JENN

Training and Consultancy

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SUBJECT: MATHEMATICS

GRADE 12

2024 AUTUMN CLASSES

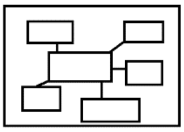
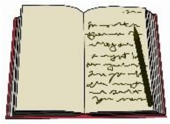
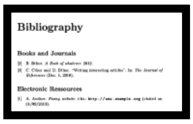
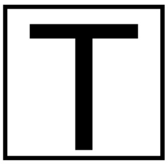
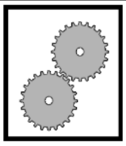
LEARNER CONTENT AND ACTIVITY MANUAL

TRIGONOMETRY

N.B: CALCULATOR MUST BE IN DEGREE MODE

CONTENTS PAGE

ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

FORMULAE FOR TRIGONOMETRY

Provided in the Information Sheet

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

Not provided in the Information Sheet

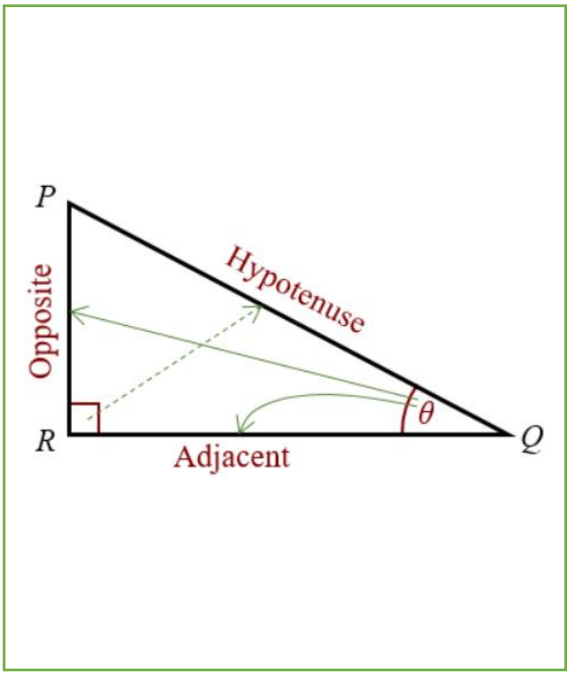
$\boldsymbol{sin^2\alpha + cos^2\alpha = 1}$ $\qquad \tan\alpha = \frac{\boldsymbol{sin\alpha}}{\boldsymbol{cos\alpha}}$

$\boldsymbol{sin^2\alpha = 1 - cos^2\alpha}$
$\boldsymbol{cos^2\alpha = 1 - sin^2\alpha}$

DEFINITIONS OF TRIGONOMETRIC RATIOS

Right-Angled Triangles

Soh Cah Toa



1.

$$\text{sine of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2.

$$\text{cosine of an angle } \theta = \frac{\text{length of the side adjacent to angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3

$$\text{tangent of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the side adjacent to angle } \theta}$$

$$\therefore \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

N.B: The Pythagoras Theorem

$$PQ^2 = PR^2 + QR^2$$

(THE HYPOTENUSE SQUARED IS EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES)

Worked Examples

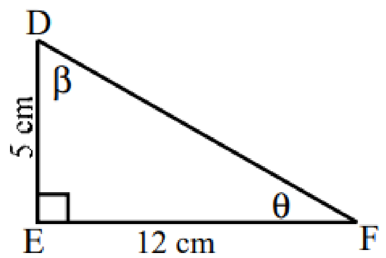
Example 1

In ΔDEF, $DE = 5$, $EF = 12$, $\hat{E} = 90°$, $\hat{D} = \beta$ and $\hat{F} = \theta$.

(a) Determine the length of the hypotenuse DF.

(b) Write the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

(c) Write the value of $\sin\beta$, $\cos\beta$ and $\tan\beta$.

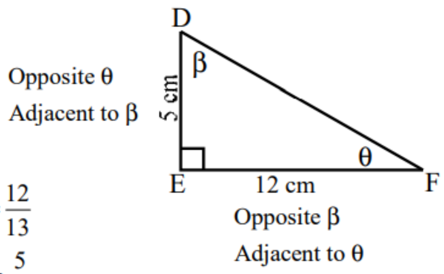


Solutions

(a) $DF^2 = 5^2 + 12^2$ [Pythagoras]

$\therefore DF^2 = 169$

$\therefore DF = 13$ cm



(b) $\sin\theta = \frac{opp}{hyp} = \frac{5}{13}$

$\cos\theta = \frac{adj}{hyp} = \frac{12}{13}$

$\tan\theta = \frac{opp}{adj} = \frac{5}{12}$

(c) $\sin\beta = \frac{opp}{hyp} = \frac{12}{13}$

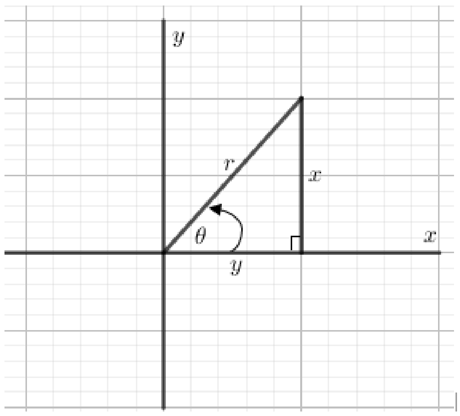
$\cos\beta = \frac{adj}{hyp} = \frac{5}{13}$

$\tan\beta = \frac{opp}{adj} = \frac{12}{5}$

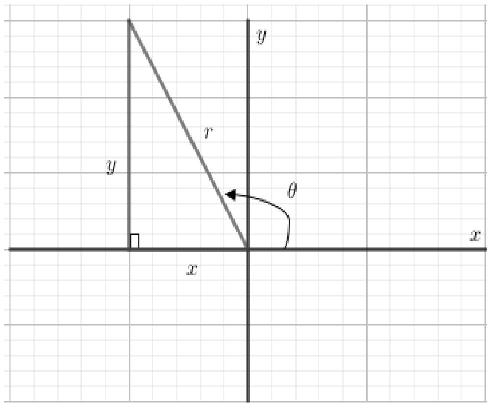
In terms of coordinates

Cartesian Plane

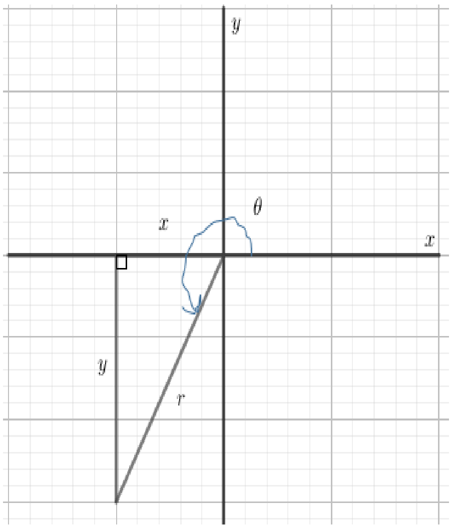
Quadrant 1



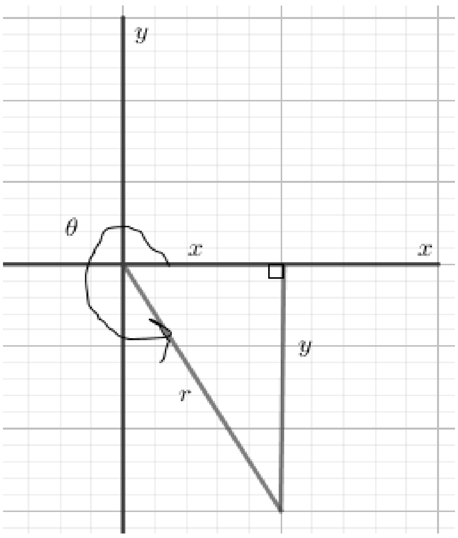
Quadrant 2



Quadrant 3



Quadrant 4



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

N.B: Pythagoras Theorem (in terms of x, y *and* r)

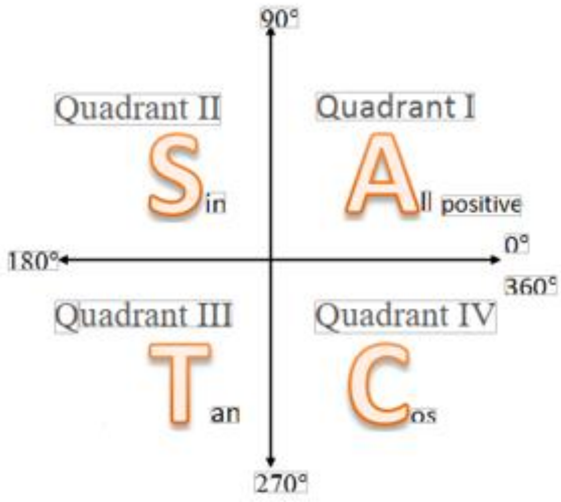
$x^2 + y^2 = r^2$

(Examples to becovered under the section ***CARTESIAN PLANE***)

REDUCTION FORMULAE

Reduction formulae are used to reduce the trigonometric ratio of any angle to the trigonometric ratio of an acute angle. The formulae you will use are $180° \pm \theta$, $360° \pm \theta$, $90° \pm \theta$ and $(-\theta)$.

Remember the **ASTC rule**



Quadrant 1	**Quadrant 2**	**Quadrant 3**	**Quadrant 4**
Sine +	Sine +	Sine –	Sine –
Cosine +	Cosine –	Cosine –	Cosine +
Tangent +	Tangent –	Tangent +	Tangent –
All	**Students (sin)**	**Take (tan)**	**Coffee (cos)**

Mnemonic to help you remember.

The reduction formulae

Cofunctions

$sin(90° + A) = cosA$

$cos(90° + A) = -sinA$

90°

2nd Quadrant

$\sin(180° - A) = \sin A$

$\cos(180° - A) = -\cos A$

$\tan(180° - A) = -\tan A$

$\sin(90° + A) = \cos A$

$\cos(90° + A) = -\sin A$

1st Quadrant

$\sin(90° - A) = \cos A$

$\cos(90° - A) = \sin A$

$\sin(360° + A) = \sin A$

$\cos(360° + A) = \cos A$

$\tan(360° + A) = \tan A$

Cofunctions

$sin(90° - A) = cosA$

$cos(90° - A) = sinA$

180° 360°

3rd Quadrant

$\sin(180° + A) = -\sin A$

$\cos(180° + A) = -\cos A$

$\tan(180° + A) = \tan A$

4th Quadrant

$\sin(360° - A) = -\sin A$

$\cos(360° - A) = \cos A$

$\tan(360° - A) = -\tan A$

$\sin(-A) = -sinA$
$\cos(-A) = cosA$
$\tan(-A) = -tanA$

270°

N.B

1. For angles greater than 360° you can subtract multiples of 360° until you get an angle between 0° and 360°.
   e.g Simplify $sin750°$
   $= sin30°$ { **30° = 750° – 360° -360°** }
   $= \frac{1}{2}$

2. For angles less than 0° you can add multiples of 360° until you get an angle between 0° and 360°.
   e.g Simplify $sin - 315°$
   $= sin45°$ { **45° = -315° + 360°** }
   $= \frac{\sqrt{2}}{2}$

Worked Examples

Example 1

Simplify fully: $\sin(90^\circ - x).\cos(180^\circ + x) + \tan x.\cos x.\sin(x - 180^\circ)$

Solution

$$\sin(90^\circ - x).\cos(180^\circ + x) + \tan x.\cos x.\sin(x - 180^\circ)$$

$$= \cos x.(-\cos x) + \frac{\sin x}{\cos x}.\cos x.(-\sin x)$$

$$= -\cos^2 x - \sin^2 x$$

$$= -(\cos^2 x + \sin^2 x)$$

$$= -1$$

$\sin(x - 180^\circ) = \sin(-(180^\circ - x))$
$= -\sin(180^\circ - x)$
$= -sinx$

OR

$\sin(x - 180^\circ)$
$= \sin(x - 180^\circ + 360^\circ)$
$= \sin(180^\circ + x)$
$= -sinx$

Example 2

Simplify fully:

$$\frac{\sin(180^\circ - x).\cos(360^\circ + x).\tan(-x)}{\cos(90^\circ - x).\cos(180^\circ + x).\tan(180^\circ + x)}$$

Solution

$$\frac{\sin(180^\circ - x).\cos(360^\circ + x).\tan(-x)}{\cos(90^\circ - x).\cos(180^\circ + x).\tan(180^\circ + x)}$$

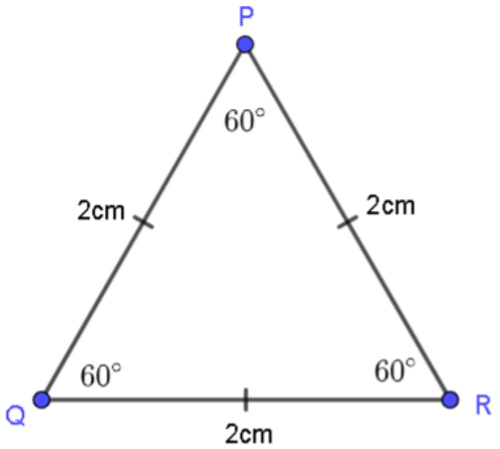
$$= \frac{(+\sin x).(+\cos x).(-\tan x)}{(+\sin x).(-\cos x).(+\tan x)}$$

$$= +1$$

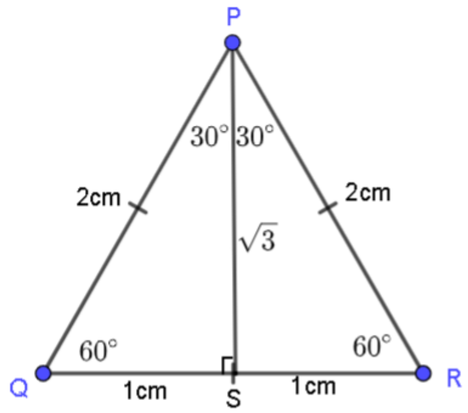
SPECIAL ANGLES

The following are ratios of the special angles (30°, 45° and 60°)

From the equilateral triangle:



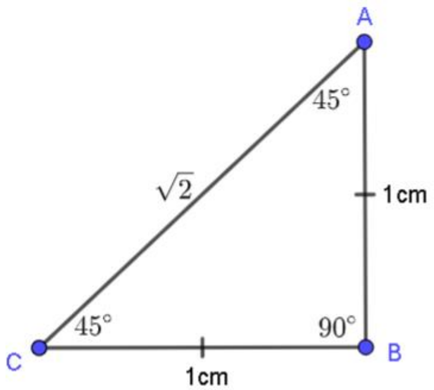
then drop the perpendicular PS



$PS \perp QR$

$PS = \sqrt{3}(from Pythagoras theorem)$

From the isosceles triangle:

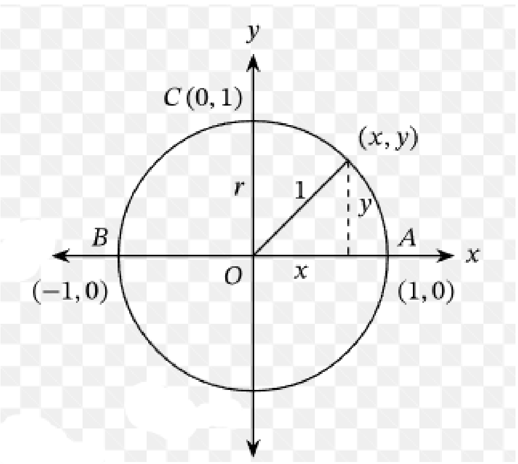


$AC = \sqrt{2}(from Pythagoras theorem)$

If you find it difficult to remember the diagrams, then learn this summary of the special angles.

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

The following are ratios of the special angles (0°, 90°, 180°, and 360°)



Angle A is at position 0°/360°

Angle C is at position 90°

Angle B is at position 180°

N.B: Remember that on the cartesian plane $sin\alpha = \frac{y}{r}$, $cos\alpha = \frac{x}{r}$ *and* $tan\alpha = \frac{y}{x}$

α	0° / 360°	90°	180°
$sin\alpha$	0	1	0
$cos\alpha$	1	0	-1
$tan\alpha$	0	UNDEFINED	0

Worked Examples

Example 1

Prove, WITHOUT using a calculator, that

$$\frac{\sin 315°.\tan 210°.\sin 190°}{\cos 100°.\sin 120°} = \frac{-\sqrt{2}}{3}$$

Use the reduction formula : (360°-A)

$sin315° = \sin(360° - 45°)$
$= (-sin45°)$

Solution

$$LHS = \frac{\sin 315°.\tan 210°.\sin 190°}{\cos 100°.\sin 120°}$$

$$= \frac{(-\sin 45°)(\tan 30°)(-\sin 10°)}{(-\sin 10°)(\sin 60°)}$$

$$= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}}$$

$$= -\frac{\sqrt{2}}{3}$$

Use the reduction formula : (90+-A)

$cos100° = \cos(90° + 10°)$
$= (-sin10°)$

Example 2

Simplify WITHOUT using a calculator:

$$\frac{\sin 120°.\cos 210°.\tan 315°.\cos 27°}{\sin 63°.\cos 540°}$$

Solution

$$\text{LHS} = \frac{\sin 120°.\cos 210°.\tan 315°.\cos 27°}{\cos 540°.\sin 63°}$$

$$= \frac{\sin 60°.(-\cos 30°)(-\tan 45)°.\sin 63°}{\cos 180°.\sin 63°}$$

$$= \frac{\frac{\sqrt{3}}{2}.\frac{-\sqrt{3}}{2}(-1)}{-1}$$

$$= -\frac{3}{4}$$

Example 3

Simplify fully, WITHOUT the use of a calculator:

$$\frac{\cos(-225°).\sin 135° + \sin 330°}{\tan 225°}$$

$\cos(-225°) = \cos 225°$
$= \cos(180° + 45°)$

Or

Add 360° to -225°

$\cos(-225°) = \cos 135°$
$= \cos(180° - 45°)$
$= -\cos 45°$

Solution

$$\frac{\cos(180° + 45°)\sin(180° - 45°) + \sin(360° - 30°)}{\tan(180° + 45°)}$$

$$= \frac{(-\cos 45°).(\sin 45°) - \sin 30°}{\tan 45°}$$

$$= \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1}$$

$$= -1$$

TRIGONOMETRIC IDENTITIES

In this topic we will revise trigonometric identities learnt in Grade 11. We will further have a look at two more identities in Grade 12. The examples that will follow will require you to use the information of the topics already covered above (special angles and reduction formulae)

Grade 11 revision

The square identity formula	The quotient identity formula
$sin^2\alpha + cos^2\alpha = 1$ $sin^2\alpha = 1 - cos^2\alpha$ $cos^2\alpha = 1 - sin^2\alpha$	$\tan\ \alpha = \dfrac{\sin\alpha}{\cos\alpha}$

N.B: The above formulae will not be given on the information sheet; you must learn them by heart.

