Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ACTIVITY MANUAL

GRADE 12

2022

TRIGONOMETRY

CONTENTS PAGE

NUMBER PATTERNS, SEQUENCE AND SERIES

Outcomes:

Investigate number patterns leading to those where there is constant difference between consecutive terms, and the general term is therefore linear.	Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is the quadratic	Identify and solve problems involving number patterns that lead to arithmetic and geometric sequences and series, including infinite geometric series.

(SOURCE: CURRICULUM AND ASSESSMENT POLICY STATEMENT(CAPS)SENIOR PHASE GRADES (10 – 12) MATHEMATICS)

KEY CONCEPTS

Number Patterns, Sequences and series

a. Quadratic sequences
   i. First differences form arithmetic/linear sequence/series
      1. Determining the quadratic sequence when given sequence of first differences and one term of quadratic sequence
      2. e.g Between which TWO terms of the quadratic pattern will the first difference be 599?
   ii. Second difference constant
   iii. Different methods of determining the general term /n^{th} term/conjecture
      1. Standard method
      2. Solving simultaneous equations
   iv. Geometric patterns of quadratic sequences
   v. Combination of quadratic sequences and sequences done in previous grades
   vi. Determining number of terms
   vii. Quadratic pattern of unknowns

b. Arithmetic and geometric sequences
   i. General term/n^{th} term/conjecture
   ii. Number of terms
   iii. Arithmetic and geometric sequences of unknowns
   iv. Inequality form
   v. Finding parameters using simultaneous equations

c. Sigma notation (summing the series of manageable number of terms)
    i. Given a series in sigma notation
    ii. Writing a series in sigma notation
    iii. Number of terms
d. Arithmetic and geometric series
    i. Proofs
    ii. The last term of an arithmetic series
    iii. Number of terms
    iv. Combinations
    v. Finding previous and succeeding terms
    vi. Finding parameters using simultaneous equations
e. Sequences and series in terms of variable
f. Sum to infinity (exists for convergent geometric series)
    i. Proof (our own)
    ii. Convergence
        1. Conditions for convergence
    iii. Convert rational number with recurring decimals to common fractions
    iv. Finding parameters using simultaneous equations
g. The use of $T_n = S_n - S_{n-1}$
h. Inequalities in the context of sequences and series
i. The use of discrete functions in the context of sequences and series
j. Applications of sequences and series
k. Mixed sequences and/or series

Part 1

QUESTION 1

1.1 Do NOT use a calculator to answer this question. Show ALL calculations.

Prove that:

1.1.1 $$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°} = \frac{3}{2}.$$

1.1.2 $$\cos 75° = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

1.2 Prove that $\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x) = 3\sin^2 x$

1.3 Prove that $(\tan x - 1)(\sin 2x - 2\cos^2 x) = 2(1 - 2\sin x\cos x)$

1.4 Simplify completely:

$$\sin(90° - x)\cos(180° - x) + \tan x.\cos(-x)\sin(180° + x)$$

1.5 Prove, without the use of a calculator, that $\dfrac{\sin 190°\cos 225°\tan 390°}{\cos 100°\sin 135°} = -\dfrac{1}{\sqrt{3}}$

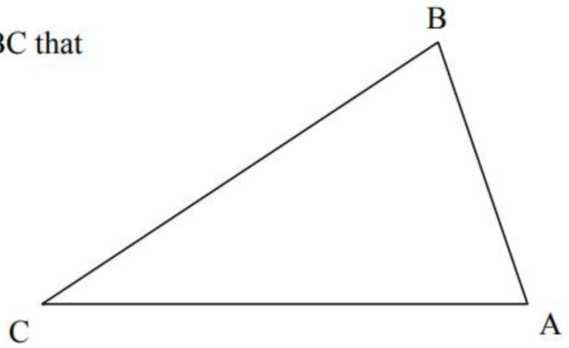
QUESTION 2

2.1 Using the expansions for sin(A + B) and cos(A + B), prove the identity of:

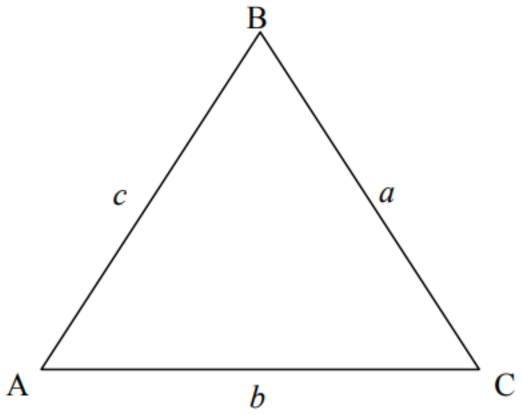
$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

2.2 If $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)}$, prove in any ΔABC that

$\tan A.\tan B.\tan C = \tan A + \tan B + \tan C.$



2.3 Triangle ABC is isosceles with AB = BC.



Prove that $\cos B = 1 - \dfrac{b^2}{2a^2}$ **3**

2.4 Simplify the following to a single trigonometric ratio:

$$\frac{\sin(90° - x)\tan(360° - x)}{\cos(180° - x)}$$

2.5 Simplify completely, **without the use of a calculator**:

$$\frac{\cos(-60°) + \tan 135°}{\tan 315° + \cos 660°}$$

2.6 Simplify completely:

$$\frac{\sin(90° + \theta) + \cos(180° + \theta)\sin(-\theta)}{\sin 180° - \tan 135°}$$

2.7 Prove that for any angle A:

$$\frac{4\sin A\cos A\cos 2A\sin 15°}{\sin 2A\left(\tan 225° - 2\sin^2 A\right)} = \frac{\sqrt{6} - \sqrt{2}}{2}$$

2.8 Prove that, if $\cos(\alpha - x) \neq 0$,

$$\frac{\sin(x + 450° - \alpha)}{\cos(\alpha - x)} = 1.$$

2.9 2.9.1 Prove that, for angles A and B,

$$\frac{\sin A}{\sin B} - \frac{\cos A}{\cos B} = \frac{2\sin(A - B)}{\sin 2B}$$

2.9.2 Hence, or otherwise, **without using a calculator**, show that:

(a) $$\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = 4\cos 2B$$

(b) $$\frac{1}{\sin 18^\circ} = 4\cos 36^\circ$$

(c) $\sin 18^\circ$ is a solution of the cubic equation $8x^3 - 4x + 1 = 0$

2.10 Prove that: $\dfrac{1-\cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$

2.11 Simplify each of the following to a single trigonometric ratio: (Show ALL the calculations.)

2.11.1 $$\frac{\tan(180^\circ + x)\cos(360^\circ - x)}{\sin(180^\circ - x)\cos(90^\circ + x) + \cos(540^\circ + x)\cos(-x)}$$

2.11.2 $$\frac{1-\cos 2x - \sin x}{\sin 2x - \cos x}$$

QUESTION 3

3.1 3.1.1 Show that: $\dfrac{\cos(360^\circ - x)\tan^2 x}{\sin(x-180^\circ)\cos(90^\circ + x)} = \dfrac{1}{\cos x}$

3.1.2 Hence, calculate without the use of a calculator, the value of:

$$\frac{\cos 330^\circ \tan^2 30^\circ}{\sin(-150^\circ)\cos 120^\circ}$$ (Leave your answer in surd form.)

3.2 Prove without the use of a calculator, that if $\sin 28^\circ = a$ and $\cos 32^\circ = b$, then $b\sqrt{1-a^2} - a\sqrt{1-b^2} = \dfrac{1}{2}$.

3.3 Evaluate each of the following without using a calculator. Show ALL working.

3.3.1 $$\frac{\sin 130^\circ . \tan 60^\circ}{\cos 540^\circ . \tan 230^\circ . \sin 400^\circ}$$

3.3.2 $$(1-\sqrt{2}\sin 75^\circ)(\sqrt{2}\sin 75^\circ + 1)$$

3.4 Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$$

3.5 Determine the value of:

$$\tan 1° \times \tan 2° \times \tan 3° \times \tan 4° \times ... \times \tan 87° \times \tan 88° \times \tan 89° .$$

3.6 Simplify as far as possible: $1 - \sin^2 \theta + 3 - \cos^2 \theta$

3.7 Simplify WITHOUT the use of a calculator: $\sqrt{4^{\sin 150°} \times 2^{3 \tan 225°}}$

3.8 Prove that $\dfrac{\cos^2 x \sin^2 x + \cos^4 x}{1 - \sin x} = 1 + \sin x$

3.9 Prove that for any angle θ, $\cos 3\theta = 4\cos^3 \theta - 3\cos\theta$.
(Hint: $3\theta = \theta + 2\theta$)

3.10 If $x = \cos 20°$, use QUESTION 3.9 to show that $8x^3 - 6x - 1 = 0$.

3.11 Simplify to ONE trigonometric function WITHOUT using a calculator:

$$\frac{\cos 160° \tan 200°}{2\sin(-10°)}$$

3.12 Consider $\cos(x + 45°)\cos(x - 45°)$.

3.12.1 Show that $\cos(x + 45°)\cos(x - 45°) = \frac{1}{2}\cos 2x$.

3.13 WITHOUT using a calculator, determine the value of the following expression:

$$\cos 350° \sin 40° - \cos 440° \cos 40°$$

3.14 3.14.1 Show that $1 - \cos 2Q = 2\sin^2 Q$.

