a diameter.

Diagram



Mathematical Statement: if $\hat{B} = 90^0$, then $\therefore AC\ is\ a\ diameter$

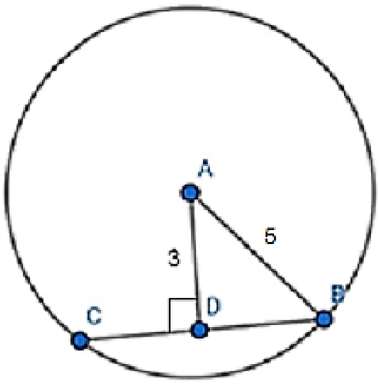
Acceptable Reason: converse $\angle s\ in\ semi - circle$

WORKED EXAMPLES

Example 1

1.1

A *is the center, Calculate the following with reasons*:



1.1.1 DB

1.1.2 CB

Solution

1.1.1 $AB^2 = AD^2 + DB^2 (Pythagoras)$

$(5)^2 = (3)^2 + DB^2$

$25 - 9 = DB^2$

$\sqrt{16} = \sqrt{DB^2}$

$DB = 4$

1.1.2 $AD \perp CB (Given)$

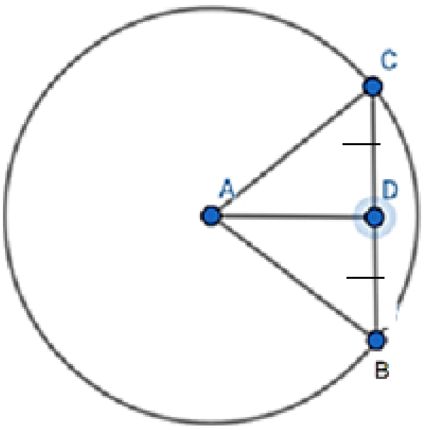
$CD = DB (Line from center \perp to chord)$

$CB = 2DB$

$CB = 2 \times 4 = 8$

Example 2

2.1 A *is the center* $CB = 12$, $AD = 8$. Calculate the length of the radius



Solution

$DC = DB \quad (Given)$

$CD = 6$ *units*

$AD \perp BC (Line from center to midpoint of chord)$

$In \triangle ADC$

$AC^2 = CD^2 + DA^2 \qquad (Pythagoras)$

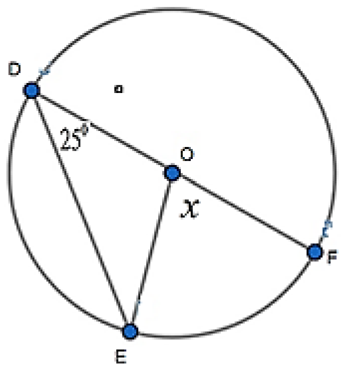
$AC^2 = (6)^2 + (8)^2$

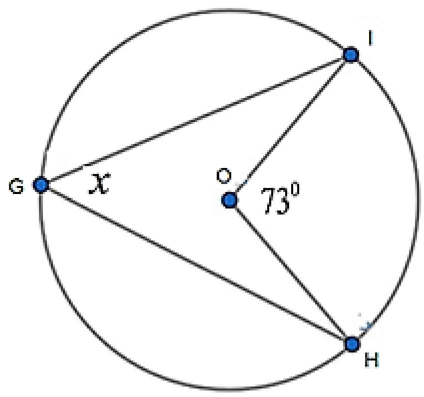
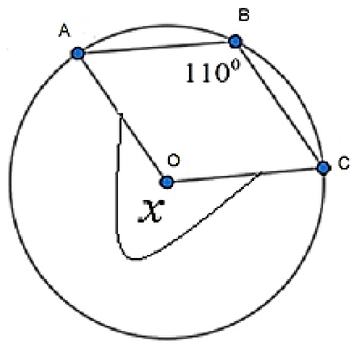
$\sqrt{AC^2} = \sqrt{100}$

$AC = 10$ *units*

Example 3

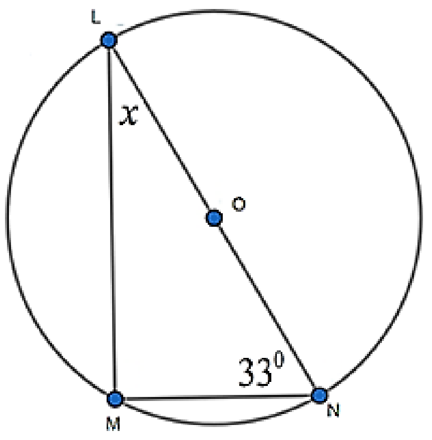
O is the Centre of the circle. Calculate x with reasons in each case.

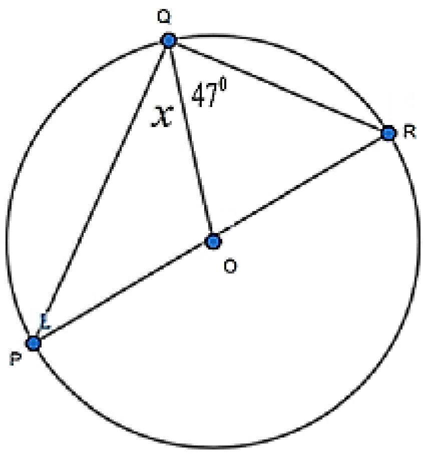
3.1  	*Solution* $x = 2 \times 25^0 (\angle atCenter = 2 \times \angle atCircumference)$ $x = 50°$

3.2  	*Solution* $73^0 = 2 \times x \qquad (\angle at\,Center = 2 \times \angle at\,Circumference)$ $\frac{73^0}{2} = x$ $36.5^0 = x$
3.3  	*Solution* $x = 2 \times 110^0 = 220^0 (\angle atCenter = 2 \times \angle atCircumference)$

Example 4

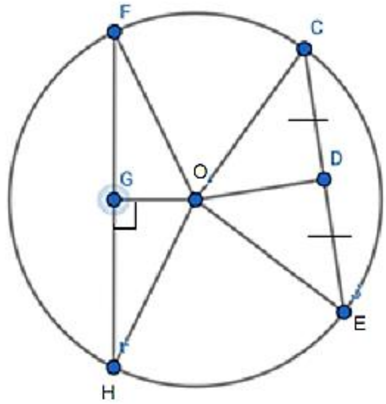
Calculate x with reasons in each case. O is the Center.

	*Solution* $\hat{M} = 90^0 \quad (\angle \, in \, semi-circle)$ $In \triangle \, LMN$ $x + 90^0 + 33^0 = 180^0 \quad (sum \, of \, \angle s \, of \, \triangle = 180^0)$ $x = 57^0$

	*Solution* $in \triangle \, PQR$ $x + 47^0 = 90^0 \quad (\angle \, in \, semi-circle)$ $x = 43^0$

ACTIVITIES

<table>
<tr>
<td>
1.1 $O\ is\ the\ center, OD = 4\ units, DC = 6\ units$



$\mathbf{1.1.1}$ $Calculate\ OH\ with\ reasons$

$\mathbf{1.1.2}$ $Calculate\ FH\ if\ OG = \frac{1}{2}OH.$
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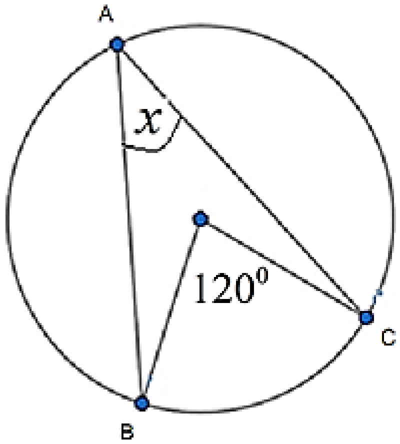
</table>

$\mathbf{1.2}$ $O\ is\ the\ centre\ of\ the\ circle. Calculate\ the\ value\ of\ x\ in\ each\ of\ the\ following:$

<table>
<tr>
<td>
1.2.1



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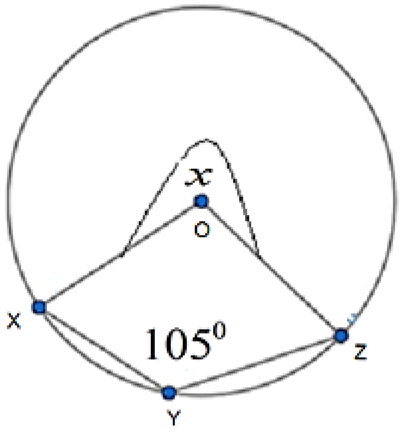
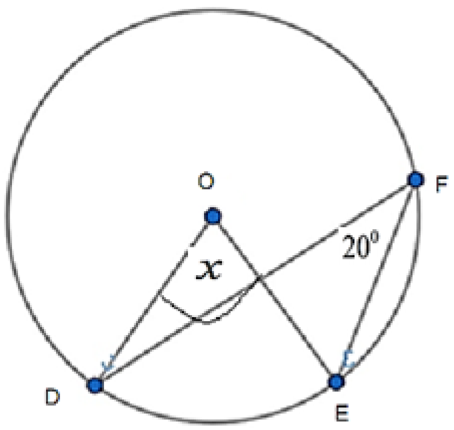
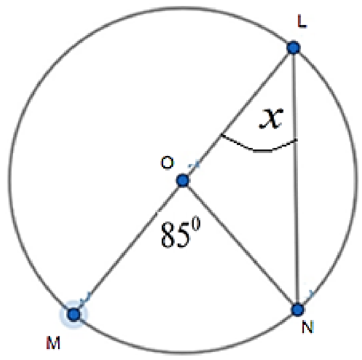
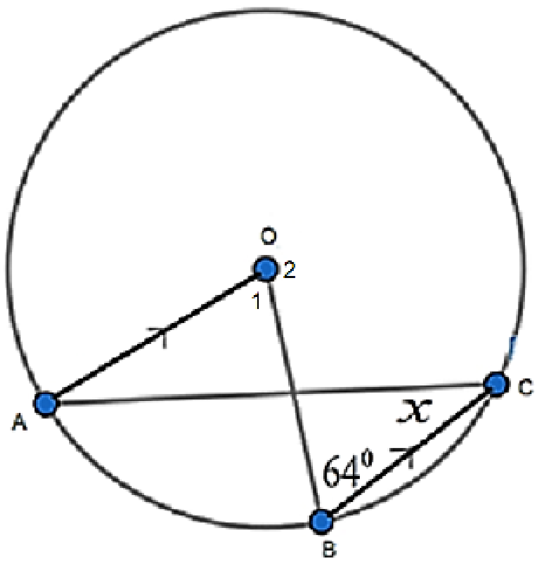
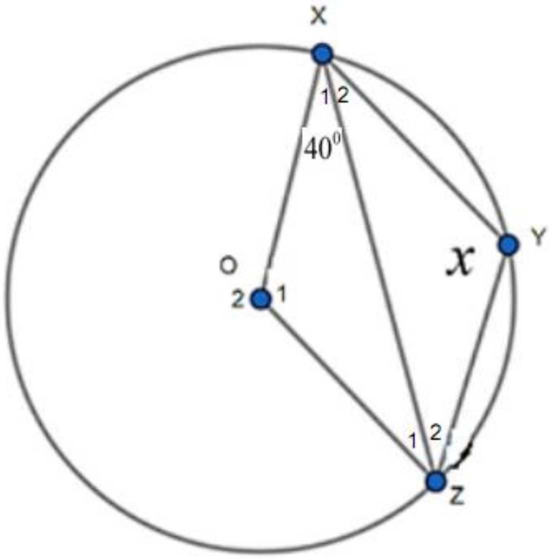
1.2.2  x O X 105^0 Z Y	
1.2.3  O F x 20^0 D E	
1.2.4  L x O 85^0 M N	

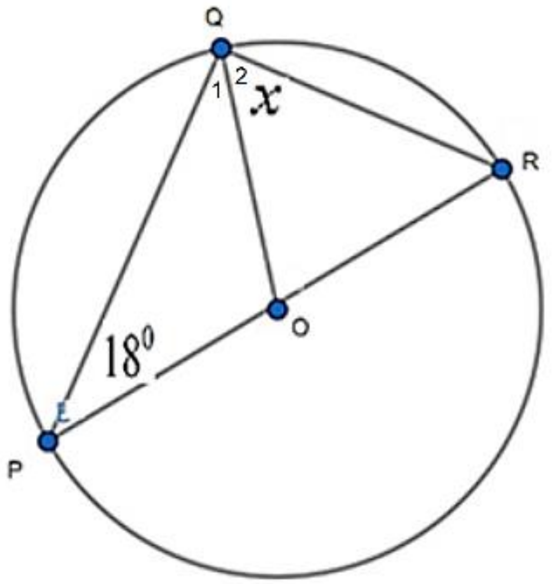
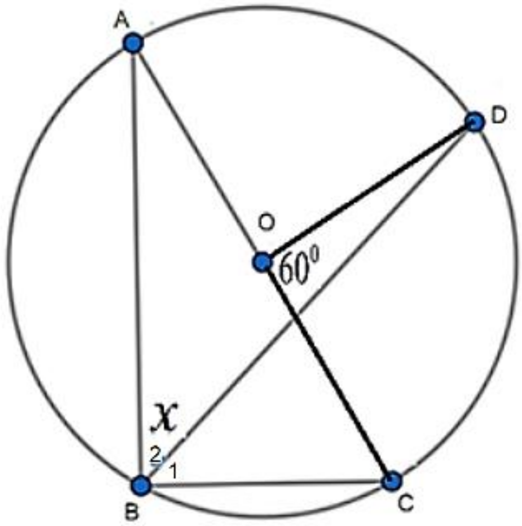
1.2.5



1.2.6



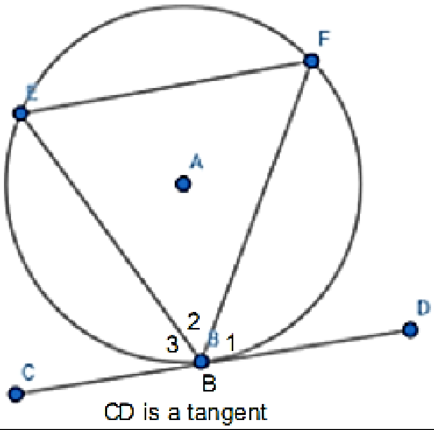
1.3 *Calculate the value of x in each of the following. O is the center.*

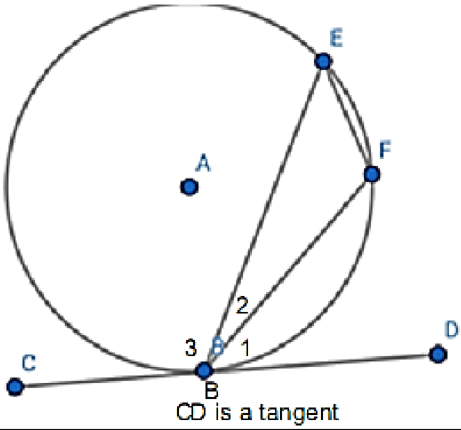
1.3.1 	
1.3.2 	

Theorems focused on the Tangents

Theorem statement: The angle between the tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment.