Grade 12 Identities (provided on the information sheet)

Compound Angle Formulas	$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$
Double Angle Formulas	$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$	$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

N.B: if you come across questions with double or compound angles with the tangent ratio, use the quotien identity formula:

1. $tan2\alpha = \dfrac{sin2\alpha}{cos2\alpha}$
2. $\tan\ (A - B) = \dfrac{\sin(A-B)}{\cos(A-B)}$

Worked Examples

Example 1

Without using a calculator, evaluate

Cos 79° cos 311° + sin 101° sin 49°

Solution

$\cos 79° \cos 311° + \sin 101° \sin 49°$

$= \cos 79° \cos 49° + \sin 79° \sin 49°$

$= \cos(79° - 49°)$

$= \cos 30°$

$= \frac{\sqrt{3}}{2}$

$\cos 311° = \cos(360° - 49°)$
$= \cos 49°$

$\sin 101° = \sin(180° - 79°)$
$= \sin 79°$

Example 2

Given: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$

Prove that $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$

Solution

LHS: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x}$

$= \frac{\cos x}{2\sin x \cos x} - \frac{1 - 2\sin^2 x}{2\sin x}$

$= \frac{1}{2\sin x} - \frac{(1 - 2\sin^2 x)}{2\sin x}$

$= \frac{1 - 1 + 2\sin^2 x}{2\sin x}$

$= \frac{2\sin^2 x}{2\sin x}$

$= \sin x$

=RHS

Double angel($\sin 2x$) – immediately apply the formula as there is a single angle ($\sin x$) in the denominator

Double angel ($\cos 2x$) – immediately apply the formula as there is a single angle ($\cos x$) in the numerator

Example 3

Simplify the following expression.

$$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$

Solution

$$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$

$$=\frac{(-\sin x)(-\sin x)-\cos^2 x}{\cos 2x}$$

$$=\frac{\sin^2 x-\cos^2 x}{\cos^2 x-\sin^2 x} \quad \textbf{OR} \quad \frac{-\cos 2x}{\cos 2x}$$

$$=\frac{-(\cos^2 x-\sin^2 x)}{\cos^2 x-\sin^2 x} \qquad =-1$$

$$=-1$$

$\cos^2(180°-x)$
$=[\cos(180°-x)]^2$
$=[-\cos x]^2$
$=\cos^2 x$

Example 4

Prove that $\dfrac{\sin 2x+\sin x}{\cos 2x+\cos x+1}=\tan x$

Solution

$$LHS=\frac{\sin 2x+\sin x}{\cos 2x+\cos x+1}$$

$$=\frac{2\sin x\cos x+\sin x}{2\cos^2 x-1+\cos x+1}$$

$$=\frac{\sin x(2\cos x+1)}{2\cos^2 x+\cos x}$$

$$=\frac{\sin x(2\cos x+1)}{\cos x(2\cos x+1)}$$

$=\tan x$
= RHS

Double angel($\sin 2x$) – immediately apply the formula as there is a single angle ($\sin x$) in the numerator

Double angel($\cos 2x$) – immediately apply the formula as there is a single angle ($\cos x$). N.B apply the formula that has (-1) to eliminate (+1) in the denominator

Example 5

Prove that $\dfrac{\sin^3 x + \sin x \cos^2 x}{\cos(360° - x)} = \tan x$

Solution

$$\frac{\sin^3 x + \sin x \cos^2 x}{\cos(360° - x)}$$

$$= \frac{\sin x(\sin^2 x + \cos^2 x)}{\cos x}$$

$$= \frac{(\sin x)(1)}{\cos x} = \tan x$$

Use the square identity formula:

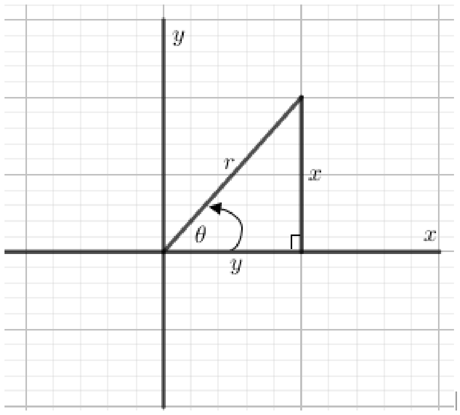
$$\boldsymbol{sin^2 x + cos^2 x = 1}$$

CARTESIAN PLANE (USING THE DIAGRAM)

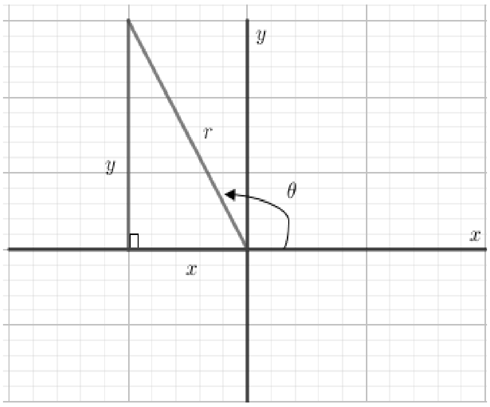
In the exam or test you will need to use the aid of the diagram to answer some questions. The following diagrams will show us how to draw our right-angled triangles in different quadrants.

N.B $\boldsymbol{\theta}$ ***is an angle measured from the positive x − axis to the termila arm (radius)***

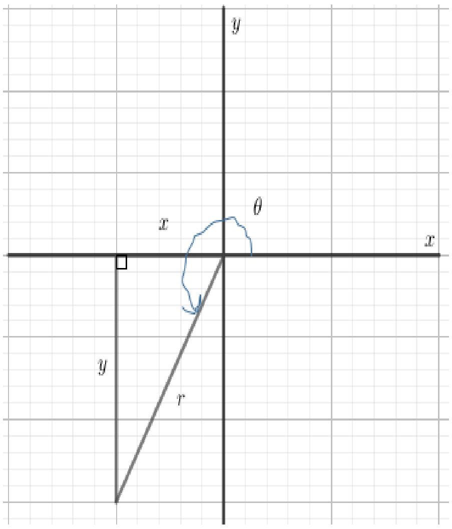
Quadrant 1



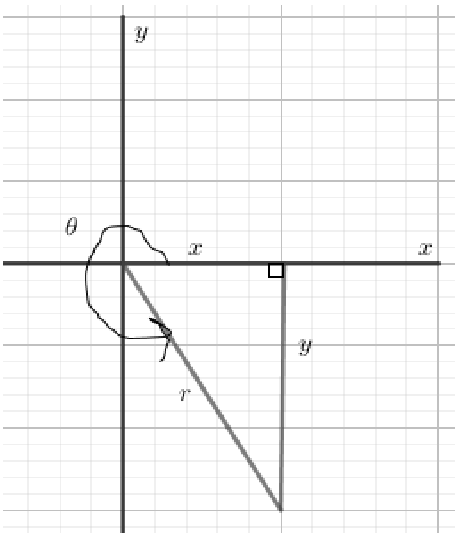
Quadrant 2



Quadrant 3



Quadrant 4



In this section you will represent the trigonometric ratios in terms of $x, y\ and\ r$, since you will be working in a cartesian plane.

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

N.B: Pythagoras Theorem

$x^2 + y^2 = r^2$

Below are examples on how to draw the right-angled triangle in the correct quadrant:

Worked Examples

Example 1

Draw the right-angled triangle under the conditions below:

a. $\sin\theta < 0$ and $90 < \theta < 270$.
b. $\cos\theta > 0$ and $\tan\theta < 0$.

Draw the right-angle triangle in the correct quadrant and find the values of $x, y, and\ r$:
c. $7\tan\theta + 3 = 0$ and $\theta \in [90°: 270°,]$

Solutions

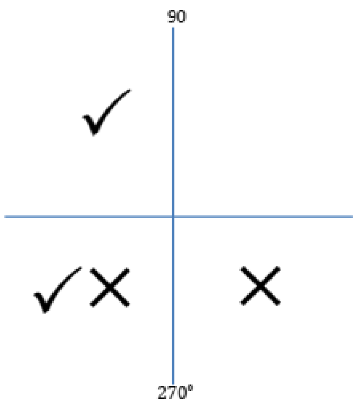
a. $\sin\theta < 0$ and $90 < \theta < 270$.

Step 1:
→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

Sine is negative in the 3rd and 4th quadrants.

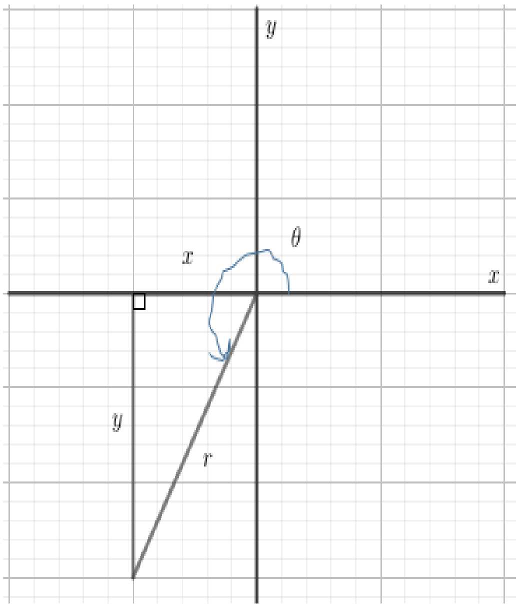
This interval represents 2nd and 3rd quadrants

Given that $\sin\theta < 0$ and $90 < \theta < 270$.



Where **X** represents the quadrant we obtained from sin and √ is the quadrants obtained from the interval. We can notice the 3rd quadrant it satisfy both conditions , hence the terminal arm will be in the 3rd quadrant.

→ Step 2: Complete the right-angled triangle.



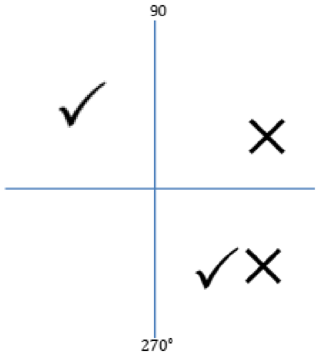
b. $\cos\theta > 0$ and $\tan\theta < 0$

Step 1:
→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

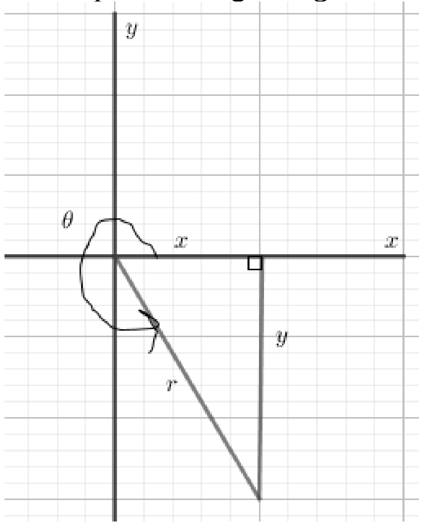
cosine is positive in the 1st and 4th quadrants.

tangent is negative in the 2nd and 4th quadrants

Given that $\cos\theta > 0$ and $\tan\theta < 0$.



Where X represents the quadrant we obtained from cos and √ is the quadrants obtained from the tan . We can notice the 4th quadrant it satisfy both conditions , hence the terminal arm will be in the 4th quadrant.
→ Complete the right-angled triangle.



c. $7\tan\theta + 3 = 0$ and $\theta \in [90°: 270°]$

Step1:
Simplify the first trig equation in such a way that we only have trig ratio(tan θ) on the left hand side.

$7\tan\theta + 3 = 0$

$7\tan\theta = -3$

$\tan\theta = -\frac{3}{7}$

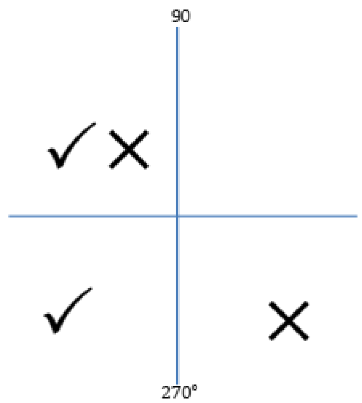
Step 2:

→use conditions to identify the quadrants and tick them with different symbols on the Cartesian plane .

tangent is negative in the 2^{nd} and 4^{th} quadrants.

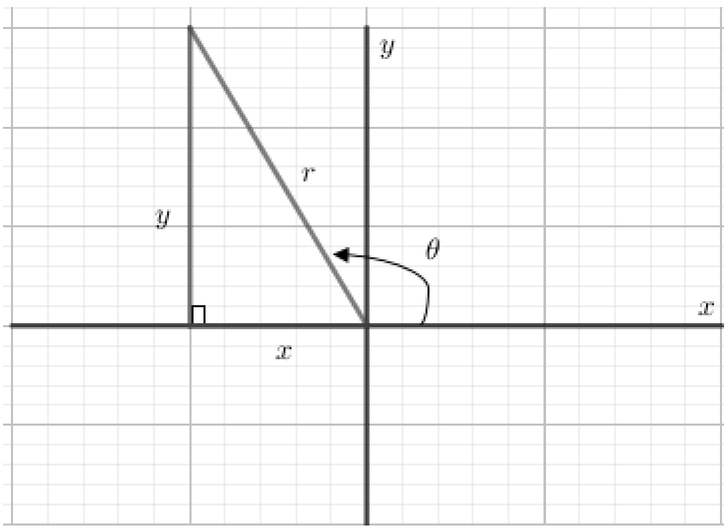
This interval represents 2^{nd} and 3^{rd} quadrants

After simplifying $\tan\theta = -\frac{3}{7} = \frac{y}{x}$ and $\theta \in [90°; 270°]$



Where X represents the quadrant we obtained from tan and tick is the quadrants obtained from the interval. We can notice the 2^{nd} quadrant satisfies both conditions , hence the terminal arm will be in the 2^{nd} quadrant.

→ Complete the right-angled triangle.



Now determine the values of $x, y, and\ r$:

$x^2 + y^2 = r^2 \qquad (Pythagoras)$

$(-7)^2 + (3)^2 = r^2$

$r^2 = 58$

$r = \sqrt{58}$

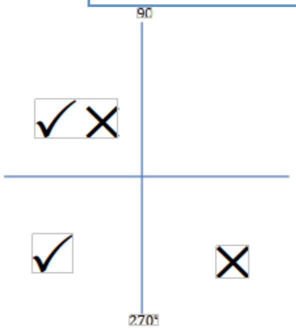
N.B: Remember hypotenuse (r) is ALWAYS positive, and for the values of x and y, they can be positive or negative depending on the quadrant you are working in.

Example 2

Given that $\sin\theta = \frac{3}{5}$, and $\theta \in [90°: 360°,]$ find the value of the following *without the use of a calculator*.

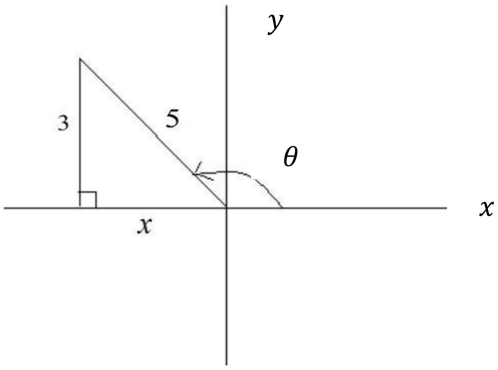
without a calculator– means you cannot use the calculator to calculate the angle and then use the angle to calculate the values. One way of approaching this question is to draw a right-angle triangle in the correct quadrant.

Solutions:



We are going to draw the triangle in a 2nd quadrant.

$\sin\theta = \frac{3}{5} = \frac{y}{r}$,



In the triangle above we do not have the value of x, then we will calculate it using **theorem of Pythagoras**:

$$x^2 + y^2 = r^2 \quad (Pythagoras)$$
$$x^2 + (3)^2 = (5)^2$$
$$x^2 + 9 = 25$$
$$x^2 = 25 - 9$$
$$x^2 = 16$$
$$x = \sqrt{16}$$
$$x = \pm 4$$
$x = -4$, because the x value in the 2nd quadrant, x is negative.