3.14.2 Given: $\hat{P} + \hat{Q} + \hat{R} = 180°$

(a) Show that $\sin 2R = -\sin(2P + 2Q)$.

(b) Hence, show that
$\sin 2P + \sin 2Q + \sin 2R = 4 \sin P \sin Q \sin R$.

3.15 Simplify to a single ratio:

$$\frac{\tan(360° - x).\, sin\ (90^0 + x)}{\sin(-x)}$$

3.16 Simplify to a single trigonometric ratio of x:

$$\frac{\cos^2 225.\tan(180°+x).\cos(90°+x)}{\sin(-x)}$$

3.17 Given the equation $2\cos\theta = \sin(\theta + 30°)$.

(a) Show that $2\cos\theta = \sin(\theta + 30°)$ is equivalent to $\sqrt{3}\sin\theta = 3\cos\theta$.

3.18 Determine the value of each of the following expressions:

(a) $$\frac{\cos(90° - 2\theta).\sin\theta}{\sin^2(180° + \theta).\cos(720° + \theta)}$$

(b) $$\frac{1}{\sin^2 2x} - \frac{1}{\tan^2 2x}$$

Part 2

QUESTION 1

1.1 If $\cos\beta = \frac{p}{\sqrt{5}}$ where $p < 0$ and $\beta \in [180°; 360°]$, determine, using a diagram, an expression in terms of p for:

1.1.1 $\tan\beta$

1.1.2 $\cos 2\beta$

QUESTION 2

Given: $\sin\alpha = \frac{8}{17}$ where $90° \leq \alpha \leq 270°$

With the aid of a sketch and without the use of a calculator, calculate:

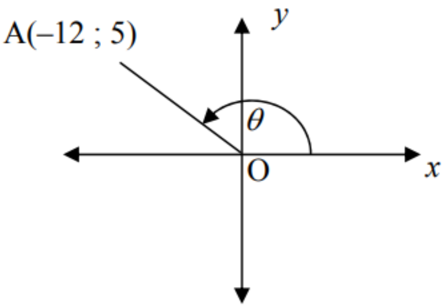
2.1 $\tan\alpha$

2.2 $\sin(90° + \alpha)$

2.3 $\cos 2\alpha$

QUESTION 3

3.1 In the Cartesian plane below, the point $A(-12\ ;\ 5)$ and the angle θ are shown.



Determine, writing your answer as a single fraction:

3.1.1 $\tan\theta$

3.1.2 $\cos\theta\sin\theta$

QUESTION 4

Given: $\tan\alpha = \frac{3}{4}$; where $\alpha \in [0° \;;\; 90°]$

With the use of a sketch and without the use of a calculator, calculate:

4.1 $\sin\alpha$

4.2 $\cos^2(90° - \alpha) - 1$

4.3 $1 - \sin 2\alpha$

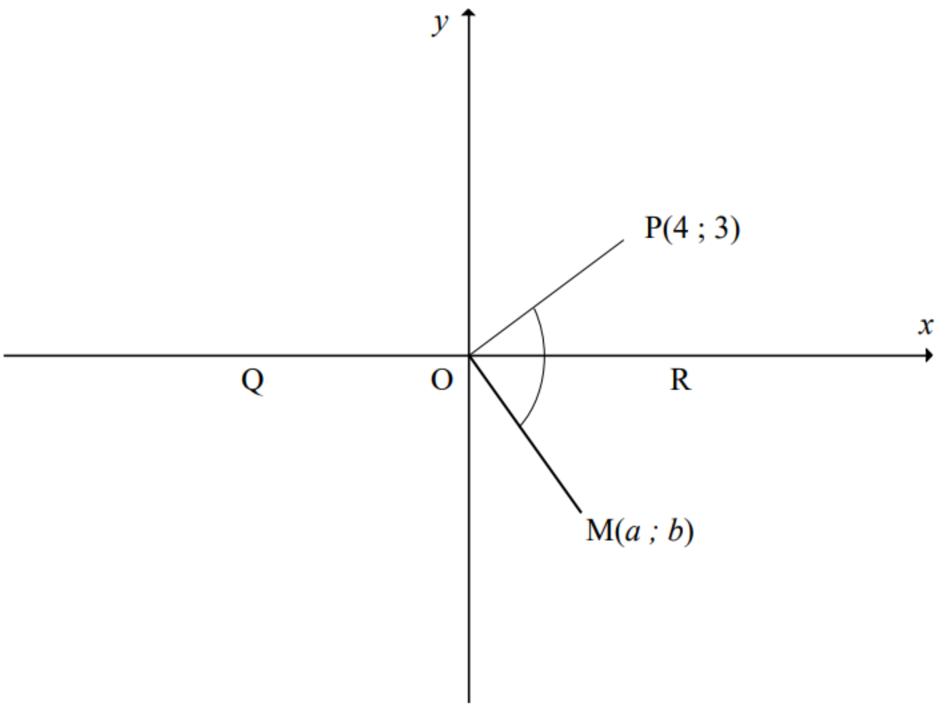
QUESTION 5

5.1 If $\tan A = \frac{3}{\sqrt{40}}$ and $0° < \hat{A} < 90°$, determine the values of the following with the aid of a sketch and without using a calculator. Leave your answers in surd form, if necessary.

5.1.1 $\cos A$

5.1.2 $\sin(180° + A)$

5.2 P(4 ; 3) and M(a ; b) are points on a circle with the origin as centre.
Q and R are x-intercepts of the circle.



5.2.1 Write down the numerical value of $\sin R\hat{O}P$.

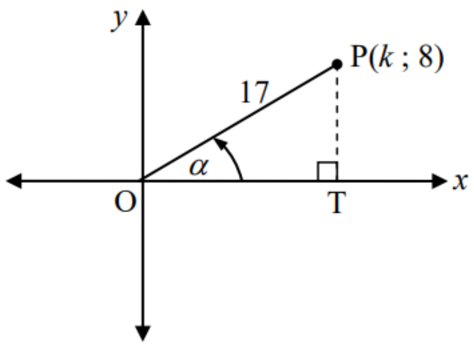
5.2.2 Calculate the size of $Q\hat{O}P$.

5.2.3 If obtuse $P\hat{O}M = 115°$, calculate the value of a, the x-coordinate of M, correct to TWO decimal places.

QUESTION 6

Answer this question WITHOUT using a calculator.

6.1 The point P(k ; 8) lies in the first quadrant such that OP = 17 units and $T\hat{O}P = \alpha$ as shown in the diagram alongside.



6.1.1 Determine the value of k.

6.1.2 Write down the value of $\cos\alpha$.

6.1.3 If it is further given that $\alpha + \beta = 180°$, determine $\cos\beta$.

6.1.4 Hence, determine the value of $\sin(\beta - \alpha)$.

6. 2 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°\,; 270°]$ and $\beta \in [90°\,; 270°]$. Determine the values of the following:

6.2.1 $\cos\alpha$

6.2.2 $\cos(\alpha + \beta)$

QUESTION 7

7.1 If $4\tan\theta = 3$ and $180° < \theta < 360°$, determine with the aid of a diagram:

7.1.1 $\sin\theta + \cos\theta$

7.1.2 $\tan 2\theta$

QUESTION 8

8.1 If $\sin A = \frac{3}{5}$ and $\cos A < 0$, determine, WITHOUT using a calculator, the value of:

8.1.1 $\sin(-A)$

8.1.2 $\tan A$

8. 2 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°; 270°]$ and $\beta \in [90°; 270°]$. Determine, without using a calculator, the values of the following:

8.2.1 $\cos \alpha$

8.2.2 $\cos(\alpha + \beta)$

Part 3

QUESTION 1

1.1 If $\sin 24° = p$, express the following in terms of p, **without the use of a calculator**:

1.1.1 $\cos 24°$

1.1.2 $\sin 12° \cos 12° - \sin(-66°)\tan 204°$

QUESTION 2

2.1 If $\sin 23° = p$, write down the following in terms of p. Do NOT use a calculator.

2.1.1 $\cos 113°$

2.1.2 $\cos 23°$

2.1.3 $\sin 46°$

QUESTION 3

3.1 If $\sin 61° = \sqrt{p}$, determine the following in terms of p:

3.1.1 $\sin 241°$

3.1.2 $\cos 61°$

3.1.3 $\cos 122°$

3.1.4 $\cos 73° \cos 15° + \sin 73° \sin 15°$

3.2 If $\cos 34° = p$, WITHOUT using a calculator, write down the following in terms of p:

3.2.1 $\cos 214°$

3.2.2 $\cos 68°$

3.2.3 $\tan 56°$

QUESTION 4

4.1 If $\sin 28° = a$ and $\cos 32° = b$, determine the following in terms of a and/or b:

4.1.1 $\cos 28°$

4.1.2 $\cos 64°$

4.1.3 $\sin 4°$

Part 4

QUESTION 1

(a) Determine the general solution for: $\frac{\tan x - 1}{2} = -3$ correct to ONE

(b) Determine the general solution of $\sin x + 2\cos^2 x = 1$.

(c) Find the general solution of $\frac{1}{2}\sin x = -0{,}243$.

(d) Find a value for x if $\cos x$; $\sin x$; $\sqrt{3}\sin x$ is a geometric sequence.

(e) Determine the general solution of:

$$6\cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0$$

(f) Determine the general solution of $\cos 2x = 1 - 3\cos x$.