Diagram



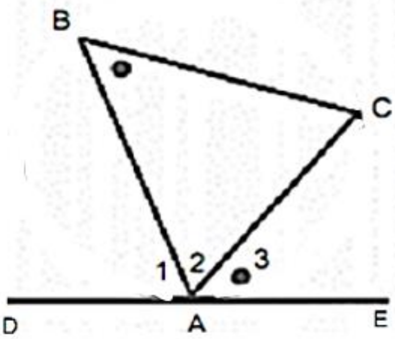


Mathematical Statement: $\hat{B}_3 = \hat{F}$

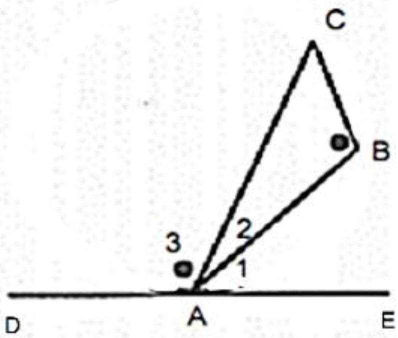
Acceptable Reason: tan chord theorem

Theorem statement: If a line is drawn through the endpoint of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.

Diagram

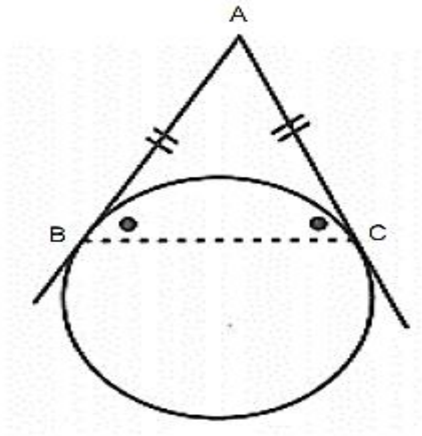


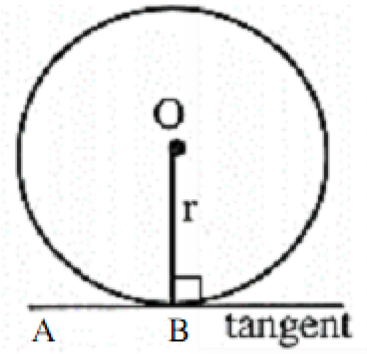
$Given: \hat{B} = \hat{A}_3$

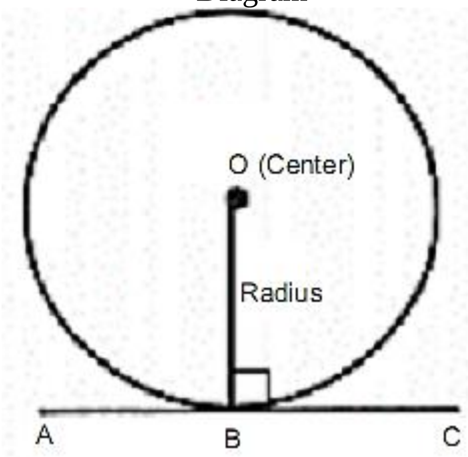


Mathematical Statement: DAE is a tangent

Acceptable Reason: Converse of tan chord theorem

Theorem statement: Two Tangents drawn to a circle from the same point outside the circle are equal in length
Diagram 
Mathematical Statement: $AB = AC$
Acceptable Reason: Tans from same point

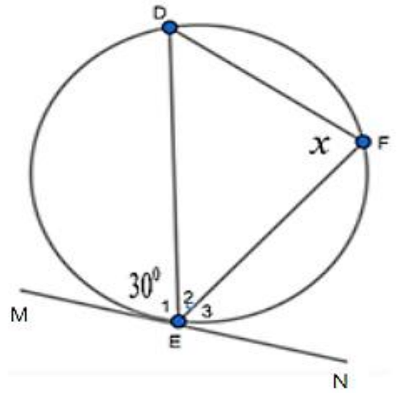
Theorem statement: The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.
Diagram 
Mathematical Statement: $OB \perp AB$
Acceptable Reason: tan $\perp$ *radius*

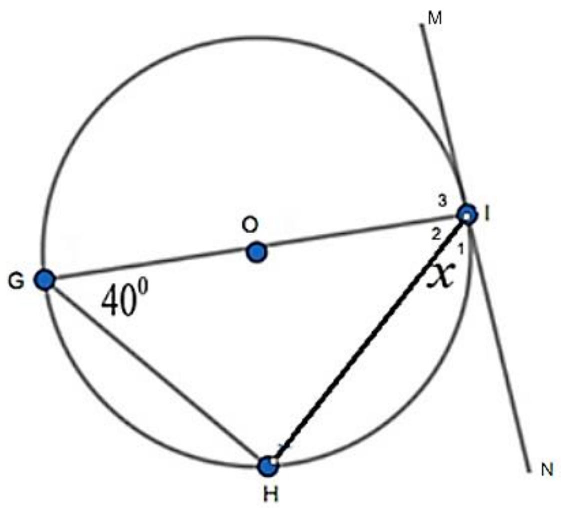
Theorem statement: If a line is drawn perpendicular to a radius/diameter at the point : where the radius/diameter meets the circle, then the line is a : tangent to the circle.
Diagram 
Mathematical Statement: $OB \perp AB$ (given) : ABC is a tan to the circle with center O
Acceptable Reason: $converse \tan \perp radius$

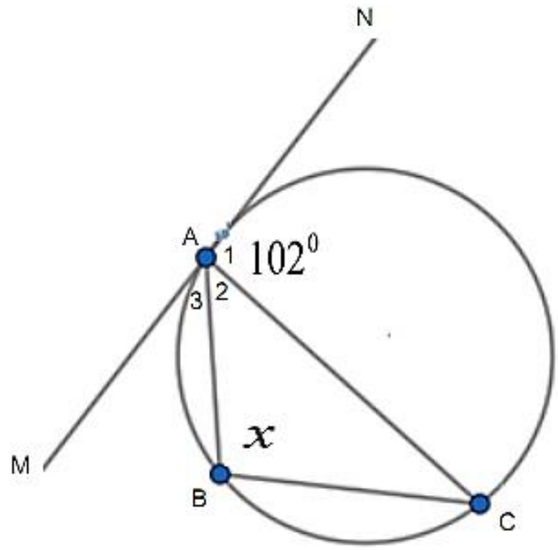
WORKED EXAMPLES

Calculate the values of x, y and z with reasons in each of the following.

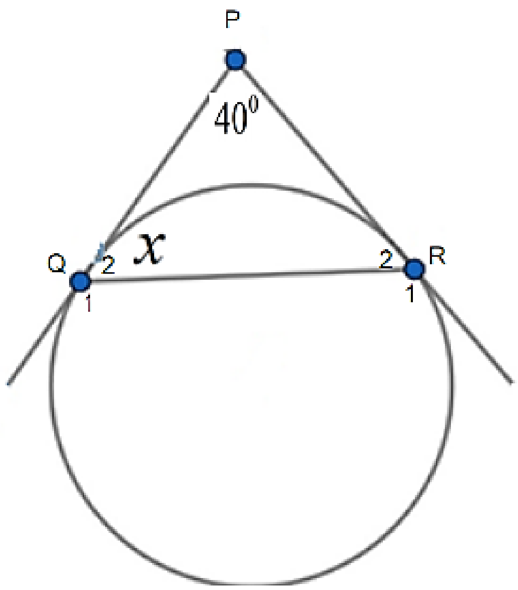
Example 1

1.1 *MN is a* tan *gent to circle DEF* 	*Solution* $\hat{F} = \hat{E}_1$ (tan *chord theorem*) $x = 30^0$

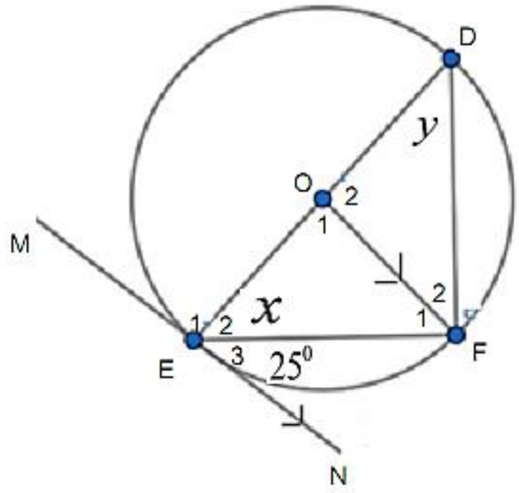
1.2 *O is the center, MN is a* tan *gent to circle GHI* 	*Solution* $\hat{I}_1 = \hat{G}$ (tan *chord theorem*) $x = 40^0$

1.3 MN *is a* tan*gent to circle* ABC	*Solution*
	$\hat{B} = \hat{A}_1$ (tan *chord theorem*) $x = 102^0$

Example 2

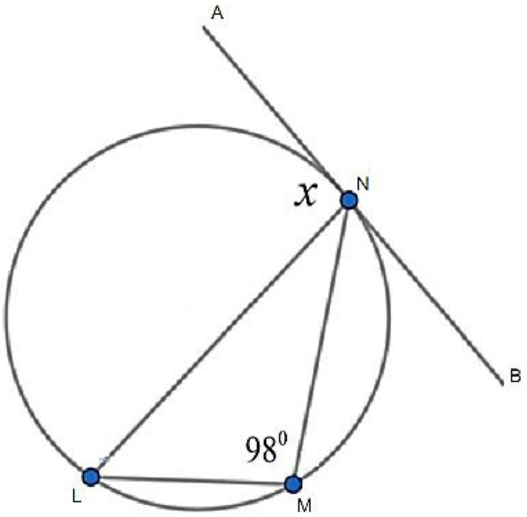
2.1 PQ *and* PR *are* tan*gents.*	*Solution*
	$PQ = PR$ (*tans from same po*int) *thus* $\hat{R}_2 = \hat{Q}_2 = x$ ($\angle s$ *opp* $=$ *sides*) *In* $\triangle PQR$ $x + x + 40^0 = 180^0$ (*Int* $\angle s$ *of* $\triangle$) $2x = 180^0 - 40^0$ $x = \frac{140^0}{2} = 70^0$

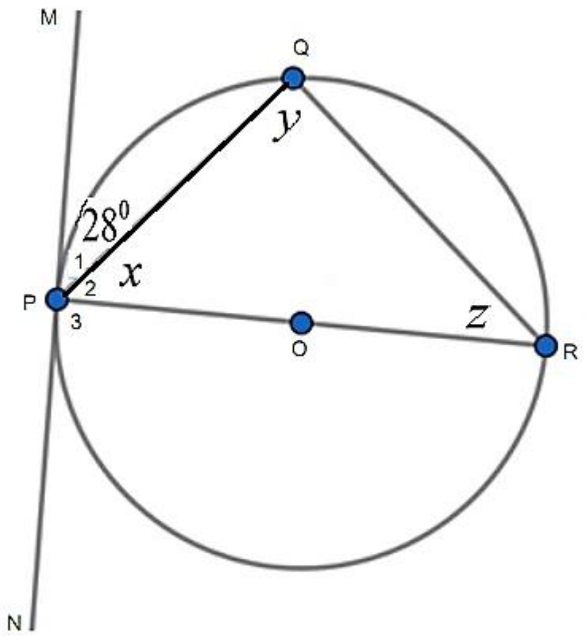
Example 3

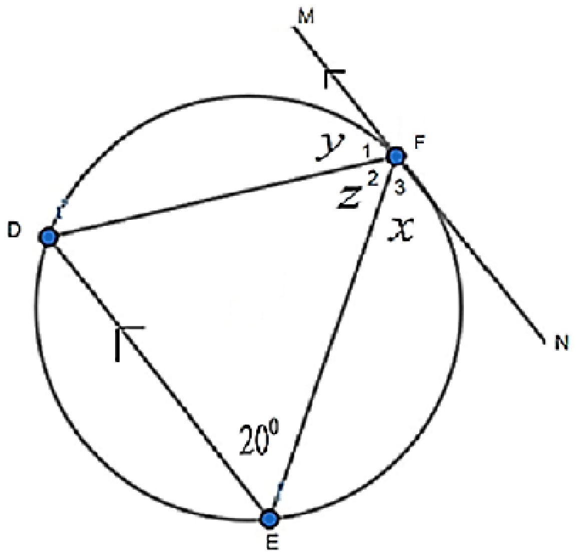
3.1	*Solution*
$O\, is\, the\, center, MN\, is\, a\, \tan gent\, to\, circle\, DEF$ 	$\hat{E}_2 = \hat{E}_3$ $x + 25^0 = 90^0$ (tan $\perp$ *radius*) $x = 90^0 - 25^0 = 65^0$ $\hat{D} = \hat{E}_3$ (tan *chord theorem*) $y = 25^0$

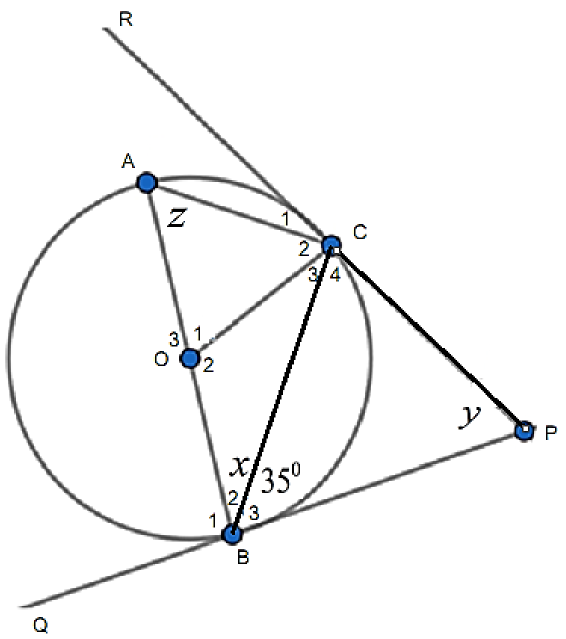
ACTIVITIES

1.4 ***Calculate the values of x, y and z with reasons in each of the following.***

1.4.1 *AB is a* tan *gent to circle NML* 	

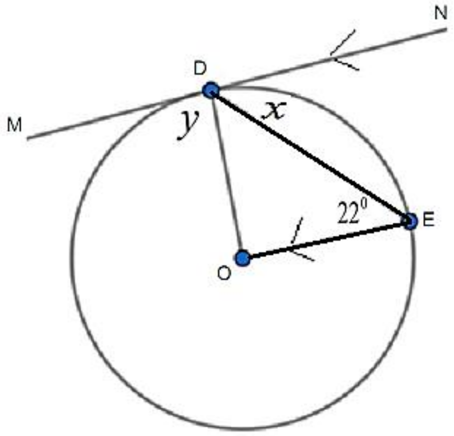
1.4.2 *O is the center and MN is a* tan *gent to circle PQR* 	

1.4.3 *MN is a* tan *gent to circle DEF and DE* || *MN* 	

1.4.4 *RP and PQ are* tan *gents to circle ABC* 	

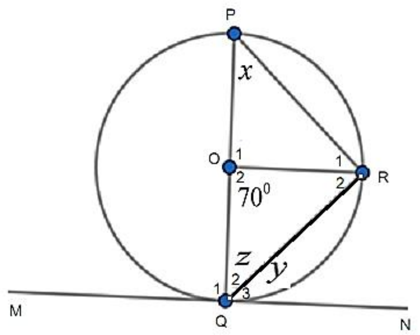
1.4.5

O is the center, MN is a tangent and MN || OE

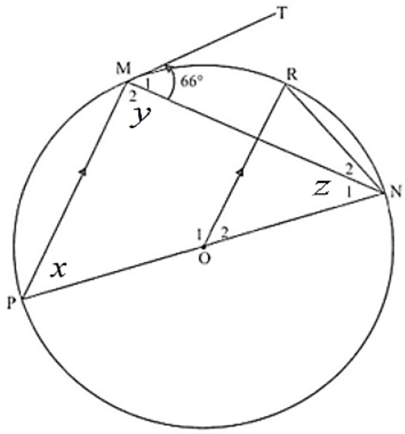


1.4.6

O is the center and MN is a tangent to circle PQR



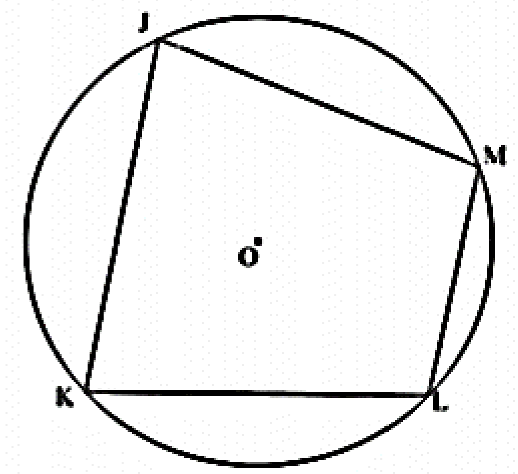
1.4.7 *PN is a diameter, MT is a tangent to circle MPNR and PM || OR*. O is the center



Theorems focused on the Cyclic Quadrilateral

Theorem statement: The opposite angles of a cyclic quadrilateral are supplementary.