Now Evaluate:

$a.\quad \boldsymbol{cos\ \theta}$
$b.\quad \boldsymbol{tan(180° - \theta)}$
$c.\quad \boldsymbol{sin(-\theta)}$
$d.\quad \boldsymbol{sin\ (\theta - 180°)}$
$e.\quad \boldsymbol{cos\ (450° + 2\theta)}$
$f.\quad \boldsymbol{sin\ (45° - \theta)}$

Solutions

a. $\cos\theta = \frac{x}{r}$
$= \frac{-4}{5}$
$= -\frac{4}{5}$

b. $\tan(180 - \theta)$
$= -\tan\theta$
And $\tan\theta = \frac{y}{x}$
$= -(\frac{3}{-4})$
$= \frac{3}{4}$

c. $\sin(-\theta)$
$= -\sin\theta$
$= -\frac{3}{5}$

d. $\sin(\theta - 180°)$
$= \sin(-180° + \theta)$
$= \sin[-(180° - \theta)]$
$= -\sin(180° - \theta)$
$= -\sin\theta$
$= -\frac{3}{5}$

e. $\cos(450 + 2\theta)$
$= \cos(450°-360° + 2\theta)$
$= \cos(90° + 2\theta)$
$= -\sin 2\theta$
$= -2\sin\theta.\cos\theta$
$= -2(\frac{3}{5}).(-\frac{4}{5})$
$= \frac{24}{25}$

f. $\sin(45° - \theta)$
$= \sin(45° - \theta°)$
$= \sin 45°.\cos\theta° - \cos 45°.\sin\theta°$
$= \frac{1}{\sqrt{2}}.\left(-\frac{4}{5}\right) - \frac{1}{\sqrt{2}}.\frac{3}{5}$
$= -4/(5\sqrt{2}) - \frac{3}{5\sqrt{2}}$
$= -\frac{7}{5\sqrt{2}}$
$= -7/(5\sqrt{2}) \times \frac{\sqrt{2}}{\sqrt{2}}$
$= -\frac{7\sqrt{2}}{10}$

Example 3

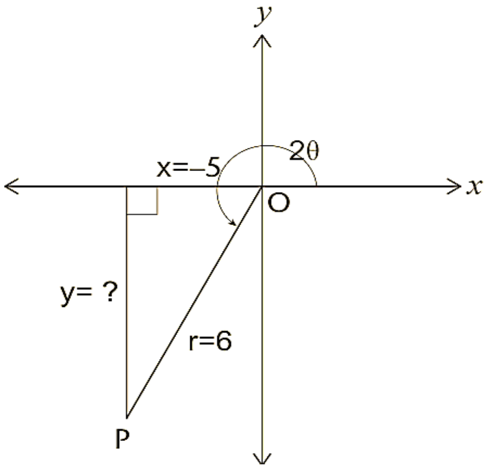
If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator,** the values in simplest form of:

$a.\ \boldsymbol{sin2\theta}$

$b.\ \boldsymbol{sin^2\theta}$

$c.\ \boldsymbol{tan2\theta}$

Solutions



Before we can answer the questions, we can note that in the triangle we do not have y value.

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = (6)^2$

$y^2 = 36 - 25$

$y^2 = 11$

$y = \sqrt{11}$

$y = \pm\sqrt{11}$

$y = -\sqrt{11}$ **, because the y value in the 3rd quadrant is negative.**

a. Sin 2θ

$= \frac{-\sqrt{11}}{6}$

b. $\sin^2\theta$

Use the double angle identity formula:

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = -\frac{5}{6}$

$1 - 2\sin^2\theta = -\frac{5}{6}$

$2\sin^2\theta = 1 + \frac{5}{6}$

$2\sin^2\theta = \frac{11}{6}$

$\sin^2\theta = \frac{11}{6} \div 2$

$\sin^2\theta = \frac{11}{6} \times \frac{1}{2}$

$\sin^2\theta = \frac{11}{12}$

c. $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

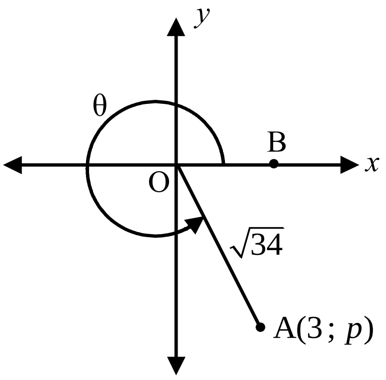
$= -\frac{\sqrt{11}}{6} \Big/ -\frac{5}{6}$

$= -\frac{\sqrt{11}}{6} \times \left(-\frac{6}{5}\right)$

$= \frac{\sqrt{11}}{5}$

Example 4

In the diagram below, $A(3; p)$ is a point in the Cartesian plane $OA = \sqrt{34}$ and $B\hat{O}A = \theta$, which is a reflex angle.



Without using a calculator, determine:

a. the value of p.

b. $\cos(60° + \theta)$

c. $\tan(90° - \theta)$

Solution

a. $x^2 + y^2 = r^2 \quad (Pythagoras)$

$$(3)^2 + (p)^2 = (\sqrt{34})^2$$
$$\therefore 9 + p^2 = 34$$
$$\therefore p^2 = 25$$
$$\therefore p = -5$$

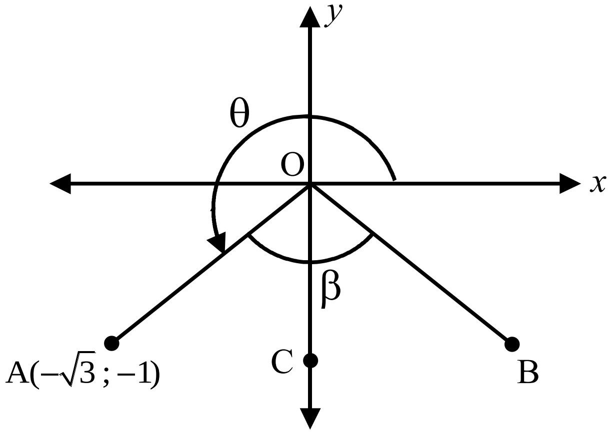
b.

$$\cos(60° + \theta)$$
$$= \cos 60° \cos\theta - \sin 60° \sin\theta$$
$$= \left(\frac{1}{2}\right)\left(\frac{3}{\sqrt{34}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-5}{\sqrt{34}}\right)$$
$$= \frac{3}{2\sqrt{34}} + \frac{5\sqrt{3}}{2\sqrt{34}}$$
$$= \frac{3 + 5\sqrt{3}}{2\sqrt{34}}$$

c. $\tan(90° - \theta) = \dfrac{\sin(90°-\theta)}{\cos(90°-\theta)} = \dfrac{cos\theta}{sin\theta} = \dfrac{\left(\frac{3}{\sqrt{34}}\right)}{\left(\frac{-5}{\sqrt{34}}\right)} = \left(\frac{3}{\sqrt{34}}\right) \times \left(\frac{\sqrt{34}}{-5}\right) = -\frac{3}{5}$

Example 5

In the diagram, A is the point $(-\sqrt{3}; -1)$.



Without using a calculator, determine the value of each of the following:

1.1.1 $tan\theta$

1.1.2 $\cos\theta$

1.1.3 the size of β, if OB is a reflection of OA about the y-axis.

Solution

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 180 + 30° = 210°$$

$$\therefore A\hat{O}C = 60°$$

$$\therefore C\hat{O}B = 60°$$

$$\therefore \beta = 120°$$

Example 6

It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°; 270°]$ and $\beta \in [90°; 270°]$. Determine, without using a calculator, the values of the following:

a) $\cos\alpha$
b) $\cos(\alpha + \beta)$

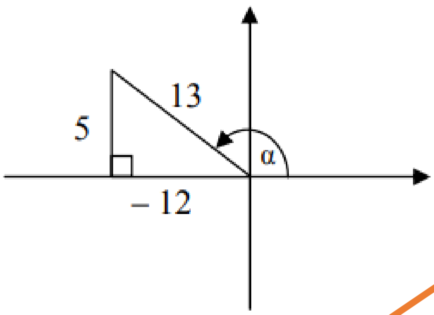
Solution

$\sin\alpha = \frac{5}{13}$

$y_\alpha = 5 \quad r_\alpha = 13$

$x_\alpha = -12$

$\cos\alpha = -\frac{12}{13}$

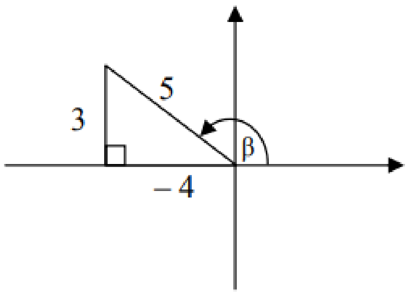


$x^2 + y^2 = r^2$
$x^2 + 5^2 = 13^2$
$x^2 = 144$
$x = \pm 12$
$\therefore x = -12$

$\tan\beta = -\frac{3}{4}$

$y_\beta = 3 \quad x_\beta = -4$

$r = 5$



$x^2 + y^2 = r^2$
$(-4)^2 + 3^2 = r^2$
$r^2 = 25$
$\therefore r = 5$

$\cos(\alpha + \beta)$

$= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{48 - 15}{65}$

$= \frac{33}{65}$

RATIOS IN TERMS OF A LETTER

In this section we focus on questions where the angle of a ratio is given, and we have to simplify in terms of the given letter.

Worked Examples

Example 1

If, $sin31° = p$ determine, without using a calculator, the following in terms of p:

a. $\sin 149°$
b. $\cos(-59°)$
c. $cos62°$
d. $sin59°$

Use the reduction formula:
$sin(180° - \alpha) = sin\alpha$
$sin149° = sin(180° - 31°)$
$= sin31°$

Solution

Method 1

a.
$\sin 149° = \sin 31° = p$

b.
$\cos(-59°) = \cos 59° = \sin 31° = p$

c.
$\cos 62° = 1 - 2\sin^2 31° = 1 - 2p^2$

Use the reduction formula:
$cos(90° - \alpha) = sin\alpha$
$\therefore$
$cos59° = \cos(90° - 31°)$
$= sin31°$

Use the double angle formula
$cos2\alpha = 1 - 2\sin^2\alpha$
$\therefore$
$cos62° = cos2(31°)$
$= 1 - 2\sin^2 31°$

d.
$sin^2 59° + cos^2 59° = 1$
$sin^2 59 = 1 - cos^2 59$
$sin59° = \sqrt{1 - cos^2 59}$
$sin59° = \sqrt{1 - p^2}$

We have already shown in b.
that, $cos59° = sin31° = p$
$\therefore cos^2 59° = p^2$

Method 2

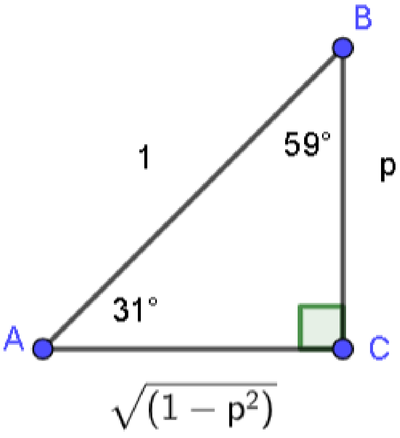
a.
$\sin 149° = \sin 31° = p$

b.
$\cos(-59°) = \cos 59° = \sin 31° = p$

c.
$\cos 62° = 1 - 2\sin^2 31° = 1 - 2p^2$

d. (solve the right triangle)

$\boldsymbol{sn31° = \frac{p}{1}} \left(\frac{\textbf{opposite}}{\textbf{hypotenuse}}\right)$



Step 1: fill the right triangle with given information (31°, p, and 1)

Step 2: find the missing side using Pythagoras theorem:

$AC^2 + BC^2 = AB^2 \qquad (Pythagoras)$
$AC^2 + p^2 = 1$
$AC^2 = 1 - p^2$
$AC = \sqrt{1 - p^2}$ {no need for $\pm$ *as the length of a triangle is alwasy*+}

Step 3: calculate the size of the missing angle (angle B) using sum of angles in a triangle theorem:
$\hat{B} + 31° + 90° = 180° \qquad (sum\ of\ angles\ in\ a\ triangle)$
$\hat{B} = 59°$

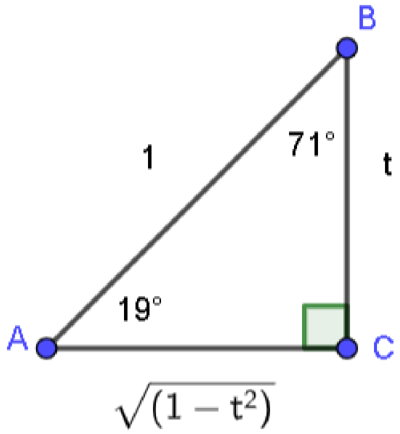
$sin\ 59° = \frac{\sqrt{1 - p^2}}{1} = \sqrt{1 - p^2}$

N.B: for examples 2 and 3, we will use only one method, however, you are free to use any method you are comfortable with.

Example 2

$If\ sin161° = t, express\ tan71°\ in\ terms\ of\ t.$

Solution



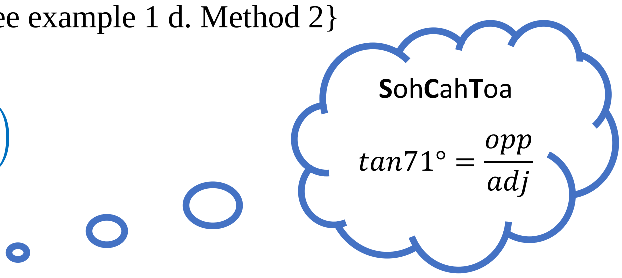
$sin161° = \sin(180° - 19°)$
$= sin19°$

$AC = \sqrt{1 - t^2}$ {see example 1 d. Method 2}

$\hat{B} = 71°$

$\therefore sin19° = \frac{t}{1}\left(\frac{opp}{hyp}\right)$

$tan71° = \frac{\sqrt{1 - t^2}}{t}$



Example 3

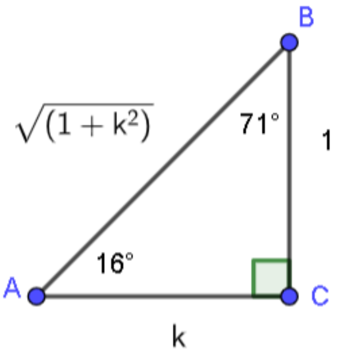
If $\sin 16° = \dfrac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a**

a. $tan16°$

b. $cos32°$

Solution

a.



$sin16° = \dfrac{1}{\sqrt{1+k^2}} \quad \left(\dfrac{opp}{hyp}\right)$

$AC^2 + BC^2 = AB^2 \qquad (Pythagoras)$

$AC^2 + 1 = 1 + k^2$

$Ac^2 = k^2$

$\therefore AC = k$

$tan16° = \dfrac{1}{k}$

b.

$cos32° = cos2(16°)$

$= 1 - 2\sin^2 16°$

$= 1 - 2\left(\dfrac{1}{\sqrt{1+k^2}}\right)^2$

$= 1 - 2\left(\dfrac{1}{1+k^2}\right)$

TRIGONOMETRIC EQUATIONS

Use the following formulae to determine the general solutions of trigonometric equations:

If $sin\alpha = m$

> **Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $sin\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$ (see the mother graph sine on page 39). BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.c on page 34).**

Formula	$\alpha = \sin^{-1}(m) + k.360°$ or $\alpha = 180° - \sin^{-1}(m) + k.360°$ $, k \in Z$
Example 1, determine the genral solution of: $sinx = \frac{1}{2}$	$x = \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(\frac{1}{2}\right) + k.360°$ $x = 30° + k.360°$ or $x = 150° + k.360°$ $, k \in Z$
Example 2, determine the genral solution of: $sinx = -\frac{1}{2}$	$x = \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ or $x = 180° - \sin^{-1}\left(-\frac{1}{2}\right) + k.360°$ $x = -30° + k.360°$ or $x = 210° + k.360°$ $, k \in Z$

If $\cos\alpha = m$

> **Take note that, when solving trigonometric equations, after isolating the trigonometric ratio, in this case $cos\alpha = m$, then you will only have solutions when $-1 \leq m \leq 1$ (see the mother graph of cosine on page 41). BUT, when $m > 1$ or $m < -1$, there will be no solution (see example 7.b on page 33).**

Formula	$\alpha = cos^{-1}(m) + k.360°$ or $\alpha = -cos^{-1}(m) + k.360°$ $, k \in Z$
Example 3, determine the genral solution of: $cosx = 1$	$x = cos^{-1}(1) + k.360°$ or $x = -\cos^{-1}(1) + k.360°$ $x = 0° + k.360°$ or $x = 0° + k.360°$ $\therefore x = 0° + k.360°$ $, k \in Z$
Example 4, determine the genral solution of: $cosx = -1$	$x = cos^{-1}(-1) + k.360°$ or $x = -\cos^{-1}(-1) + k.360°$ $, k \in Z$ $x = 180° + k.360°$ or $x = -180° + k.360°$ $, k \in Z$

If $\tan\alpha = m$

Formula	$\alpha = tan^{-1}(m) + k.180°$ $, k \in Z$
Example 5, determine the genral solution of: $tanx = 3$ **(Round your answer to TWO decimal places)**	$x = tan^{-1}(3) + k.180°$ $x = 71{,}57° + k.180°$ $, k \in Z$
Example 6, determine the genral solution of: $tanx = -3$ **(Round your answer to TWO decimal places)**	$x = tan^{-1}(-3) + k.180°$ $x = -71{,}57° + k.180°$ $, k \in Z$

N.B: You may use other methods to determine the general solutions, however, the above method is highly recommeded

Example 7

Determine the general solutions of the following (round off answers to TWO decimal places where necessary:

$\boldsymbol{a}.$ $\sin^2 x - sinxcosx = 0$
$\boldsymbol{b}.$ $\cos^2 x - 2cosx - 3 = 0$
$\boldsymbol{c}.$ $2\cos^2 x + 7sinx = 5$
$\boldsymbol{d}.$ $\cos(x - 45°) = \sin 15°$
$\boldsymbol{e}.$ $sin3x = cosx$

Solution

Take out the common factor (sinx)

$\boldsymbol{a}.$ $\sin^2 x - sinxcosx = 0$
$sinx(sinx - cosx) = 0$
$sinx = 0 \quad or \quad sinx - cosx = 0$
$sinx = cosx$
$\frac{sinx}{cosx} = 1$
$tanx = 1$
Now determine the general solutions

$$\boldsymbol{sinx = 0}$$
$$\boldsymbol{x = \sin^{-1}(0) + k.360° \quad or \quad x = 180° - sin^{-1}(0) + k.360°}$$
$$\boldsymbol{x = 0° + k.360° \quad or \quad x = 180° + k.360° \quad , k \in Z}$$

OR

$$\boldsymbol{tanx = 1}$$
$$\boldsymbol{x = \tan^{-1}(1) + k.180°}$$
$$\boldsymbol{x = 45° + k.180° \quad , k \in Z}$$

b.