(g) Consider the expression: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$

1. The above expression is undefined if $\sin 2x - \cos x = 0$. Solve this equation in the interval $0° \leq x \leq 360°$

(h) Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$

(i) Consider: $\dfrac{\cos 2x.\tan x}{\sin^2 x}$

.1 For which values of x, $x \in [0°\,;180°]$, will this expression be undefined?

.2 Prove that $\dfrac{\cos 2x.\tan x}{\sin^2 x} = \dfrac{\cos x}{\sin x} - \tan x$ for all other values of x.

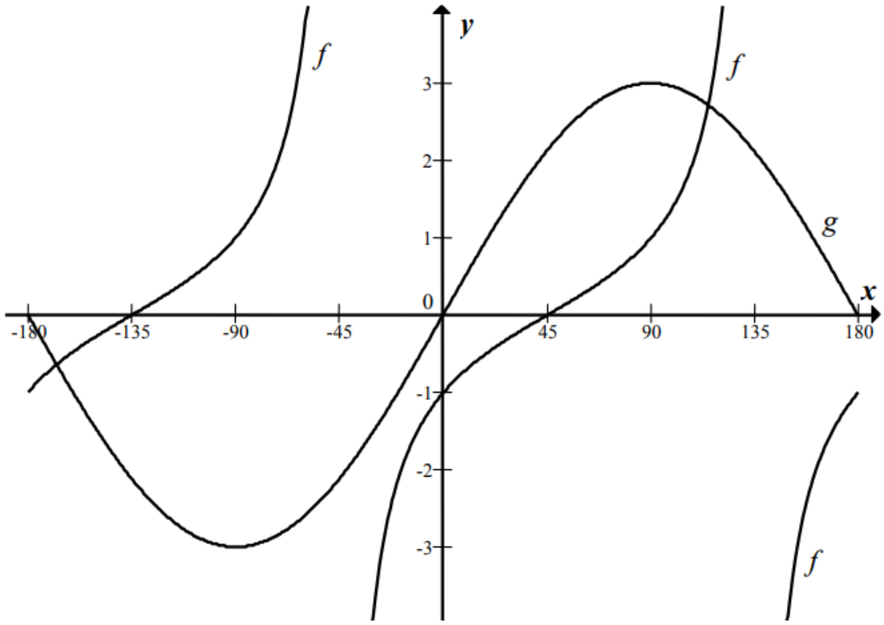
(j) Consider $\cos(x+45°)\cos(x-45°)$.

.1 Show that $\cos(x+45°)\cos(x-45°) = \dfrac{1}{2}\cos 2x$.

.2 Hence, determine a value of x in the interval $0° \leq x \leq 180°$ for which $\cos(x+45°)\cos(x-45°)$ is a minimum.

QUESTION 2

Sketched below are the graphs of the functions $f(x) = \tan(x-45°)$ and $g(x) = 3\sin x$ for $x \in [-180°\,;180°]$.



2.1 Write down the equations of the asymptotes of $y = f(x)$ for $x \in [-90°\,;180°]$.

2.2 Describe the transformation of the graph of f to h if $h(x) = \tan(45° - x)$.

2.3 The period of g is reduced to 180° and the amplitude and y-intercept remain the same. Write down the equation of the resulting function.

QUESTION 3

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90°\,;180°]$.

3.1 Solve for x if $f(x) = g(x)$.

3.2 Sketch the graphs of f and g on the system of axes on DIAGRAM SHEET 1 for $x \in [-90°\,;180°]$.

3.3 Solve for x if $f(x) \leq g(x)$ where $x \in [-90°\,;0°]$.

QUESTION 4

Given: $g(x) = 2\cos(x - 30°)$

4.1 Sketch the graph of g for $x \in [-90°; 270°]$ on DIAGRAM SHEET 1.

4.2 Use the symbols A and B to plot the two points on the graph of g for which $\cos(x - 30°) = 0{,}5$

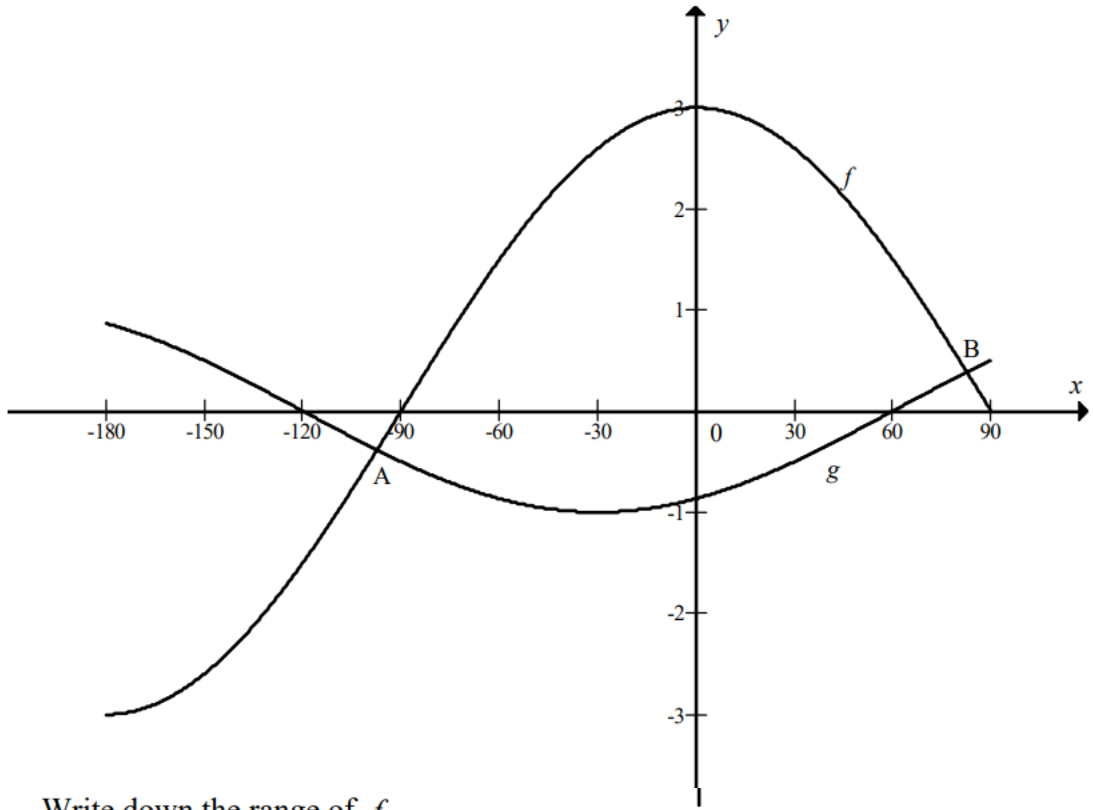
4.3 Calculate the x-coordinates of the points A and B.

4.4 Write down the values of x, where $x \in [-90°; 270°]$ and $g'(x) = 0$.

4.5 Use the graph to solve for x, $x \in [-90°; 270°]$, where $g(x) < 0$

QUESTION 5

The graphs of $f(x) = 3\cos x$ and $g(x) = \sin(x - 60°)$ are sketched below for $x \in [-180°; 90°]$.



5.1 Write down the range of f.

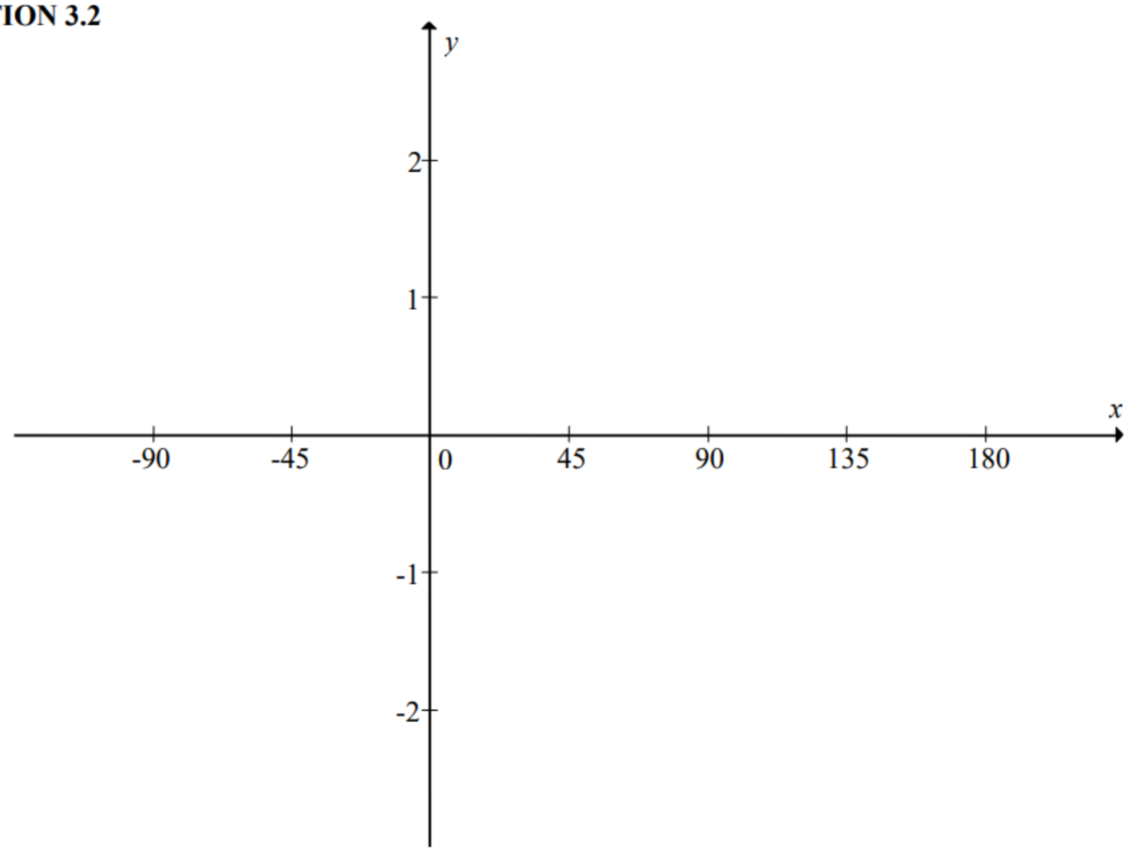
5.2 If A$(-97{,}37°; -0{,}38)$, write down the coordinates of B.