Diagram



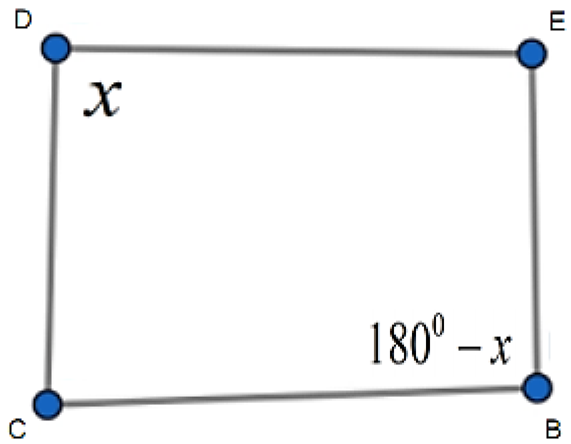
Mathematical Statement: $\hat{J}+\hat{L}=180^0$

$\hat{M}+\hat{K}=180^0$

Acceptable Reason: *opp ∠s of a cyclic quad*

Theorem statement: If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

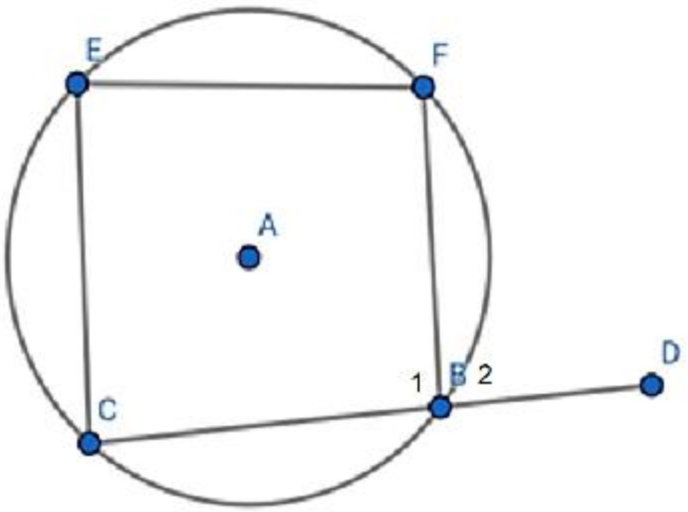
Diagram

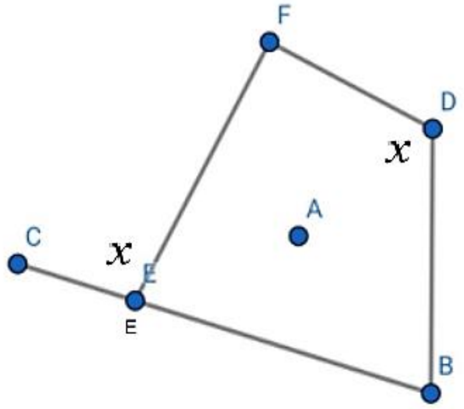


Mathematical Statement: $\hat{D}+\hat{B}=x+(180-x)=180$ (Given)

$\therefore DCBE is cyclic$

Acceptable Reason: *converse opp ∠s of cyclic quad*

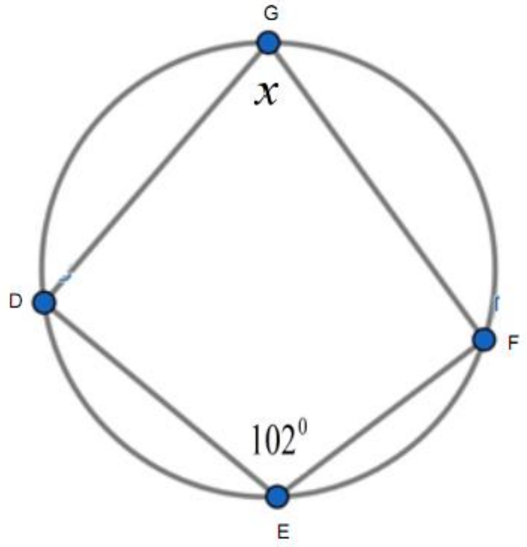
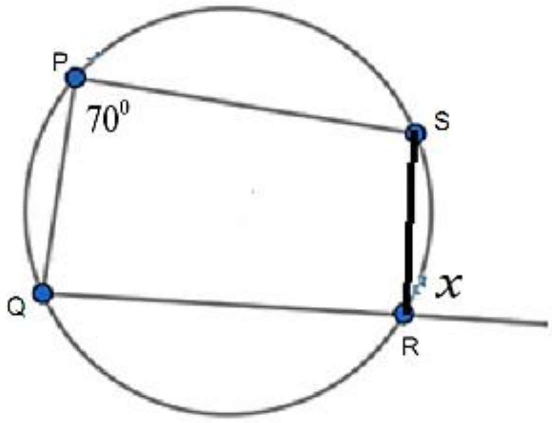
Theorem statement: The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
Diagram 
Mathematical Statement: $\hat{B}_2 = \hat{E}$
Acceptable Reason: *ext ∠ of cyclic quad*

Theorem statement: If the exterior angle of a quadrilateral is equal to the interior : opposite angle of the quadrilateral, then the quadrilateral is : concyclic.
Diagram

Mathematical Statement: $C\hat{E}F = \hat{D}$ $\therefore BEFD \text{ is a cyclic quadrilateral}$
Acceptable Reason: *Converse ext ∠ of cyclic quad*

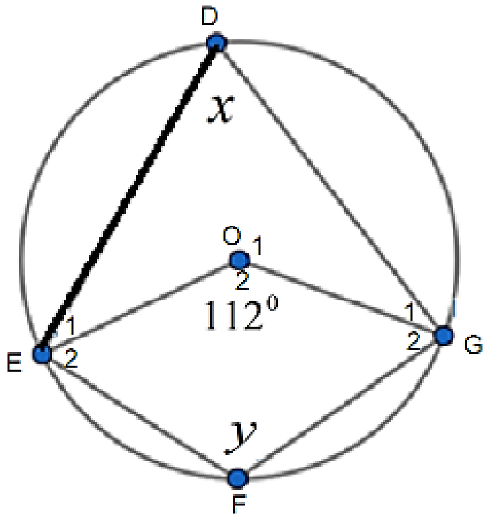
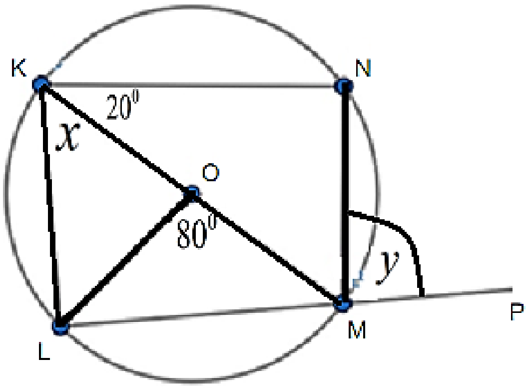
WORKED EXAMPLES

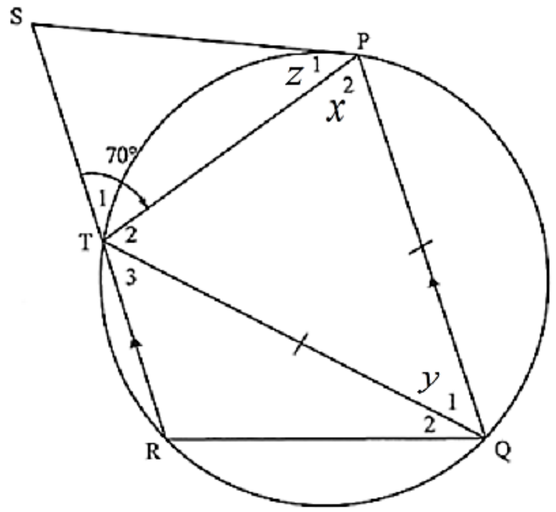
Example 1

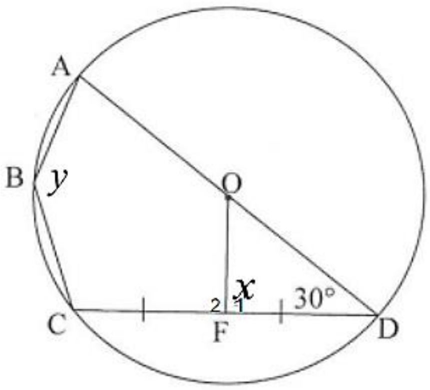
Calculate the values of x, y and z with reasons in each of the following.

1.1 	*Solution* $x + 102^0 = 180^0$ $(opp\angle s\ of\ cyclic\ quad)$ $x = 180^0 - 102^0 = 78^0$
1.2 	*Solution* $x = 70^0 (ext\angle\ of\ cyclic\ quad)$

ACTIVITIES: Calculate the x *and* y

1.5.1 *O is the center of circle DEFG* 	
1.5.2 *O is the center of circle KLMN* 	

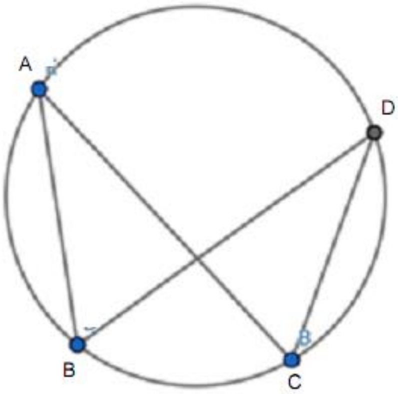
1.5.3 $RT \parallel QP$, SP is a tangent to circle $TRQP$ and $TQ = TP$. 	

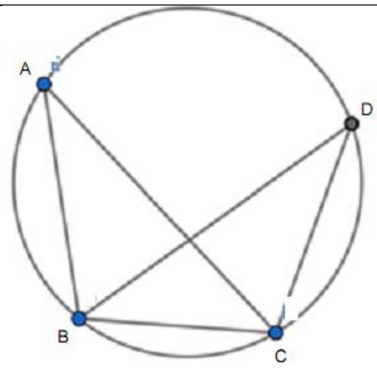
1.5.4 O is the center of circle $ABCD$ 	

Theorems that are classed under the keyword SUBTEND

Theorem statement: Angles subtended by a chord/segment of a circle, on the same
: side of the chord/segment are equal.

Diagram





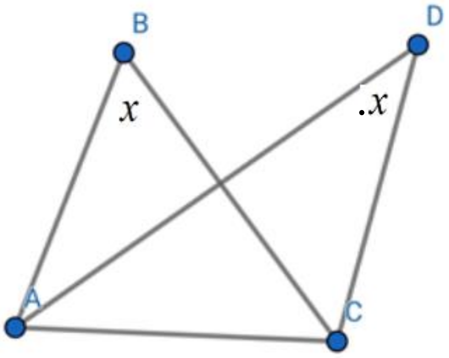
Mathematical Statement: $\hat{A} = \hat{D}$

$\hat{B} = \hat{C}$

Acceptable Reason: $\angle s$ *in the same segment*

Theorem statement: If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.

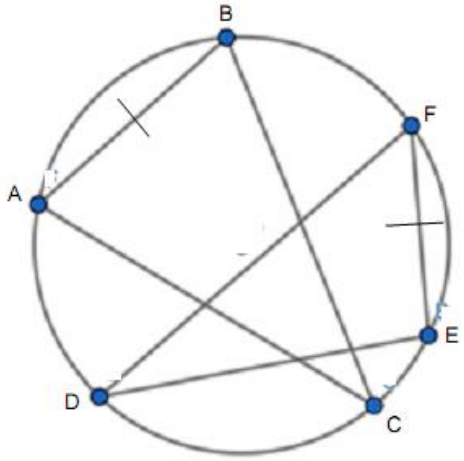
Diagram

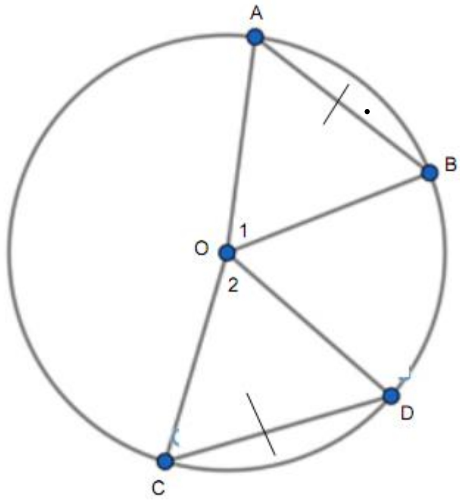


Mathematical Statement: If $\hat{B} = \hat{D}$

$then\ A, B, D, and\ C are concyclic$

Acceptable Reason: *converse* $\angle s$ *in the same segment*

Theorem statement: Equal chords subtend equal angles at the circumference of the circle.
Diagram 
Mathematical Statement: $AB = EF(given)$ $\therefore \hat{C} = \hat{D}$
Acceptable Reason: *equal chords equal* $\angle s$

Theorem statement: Equal chords subtend equal angles at the center of the circle.
Diagram *O is the center* 
Mathematical Statement: $AB = CD(given)$ $\therefore \hat{O}_1 = \hat{O}_2$
Acceptable Reason: *equal chords equal* $\angle s$

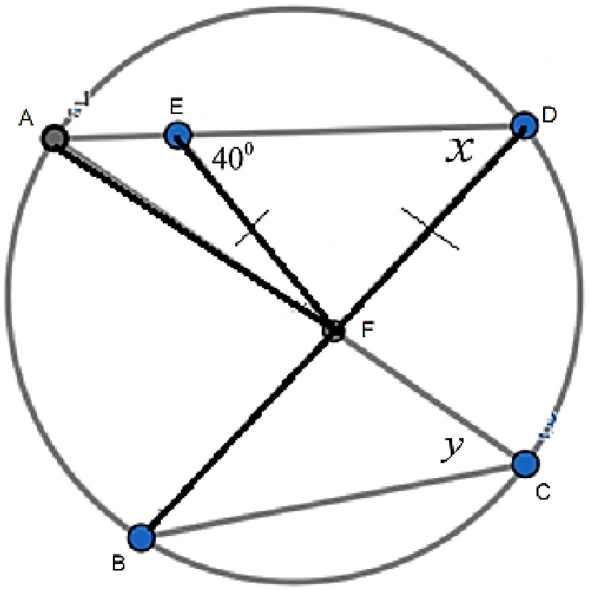
WORKED EXAMPLES

Calculate the values of x, y and z with reasons in each of the following.