Quadratic equation

$\cos^2 x - 2cosx - 3 = 0$
$\cos^2 x - 2cox - 3 = 0$
$(cosx + 1)(cosx - 3) = 0$
$cosx = -1 \quad or \quad cosx = 3$

Now determine the general solutions

$$\boldsymbol{cosx = -1}$$
$$\boldsymbol{x = \cos^{-1}(-1) + k.360° \quad or \quad x = -cos^{-1}(-1) + k.360°}$$
$$\boldsymbol{x = 180° + k.360° \quad or \quad x = -180° + k.360° \quad , k \in Z}$$

OR

$$\boldsymbol{cosx = 3}$$
$$\boldsymbol{No\ Solution\ (-1 \leq cosx \leq 1)}$$

c.

$2\cos^2 x + 7sinx = 5$
$2\cos^2 x + 7sinx - 5 = 0$
$2(1 - \sin^2 x) + 7sinx - 5 = 0$
$-2\sin^2 x + 7sinx - 3 = 0$
$2\sin^2 x - 7sinx + 3 = 0$
$(2sinx - 1)(sinx - 3) = 0$
$sinx = \frac{1}{2} \quad or \quad sinx = 3$

Quadratic equation

Now determine the general solutions

$\boldsymbol{sinx = \frac{1}{2}}$
$\boldsymbol{x = \sin^{-1}\left(\frac{1}{2}\right) + k.360° \quad or \quad x = 180° - sin^{-1}\left(\frac{1}{2}\right) + k.360°}$
$\boldsymbol{x = 30° + k.360° \quad or \quad x = 150° + k.360° \quad , k \in Z}$

OR

$\boldsymbol{sinx = 3}$
$\boldsymbol{No\ Solution}$ $\boldsymbol{(-1 \leq sinx \leq 1)}$

d.

$\cos(x - 45°) = \sin 15°$
$\cos(x - 45°) = \sin(90° - 75°)$
$\cos(x - 45°) = cos75°$

Cofunctions

Now determine the general solutions
$x - 45° = \cos^{-1}(cos75°) + k.360° \quad or \quad x - 45° = -\cos^{-1}(cos75°) + k.360°$
$x - 45° = 75° + k.360° \quad or \quad x - 45° = -75° + k.360°$
$x = 120° + k.360° \quad or \quad x = -30° + k.360° \quad , k \in Z$

e.

$sin3x = cosx$
$sin3x = \sin(90° - x)$

Cofunctions

Now determine the general solutions
$3x = \sin^{-1}[\sin(90° - x)] \quad or \quad 3x = 180° - \sin\text{^}(-1)\,[\sin(90° - x)]$
$3x = 90° - x + k.360° \quad or \quad 3x = 180 - (90° - x) + k.360°$
$4x = 90° + k.360° \quad or \quad 3x = 90° + x + k.360°$
$4x = 90° + k.360° \quad or \quad 2x = 90° + k.360$
$x = 22{,}5° + k.90° \quad or \quad x = 45 + k.180° \quad , k \in Z$

Example 8 (restrictions on identities)

- **An identity with the function $\boldsymbol{tanx}$** is undefined for $\boldsymbol{x = 90° + k.180° \ \ , k \in Z}$.
- **An identity is undefined when any denominator is zero.**

a. For which values of B is the identity $\frac{cosB}{1+sinB} = \frac{1-sinB}{cosB}$ undefined?

b. For which values of A is the identity $\frac{1}{cosA} + tanA = \frac{cosA}{1-sinA}$ undefined?

Solution

a.

The indentity will be undefined when:

$1 + sinB = 0 \quad or \quad cosB = 0$

$\boldsymbol{1 + sinB = 0}$

$\boldsymbol{sinB = -1}$

$\boldsymbol{B = -90° + k.360° \quad or \quad B = 270° + k.360° \, , k \in Z}$

OR

$\boldsymbol{cosB = 0}$

$\boldsymbol{B = 90° + k.360° \quad or \quad B = -90° + k.360° \, , k \in Z}$

b.

The indentity will be undefined when:

$(\boldsymbol{we\ have\ tanA})$

$\therefore \boldsymbol{A = 90° + k.180° \ \ , k \in Z}$

OR

$\boldsymbol{cosA = 0}$

$\boldsymbol{A = 90° + k.360° \quad or \quad A = -90° + k.360° \, , k \in Z}$

OR

$\boldsymbol{1 - sinA = 0}$

$\boldsymbol{sinA = 1}$

$\boldsymbol{A = 90° + k.360° \quad or \quad A = 90° + k.360° \, , k \in Z}$

Example 9 (finding the size of the angle): Round off answers to Two decimals places where needed

Solve for x:

a. $3\tan(x-20°) = -4{,}456 \qquad x \in [-90°; 270°]$
b. $\sin^2 x = sinx \qquad -180° \leq x \leq 270°$
c. $tan2x + 3 = 7{,}5$

Solution

a.
$3\tan(x-20°) = -4{,}456$
$\tan(x-20°) = -\dfrac{557}{375}$
first determine the general solution

$x - 20° = -56{,}05° + k.180°$
$x = -36{,}05° + k.180° \quad ,k \in Z$

Then solve for x

$x = -36{,}05° + k.180° \quad ,k \in Z \qquad\qquad x \in [-90°; 270°]$

k	-3	-2	-1	0	1	2
x	X	X	~~-216,05°~~	-36,05°	143,95°	~~323,95°~~

$$\therefore \boldsymbol{x = -36{,}05°\,;143{,}95°}$$

b.

$sin^2x = sinx$
$sin^2x - sinx = 0$
$sinx(sinx - 1) = 0$
$sinx = 0 \quad or\ sinx = 1$

first determine the general solutions

$sinx = 0$

$\boldsymbol{x = 0° + k.360° \quad or \quad x = 180° + k.360° \quad ,k \in Z}$

OR

$sinx = 1$
$\boldsymbol{x = 90° + k.360° \quad ,k \in Z}$ **{no need to repeat the equation (as 180°-90° gives the same equation)}**

Then solve for x (for each general solution)

$x = 0° + k.360° \quad , k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x			~~-360°~~	0°	~~360°~~	

$$\therefore \boldsymbol{x = 0°}$$

OR

$x = 180° + k.360° \quad , k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x		~~-540°~~	-180°	180°	~~540°~~	

$$\therefore \boldsymbol{x = -180°; 180°}$$

OR

$x = 90° + k.360° \quad , k \in Z \qquad\qquad -180° \leq x \leq 270°$

k	-3	-2	-1	0	1	2
x			~~-270°~~	90°	~~450°~~	

$$\therefore \boldsymbol{x = 90°}$$

c.

$tan2x + 3 = 7,5$
$tan2x = 4,5$
$2x = 77,47° + k.180°$
$x = 38,74° + k.90° \ , k \in Z$

{Note that we were not given the interval, therefore we will stop ater finding the general solution}

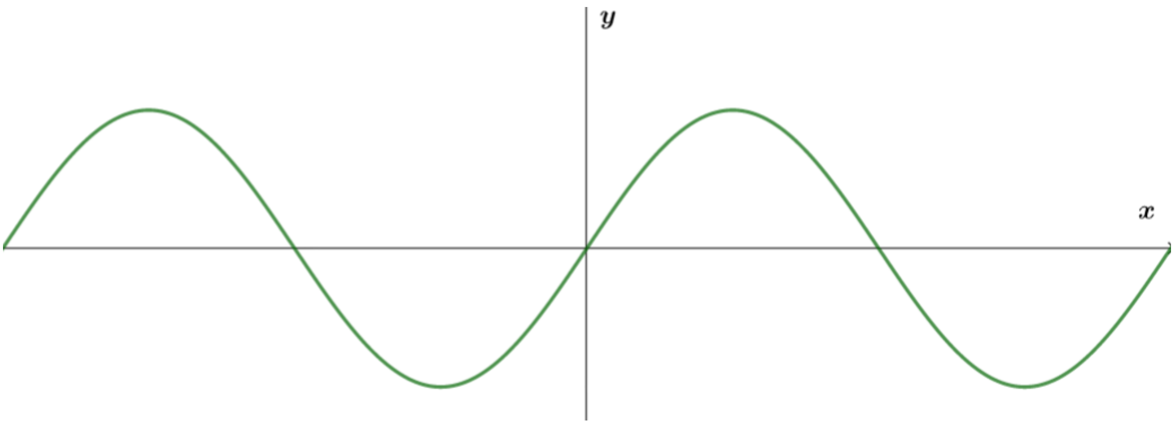
TRIGONOMETRIC FUNCTIONS (GRAPHS)

Summary of The sine function $y = a\sin k(x + p) + q$

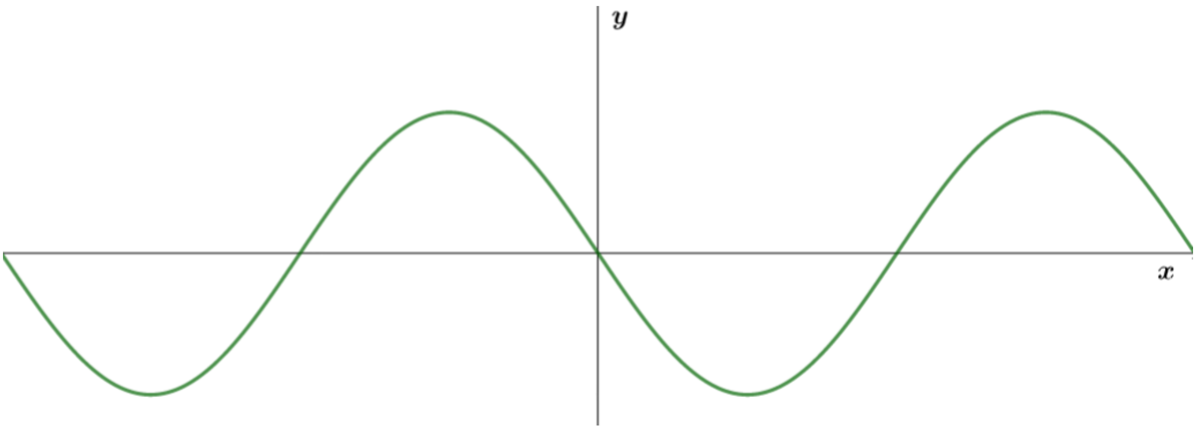
helps to find amplitude (a) **helps with period** (k) **horizontal translation** (p) **Vertical translation** (q)

- **Shape**

$$a > 0$$



$$a < 0$$



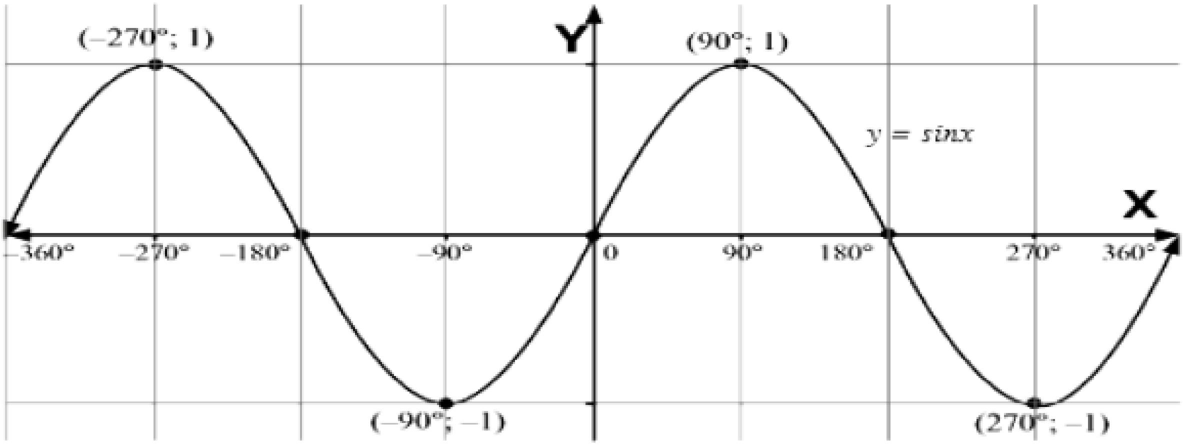
- **Amplitude** – halfway between the maximum and the minimum → $\frac{max - min}{2}$.
  - If $y = 2\sin x$, then the amplitude is 2
  - If $y = -3\sin x$, then the amplitude is 3
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow$ *the horizontal shift*
  - If $y = \sin(x + 45°)$ $\rightarrow$ *shifts* $45°$ *to the left*
  - If $y = \sin(x - 30°)$ $\rightarrow$ *shifts* $30°$ *to the right*
- $\boldsymbol{q} \rightarrow$ *the vertical shift*
  - If $y = \sin x + 3$ $\rightarrow$ *shifts* 3 *units up*
  - If $y = \sin x - 2$ $\rightarrow$ *shifts* 2 *units down*

Example 1

$sketch\ the\ graph\ y = sinx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0



Take note of the following key aspects of the graph of $y = sinx \quad for\ x \in [-360°; 360°]$

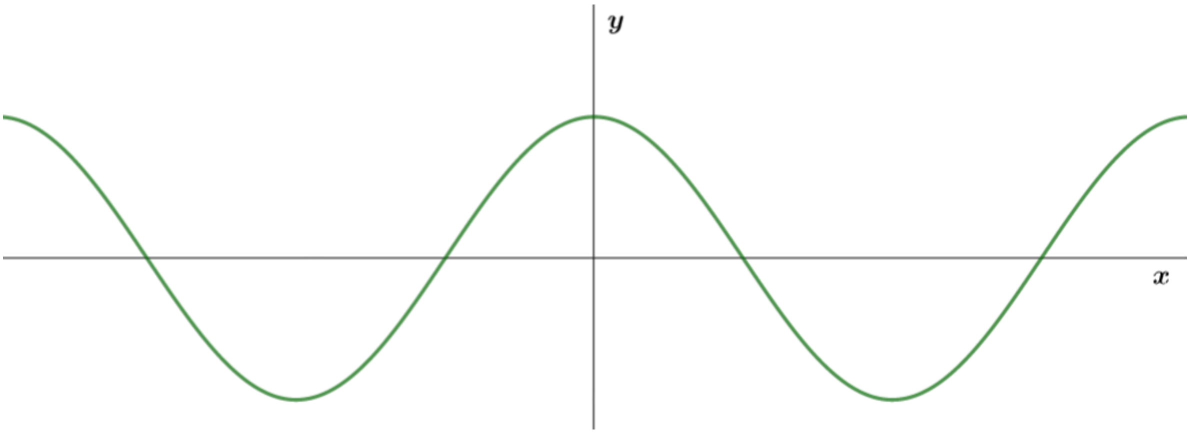
1	**Maximum Value**	**1** (at $x = -270°$ and $x = 90°$)
	Minimum Value	**-1** (at $x = -90°$ and $x = 270°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-360°, -180°, 0°, 180°, 360°$
	y-intercept	**0**
4	**Amplitude**	**1** $\left\{\frac{max-min}{2} \rightarrow \frac{1-(-1)}{2} = 1\right\}$
5	**Period**	$360°$ $\left\{period = \frac{360°}{k} \rightarrow \frac{360°}{1} = 360°\right\}$

Summary of The cosine function $y = acosk(x + p) + q$

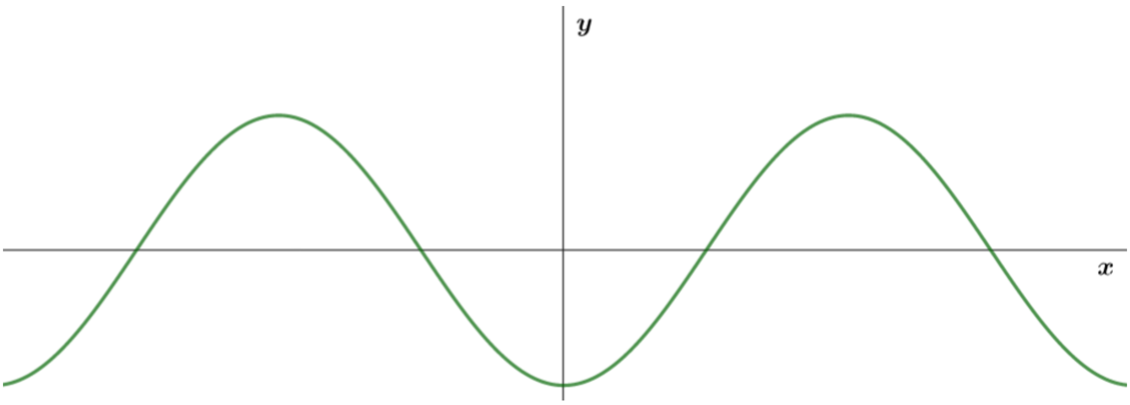
helps to find amplitude — helps with period — horizontal translation — vertical translation