5.3 Write down the period of $g(3x)$.

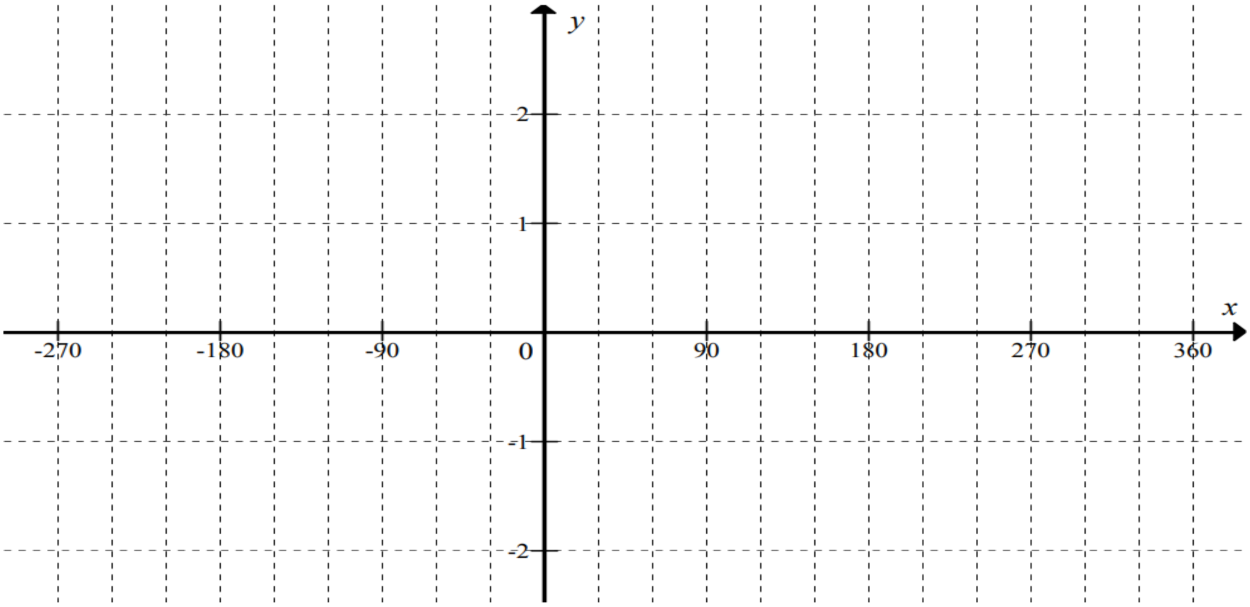
5.4 Write down a value of x for which $g(x) - f(x)$ is a maximum.

DIAGRAM SHEET 1

QUESTION 3.2

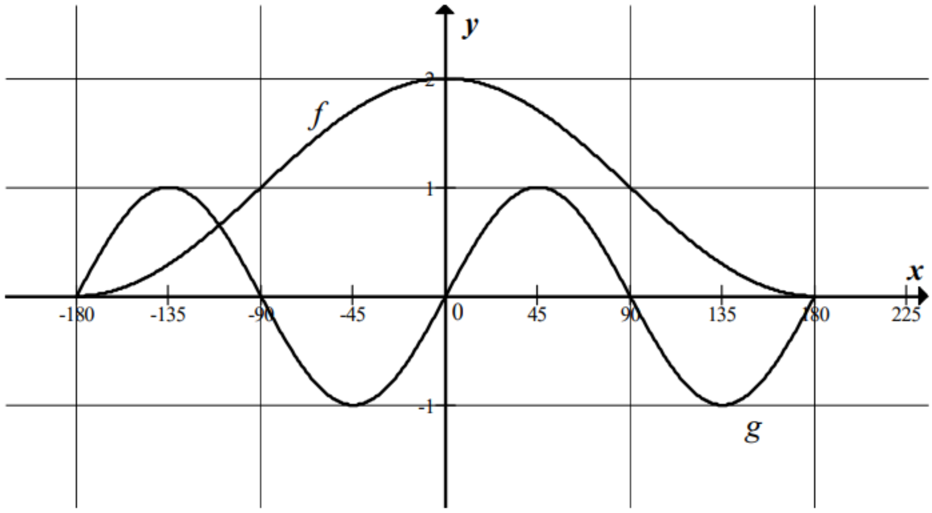


QUESTION 4.1



QUESTION 6

The sketch below shows the graphs of $f(x) = q + \cos x$ and $g(x) = \sin px$ for $x \in [-180° ; 180°]$.



6.1 Write down the values of p and q.

6.2 Write down the range of f.

6.3 Use the graphs to answer the following:

6.3.1 Explain how you would solve the equation $(2\sin x - 1)\cos x = 1$.

6.3.2 Give ONE solution to the equation in QUESTION 6.3.1.

QUESTION 7

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

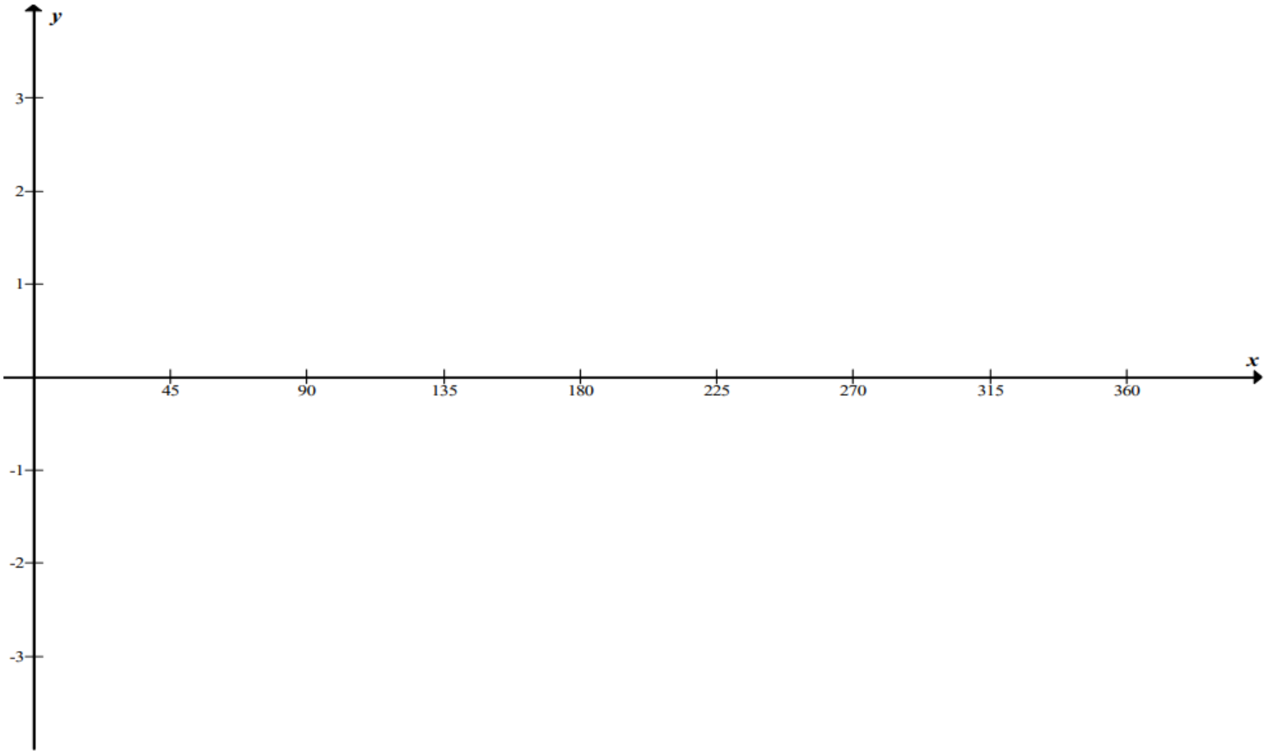
7.1 Calculate the points of intersection of the graphs f and g for $x \in [180° ; 360°]$.