Example 1

<table>
<tr>
<td>

1.1



</td>
<td>

Solution

$x = 40^0$ $(\angle s\ opp = sides)$

$y = x = 40^0$ $(\angle s\ in\ the\ same\ segment\ /\ arc)$

</td>
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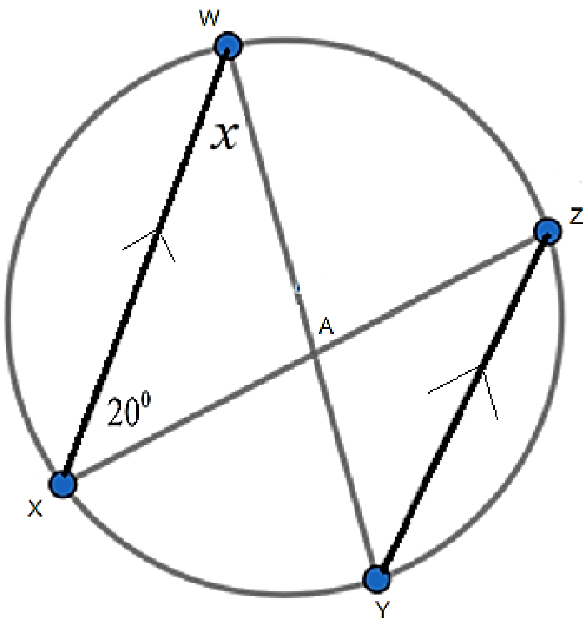
1.2



</td>
<td>

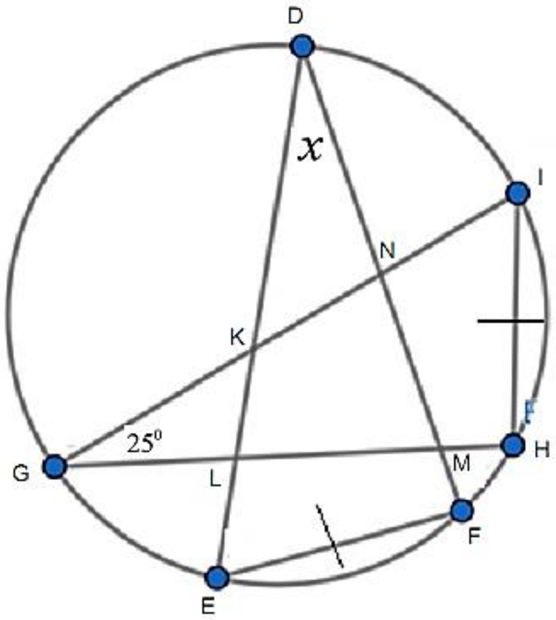
Solution

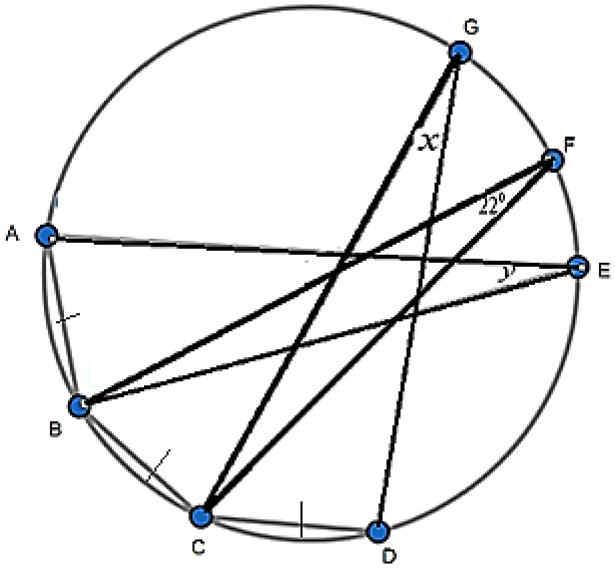
$\hat{Z} = 20^0$ $(Alternating\ \angle s, WX \parallel ZY)$

$x = \hat{Z} = 20^0$ $(\angle s\ in\ the\ same\ segment\ /\ arc)$

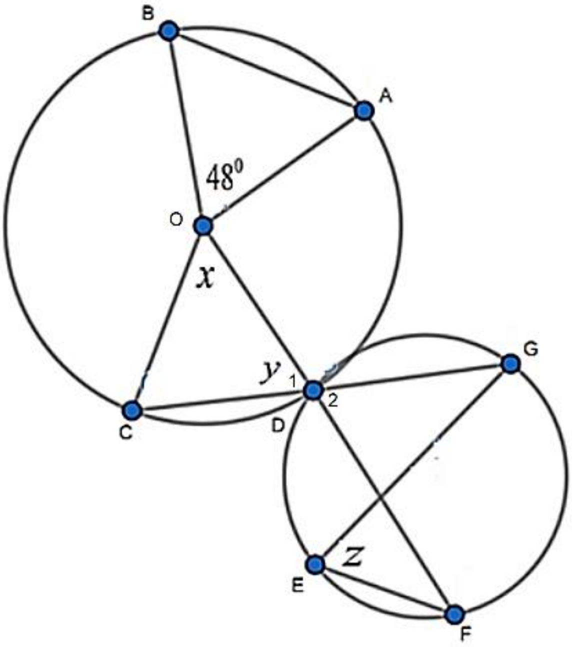
</td>
</tr>
</table>

Example 2

2.1 	*Solution* $x = 25^0$ (*equal chords; equal* $\angle s$)

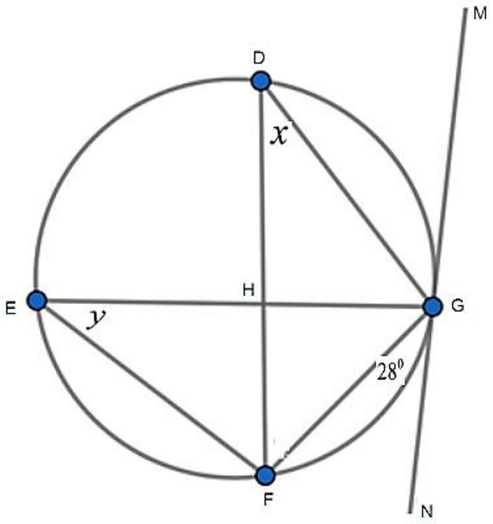
2.2 	*Solution* $x = y = 22^0$ (*equal chords; equal* $\angle s$)

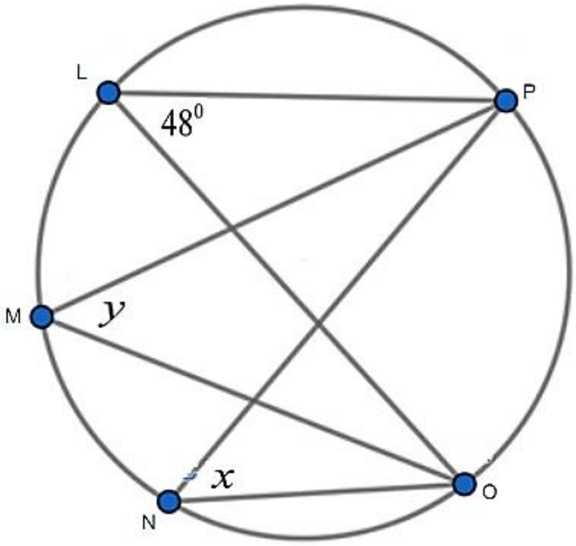
Example 3

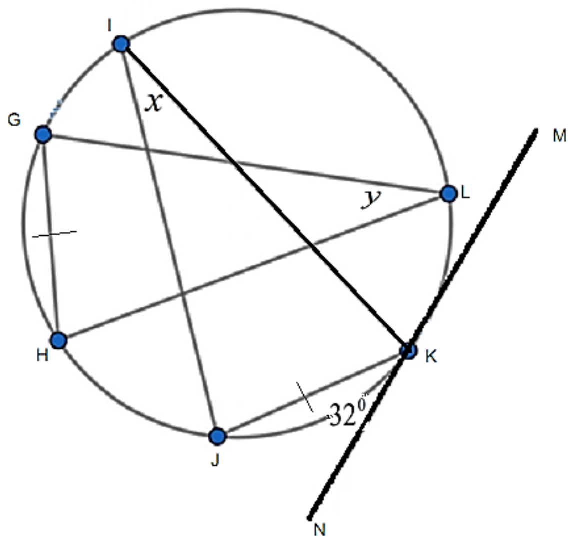
3.1 O *is the center of circle ABCD* 	*Solution* $x = 48^0$ (*equal chords equal* $\angle s$) *In* $\triangle OCD$ $y = \hat{C}$ ($\angle s$ *opp* = *sides*; *radii*) $x + y + \hat{C} = 180^0$ (*sum of* $\angle s$ *in a* $\triangle$) $48^0 + y + y = 180^0$ $2y = 180^0 - 48^0$ $y = \dfrac{132^0}{2} = 66^0$ $\hat{D}_2 = y = 66^0$ (*vert opp* $\angle s$) $z = \hat{D}_2 = 66^0$ ($\angle s$ *in the same segment / arc*)

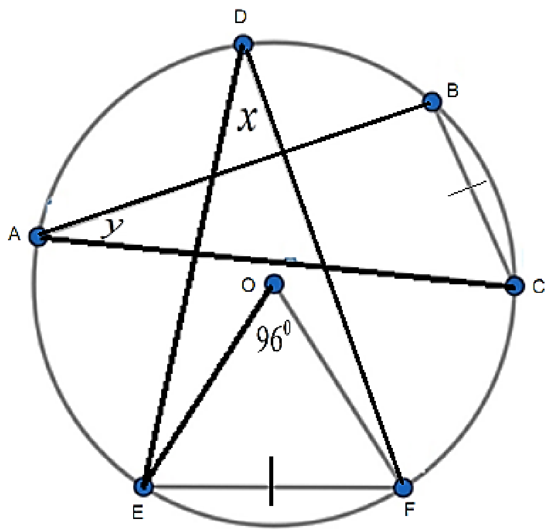
ACTIVITIES

Calculate the values of x, y and z with reasons in each of the following.

1.5.5 *MN is a* tan *gent to circle DEFG* 	

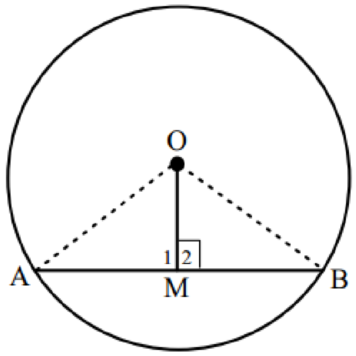
1.5.6 	

1.5.7 MN is a tangent	
	

1.5.8	
	

Proofs of examinable theorems:
source (Mind Action Series Mathematics Grade 11 Text Book)

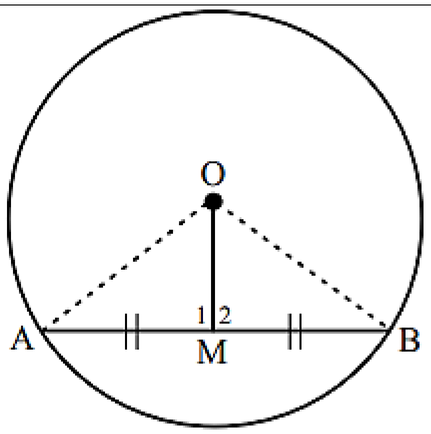
1. The line drawn from the centre of a circle perpendicular to a chord bisects the chord;



Given: Circle with centre O with $OM \perp AB$. AB is a chord
Required to prove: $AM = MB$.

Proof
Join OA and OB.
In ΔOAM and ΔOBM :
(a) $OA = OB$ ..... radii
(b) $\hat{M}_1 = \hat{M}_2 = 90°$ ..... given
(c) $OM = OM$ ..... common
$\therefore \Delta OAM \equiv \Delta OBM$ ..... RHS
$\therefore AM = MB$

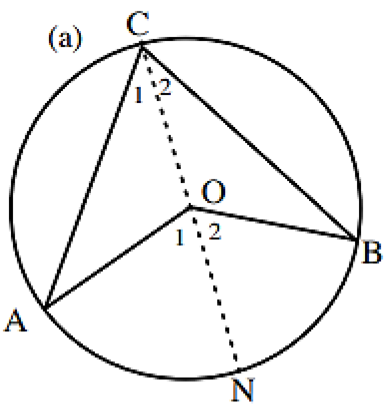
2. The line drawn from the centre of the circle to the midpoint of the chord is perpendicular the chord.



Given: Circle with centre O.
Required to prove: $OM \perp AB$

Proof
Join OA and OB
In ΔOAM and ΔOBM :
(a) $OA = OB$ ..... radii
(b) $AM = BM$ ..... given
(c) $OM = OM$ ..... common
$\therefore \Delta OAM \equiv \Delta OBM$ ..... SSS
$\therefore \hat{M}_1 = \hat{M}_2$
But AMB is a straight line
$\therefore \hat{M}_1 = \hat{M}_2 = 90°$ ..... Adjacent supplementary $\angle s$

3. The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle



Given: Circle with centre O :

Required to prove: $A\hat{O}B = 2A\hat{C}B$

Proof:

Join CO and produce to N.

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ ….. Ext $\angle$ of ΔOAC

But $\hat{C}_1 = \hat{A}$ ….. $OA = OC$, Radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

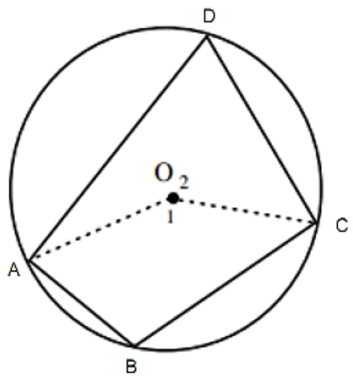
Similarly, in ΔOCB $\hat{O}_2 = 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{C}_1 + \hat{C}_2\right)$

$\therefore A\hat{O}B = 2A\hat{C}B$

4. The opposite angles of a cyclic quadrilateral are supplementary



Required to prove: $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$

Proof

Join AO and OC.