- **Shape**

$$a > 0$$



$$a < 0$$



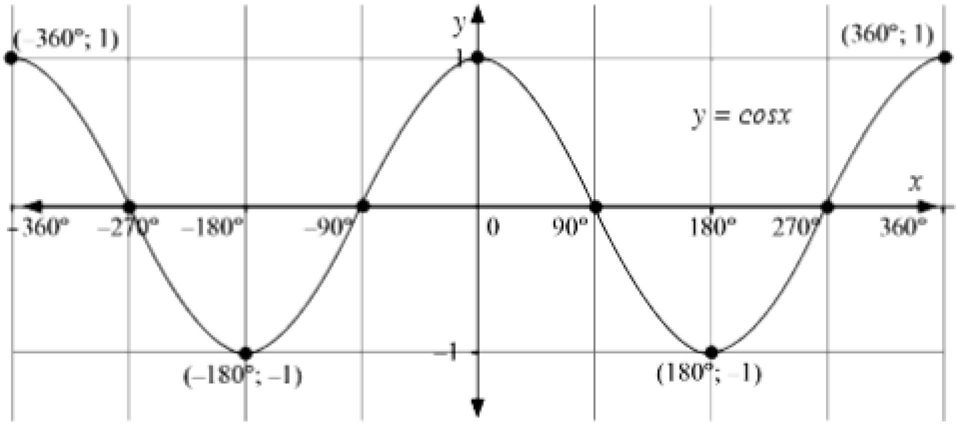
- **Amplitude** – halfway between the maximum and the minimum → $\frac{max - min}{2}$.
  - If $y = 2cosx, then\ the\ amplitude\ is\ 2$
  - If $y = -3cosx, then\ the\ amplitude\ is\ 3$
- $\boldsymbol{Period} = \frac{360°}{k}$
- $\boldsymbol{p} \rightarrow the\ horizontal\ shift$
  - $y = \cos(x + 45°) \rightarrow shifts\ 45°\ to\ the\ left$
  - $y = \cos(x - 30°) \rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow the\ vertical\ shift$
  - $y = cos\ x + 3 \rightarrow shifts\ 3\ units\ up$
  - $y = \cos\ x - 2 \rightarrow shifts\ 2\ units\ down$

Example 2

$sketch\ the\ graph\ y = cosx \quad for\ x \in [-360°; 360°]$

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1



Take note of the following key aspects of the graph of $y = cosx \quad for\ x \in [-360°; 360°]$

1	**Maximum Value**	**1** (at $x = -360°, x = 0°$ and $x = 90°$)
	Minimum Value	-1 (at $x = -180°$ and $x = 180°$)
2	**Domain**	$x \in [-360°; 360°], x \in R$
	Range	$y \in [-1; 1], y \in R$
3	**x-intercepts**	$-270°, -90°, 90°, 270°$
	y-intercept	**1**
4	**Amplitude**	**1** $\{\frac{max-min}{2} \rightarrow \frac{1-(-1)}{2} = 1\}$
5	**Period**	$360°$ $\{period = \frac{360°}{k} \rightarrow \frac{360°}{1} = 360°\}$

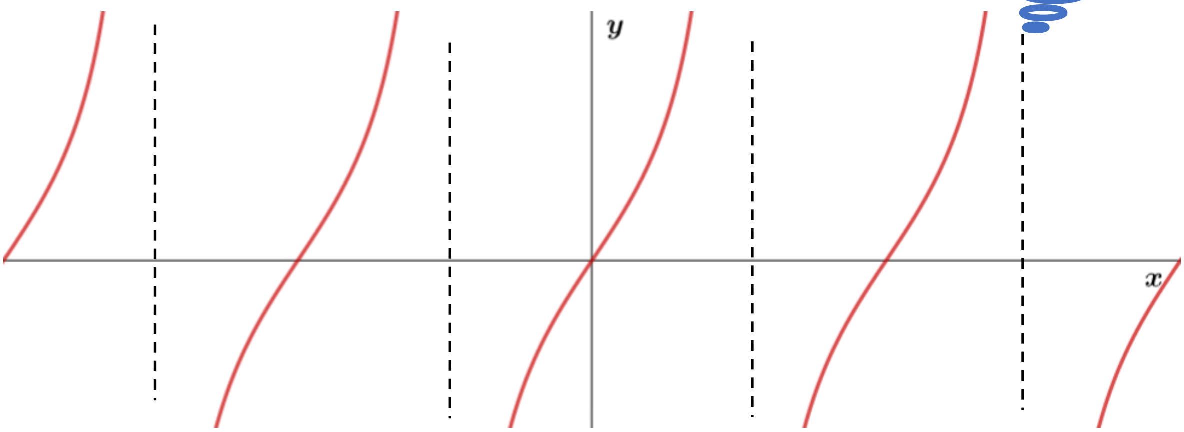
Summary of The cosine function $y = atank(x+p)+q$

helps with slope point — helps with period — horizontal translation — vertical translation

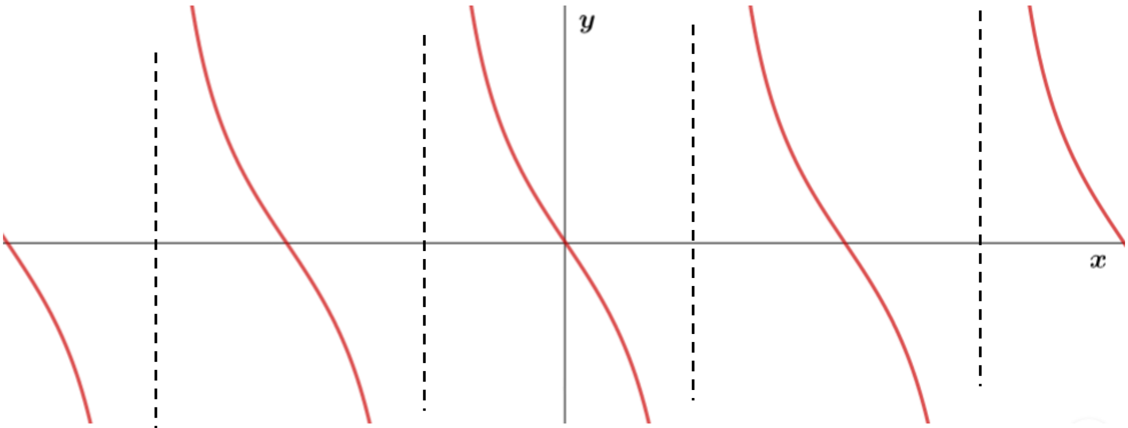
- **Shape**

Asymptote line

$a > 0$



$a < 0$



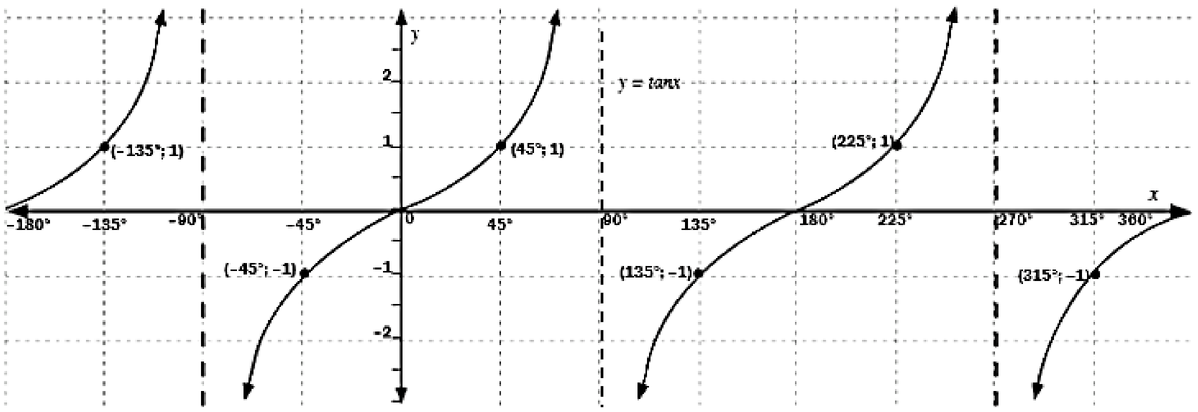
- **Amplitude** – tangent graph does not have maximum or the minimun VALUE, thus **THERE IS NO AMPLITUDE** for the tangent function.
- **Asymptotes**
  - Positions of the first asymptotes are at $\mathbf{0°} \pm \frac{\boldsymbol{period}}{\mathbf{2}}$
  - Then, other asymptotes are found every period.
- $\boldsymbol{Period} = \frac{180°}{k}$
- $\boldsymbol{p} \rightarrow the\ horizontal\ shift$
  - $y = \tan\,(x+45°)\ \rightarrow shifts\ 45°\ to\ the\ left$
  - $y = \tan\,(x-30°)\ \rightarrow shifts\ 30°\ to\ the\ right$
- $\boldsymbol{q} \rightarrow the\ vertical\ shift$
  - $y = tan\ x + 3\ \rightarrow shifts\ 3\ units\ up$
  - $y = \tan\ x - 2 \rightarrow shifts\ 2\ units\ down$

Example 3

sketch the graph $y = tanx \quad for\ x \in [-180°; 360°]$

Solution

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
y	0	1	unde-fined	-1	0	1	unde-fined	-1	0	1	unde-fined	-1	0



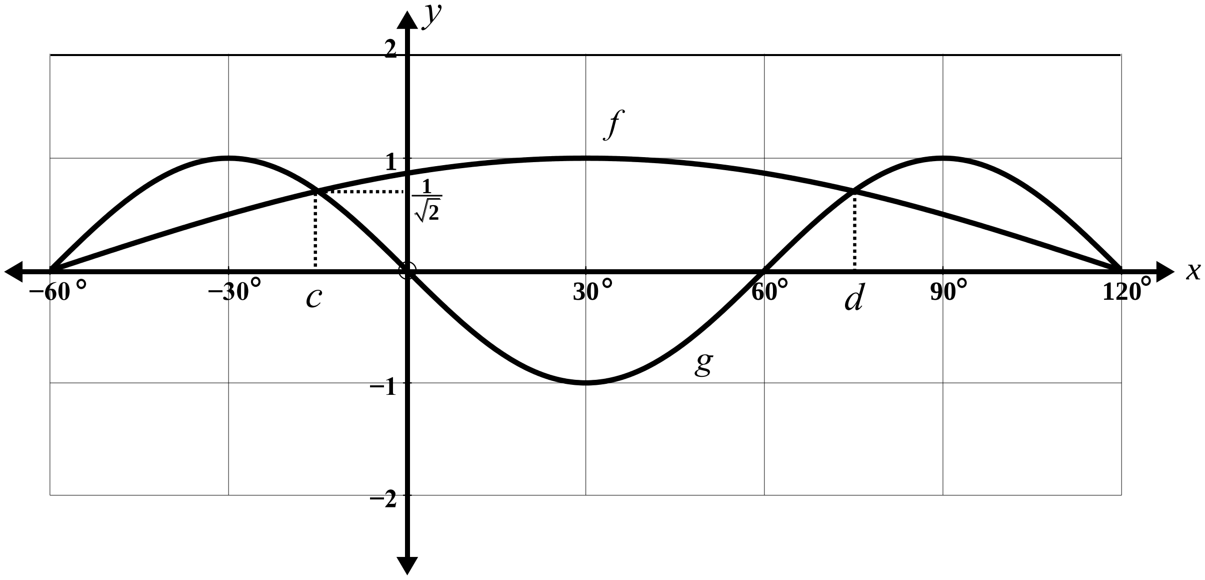
Take note of the following key aspects of the graph of $\boldsymbol{y = tanx \quad for\ x \in [-180°; 360°]}$

1	**Maximum Value**	**N/A**
	Minimum Value	**N/A**
2	**Domain**	$\boldsymbol{x \in [-180°; 360°], \quad but\ x \neq -90°, 90°, 270°}$
	Range	$\boldsymbol{y \in (-\infty; \infty), y \in R}$
3	**x-intercepts**	$\boldsymbol{-180°, 0°, 180°, 360°}$
	y-intercept	**0**
4	**Amplitude**	**N/A**
5	**Period**	$\boldsymbol{180° \quad \{period = \frac{180°}{k} \rightarrow \frac{180°}{1} = 180°\}}$
6	**Equations of asymptotes**	$\boldsymbol{x = -90°, x = 90°, \quad and\ x = 270}$
7	**Slope points**	$\boldsymbol{(-135°; 1), (-45°; -1), (45°; 1), (135°; -1), (225°; 1), (315°; -1)}$

Worked Examples (sine, cosine, and tangent functions)

Example 4

In the diagram below, the graphs of $f(x)=\cos(x+p)$ and $g(x)=a\sin bx$ are shown in the interval $-60^\circ \le x \le 120^\circ$. The *y*-value at *c* is $\frac{1}{\sqrt{2}}$. The graphs intersect at *c* and *d*.



4.1 Determine the values of *a*, *b* and *p*.

4.2 Calculate the values of *c* and *d*.

4.3 Determine graphically the value(s) of *x* in the interval $-60^\circ \le x \le 120^\circ$ for which:

4.3.1 $f(x)-g(x)\ge 0$

Solutions

4.1 $f(x)=\cos(x+p)$

$p=-30^\circ$

$g(x)=a\sin bx$:

Period:

$$\frac{360^\circ}{b}=120^\circ$$

$\therefore b=3$

Also $a=-1$

4.2 $\frac{1}{\sqrt{2}} = -sin\ 3\,x$

$\therefore sin\ 3\,x = -\frac{1}{\sqrt{2}}$

$\therefore 3x = -45° + k.360 \quad or \quad 3x = 225° + k.360°, \qquad k \in Z$

$x = -15° \qquad or \quad x = 75°$

$\therefore c = -15°$

$\therefore d = 75°$

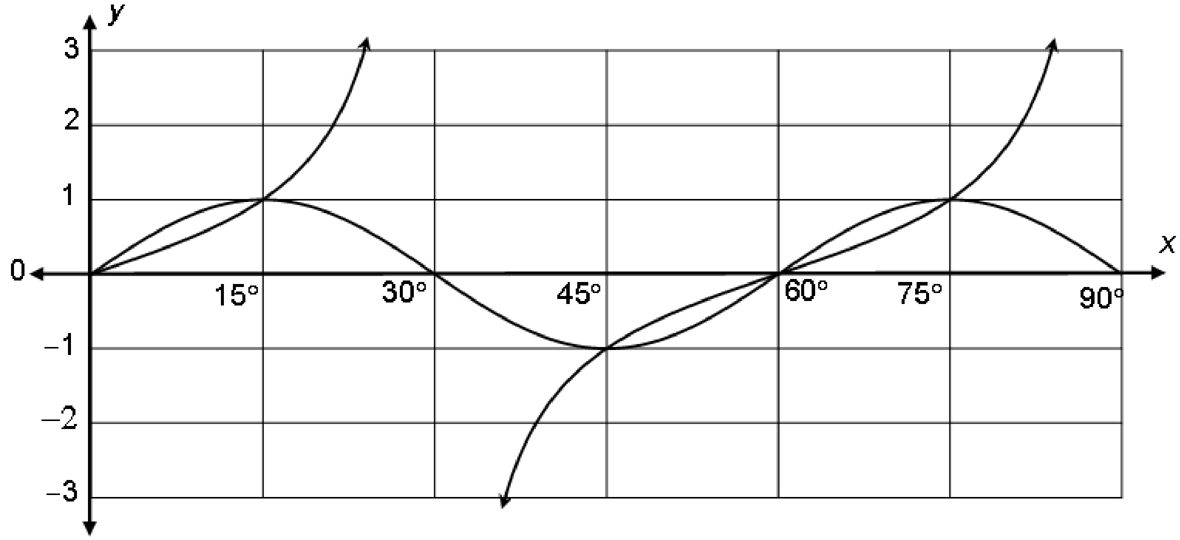
4.3.1 $f(x) - g(x) \geq 0$

$f(x) \geq g(x)$

$\therefore -15° \leq x \leq 75°" \ or\ "x = -60°" \ or\ "x = 120°$

Example 5

On the set of axes, the graphs of $f(x) = \tan 3x$ and $g(x) = \sin 6x$ are shown for the interval $x \in [0°; 90°]$.