7.2 Draw sketch graphs of f and g for $x \in [180° ; 360°]$ on the same system of axes provided on DIAGRAM SHEET 2.

7.3 For which values of x will $f(x) \leq g(x)$ for $x \in [180° ; 360°]$?

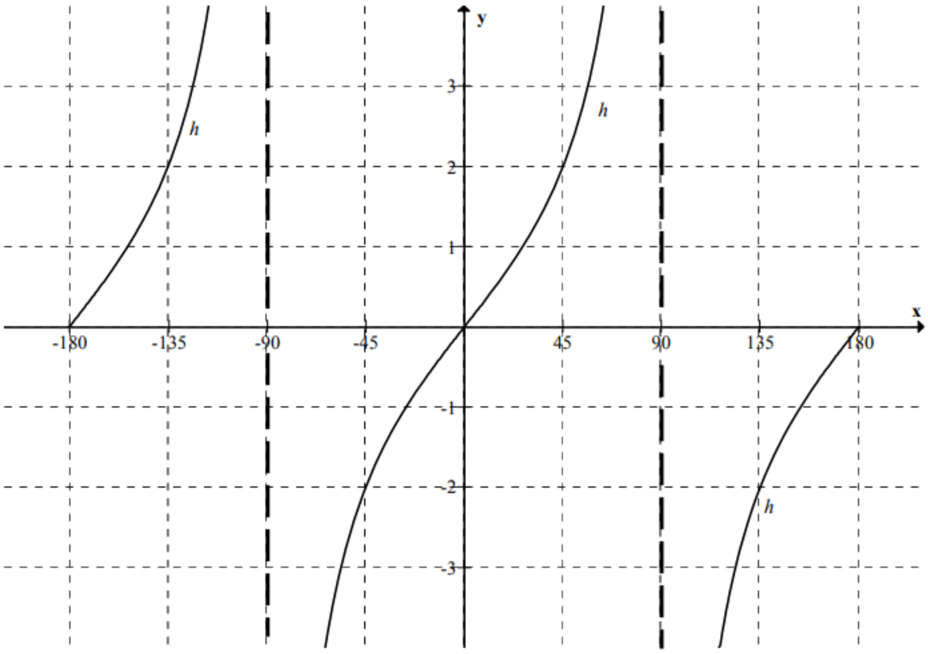
DIAGRAM SHEET 2

QUESTION 7.2



QUESTION 8

The graph of $h(x) = a \tan x$ for $x \in [-180°\,;180°]$, $x \neq -90°, x \neq 90°$, is sketched below.



8.1 Write down the value of a.

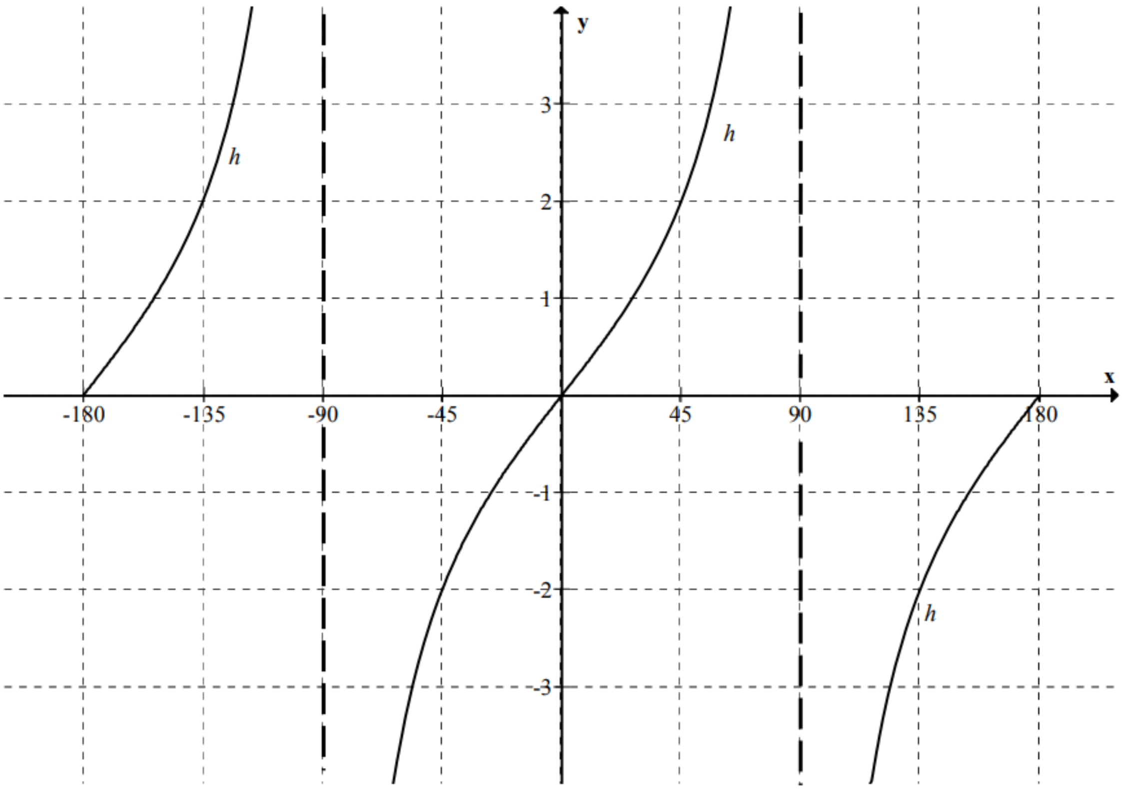
8.2 If $f(x) = \cos(x + 45°)$, sketch the graph of f for $x \in [-180°\,;180°]$ on the same system of axes as h on DIAGRAM SHEET 3 (attached).

8.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°\,;180°]$?

8.4 Let θ be the smallest positive value of x at which h and f intersect. Is θ bigger than 14,5° or smaller than 14,5°? Give a reason for your answer.

DIAGRAM SHEET 3

QUESTION 8.2



QUESTION 9

Given: $f(x) = 2\cos x$ and $g(x) = \tan 2x$

9.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90°\ ;\ 90°]$

9.2 Solve for x if $2\cos x = \tan 2x$ and $x \in [-90°\ ;\ 90°]$. Show ALL working details.

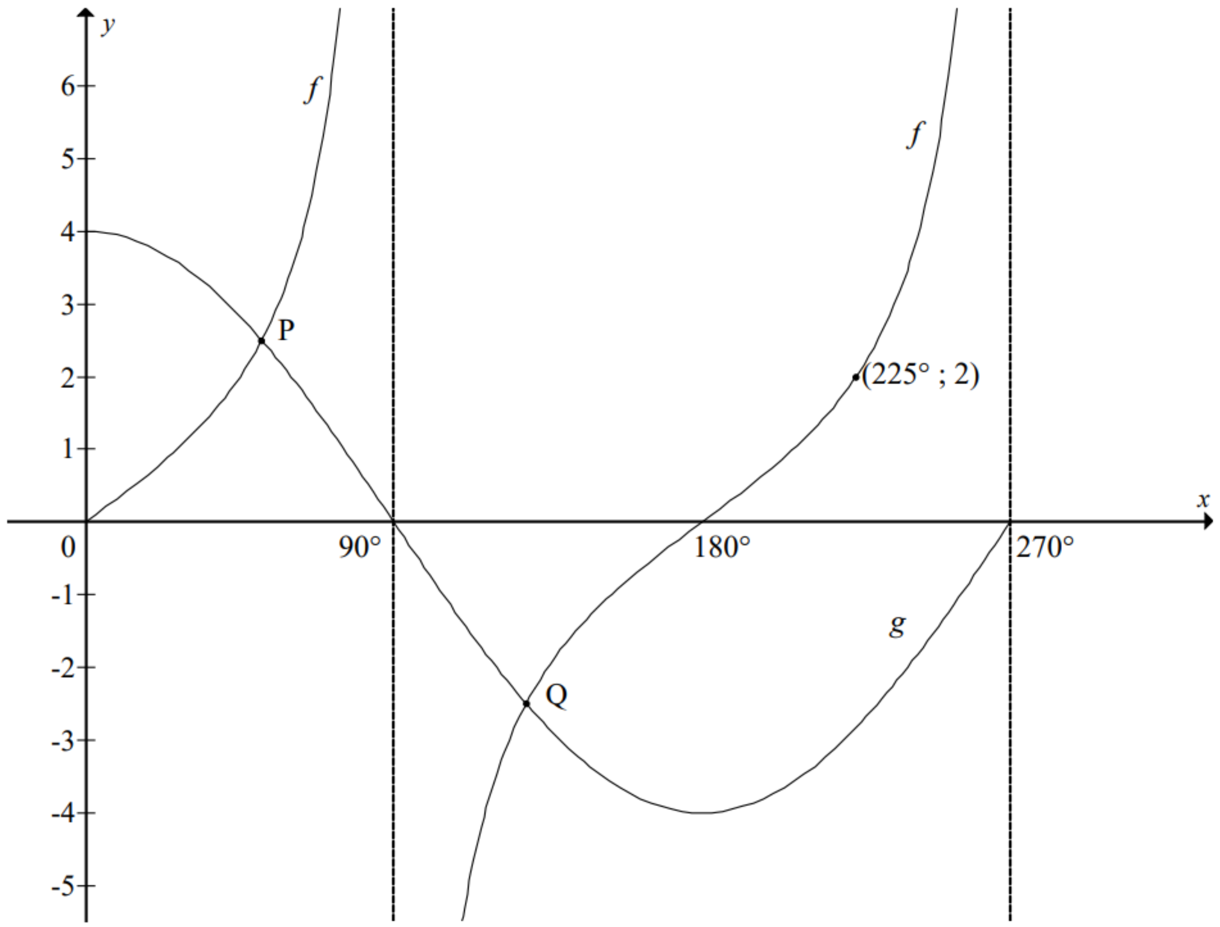
9.3 Use the graph to solve for x: $2\cos x . \tan 2x > 0$.