$\hat{O}_1 = 2\hat{D}$ ….. $\angle$ at centre $= 2\times\angle$ at circum

$\hat{O}_2 = 2\hat{B}$ ….. $\angle$ at centre $= 2\times\angle$ at circum

$\hat{O}_1 + \hat{O}_2 = 2\hat{D} + 2\hat{B}$

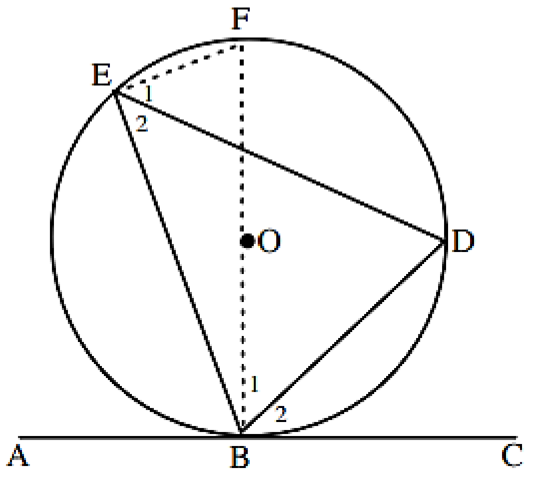
And $\hat{O}_1 + \hat{O}_2 = 360°$ ….. $\angle$'s at a point

$\therefore 360° = 2\left(\hat{D} + \hat{B}\right)$

$\therefore 180° = \hat{D} + \hat{B}$

Similarly, by joining BO and DO, it can be proven that $\hat{A} + \hat{C} = 180°$

5. The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;



Given: Tangent ABC

Required to prove: $C\hat{B}D = B\hat{E}D$

Proof:

Draw diameter BOF and join EF

$\hat{B}_1 + \hat{B}_2 = 90°$ ….. tan $\perp$ rad

$\hat{E}_1 + \hat{E}_2 = 90°$ ….. $\angle$ in semi-circle

But $\hat{B}_1 = \hat{E}_1$ ….. FD subt $= \angle s$

$\therefore \hat{B}_2 = \hat{E}_2$

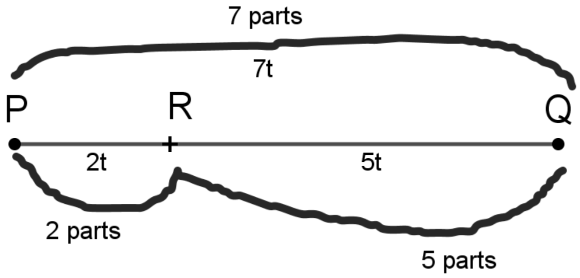
$\therefore C\hat{B}D = B\hat{E}D$

SECTION4: SIMILARITY AND PROPORTIONALITY

Ratios and Proportionality

Ratios

Consider line segment PQ. If R divides line segment PQ in a ratio 2:5,



let PR = 2t, RQ = 5t and PQ = 7t. Therefore:

(a) $\frac{PR}{PQ} = \frac{2t}{7t} = \frac{2}{7}$

(b) $\frac{RQ}{PQ} = \frac{5t}{7t} = \frac{5}{7}$

(c) $\frac{PQ}{RQ} = \frac{7t}{5t} = \frac{7}{5}$

(d) $\frac{PQ}{PR} = \frac{7t}{2t} = \frac{7}{2}$

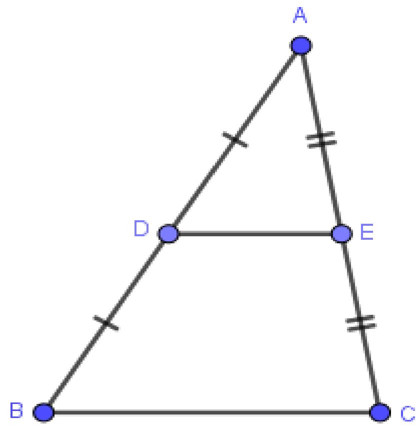
Ratios

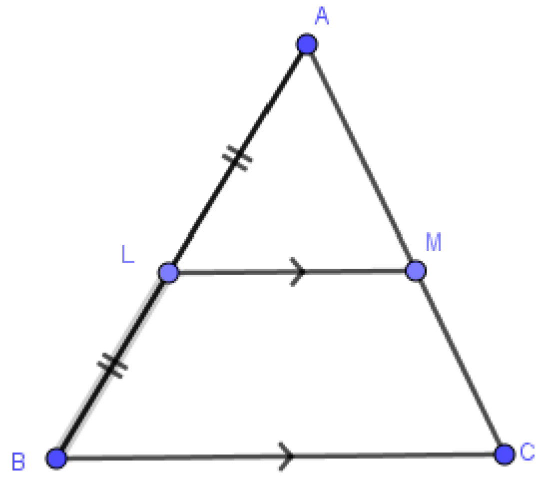
Proportionality

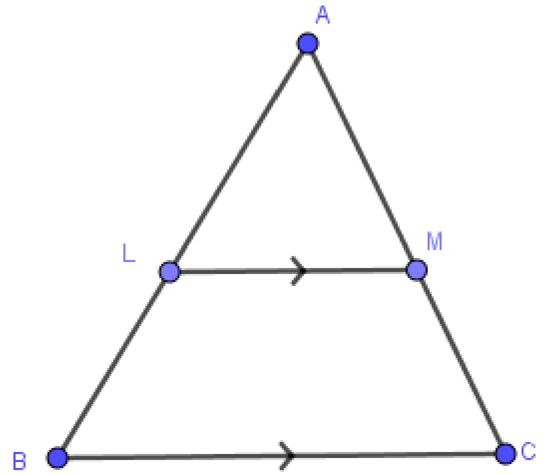
Proportion – the mathematical concept that tells us that the two ratios are equal. e.g. $\frac{1}{2} = \frac{3}{6}$

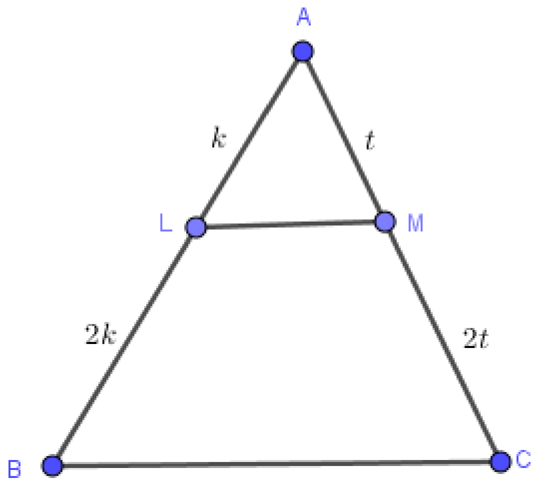
Theorem Statements

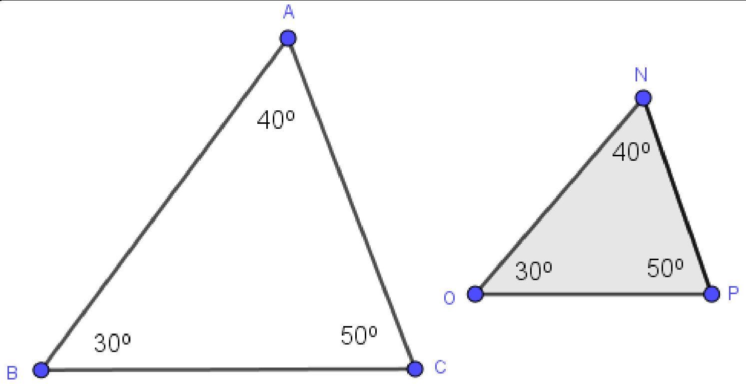
N.B All the theorem statements were taken from 2021 Grade 12 mathematics examination guidelines

1.	Theorem statement	The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side
	Diagram	
	Mathematical statement	If AD = DB and AE = EC, then DE || BC and DE = $\frac{1}{2}$ BC
	Reason	Midpt Theorem

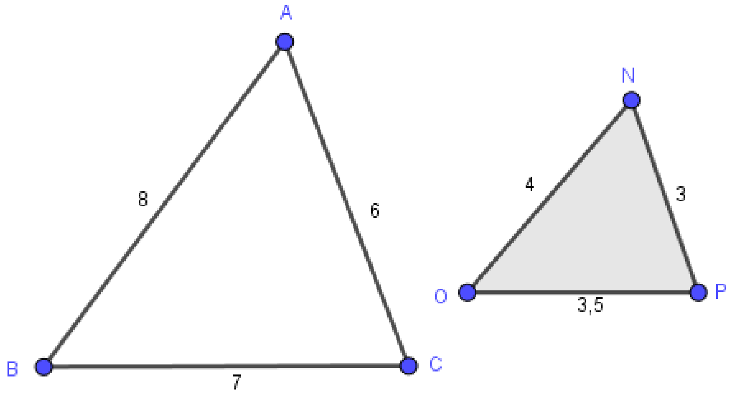
2.	Theorem statement	The line drawn from the midpoint of one side of the triangle, parallel to another side, bisects the third side
	Diagram	
	Mathematical statement	If AL = LB and LM || BC, then AM = MC
	Reason	Line through midpt || 2^{nd} side

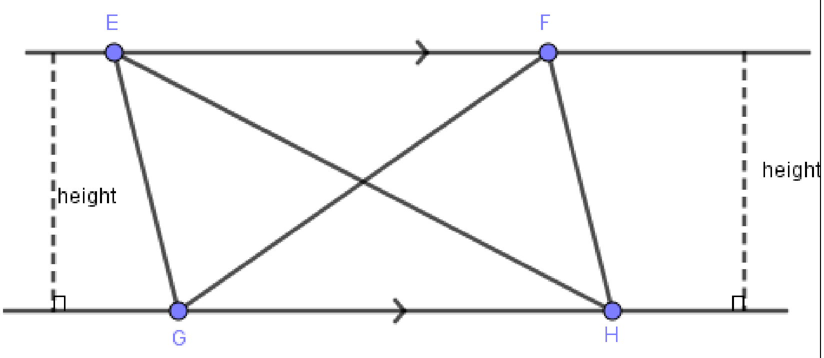
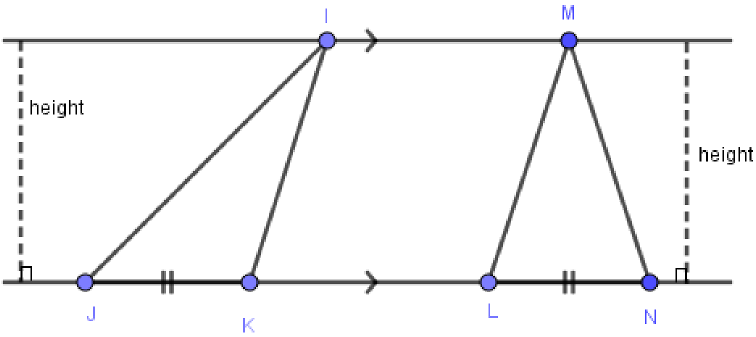
3.	Theorem statement	A line drawn parallel to one side of a triangle divides the other two sides proportionally
	Diagram	
	Mathematical statement	if LM||BC, then $\frac{AL}{LB}=\frac{AM}{MC}$
	Reason	Prop theorem; LM||BC

4.	Theorem statement	If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side
	Diagram	 AL = k LB = $2k$ AM = t MC = $2t$
	Mathematical statement	If $\frac{AL}{LB}=\frac{AM}{MC}$, then LM || BC
	Reason	Line divides two sides of Δ in prop

5.	Theorem statement	If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar)
	Diagram	

	Mathematical statement	If $\hat{A} = \hat{N}, \hat{B} = \hat{O}$, and $\hat{C} = \hat{P}$, *then* $\frac{AB}{NO} = \frac{AC}{NP} = \frac{BC}{OP}$ and $\Delta ABC \,|||\, \Delta NOP$
	Reason	$|||\Delta s$

6.	Theorem statement	If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar)
	Diagram	
	Mathematical statement	If $\frac{AB}{NO} = \frac{AC}{NP} = \frac{BC}{OP}$, then $\hat{A} = \hat{N}, \hat{B} = \hat{O}$, and $\hat{C} = \hat{P}$ and $\Delta ABC \,|||\, \Delta NOP$
	Reason	Sides of Δ in prop

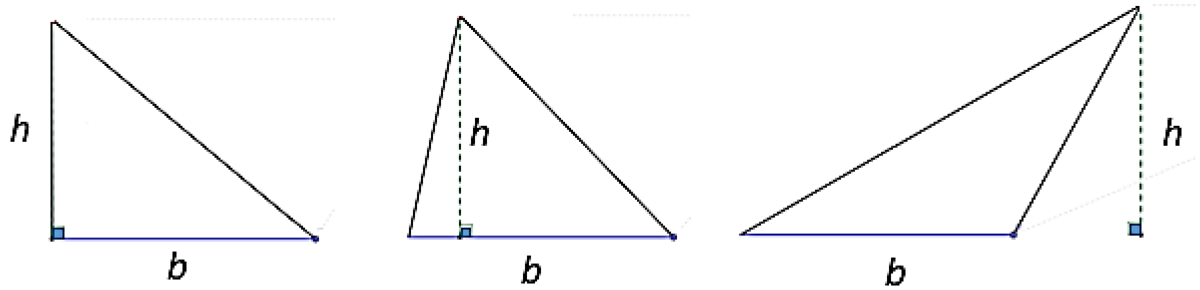
<table>
<tr><td>7.</td><td>Theorem statement</td><td>If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallel grams) have equal areas.</td></tr>
<tr><td></td><td colspan="2">Triangles with same base</td></tr>
<tr><td></td><td>Diagram</td><td>
</td></tr>
<tr><td></td><td>Mathematical statement</td><td>Area ΔEGF = Area ΔFGH</td></tr>
<tr><td></td><td>Reason</td><td>same base; same height</td></tr>
<tr><td></td><td colspan="2">Triangles with bases of equal length</td></tr>
<tr><td></td><td>Diagram</td><td>
</td></tr>
<tr><td></td><td>Mathematical statement</td><td>Area ΔIJK = Area ΔLMN</td></tr>
<tr><td></td><td>Reason</td><td>equal bases; equal height</td></tr>
</table>

Area of Triangles

Right angled triaangle (original or by construction)

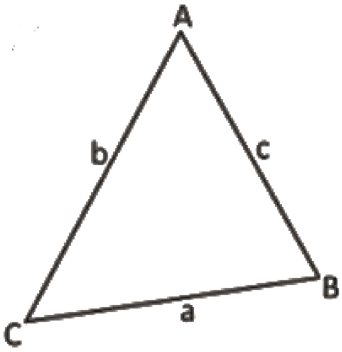
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$A = \frac{1}{2} bh$$



(SOURCE: https://www.google.co.za/imgres?imgurl=https%3A%2F%2Fwww.onlinemathlearning.com%2Fimage-files%2Farea-triangle.png&imgrefurl=https%3A%2F%2Fwww.onlinemathlearning.com%2Farea-triangles.html&tbnid=-8Usqt8fWQ6hM&vet=12ahUKEwidzYil_aTuAhVYgqQKHYBtCOYQMygBegUIARDHAQ..i&docid=Xkrjbsh1QYI10M&w=648&h=359&q=area%20of%20triangles&ved=2ahUKEwidzYil_aTuAhVYgqQKHYBtCOYQMygBegUIARDHAQ)

Area Rule

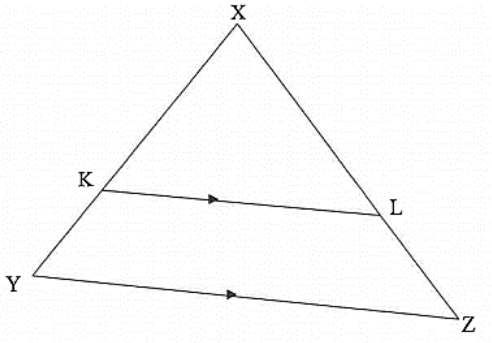


$$\text{Area of } \triangle \text{ ABC} = \frac{1}{2} a.b.SinC$$
$$= \frac{1}{2} a.c.SinB$$
$$= \frac{1}{2} b.c.SinA$$

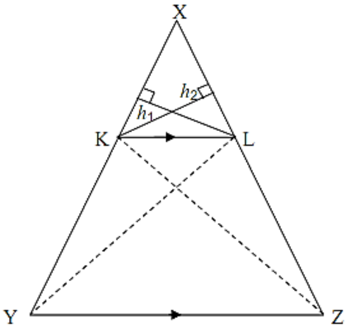
Proofs of examinable theorems:

1. A line drawn parallel to one side of a triangle divides the other two sides proportionally.

If $KL \parallel YZ$, prove that $\frac{XK}{KY} = \frac{XL}{LZ}$



Proof:



Constr: Join KZ and LY and draw h_1 from $K \perp XL$ and h_2 from $L \perp XK$

Proof :

$$\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2} XK \times h_1}{\frac{1}{2} KY \times h_1} = \frac{XK}{KY}$$

$$\frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ} = \frac{\frac{1}{2} XL \times h_2}{\frac{1}{2} LZ \times h_2} = \frac{XL}{LZ}$$

area ΔXKL = area ΔXKL [common]

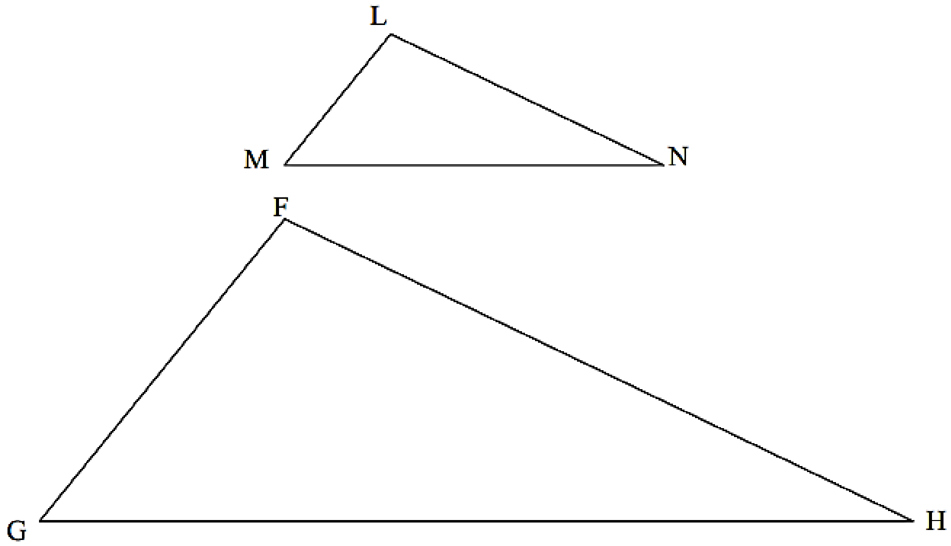
But area ΔLYK = area ΔKLZ [same base & height ; $LK \parallel YZ$]

$$\therefore \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\text{area } \Delta XKL}{\text{area } \Delta KLZ}$$

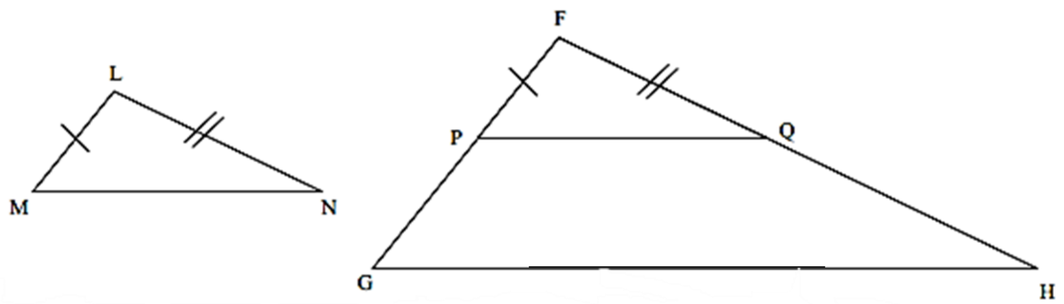
$$\therefore \frac{XK}{KY} = \frac{XL}{LZ}$$

2. The corresponding sides of two equiangular triangles are in the same proportion and therefore the triangles are similar

If in Δ LMN and Δ FGH it is given that $\hat{L} = \hat{F}$ and $\hat{M} = \hat{G}$, prove the theorem that states $\frac{LM}{FG} = \frac{LN}{FH}$.



Proof:



Draw a point P on FG such that FP = LM and a point Q on FH such that FQ = LN.

In ΔFPQ and ΔLMN

1. $\hat{F} = \hat{L}$ (given)
2. FP = LM (construction)
3. FQ = LN (construction)

$\therefore \Delta FPQ \equiv \Delta LMN$ (SAS)

$F\hat{P}Q = L\hat{M}N$ ($\equiv\Delta$s)

But $F\hat{G}H = L\hat{M}N$ (given)

$F\hat{P}Q = F\hat{G}H$

PQ || GH (corresponding angles =)

$\frac{FP}{FG} = \frac{FQ}{FH}$ (PQ || GH ; Prop Th)

$\frac{LM}{FG} = \frac{LN}{FH}$

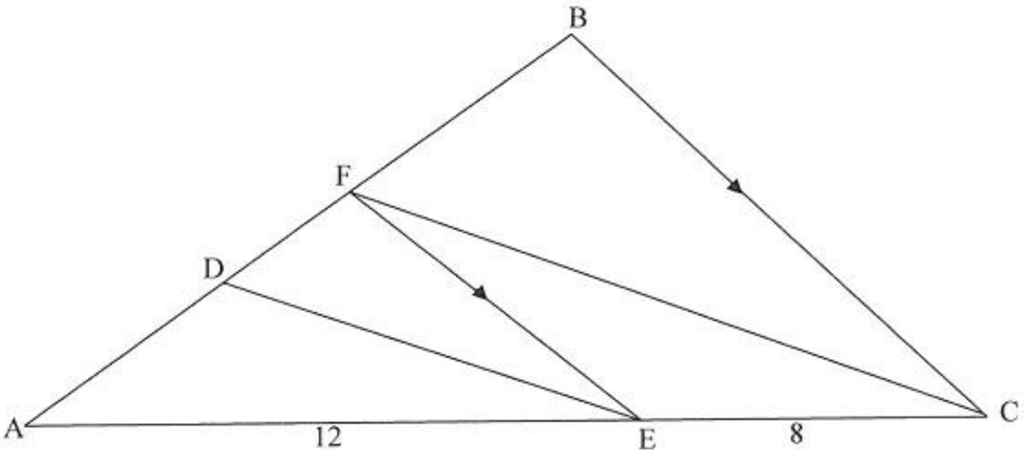
WORKED EXAMPLES

Example 1

1 Complete the following statement of the theorem in the ANSWER BOOK:

If a line divides two sides of a triangle in the same proportion, then ...

2 In the diagram ABC is a triangle with F on AB and E on AC. BC | | FE. D is on AF with $\frac{AD}{AF}=\frac{3}{5}$. AE = 12 units and EC = 8 units.



2.1 Prove that DE | | FC.

2.2 If AB = 14 units, calculate the length of BF.

Solution:

1 then the line is **parallel** to the **third side**

2.1 $\frac{AE}{AC}=\frac{12}{20}=\frac{3}{5}$

$\frac{AD}{AF}=\frac{3}{5}$

$\therefore \frac{AE}{AC}=\frac{AD}{AF}$

$\therefore DE \parallel FC$ (line divides two sides of Δ in prop)

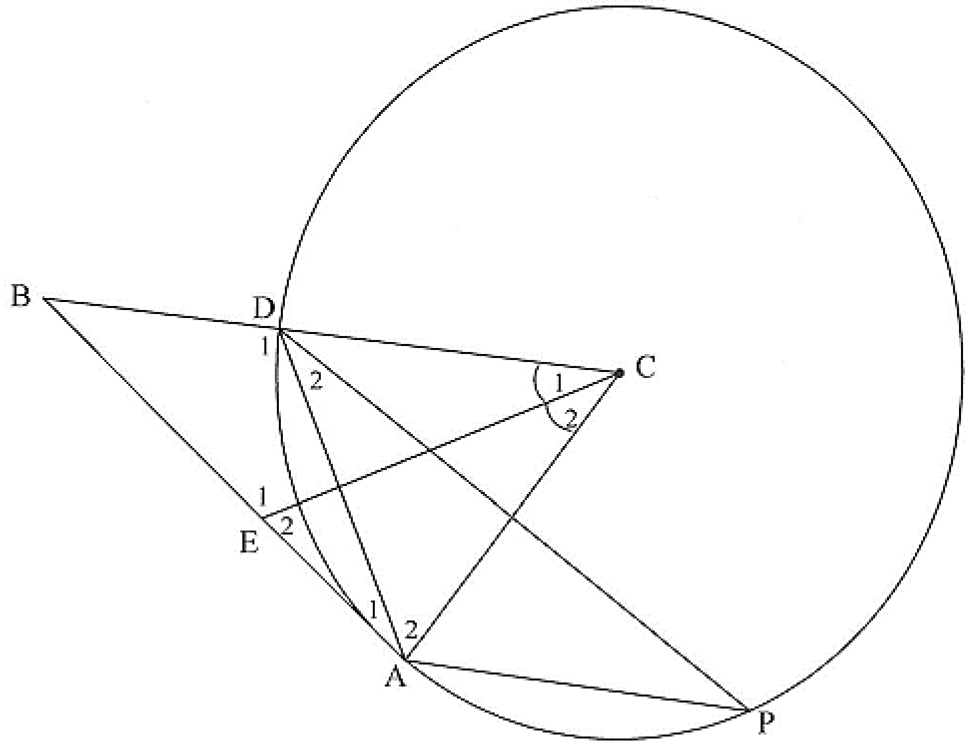
2.2 $\frac{BF}{BA}=\frac{8}{20}$ (prop theorem BC ∥ FE)

$\therefore BF=\frac{8}{20}(14)$

$\therefore BF=\frac{28}{5}$ **OR** $FB=5\frac{3}{5}$ **OR** $FB=5{,}6$

Example 2

In the diagram C is the centre of the circle DAP. BA is a tangent to the circle at A. CD is produced to meet the tangent to the circle at B. DP and DA are drawn. E is a point on BA such that EC bisects $D\hat{C}A$. Let $\hat{C}_1 = x$.



1 Prove that ΔBAD ||| ΔBCE.

2 If it is also given that AB = 8 units and AC = 6 units, calculate:

(a) The length of BD

(b) The length of BE

(c) The size of x

Solution:

1 $D\hat{C}A = 2x$ (EC bisector)

$\hat{P} = x$ ($\angle$ at centre $= 2 \times \angle$ at circumference)

$\hat{A}_1 = \hat{P} = x$ (tangent-chord theorem)

In ΔBAD and ΔBCE:

$\hat{B} = \hat{B}$ (common)

$\hat{A}_1 = \hat{C}_1$ (proven above)

$\therefore \Delta BAD \,|||\, \Delta BCE$ ($\angle\angle\angle$)

2(a) $B\hat{A}C = 90°$ (tangent $\perp$ radius)

$\therefore BC^2 = 8^2 + 6^2 = 100$ (Pythagoras theorem/)

$BC = 10$

$AC = DC = 6$ (radii)

$\therefore BD = 10 - 6 = 4$ units

2(b) $\dfrac{BA}{BC} = \dfrac{BD}{BE}$ ($\Delta BAD \,|||\, \Delta BCE$)

$\therefore \dfrac{8}{10} = \dfrac{4}{BE}$

$\therefore BE = 5$ units

2(c) $AE = 3$

In ΔACE:

$\tan x = \dfrac{3}{6}$

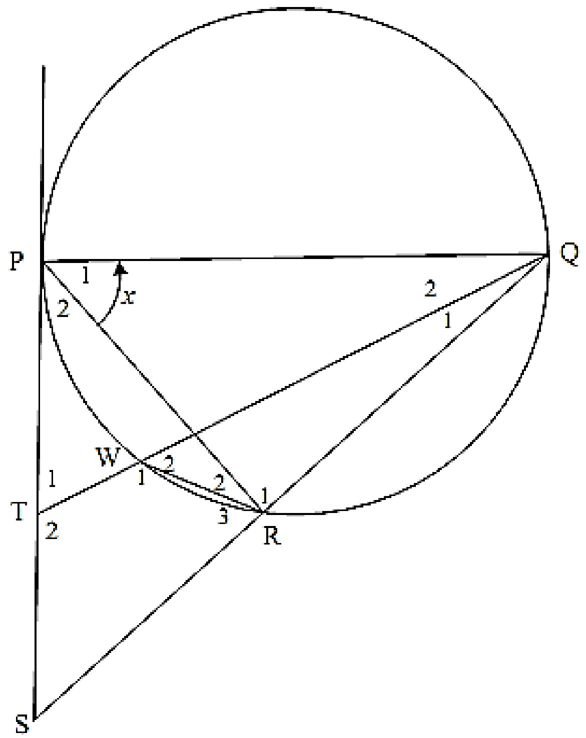
$\therefore x = 26{,}57°$

ACTIVITIES

QUESTION 1

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

Let $\hat{P}_1 = x$



1.1 Why is $P\hat{R}Q = 90°$?

1.2 Prove that $\hat{P}_1 = \hat{S}$.

1.3 Prove that SRWT is a cylic quadrilateral.

1.4 Prove that ΔQWR /// ΔQST.

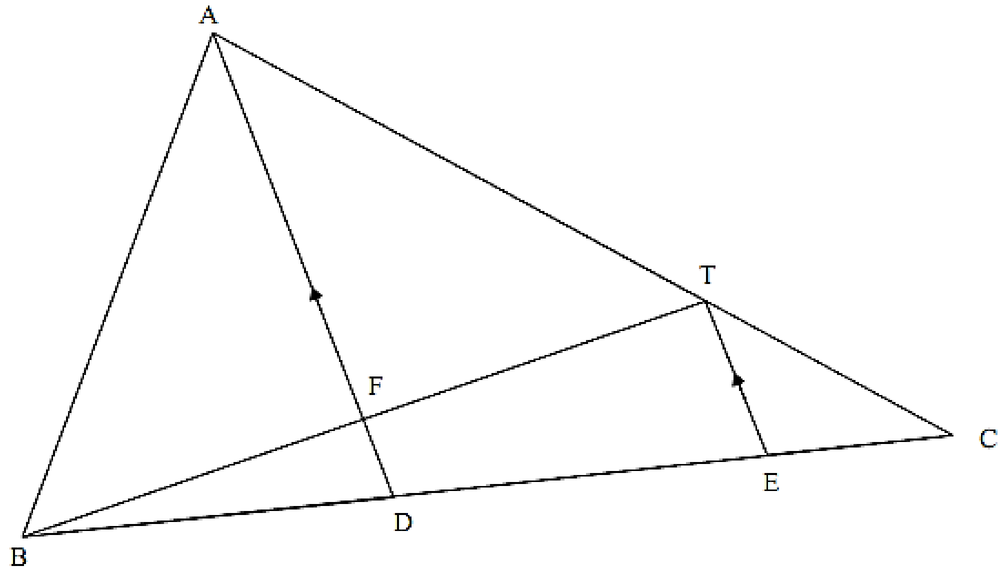
1.5 If QW = 5 cm, TW = 3 cm, QR = 4 cm and WR = 2 cm, calculate the length of:

1.5.1 TS

1.5.2 SR

QUESTION 2

In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD || TE.