5.1 Write down the period of *f*.

5.2 Determine graphically the values of *x* for which:

$$f(x) \leq g(x)$$

5.3 If the graph of *g* is shifted 2 units vertically up, write down the range of the resulting graph.

Solution

5.1 60°

5.2 $0° \leq x \leq 15° \quad \text{or} \quad 30° < x \leq 45° \quad \text{or} \quad 60° \leq x \leq 75°$

5.3 $y \in [1; 3]$

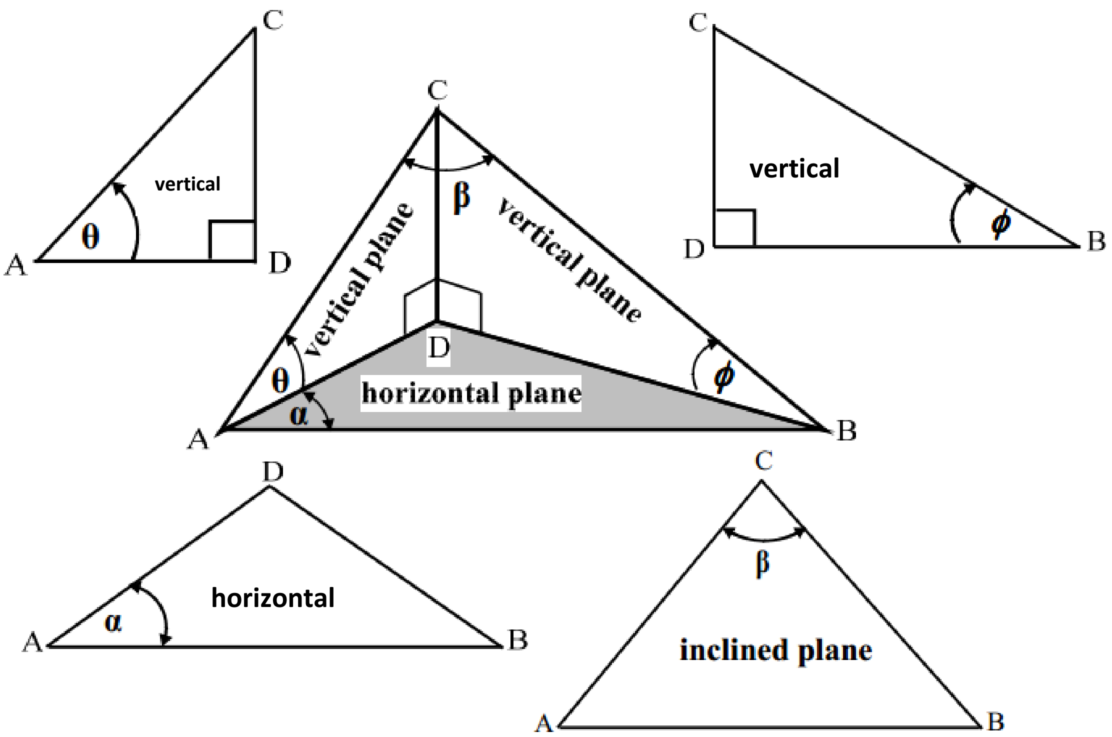
SOLUTION OF TRIANGLES: 2-D & 3-D

3-dimensional space takes up 3 planes (horizontal, vertical and inclined/slanted).

The 3-dimensional diagram below is split such that you can work separately on each 2-D plane.

The 3-D diagram below has 4 planes:

- Vertical plane → ΔADC
- Vertical plane → ΔBDC
- Horizontal plane → ΔADB
- Inclined plane → ΔABC

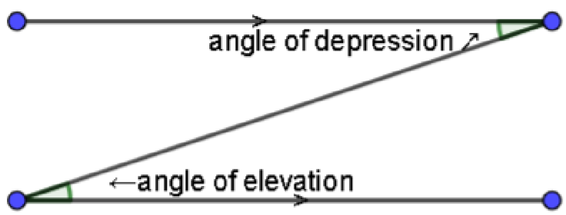


Take note that:

- You cannot add two angles from different planes to get the sum, from the 3-D diagram, $\boldsymbol{\theta + \alpha \neq C\hat{A}B}$

Angle of elevation vs Angle of depression

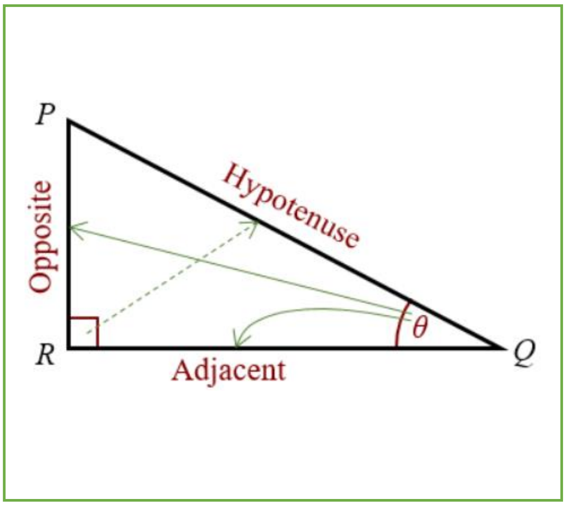
Angle of **depression** (measured from horizontal going down)



Angle of **elevation** (measured from the horizontal going up)

For right-angled triangles

Soh Cah Toa



1.

$$\text{sine of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2.

$$\text{cosine of an angle } \theta = \frac{\text{length of the side adjacent to angle } \theta}{\text{length of the hypotenuse}}$$

$$\therefore \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3

$$\text{tangent of an angle } \theta = \frac{\text{length of the side opposite angle } \theta}{\text{length of the side adjacent to angle } \theta}$$

$$\therefore \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

To calculate the area of a right-angled triangle PQR above, use the formula $Area\ \Delta = \frac{1}{2} b \times h$

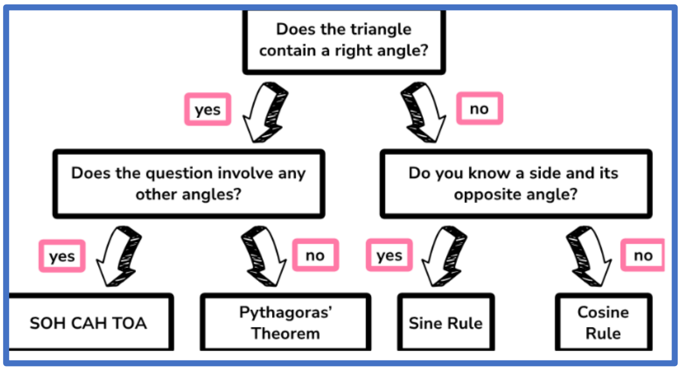
$$\therefore Area\ \Delta PQR \ = \frac{1}{2} RQ \times PR$$

For triangles that are not right-angled triangles

Rule	Formula	When to use
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Given two sides and the angle opposite one of those sides.
		One side and any two angles.
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$	Given two sides and the included angle.
		Three sides.
Area rule	$\frac{1}{2}ab\sin C$	Area is required. In order to use the formula for Area, two sides and the included angle are required.

N.B Only use area formula when you are asked to calculate the area or when you are given the area.

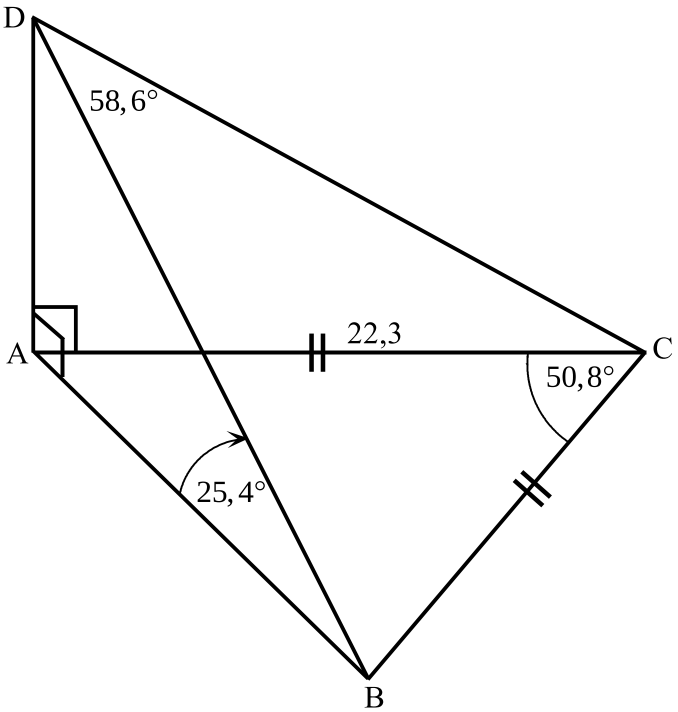
2-D & 3-D Approach



Worked Examples

Example 1

In the diagram below, A, B and C are points in the same horizontal plane with $AC = BC = 22,3$ metres. AD is a vertical tower which is anchored at B and C. The angle of elevation of D, from B is $25,4^\circ$. $B\hat{D}C = 58,6^\circ$ and $A\hat{C}B = 50,8^\circ$.



Calculate, correct to ONE decimal place:

1.1 the area of $\triangle ABC$
1.2 the length of AB

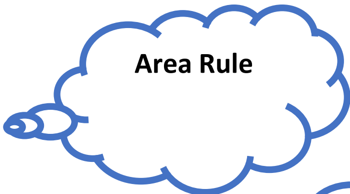
1.3 the height of the tower, AD

1.4 the length of DC if $D\hat{B}C = 67,3^\circ$

Solution

1.1 $\text{Area } \Delta ABC = \frac{1}{2}(22,3)(22,3(\sin 50,8°)$

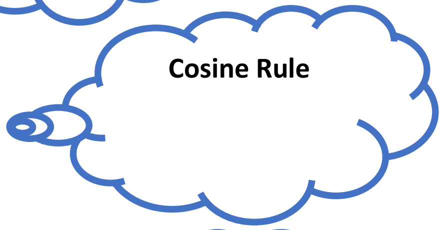
$= 192,7 \text{ units}^2$



1.2 $AB^2 = (22,3)^2 + (22,3)^2 - 2(22,3)(22,3)\cos 50,8°$

$= 365,9762962$

$\therefore AB = 19,1 \text{ units}$



1.3 $\frac{AD}{19,1} = \tan 25,4°$

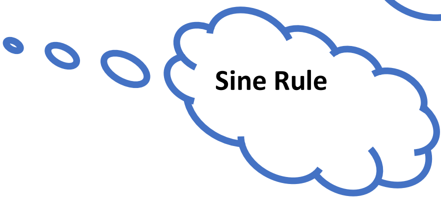
$\therefore AD = 19,1 \tan 25,4°$

$\therefore AD = 9,1 \text{ units}$

1.4 $\frac{DC}{\sin 67,3°} = \frac{22,3}{\sin 58,6°}$

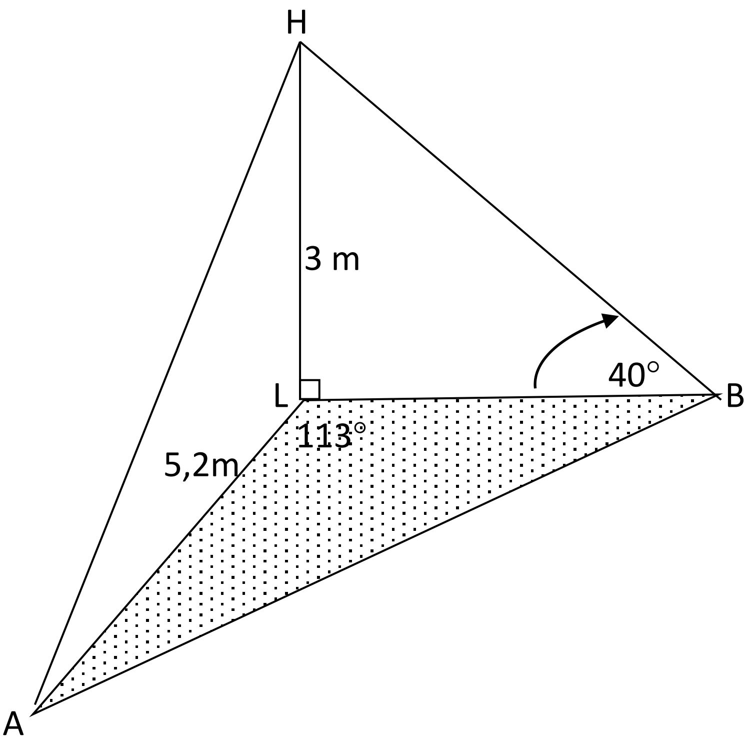
$\therefore DC = \frac{22,3 \sin 67,3°}{\sin 58,6°}$

$\therefore DC = 24,1 \text{ units}$



Example 2

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.



2.1 Calculate the length of LB.

2.2 Hence, or otherwise, calculate the length of AB.

2.3 Determine the area of ΔABL.

Solution

2.1

$$\frac{3}{LB} = \tan 40^\circ \qquad \qquad \frac{LB}{\sin 50^\circ} = \frac{3}{\sin 40^\circ}$$

$$\therefore LB = \frac{3}{\tan 40^\circ} \quad \text{or} \quad \therefore LB = \frac{3\sin 50^\circ}{\sin 40^\circ}$$

$$\therefore LB = 3,58 \text{ m} \qquad \qquad \therefore LB = 3,58 \text{ m}$$

2.2

$$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113^\circ$$

$$\therefore AB^2 = (5,2)^2 + (3,58)^2 - 2(5,2)(3,58)\cos 113^\circ$$

$$\therefore AB^2 = 54,40410138 \text{ m}^2$$

$$\therefore AB = 7,38 \text{ m}$$

2.3

$$\text{Area of } \Delta ABL = \frac{1}{2} AL.BL.\sin A\hat{L}B$$

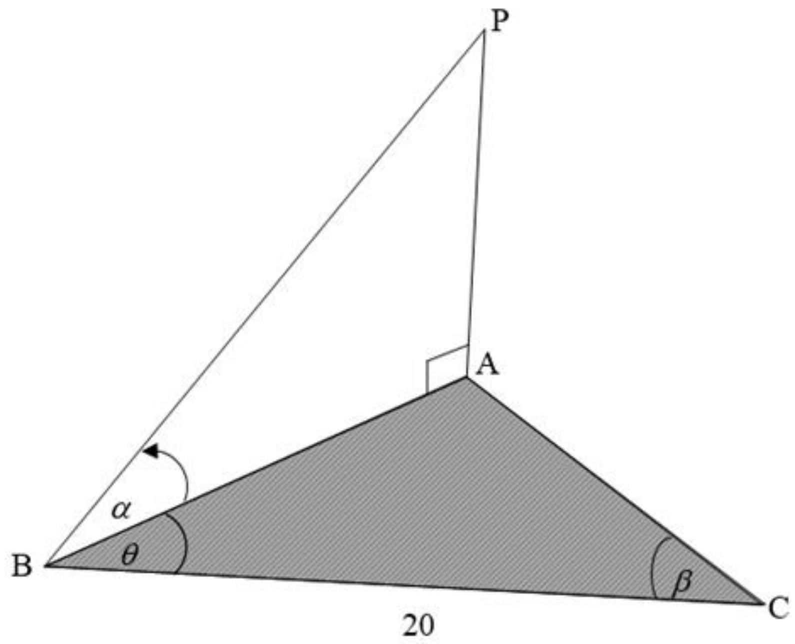
$$= \frac{1}{2}(5,2)(3,58)\sin 113^\circ$$

$$= 8.568059176$$

$$= 8,57 \text{ m}$$

Example 3

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is α. $A\hat{C}B = \beta$, $A\hat{B}C = \theta$ and BC = 20 units.



3.1 Write AP in terms of AB and α.

3.2 Prove that $AP = \dfrac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$

3.3 Given that AB = AC, determine AP in terms of α and β in its simplest form.

Solution

3.1

$$\frac{AP}{AB} = \tan\alpha$$

$$\therefore AP = AB\tan\alpha$$

3.2

$$\frac{AB}{\sin\beta} = \frac{20}{\sin[180° - (\theta+\beta)]}$$

$$\therefore \frac{AB}{\sin\beta} = \frac{20}{\sin(\theta+\beta)}$$

$$\therefore AB = \frac{20\sin\beta}{\sin(\theta+\beta)}$$

$$\therefore AP = \frac{20\sin\beta}{\sin(\theta+\beta)}\tan\alpha$$

$$\therefore AP = \frac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$$

3.3

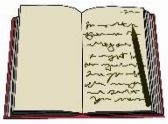
$$AP = \frac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$$

$$= \frac{20\sin\beta\tan\alpha}{\sin(\beta+\beta)}$$

$$= \frac{20\sin\beta\tan\alpha}{\sin 2\beta}$$

$$= \frac{20\sin\beta\tan\alpha}{2\sin\beta\cos\beta}$$

$$= \frac{10\tan\alpha}{\cos\beta}$$



THE FOLLOWING ACTIVITIES HAVE DIFFERENT LEVELS OF DIFFICULTY ON THE RIGHT, LEVEL 1 BEING EASY AND LEVEL 4 BEING CHALLENGING.