9.4 Write down the period of $f\left(\frac{x}{2}\right)$.

9.5 Write down the equations of the asymptotes of $g(x - 25°)$, where $x \in [-90°\ ;\ 90°]$.

QUESTION 10

The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0° \leq x \leq 270°$ are shown in the diagram below. The point (225° ; 2) lies on *f*. The graphs intersect at points P and Q.



10.1 Determine the numerical values of a and b.

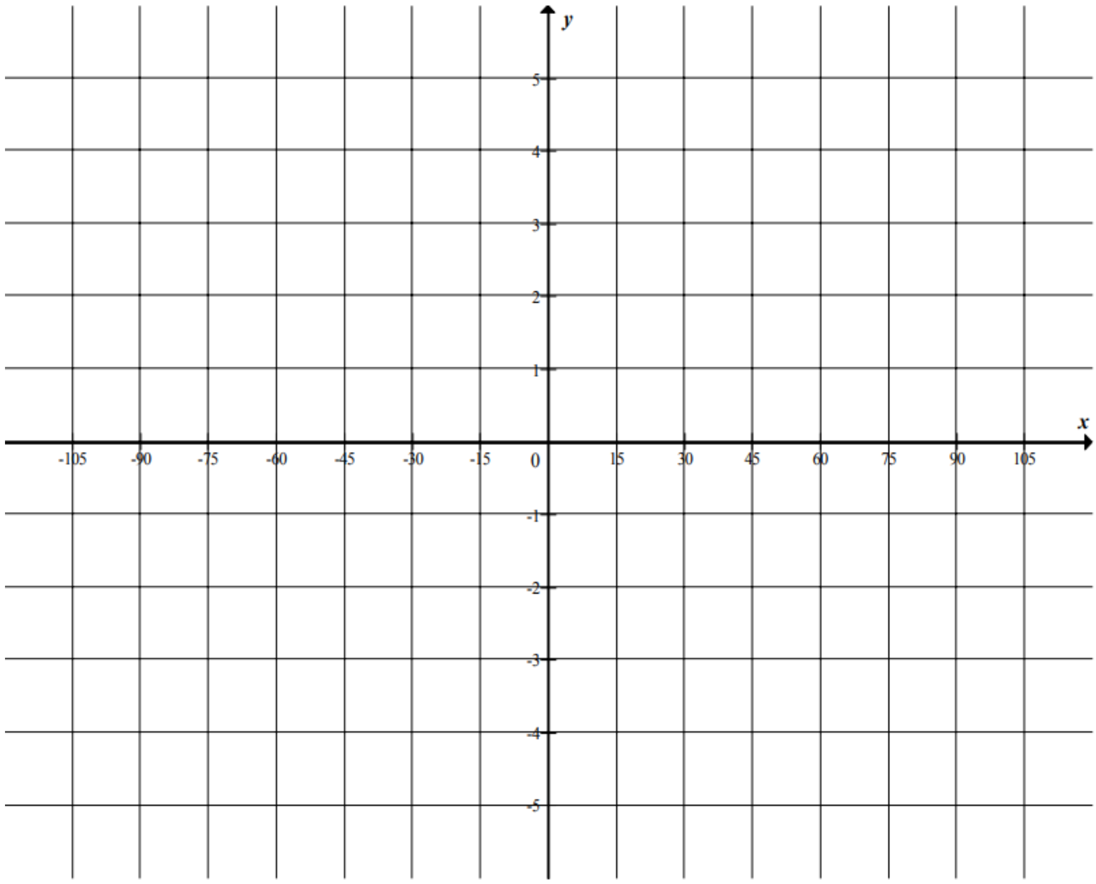
10.2 Determine the minimum value of $g(x) + 2$.

10.3 Determine the period of $f\left(\frac{1}{2}x\right)$.

10.4 Show that, if the x-coordinate of P is θ, then the x-coordinate of Q is $(180° - \theta)$.

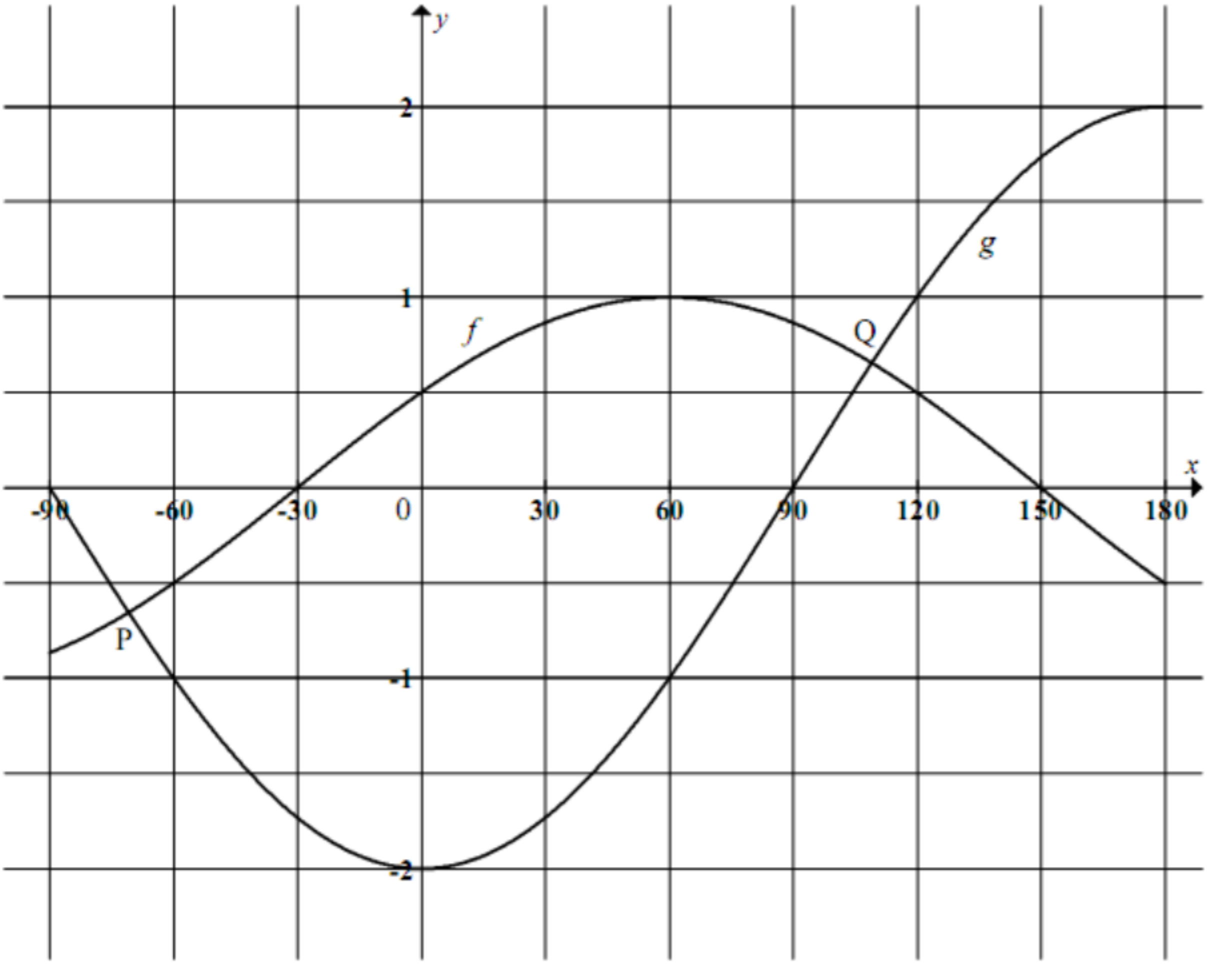
DIAGRAM SHEET 4

QUESTION 9.1



QUESTION 11

The graphs of $f(x) = \sin(x + 30°)$ and $g(x) = -2\cos x$ for $-90° \leq x \leq 180°$ are given below. The graphs intersect at point P and point Q.



11.1 Calculate $f(0) - g(0)$.

11.2 Calculate the x-coordinates of point P and point Q.

11.3 For which values of x will $f(x) \geq g(x)$?

11.4 Graph h is obtained by the following transformation of f: $h(x) = 2f(x + 60°)$. Describe the relationship between g and h.

QUESTION 12

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2}\tan x$ for $x \in [-90° ; 180°]$.