2.1 Write down the numerical value of $\dfrac{CE}{ED}$

2.2 Show that D is the midpoint of BE.

2.3 If FD = 2 cm, calculate the length of TE.

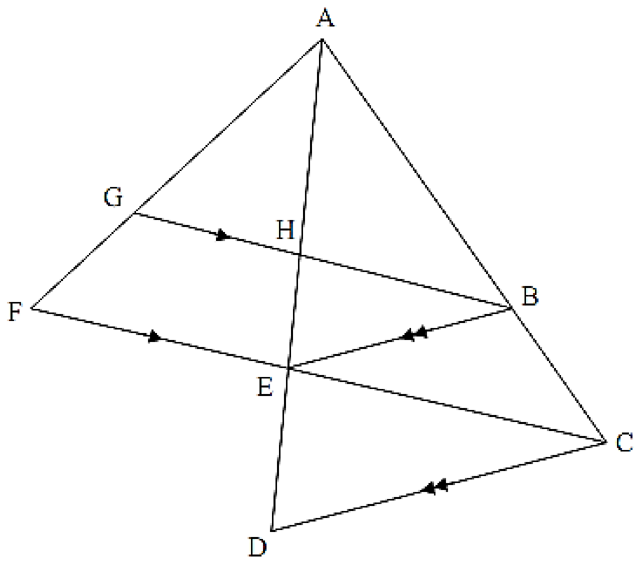
2.4 Calculate the numerical value of:

2.4.1 $\dfrac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$

2.4.2 $\dfrac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$

QUESTION 3

In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



3.1 Calculate with reasons:

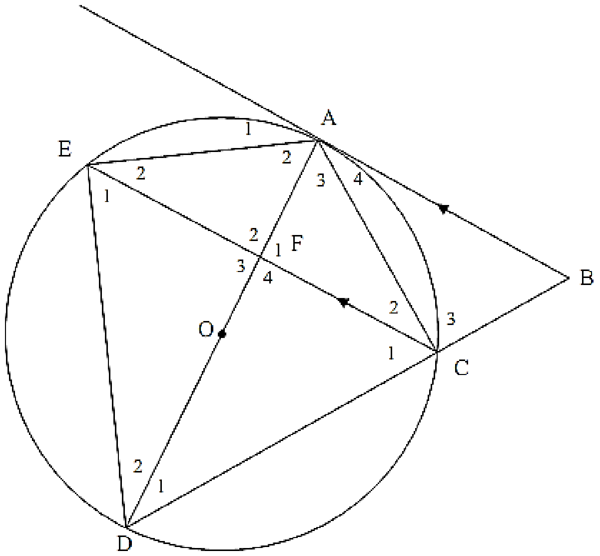
3.1.1 AH : ED

3.1.2 $\frac{BE}{CD}$

3.2 If HE = 2 cm, calculate the value of AD × HE.

QUESTION 4

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



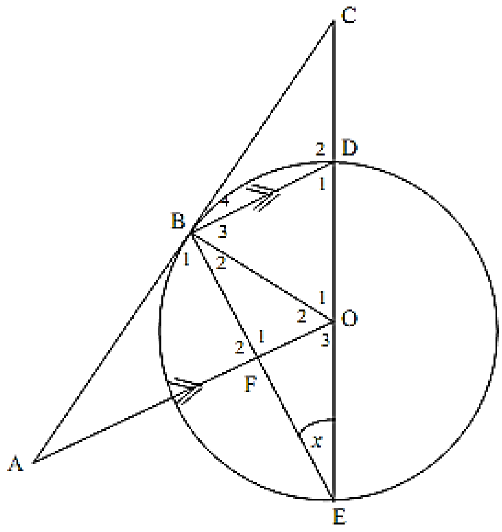
4.1 Show $\hat{C}_2 = \hat{D}_1$.

4.2 Prove that $\Delta ACF\ |||\ \Delta ADC$.

4.3 Prove that AD = 4AF.

QUESTION 5

5.1 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F. BD || AO. $\hat{E} = x$.



5.1.1 Write down, with reasons, THREE other angles equal to x.

5.1.2 Determine, with reasons, $C\hat{B}E$ in terms of x.

5.1.3 Prove that F is the midpoint of BE.

5.1.4 Prove that $\Delta CBD \parallel\!\mid \Delta CEB$.

5.1.5 Prove that 2EF.CB = CE.BD.

Solutions to Algebra activities

1.	1.1 $4+2-3\times3=4+2-9$ $=-3$ 1.2 $8\div2+2-2\times2=4+2-4$ $=2$ 1.3 $8\div2+(2-3)\times3=8\div2+(-1)\times3$ $=4-3$ $=1$ 1.4 $100\div5^2+(4-1)\times3=100\div25+(3)\times3$ $=4+9$ $=13$
2.	2.1 $5x-3y+x-8x+9y=6x+6y$ 2.2 $\frac{5}{2}x-\frac{3}{4}y+x-8x+\frac{5}{3}y=-\frac{9}{2}x+\frac{11}{12}y$ 2.3 $2(x+3)-4(5x-54)=2x+6-20x+216=-18x+222$ 2.4 $2x(x+3)-4x(5x-4)=2x^2+6x-20x^2+16x=-18x^2+22x$ 2.5 $(x+3)(2x-5)=2x^2-5x+6x-15=2x^2+x-15$ 2.6 $(x+3)(x^2+6x+6-x)=(x+3)(x^2+5x+6)$ $=x^3+5x^2+6x+3x^2+15x+18$ $=x^3+8x^2+21x+18$ 2.7 $(x^2-x+2)(2-x)=(-x+2)(x^2-x+2)=-x^3+x^2-2x+2x^2-2x+4$ $=-x^3+3x^2-4x+4$

3.	3.1 $5x = x - 8$ $4x = -8$ $x = -2$ 3.2 $\frac{5}{2}x - 8x = 3$ $-\frac{11}{2}x = 3$ $x = -\frac{6}{11}$ 3.3 $2(x+3) - 4(5x-54) = 1$ $2x + 6 + 20x + 216 = 1$ $22x = -215$ $x = -\frac{215}{22} = -9\frac{17}{22}$ 3.4 $x + 4 = -\frac{3}{x}$ $(x \neq 0)$ $x^2 + 4x + 3 = 0$ $(x+1)(x+3) = 0$ $x = -1 \quad or \quad x = -3$
4.	4.1 Solve for m: $F = \text{ma}$; $m = \frac{F}{a}$ 4.2 Solve for b: $P = 2b + 2l$; $-2b = -P + 2l$; $b = \frac{P}{2} - l$ 4.3 Solve for b: $a = b + \frac{c}{2}$; $b = a - \frac{c}{2}$

Solutions to Lines activities

1.	$G\hat{B}E = G\hat{E}B \quad (\angle s\ opp\ equal\ sides)$ $\therefore a = b$ $a + b + 48^o = 180^o \quad (sum\ of\ \angle s\ in\ \Delta)$ $but\ a = b$ $\therefore a + a + 48^o = 180^o$ $2a = 132^o$ $a = 66^o$ $b = 66^o$ $34^o + a + c = 180^o \quad (\angle s\ on\ str\ line)$ $34^o + 66^o + c = 180^o$ $c = 80^o$ $34^o + a = d \quad (alt\ \angle s; AC \parallel DF)$ $34^o + 66^o = d$ $d = 100^o$ $b + e = c \quad (alt\ \angle s; AC \parallel DF)$ $66^o + e = 80^o$ $e = 14^\circ$
2.	$\hat{C} = \hat{D} \quad (\angle s\ opp\ equal\ sides)$ $\therefore \hat{D} = 2x - 10^\circ$ $\hat{B}_1 = \hat{C} + \hat{D}$ $\hat{B}_1 = 2x - 10^\circ + 2x - 10^\circ$ $108^\circ = 4x - 20^\circ$ $x = 32^\circ$

3.	$(4)^2+(BC)^2=(5)^2$ $16+(BC)^2=25$ $(BC)^2=9$ $BC=\pm 3$ $\therefore BC=3$
4.	$(DF)^2=\left(\sqrt{113}\right)^2$ $(DF)^2=113$ $(DE)^2=8^2=64$ $(EF)^2=7^2=49$ $(DE)^2+(EF)^2=64+49=113$ $\therefore (DE)^2+(EF)^2=(DF)^2$ ΔDEF is right-angled, and $\hat{E}=90^\circ$
5.	$\hat{D}+\hat{E}+\hat{F}=180^\circ$ (sum of $\angle$s in Δ) $110^\circ+35^\circ+\hat{F}=180^\circ$ $\hat{F}=35^\circ$ $\therefore DE=DF$ (sides opp equal $\angle$s)
6.	Yes alt $\angle$s =
7.	$120^\circ+3x+31^\circ+x+35^\circ+2x=360^\circ$ ($\angle$s round a pt) $6x=174^\circ$ $x=29^\circ$
8.	$\hat{B}_1+\hat{B}_2=30^\circ+150^\circ$ $=180^\circ$ $\therefore ABC$ is a straight line (adj $\angle$s supp)
9.	9.1 Parallel 9.2 co-interior

Solutions to Circle Geometry activities

1.1.1

$OD \perp CE \qquad (line\ from\ center\ to\ midpoint\ of\ chord)$

$In \triangle ODC$

$OC^2 = DC^2 + OD^2$

$OC^2 = (6)^2 + (4)^2$

$\sqrt{OC^2} = \sqrt{52}$

$OC = 2\sqrt{13}$

$but\ OC = OH = 2\sqrt{13} \qquad (radii)$

1.1.2

$OG = \frac{1}{2}OH = \frac{1}{2} \times 2\sqrt{13} = \sqrt{13}$

$In \triangle OGH$

$OH^2 = OG^2 + GH^2$

$(2\sqrt{13})^2 = (\sqrt{13})^2 + GH^2$

$52 - 13 = GH^2$

$\sqrt{39} = \sqrt{GH^2}$

$GH = \sqrt{39}$

$but\ OG \perp FH \qquad (given)$

$thus\ FG = GH = \sqrt{39} \qquad (line\ from\ center \perp to\ chord)$

1.2.1

$$2x = 120^0 \qquad (\angle \text{ at center} = 2 \times \angle \text{ at circumference})$$
$$x = \frac{120^0}{2} = 60^0$$

1.2.2

$$x = 2 \times 105^0 = 210^0 \qquad (\angle \text{ at center} = 2 \times \angle \text{ at circumference})$$

1.2.3

$$x = 2 \times 20^0 = 40^0 \qquad (\angle \text{ at center} = 2 \times \angle \text{ at circumference})$$

1.2.4

$$2x = 85^0 \qquad (\angle \text{ at center} = 2 \times \angle \text{ at circumference})$$
$$x = \frac{85^0}{2} = 42.5^0$$

1.2.5

$$\hat{O}_1 = 64^0 \qquad (\text{alternating } \angle s, AO \parallel BC)$$
$$2x = 64^0 \qquad (\angle \text{ at center} = 2 \times \angle \text{ at circumference})$$
$$x = \frac{64^0}{2} = 32^0$$

1.2.6

$OX = OZ$ $(radii)$

$\hat{Z}_1 = 40^0$ $(\angle s\, opp = sides)$

$In \triangle OXZ$

$\hat{Z}_1 + 40^0 + \hat{O}_1 = 180^0$ $(Int \angle s \triangle)$

$40^0 + 40^0 + \hat{O}_1 = 180^0$

$\hat{O}_1 = 180^0 - 40^0 - 40^0$

$\hat{O}_1 = 100^0$

$\hat{O}_2 + \hat{O}_1 = 360^0$ $(\text{Re}\,volution)$

$\hat{O}_2 + 100^0 = 360^0$

$\hat{O}_2 = 360^0 - 100^0$

$\hat{O}_2 = 260^0$

$2x = 260^0$ $(\angle\, at\, center = 2 \times \angle\, at\, circumference)$

$x = \frac{260^0}{2} = 130^0$

1.3.1

$\hat{Q}_1 = 18^0$ $(\angle s\, opp = sides)$

$x + \hat{Q}_1 = 90^0$ $(\angle\, in\, semi\, circle)$

$x + 18^0 = 90^0$

$x = 90^0 - 18^0 = 72^0$

1.3.2

$2\hat{B}_1 = 60^0$ $(\angle\, at\, center = 2 \times \angle\, at\, circumference)$

$\hat{B}_1 = \frac{60^0}{2} = 30^0$

$x + \hat{B}_1 = 90^0$ $(\angle\, in\, semi\, circle)$

$x + 30^0 = 90^0$

$x = 90^0 - 30^0 = 60^0$

1.4.1

$x = 98^0$ $(\tan chord\, theorem)$

1.4.2

$$z = 28^0 \qquad (\tan chord\ theorem)$$
$$y = 90^0 \qquad (\angle in\ semi\ circle)$$
$$x + y + z = 180^0 \qquad (Int\ \angle s\ of\ \square)$$
$$x + 90^0 + 28^0 = 180^0$$
$$x = 180^0 - 90^0 - 28^0$$
$$x = 62^0$$

1.4.3

$$x = 20^0 \qquad (alternating\ \angle s, DE \parallel MN)$$
$$y = 20^0 \qquad (\tan chord\ theorem)$$
$$20^0 + 20^0 + z = 180^0 \qquad (Int\ \angle s\ of\ \square)$$
$$z = 180^0 - 20^0 - 20^0$$
$$z = 140^0$$

1.4.4

$$x + 35^0 = 90^0 \qquad (\tan \perp radius)$$
$$x = 90^0 - 35^0 = 55^0$$
$$z = 35^0 \qquad (\tan chord\ theorem)$$
$$\hat{C}_4 = 35^0 \qquad (\tan s\ \ from\ same\ po\text{int})$$
$$In \square PBC$$
$$y + \hat{C}_4 + 35^0 = 180^0 \qquad (Int\ \angle s\ of\ \square)$$
$$y + 35^0 + 35^0 = 180^0$$
$$y = 180^0 - 35^0 - 35^0$$
$$y = 110^0$$

1.4.5

$$x = 22^0 \qquad (alternating\ \angle s, EO \parallel ND)$$
$$y = 90^0 \qquad (\tan \perp radius)$$

1.4.6

$$2x = 70^0 \qquad (\angle\, at\, center = 2\times\angle\, at\, circumference)$$
$$x = \frac{70^0}{2} = 35^0$$
$$y = x \qquad (\tan chord\, theorem)$$
$$y = 35^0$$
$$z + y = 90^0 \qquad (\tan \perp radius)$$
$$z + 35^0 = 90^0$$
$$z = 90^0 - 35^0 = 55^0$$

1.4.7

$$x = 66^0 \qquad (\tan chord\, theorem)$$
$$y = 90^0 \qquad (\angle\, in\, semi\, circle)$$
$$In \triangle NMP$$
$$z + x + y = 180^0 \qquad (Int\, \angle s\, of \triangle)$$
$$z + 66^0 + 90^0 = 180^0$$
$$z = 180^0 - 66^0 - 90^0$$
$$z = 24^0$$

1.5.1

$$2x = 112^0 \qquad (\angle\, at\, center = 2\times\angle\, at\, circumference)$$
$$x = \frac{112^0}{2} = 56^0$$
$$y + x = 180^0 \qquad (opp\, \angle s\, of\;\, cyclic\, quad)$$
$$y + 56^0 = 180^0$$
$$y = 180^0 - 56^0 = 124^0$$

1.5.2

$$2x = 80^0 \qquad (\angle\, at\, center = 2\times\angle\, at\, circumference)$$
$$x = \frac{80^0}{2} = 40^0$$
$$y = x + 20^0 \qquad (ext\,\angle = \text{int}\, opp\,\angle)$$
$$y = 40^0 + 20^0 = 60^0$$