ACTIVITIES

PAPER	SECTION	LEVEL
	REDUCTION FORMULAE	
GP 2022	5.2 Simplify the following expression: $\cos^2(180°+x)+\cos(-x)\,.\,\tan x\,.\,\cos(90°+x)$	L2
KZN 2022	6.1 **Without using a calculator**, simplify the following expression fully: $\dfrac{\sin(180°-x).\tan(x-180°).\cos(360°+x)}{\sin^2(180°+x)+\sin^2(90°-x)}$	L2
KZN 2022	6.2 **Without using a calculator**, determine the value of: $\dfrac{\cos 330°.\tan 150°.\sin 12°}{\tan 675°.\cos 258°}$	L2
FS 2022	5.1 Simplify the following expression to a single trigonometric ratio: $\dfrac{\cos(x-180°).\tan(-x).\sin^2(90°-x)}{\sin(180°-x)}-4\cos^2 x$	L2
	TRIGONOMETRIC IDENTITIES	
GP 2022	**Prove that:** $\cos(2x+77°)\cos(x+47°)+\sin(x+47°)\sin(2x+437°)=\cos(x+30°)$	L3
KZN 2022	6.3 Given the identity: $\dfrac{\cos\alpha+\cos 2\alpha}{\sin 2\alpha-\sin\alpha}=\dfrac{\cos\alpha+1}{\sin\alpha}$	
	6.3.1 Prove the identity.	L3
	6.3.2 For which other values of α is the identity undefined?	L2
L 2022	5.4 Prove that: $\dfrac{\sin 2x-\tan x}{\cos 2x}=\tan x$	L3

5.2 Consider: $\cos(A-B)-\cos(A+B)=2\sin A\sin B$

FS 2022

5.2.1 Prove the identity. L2

5.2.2 Hence or otherwise calculate, **without using a calculator**, the value of $\cos 15° - \cos 75°$. L3

5.3 **Without using a calculator**, determine the value of:

FS 2022

$$\frac{\cos 36°}{\cos 12°}-\frac{\sin 36°}{\sin 12°}$$ L3

5.4 Consider: $\frac{2\sin^2 x+\sin 2x}{\cos 2x}=\frac{2\sin x}{\cos x-\sin x}$

FS 2022

5.4.1 Prove the identity. L3

5.4.2 For which value(s) of x in the interval $x\in[-90°;180°]$ will the identity not be valid? L2

5.3 Consider: $\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$

EC 2022

5.3.1 Prove the identity. L3

5.3.2 For which value(s) of x, in the interval $x\in[-180°;180°]$, is the identity not valid? L2

5.2 Given: $\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$

5.2.1 Prove that $\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}=1-\sin x$ L3

NOV 2023

5.2.2 For what value(s) of x in the interval $x\in[0°;360°]$ is $\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ undefined? L2

5.2.3 Write down the minimum value of the function defined by $y=\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ L2

5.3 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

NOV 2023

5.3.1 Use the above identity to deduce that $\sin(A-B)=\sin A\cos B-\cos A\sin B$ L2

5.3 Determine, **without using a calculator**, the value of:

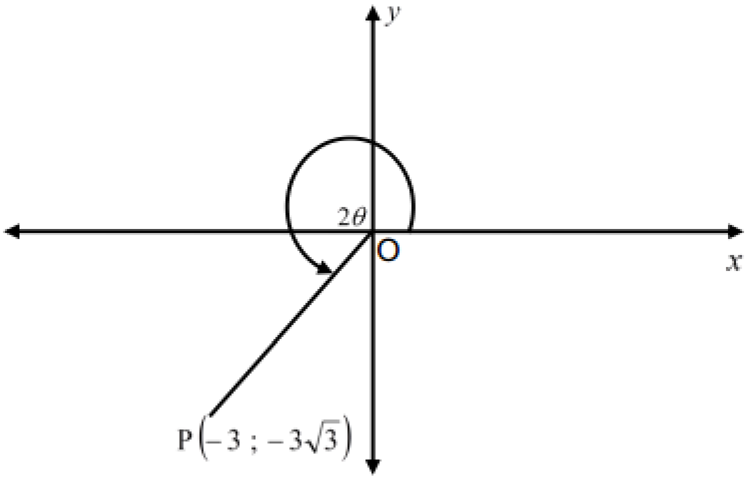
MAY 2023

$$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$$ L3

CARTESIAN PLANE

GP 2022

5.1 In the diagram below, point $P(-3\ ;-3\sqrt{3})$ and reflex angle 2θ are shown.



Determine, **without the use of a calculator**, the value of:

5.1.1 $\cos 2\theta$ **L2**

5.1.2 $\sin \theta$ **L2**

KZN 2022

5.1 If $5\cos A = 2\sqrt{6}$ where $A \in [90°;360°]$, calculate, **without using a calculator** and with the aid of a diagram, the values in simplest form of:

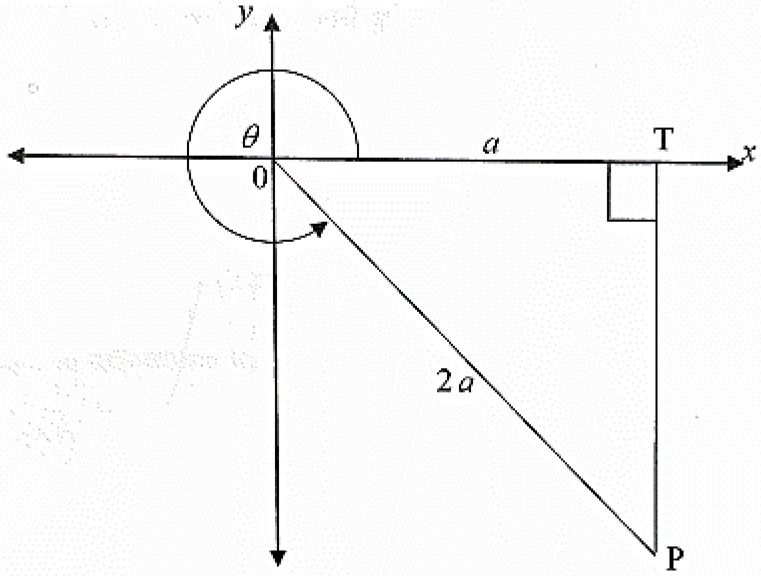
5.1.1 $-\sqrt{6}.\tan A$ **L2**

5.1.2 $\sin 2A$ **L2**

QUESTION 5

5.1 P is a point in the fourth quadrant. $OP = 2a$ and $OT = a$. $X\hat{O}P = \theta$. Use the diagram to answer the following questions.

L
2022



Determine the value of the following in terms of a:

5.1.1 PT. L2

5.1.2 $\sin^2\theta$ L2

5.1.3 $\sin(450° - \theta)$ L2

L
2022

5.3 If $\sin 2\alpha = \frac{1}{3}$ for $2\alpha \in [0°\,;\,90°]$, calculate without the use of a calculator, the value of $\cos\alpha$. L2

NOV
2023

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90°;\,270°)$.

Without using a calculator, determine each of the following:

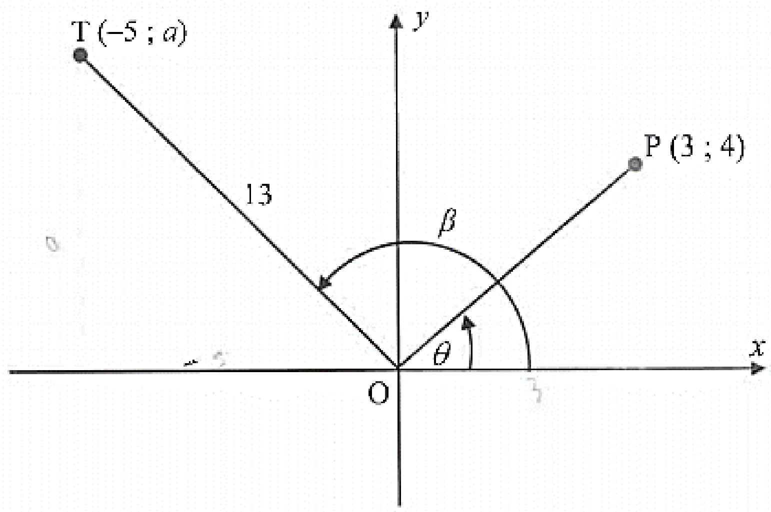
5.1.1 $\cos\beta$ L2

5.1.2 $\sin 2\beta$ L2

5.1.3 $\cos(450° - \beta)$ L2

5.2 In the diagram, OP and OT are drawn with endpoints at P(3 ; 4) and T(–5 ; a).
OT = 13 units
OP makes an angle of θ with the positive x-axis.
OT makes an angle β with the positive x-axis.

MP 2022



Determine, without using a calculator, the following:

5.2.1 The value of a. L2

5.2.2 The length OP. L2

5.2.3 $\sin(\beta-\theta)$ L2

5.2.4 $\cos 2\theta$ L2

RATIOS IN TERMS OF A LETTER

5.2 Given: $\sin 18° = p$

Without using a calculator, determine each of the following in terms of p.

KZN 2022

5.2.1 $\cos 18°$ L2

5.2.2 $\cos 48°$ L3

5.2.3 $\sin 9°$ L3

EC 2022	5.1	Given that: $\cos 26° = p$ Express each of the following in terms of p, **without using a calculator.**	
		5.1.1 $\sin 26°$	**L2**
		5.1.2 $\tan 154°$	**L2**
		5.1.3 $\sin 13°.\cos 13°$	**L2**
MP 2015	5.1	If $\cos 48° = t$ express, without the use of a calculator, each of the following in terms of t. Show all calculations.	
		5.1.1 $\cos 96°$	**L2**
		5.1.2 $\sin(-42°)$	**L2**

TRIGONOMETRIC EQUATIONS

GP 2022	5.3	Consider the equation $5\tan\theta - 6\cos\theta = 0$:	
		5.3.1 Show that the equation can be rewritten as $6\sin^2\theta + 5\sin\theta - 6 = 0$.	**L3**
		5.3.2 Determine the general solution of $5\tan\theta - 6\cos\theta = 0$.	**L2**
MAY 2023	5.5	Given: $3\tan 4x = -2\cos 4x$	
		5.5.1 **Without using a calculator**, show that $\sin 4x = -0{,}5$ is the only solution to the above equation.	**L2**
		5.5.2 Hence, determine the general solution of x in the equation $3\tan 4x = -2\cos 4x$	**L2**
FS 2014	7.2	Determine the general solution of $\frac{1}{2}\sin x = -0{,}243$.	**L2**
GP 2015	6.2.2	Show that $\cos x\left(\frac{\cos 2x}{\cos x + \sin x}\right) = \frac{1}{2}$ can be simplified to $\cos 2x = \sin 2x$.	**L3**
	6.2.3	Hence, determine the general solution of $\cos x\left(\frac{\cos 2x}{\cos x + \sin x}\right) = \frac{1}{2}$.	**L2**
MP 2015	6.1	Calculate the value(s) of x where $x \in [-90°; 270°]$ if $\sin x = \cos 2x - 1$	**L3**
MP 2022	5.3	Solve for x: $4\sin^2 x - 3\sin x - 1 = 0$ for $x \in [0°; 360°]$	**L2**

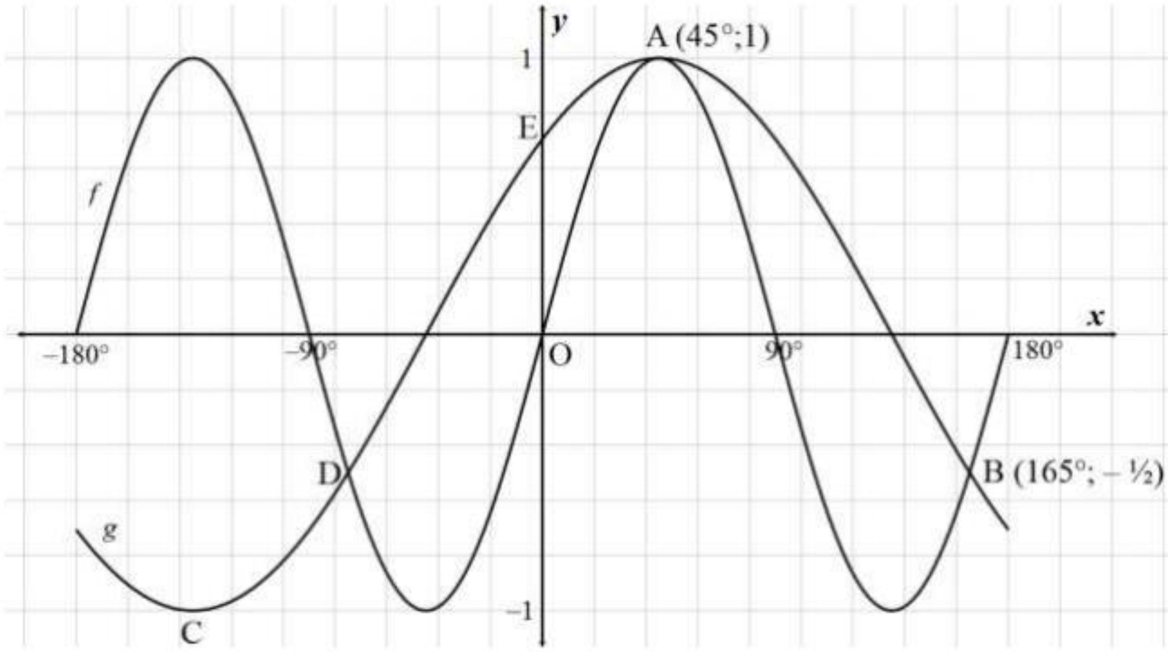
TRIGONOMETRIC FUNCTIONS

QUESTION 7

Given: $f(x) = \sin 2x$ and $g(x) = \cos(x + a)$ where $x \in [-180^\circ ; 180^\circ]$

The graphs of f and g intersect at B and D. E is the y-intercept of g, and C is a turning point of g. A is a turning point of both f and g.

KZN 2022



7.1 Write down the value of a. **L2**

7.2 State the period of f. **L1**

7.3 Determine the coordinates of C and E. **L2**

7.4 Write down the amplitude of h if $h(x) = 3f(x)$. **L2**

7.5 Determine for which value(s) of x, if $x \in [0^\circ ; 180^\circ]$, will:

7.5.1 $g(x) > f(x)$ **L2**

QUESTION 7

Consider: $f(x)=\cos(x+30°)$ and $g(x)=\sin x$

7.1 Calculate the values for which x will $g(x)=f(x)$. L2

7.2 Sketch the graphs of f and g on the same set of axes for $x\in[-180°\ ;\ 180°]$ L2

L 2022

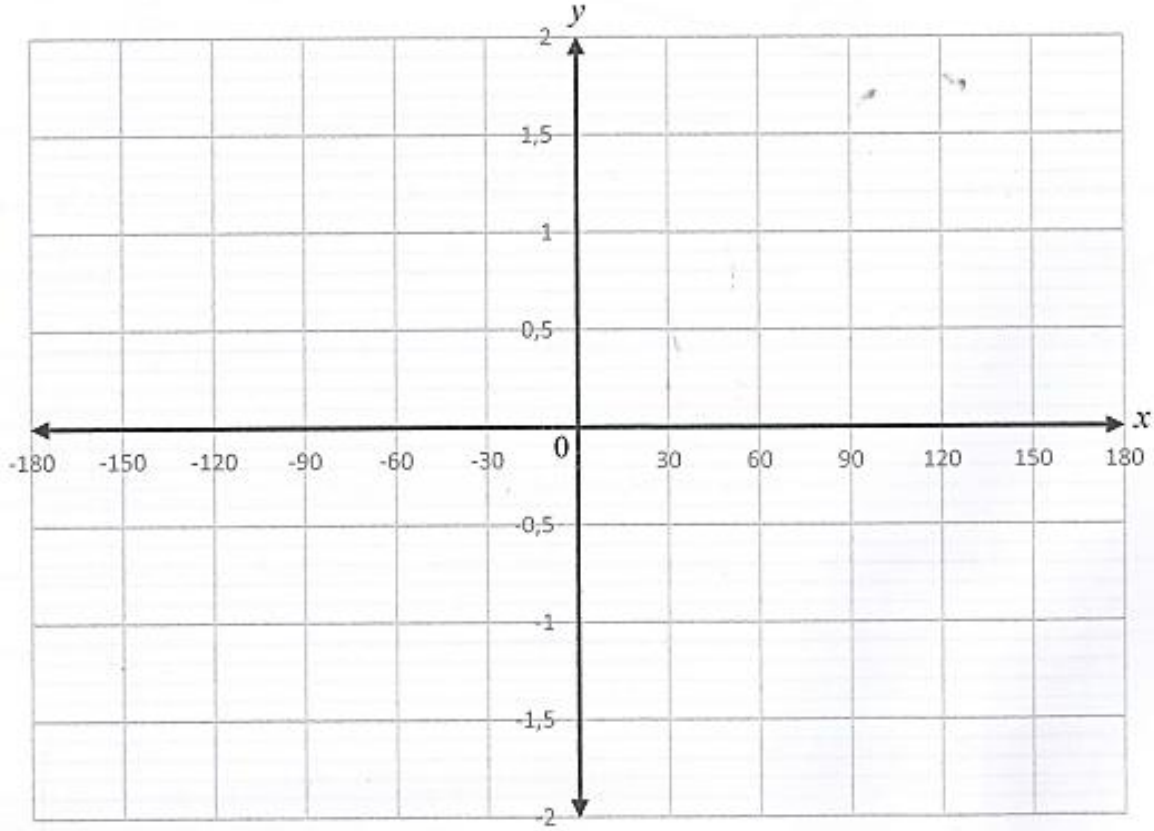
7.3 Write down the amplitude of $y = g(x)$. L1

7.4 Determine the period of $f(2x)$. L1

7.5 Use the graph to determine for which values of x will $f(x).g'(x) > 0$, for

$x\in[-90° : 90°]$ L3

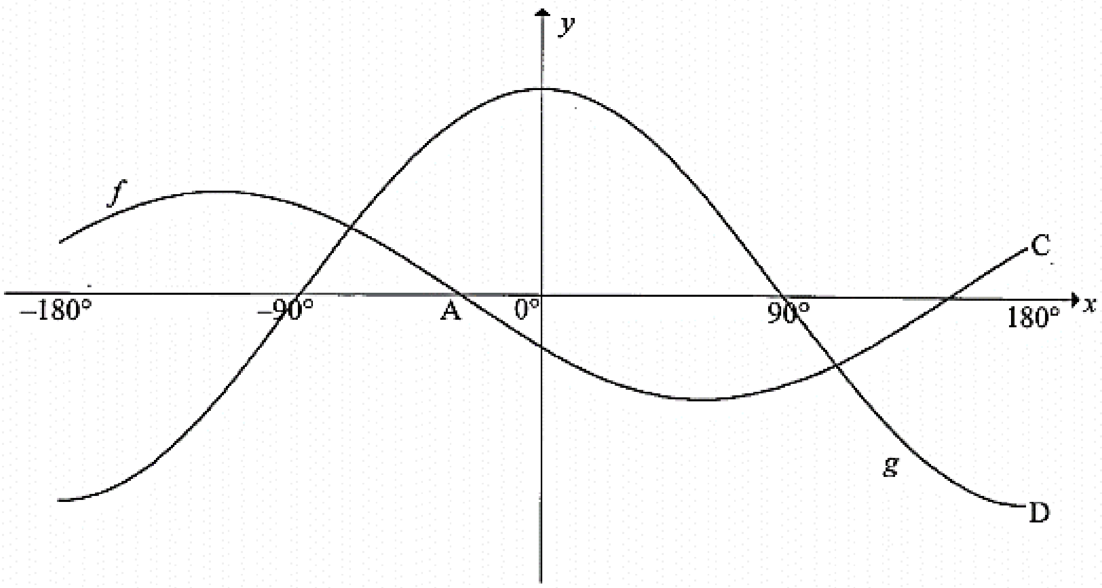
7.2 DIAGRAM SHEET



QUESTION 7

In the diagram below, the functions $f(x) = -\sin(x + 30°)$ and $g(x) = 2\cos x$ are drawn in the interval $x \in [-180° ; 180°]$. A is an x-intercept of f and C and D are the endpoints of the graphs of f and g at 180°.