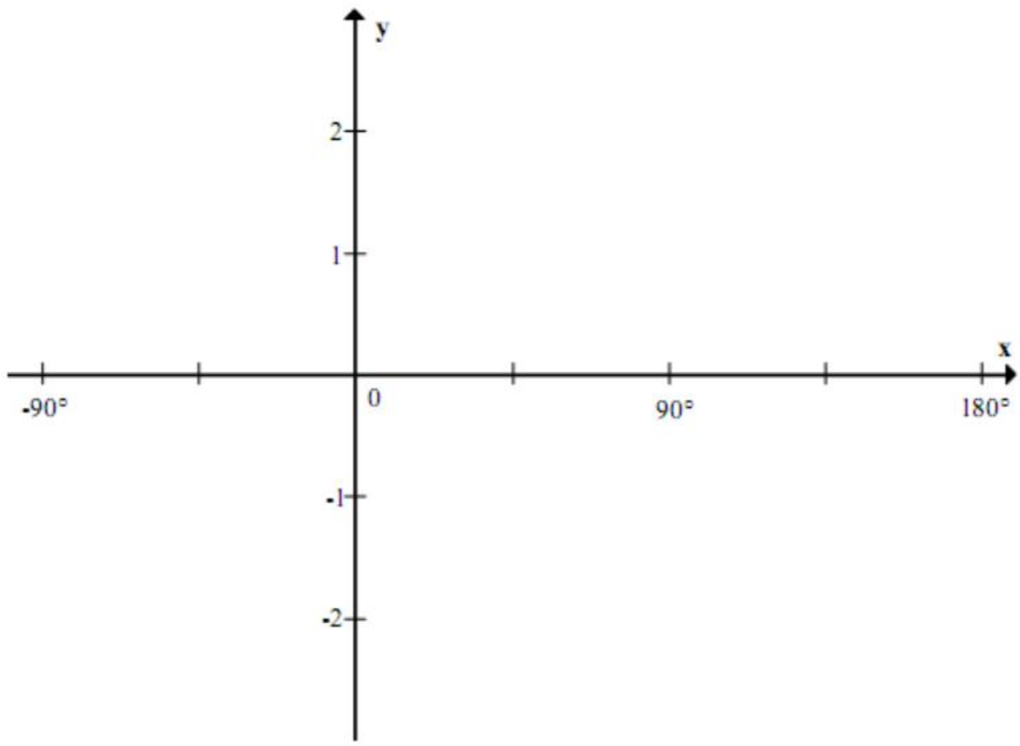
12.1 Sketch the graphs of f and g on the same system of axes on DIAGRAM SHEET 5.

12.2 Calculate the x-coordinates of the points of intersection of f and g.

12.3 Determine the values of x for which $g(x) > f(x)$.

DIAGRAM SHEET 5

QUESTION 12.1



QUESTION 13

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

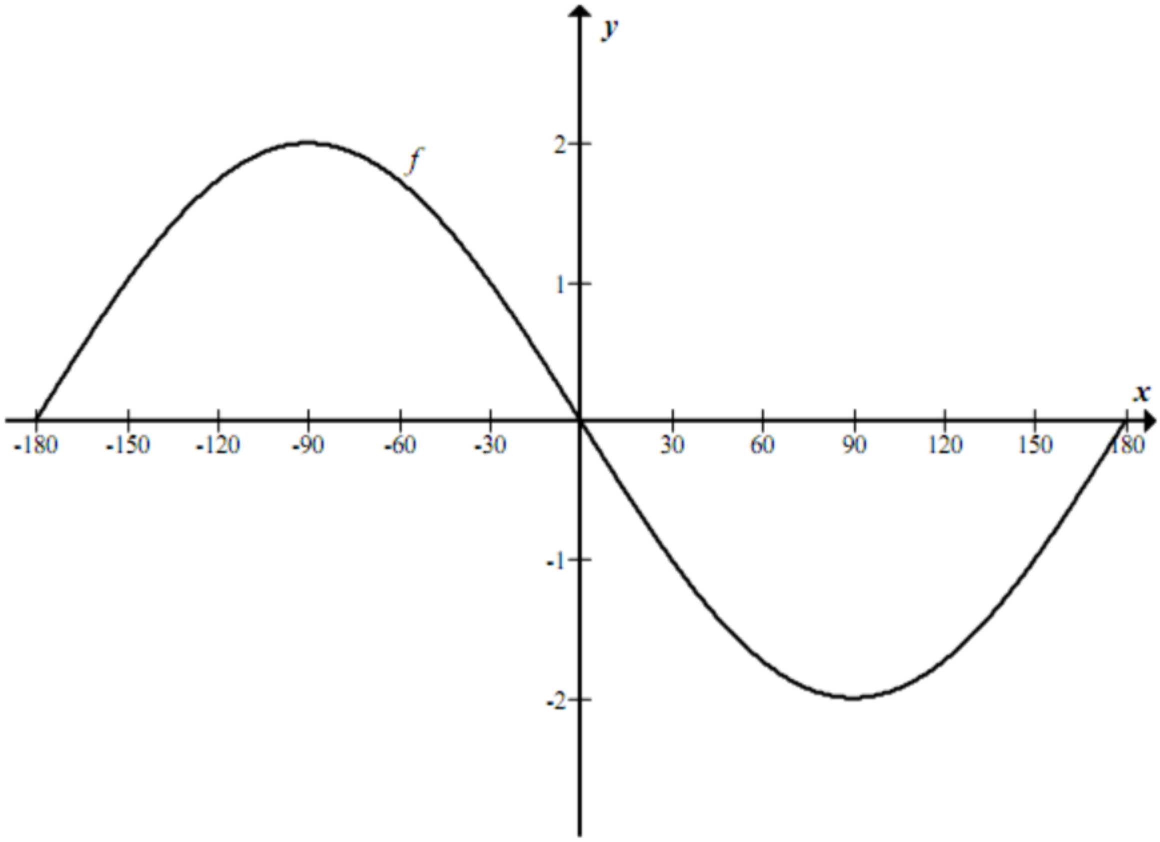
13.1 Calculate the points of intersection of the graphs f and g for $x \in [180° ; 360°]$.

13.2 Draw sketch graphs of f and g for $x \in [180° ; 360°]$ on the same system of axes provided on DIAGRAM SHEET 6.

13.3 For which values of x will $f(x) \leq g(x)$ for $x \in [180° ; 360°]$?

QUESTION 14

The graph of $f(x) = -2\sin x$ is drawn below.



14.1 Write down the period of f.

14.2 Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$.

14.3 Draw the graph of $g(x) = \cos(x - 30°)$ for $x \in [-180°\,;\,180°]$ on the grid provided on DIAGRAM SHEET 6.

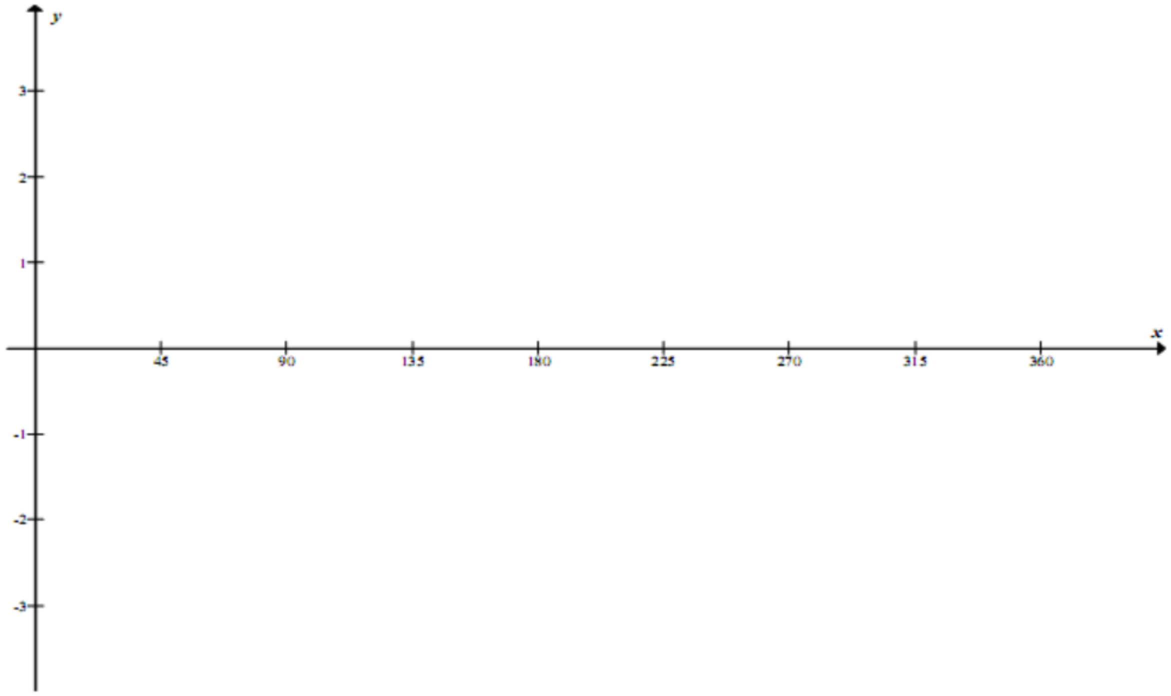
14.4 Use the graph to determine the number of solutions for $-2\sin x = \cos(x - 30°)$, $x \in [-180°\,;\,180°]$.