1.5.3

$x = 70^0$ $(Alternating \angle s, RT \parallel QP)$

$In \triangle QTP$

$\hat{T}_2 = x$ $(\angle s\ opp = sides)$

$\hat{T}_2 = 70^0$

$y + x + \hat{T}_2 = 180^0$ $(Int \angle s\ of \triangle)$

$y + 70^0 + 70^0 = 180^0$

$y = 180^0 - 70^0 - 70^0$

$y = 40^0$

$z = y$ $(\tan chord\ theorem)$

$z = 40^0$

1.5.4

$x = 90^0$ $(line\ from\ center\ to\ midpoint\ of\ chord)$

$y + 30^0 = 180^0$ $(opp \angle s\ of\ cyclic\ quad)$

$y = 180^0 - 30^0 = 150^0$

1.5.5

$x = 28^0$ $(\tan chord\ theorem)$

$y = x = 28^0$ $(\angle s\ in\ the\ same\ segment)$

1.5.6

$x = y = 48^0$ $(\angle s\ in\ the\ same\ segment)$

1.5.7

$x = 32^0$ $(\tan chord\ theorem)$

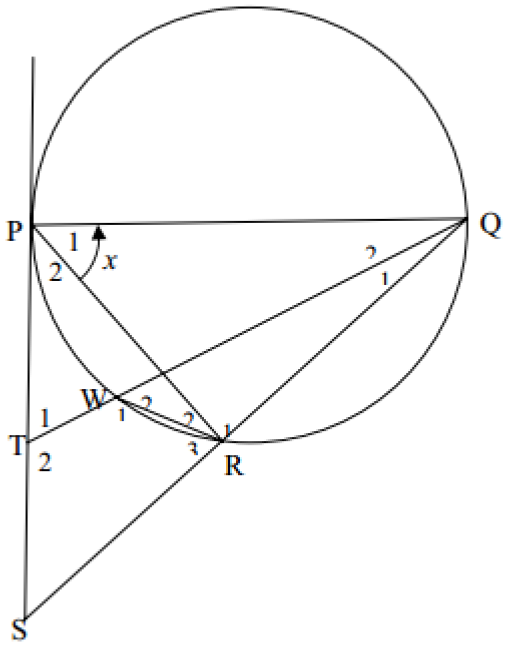
$y = x = 32^0$ $(equal\ chord; equal \angle s)$

1.5.8

$y = x = 48^0$ $(equal\ chords; equal\ \angle s)$

Solutions to Similarity and Proportionality activities

Question 1



1.1 $\hat{R}_1 = 90°$ …(angle in a semi-circle)

1.2 $\hat{P}_2 = 90° - x$ …(angle between radius and tangent)

$\hat{S} = 90° - \hat{P}_2$ …(ext. angle of Triangle)(sum of angles of triangle)

$= 90° - (90° - x) = x$

$\therefore \hat{P}_1 = \hat{S} = x$

1.3 $\hat{W}_2 = \hat{P}_1 = x$ …(angles in the same segment)

Also $\hat{S} = x$ …(proved 9.2)

$\hat{W}_2 = \hat{S}$

$\therefore$ SRWT is a cyclic quad…(ext angle = int. opposite angle)

1.4 In Δ QWR ; Δ QST

$\hat{W}_2 = \hat{S}$ ….(proved 9.3)

$\hat{Q}_1$ is common

$W\hat{R}Q = \hat{T}_2$ ….(remaining angles)

Δ QWR ||| Δ QST (AAA) or ($\angle\angle\angle$) or equiangular

1.5.1 $\frac{TS}{RW} = \frac{QT}{QR}$ ….. Δ QWR ||| Δ QST

$\therefore \frac{TS}{2} = \frac{8}{4}$

$4TS = 16$

$\therefore TS = 4\,cm$

1.5.2

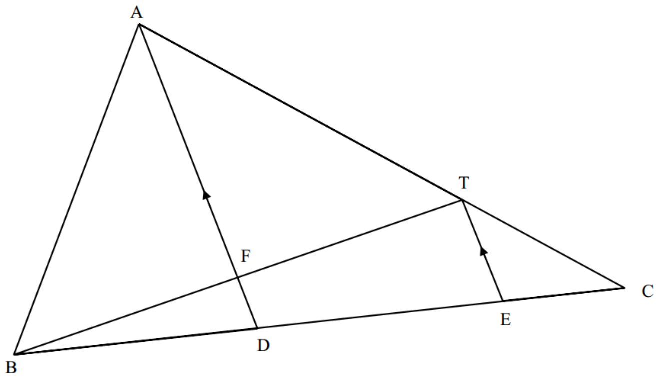
$\frac{SQ}{WQ} = \frac{TS}{RW}$

$SQ = \frac{4 \times 5}{2} = 10\,cm$

$\therefore SR = SQ - RQ$

$= 6\ cm$

Question 2



2.1

$$\frac{CE}{ED} = \frac{CT}{TA} = \frac{1}{2}$$

2.2 From 10.1 $\frac{CE}{ED} = \frac{1}{2}$

But DC = 9 cm
$\therefore$ DE = 6 cm
= BD.
$\therefore$ D is the midpoint of BE.

2.3

$$\frac{FD}{TE} = \frac{BD}{BE}$$

$$\frac{2}{TE} = \frac{6}{12}$$

$6 \times TE = 24$

$TE = 4\,cm$

ALTERNATIVE

D is the midpoint of BE. (from 10.2)
Then F is the midpoint of BT. … (sides in proportion)

$\therefore$ TE = 2FD (midpoint theorem)
= 4 cm

2.4.1 $\dfrac{\Delta ADC}{\Delta ABD}=\dfrac{3}{2}$

2.4.2

$$\frac{\Delta TEC}{\Delta ABC}=\frac{\Delta TEC}{\Delta TBC}\times\frac{\Delta TBC}{\Delta ABC}$$
$$=\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$$
$$=\frac{1}{15}$$

OR

$$\frac{\text{area }\Delta TEC}{\text{area }\Delta ABC}=\frac{\frac{1}{2}.TC.EC.\sin\hat{C}}{\frac{1}{2}.AC.BC.\sin\hat{C}}$$
$$=\frac{TC.EC}{AC.BC}$$
$$=\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$$
$$=\frac{1}{15}$$

QUESTION 3

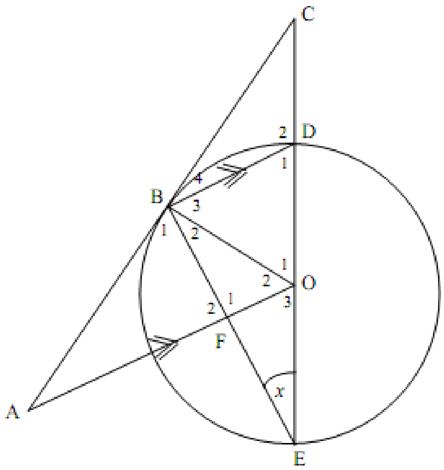
3.1.1	$\frac{AH}{HE}=\frac{2}{1}$ $AH = 2y$ $HE = y$ $\frac{AE}{ED}=\frac{2}{1}$ $ED = 1{,}5\,y$ $\frac{AH}{ED}=\frac{2}{1{,}5}$ $\frac{AH}{ED}=\frac{4}{3}$	(GHB ∥ FEC) (BE ∥ CD)
3.1.2	$\frac{BE}{CD}=\frac{4}{6}$ $=\frac{2}{3}$	(ΔAEB ∥∣ ΔADC)
3.2	$HE = 2$ cm $AH = 4$ cm $ED = 3$ cm $AD.HE = (AH + HE + ED).HE$ $= (4 + 2 + 3).(2)$ $= 18$	(given)

QUESTION 4

4.1	$\hat{D}_1 = \hat{A}_4$ $= \hat{C}_2$	(tan-chord theorem) (alt ∠'s , BA ∥ CE)

4.2	In ΔACF and ΔADC 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) $\Delta ACF \mathbin{\vert\vert\vert} \Delta ADC$ ($\angle\angle\angle$) **OR** In ΔACF and ΔADC 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) 3. $\hat{F}_1 = A\hat{C}D$ (remaining $\angle$s in triangles) $\Delta ACF \mathbin{\vert\vert\vert} \Delta ADC$
4.3	$\frac{AF}{AC} = \frac{AC}{AD}$ (sim Δ's $\therefore$ sides in proportion) $AF = \frac{AC.AC}{AD}$ $AC = AO = \frac{1}{2}AD$ (2radius = diameter) $AF = \frac{\frac{1}{2}AD.\frac{1}{2}AD}{AD}$ $AF = \frac{AD}{4}$ $4AF = AD$ **OR** ΔAOC is equilateral $\therefore A\hat{O}C = \hat{A}_3 = 60°$ $\cos 60° = \frac{AF}{AC} = \frac{1}{2}$ $AF = \frac{1}{2}AC = \frac{1}{2}AO$ $AF = \frac{1}{2}(\frac{1}{2}AD)$ (2radius = diameter) $AF = \frac{1}{4}AD$ $AD = 4AF$

QUESTION 5



5.1.1	$\hat{B}_4 = x$ (tan chord theorem) $\hat{A} = \hat{B}_4 = x$ (corres ∠; BD ∥ AO) $\hat{B}_2 = x$ (BO = EO = radii)	**Note:** If start with $\hat{A} = x$ and do not use tan ch th: max 2 marks
5.1.2	$D\hat{B}E = 90°$ (∠ in semi-circle) $C\hat{B}E = 90° + x$ **OR** $C\hat{B}O = 90°$ (rad ⊥ tan) $C\hat{B}E = 90° + x$ **OR** $\hat{O}_1 = 2x$ (∠ circ cent) $\hat{B}_3 = \hat{D}_1 = 90° - x$ (radii) $C\hat{B}E = x + (90° - x) + x$ $= 90° + x$	
5.1.3	$D\hat{B}E = 90°$ (proved in 8.2.2) $B\hat{F}O = 90°$ (co-int angles supp; BD ∥ AO) BF = FE (line from circ cent ⊥ ch bisect ch) F is the midpoint of EB	

	OR OD = OE (radii) BF = FE (BD ∥ AO) F is the midpoint of EB **OR** $B\hat{F}O = E\hat{F}O = 90°$ (BD ∥ AO) OF is common BO = OE (radii) $\Delta BOF \equiv \Delta EOF$ (90°HS) BF = FE ($\equiv \Delta s$) **OR** $\hat{B}_2 = \hat{A} = x$ (proven) $\hat{O}_2$ is common $\Delta AOB \mid\mid\mid \Delta BOF$ (AAA) $A\hat{B}O = B\hat{F}O$ $A\hat{B}O = 90°$ (proven) $A\hat{B}O = B\hat{F}O = 90°$ BF = FE (line from circ cent ⊥ ch bisects ch)

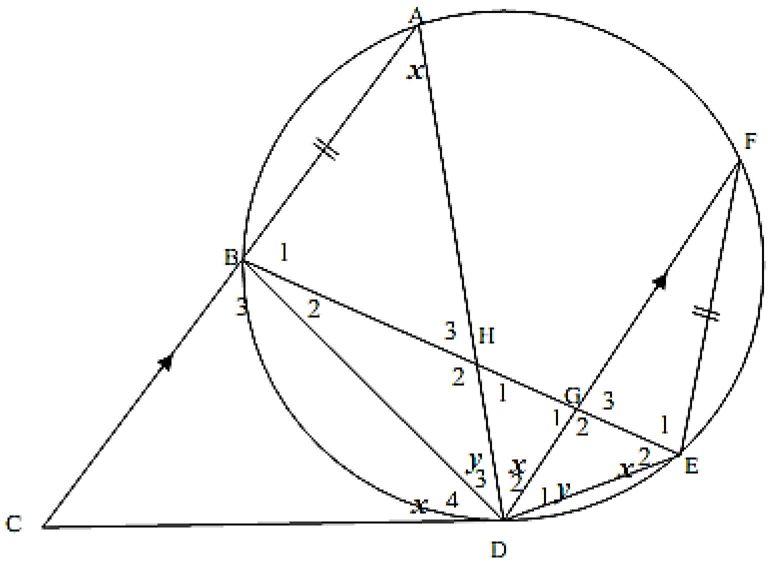
5.1.4	In ΔCBD and ΔCEB 1. $\hat{E} = \hat{B}_4 = x$ (proven in 8.2.1) 2. $\hat{C}$ is common 3. $\hat{D}_4 = C\hat{B}E = 90° + x$ $\Delta CBD \mid\mid\mid \Delta CEB$ (AAA)

5.1.5	$\frac{EB}{BD} = \frac{CE}{CB}$ (sim Δs $\therefore$ sides in proportion) EB.CB = CE.BD but EB = 2EF (F is the midpoint of BE) 2EF.CB = CE.BD

QUESTION 6

6.1	$M\hat{E}C = 90°$ (tan $\perp$ rad) $M\hat{D}C = 90°$ (line from cent bisects ch) $M\hat{E}C + M\hat{D}C = 180°$ $\therefore$ MDCE a cyclic quad (opp $\angle$s of quad supplementary) **OR** $M\hat{E}C = 90°$ (tan $\perp$ rad) $M\hat{D}A = 90°$ (line from cent bisects ch) $M\hat{E}C = M\hat{D}A$ $\therefore$ MDCE a cyclic quad (ext $\angle$ quad = int opp)
6.2	$MD^2 = MB^2 - DB^2$ (Pythagoras; ΔMBD) $MC^2 = MD^2 + DC^2$ (Pythagoras; ΔMDC) $= MB^2 - DB^2 + DC^2$
6.3	DB = 30 (given) MB = 40 (radii) $MC^2 = (40)^2 + (50)^2 - (30)^2$ $= 3\ 200$ $MC = 40\sqrt{2} = 56{,}57$ $MC^2 = ME^2 + CE^2$ (Pythagoras) $CE^2 = 3\ 200 - 1\ 600$ $CE^2 = 1\ 600$ CE = 40 mm

QUESTION 7



7.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) **OR** ($\angle$s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$s; CA ∥ DF)	✓ $\hat{A} = x$ ✓ tan ch th ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ alt $\angle$s; CA ∥ DF (6)
7.2	In ΔBHD and Δ FED 1. $\hat{B}_2 = \hat{F}$ ($\angle$s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$s) ΔBHD ||| Δ FED ($\angle\angle\angle$)	✓ $\hat{B}_2 = \hat{F}$ ✓ $\angle$s in same seg ✓ $\hat{D}_3 = \hat{D}_1$ ✓ = chs subt = $\angle$s ✓ $\angle\angle\angle$ (5)
7.3	$\frac{FE}{BH} = \frac{FD}{BD}$ (||| Δs) But FE = AB (given) $\frac{AB}{BH} = \frac{FD}{BD}$ AB.BD = FD.BH	✓ $\frac{FE}{BH} = \frac{FD}{BD}$ ✓ FE = AB (2) **[13]**

Appendix A: Examinations Guideline

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	$\angle$s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle$s supp
The adjacent angles in a revolution add up to 360°.	$\angle$s round a pt **OR** $\angle$s in a rev
Vertically opposite angles are equal.	vert opp $\angle$s =
If AB || CD, then the alternate angles are equal.	alt $\angle$s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp $\angle$s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int $\angle$s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle$s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle$s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint $\angle$s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle$s in Δ **OR** Int $\angle$s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle$s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle$s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S$\angle$S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** $\angle\angle$S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord

QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

Appendix B: Information Sheet

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad$ $A=P(1-ni)$ $\quad$ $A=P(1-i)^n$ $\quad$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$T_n=ar^{n-1}$ $\quad$ $S_n=\frac{a(r^n-1)}{r-1}$; $\quad r\neq 1$ $\quad$ $S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $\quad$ $P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad$ $y-y_1=m(x-x_1)$ $\quad$ $m=\frac{y_2-y_1}{x_2-x_1}$ $\quad$ $m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

$In\ \Delta ABC:\ \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad$ $a^2=b^2+c^2-2bc.\cos A$ $\quad$ $area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$ $\quad$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad$ $P(A\text{ or }B)=P(A)+P(B)-P(A\text{ and }B)$

$\hat{y}=a+bx$ $\quad$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

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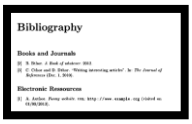
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