FS
2022



7.1 Calculate the:

7.1.1 Coordinates of A. L2

7.1.2 Distance CD. L2

7.2 Write down the period of g. L1

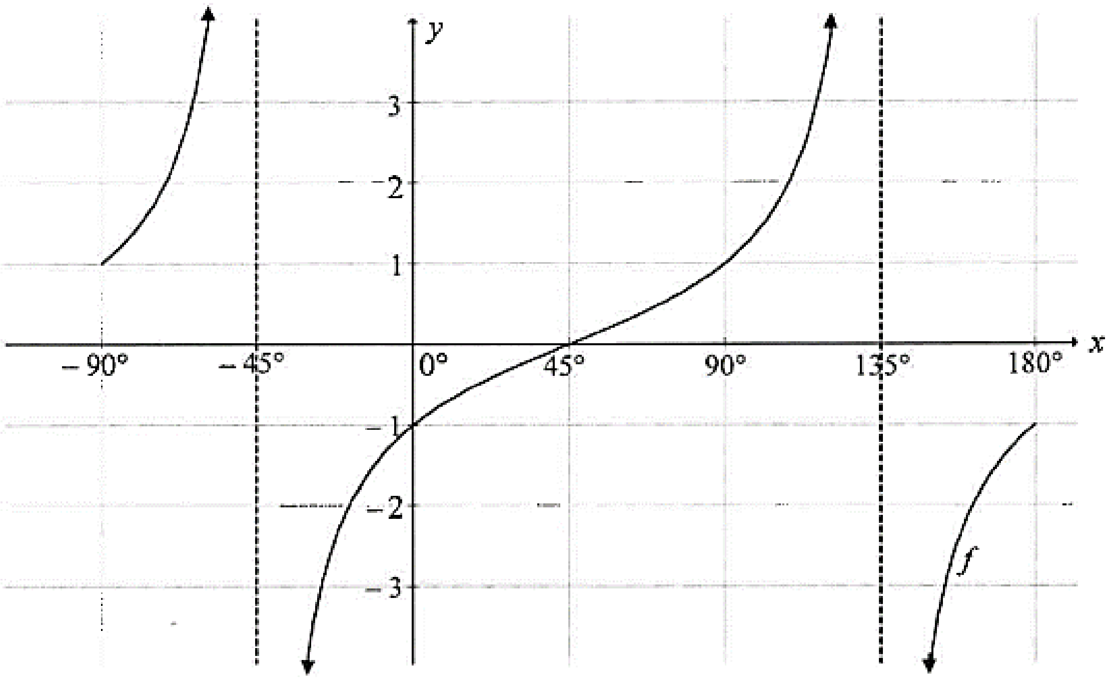
7.3 Determine the general solution of the equation $2\cos x + \sin(x + 30°) = 0$. L2

7.4 For which values of x in the interval $x \in [-180° ; 180°]$ will $2\cos(x+20°)+\sin(x+50°) > 0$? L3

QUESTION 6

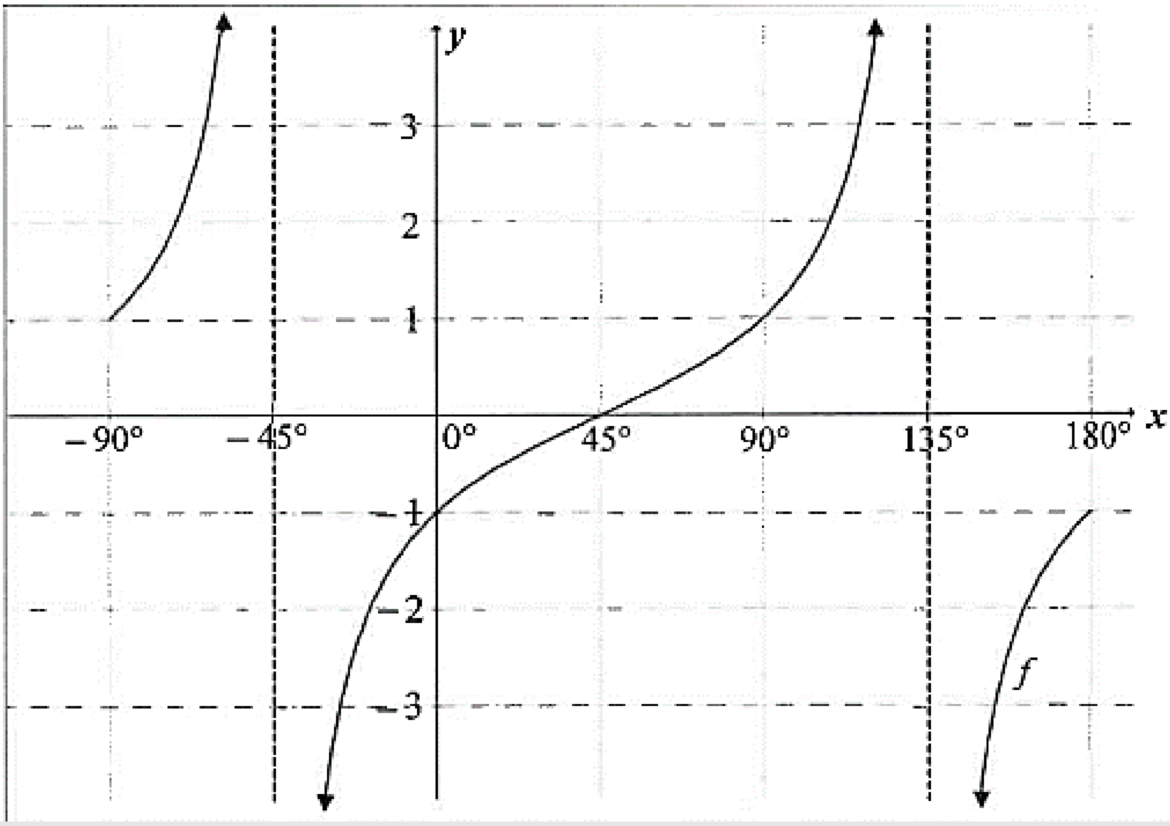
In the diagram below, the graph of $f(x) = \tan(x - 45°)$ is drawn for $x \in [-90°\,;\,180°]$.

MAY 2023



6.1	Write down the period of f.		L1
6.2	Draw the graph of $g(x) = -\cos 2x$ for the interval $x \in [-90°\,;\,180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with the axes, as well as the minimum and maximum points of the graph.		L2
6.3	Write down the range of g.		L1
6.4	The graph of g is shifted $45°$ to the left to form the graph of h. Determine the equation of h in its simplest form.		L2
6.5	Use the graph(s) to determine the values of x in the interval $x \in [-90°\,;\,90°]$ for which:		
	6.5.1	$f(x) > 1$	L2
	6.5.2	$2\cos 2x - 1 > 0$	L2

6.2 DIAGRAM SHEET

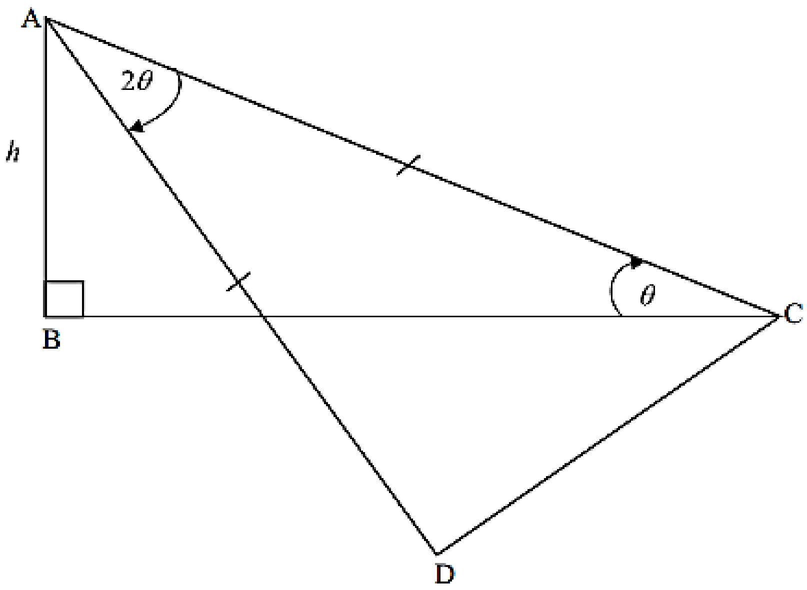


SOLUTION OF TRIANGLES: 2-D & 3-D

QUESTION 7

GP 2022

In the diagram below, AB is a pole anchored by two cables at C and D. B, C and D are in the same horizontal plane. The height of the pole is h and the angle of elevation from C to the top of the pole, A , is θ. $C\hat{A}D = 2\theta$ and AC = AD.

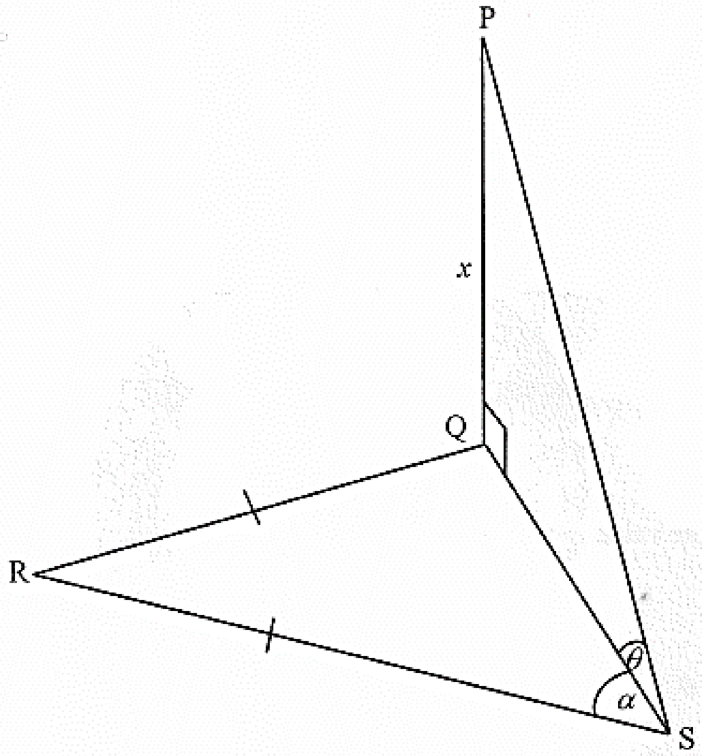


Determine CD, the distance between the two anchors, in terms of h.

QUESTION 6

PQ is a vertical flagpole with length x metres. Q is at the foot of the flagpole. R, Q and S are three points on the same horizontal surface. If RQ = RS, $Q\hat{S}R = \alpha$ and $P\hat{S}Q = \theta$:

L
2022



6.1 Show that: $QS = \dfrac{x}{\tan\theta}$ **L2**

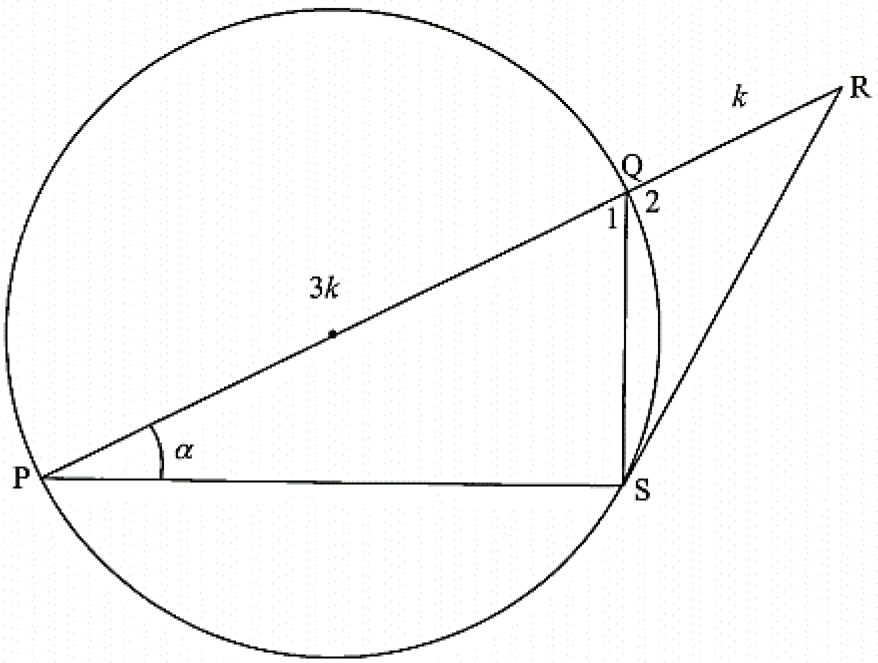
6.2 Prove that: $RS = \dfrac{x}{2\tan\theta\cos\alpha}$ **L2**

6.3 If $\theta = 45°$, $\alpha = 60°$ and $x = 4$, calculate the length of RS without using a calculator. **L2**

QUESTION 6

In the diagram below, PQ is a diameter of the circle and RS is a tangent to the circle at S. The tangent and diameter produced meet at R such that $PQ = 3k$ and $QR = k$. Chord PS is drawn. $\hat{P} = \alpha$.

FS 2022



6.1 Write $\hat{R}$ in terms of α. **L2**

6.2 Determine QS in terms of k and α. **L2**

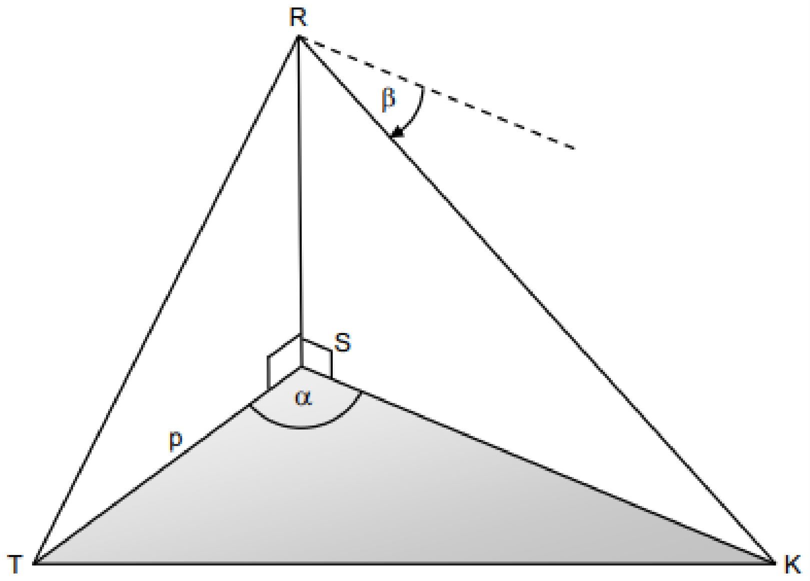
6.3 Show that $PS = \frac{4k\cos 2\alpha}{\cos\alpha}$ **L2**

6.4 Show that $\tan\alpha = \frac{3}{8}\tan 2\alpha$ **L3**

QUESTION 7

In the diagram, S, T and K lie in the same horizonal plane.
RS is a vertical tower. The angle of depression from R to K is β.
$T\hat{S}K = \alpha$, TS = p metres and the area of ΔSTK is q m^2.

NOV 2023



7.1 Determine the length of SK in terms of p, q and α. L2

7.2 Show that $RS = \dfrac{2q \tan\beta}{p \sin\alpha}$ L2

7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m, p = 80 m, q = 2 500 m^2 and $\beta = 42°$. L2

INTEGRATION OF CONCEPTS OR (SECTIONS)

L 2022

5.2 Given: $f(x) = \dfrac{1}{1+\cos x}$

If $x \in [0°\ ;\ 360°]$, determine the following:

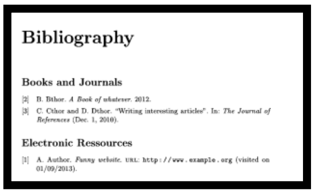
5.2.1 the value of x for which f is undefined. L2

5.2.2 the minimum value of f. L3

5.2.3 the values of x for which f is a minimum. L2

FS 2022

5.5 A line is drawn from A $(\cos\theta; \sin\theta)$ to B $(6; 7)$. If $AB = \sqrt{86}$, determine the value of $\tan\theta$. L3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