14.5 For which values of x is $g(x) \geq 0$?

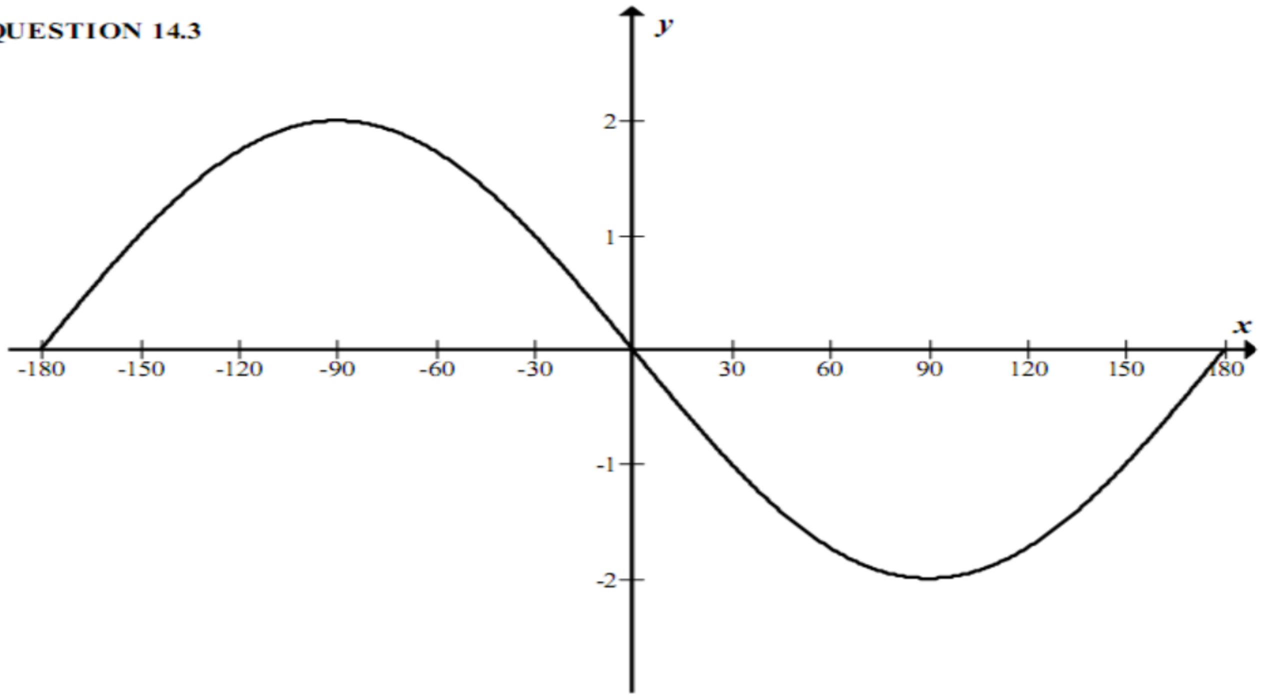
14.6 For which values of x is $f'(x) < 0$ and $g'(x) > 0$?

DIAGRAM SHEET 6

QUESTION 13.2

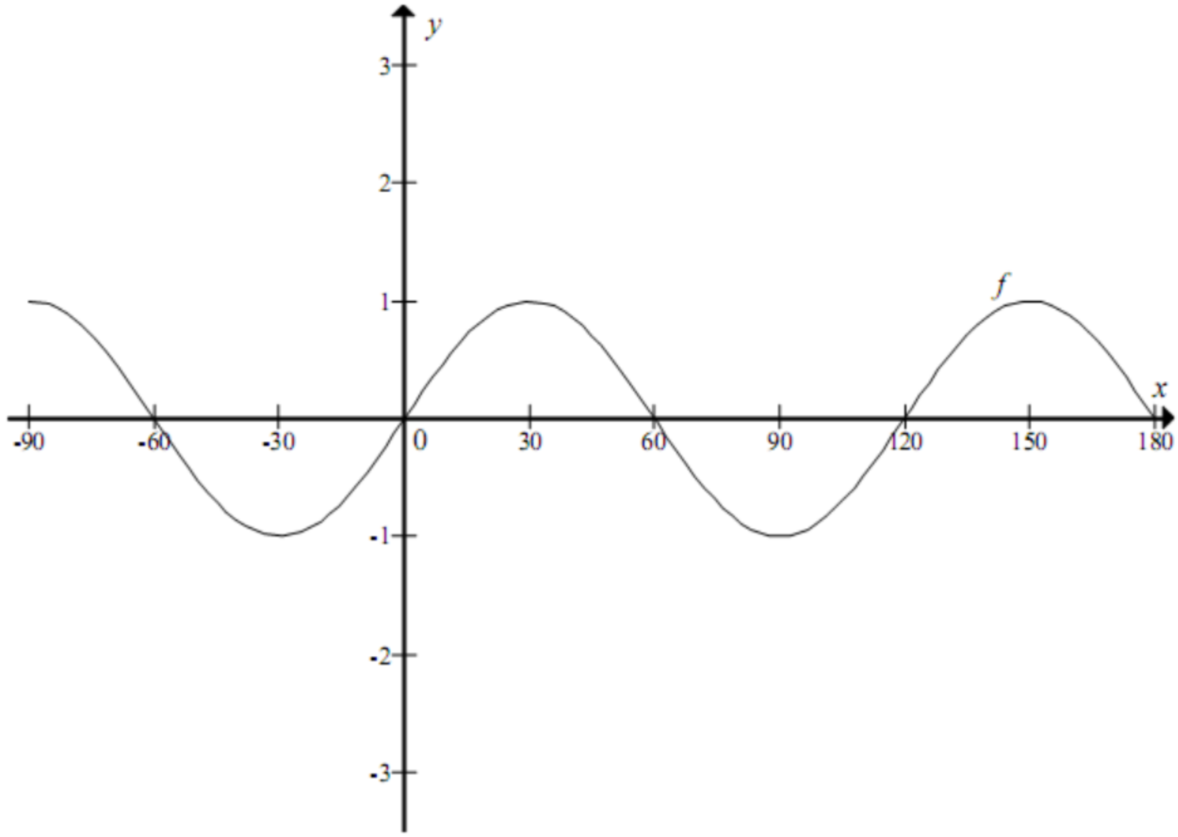


QUESTION 14.3



QUESTION 15

The graph of $f(x) = \sin 3x$ is drawn below for $x \in [-90° ; 180°]$.



15.1 Write down the period of f.

15.2 Write down the solutions for $\sin 3x = -1$ on the interval $[-90° ; 180°]$.

15.3 Give the maximum value of h if $h(x) = f(x) - 1$.

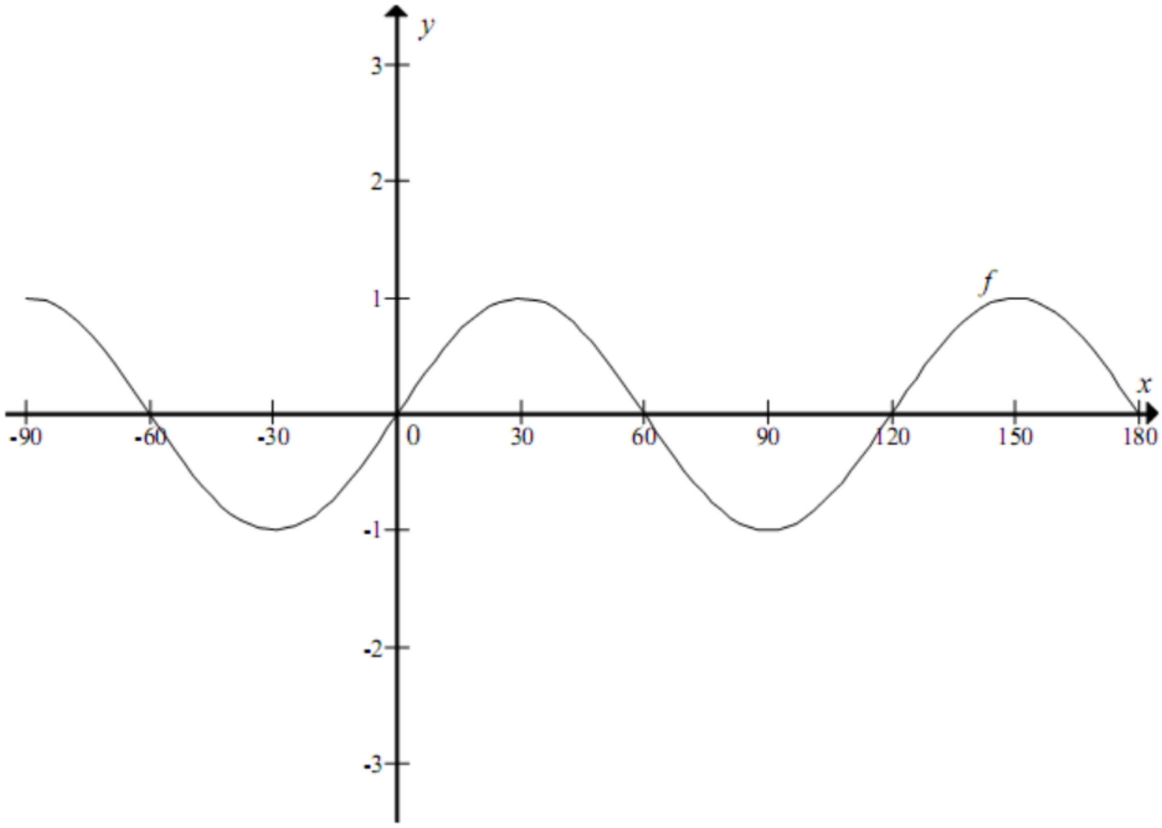
15.4 Draw the graph of $g(x) = 3\cos x$ for $x \in [-90° ; 180°]$ on the grid on DIAGRAM SHEET 7.

15.5 Use the graphs to determine how many solutions there are to the equation $\frac{\sin 3x}{3} - \cos x = 0$ on the interval $[-90° ; 180°]$.

15.6 Use the graphs to solve: $f(x).g(x) < 0$.

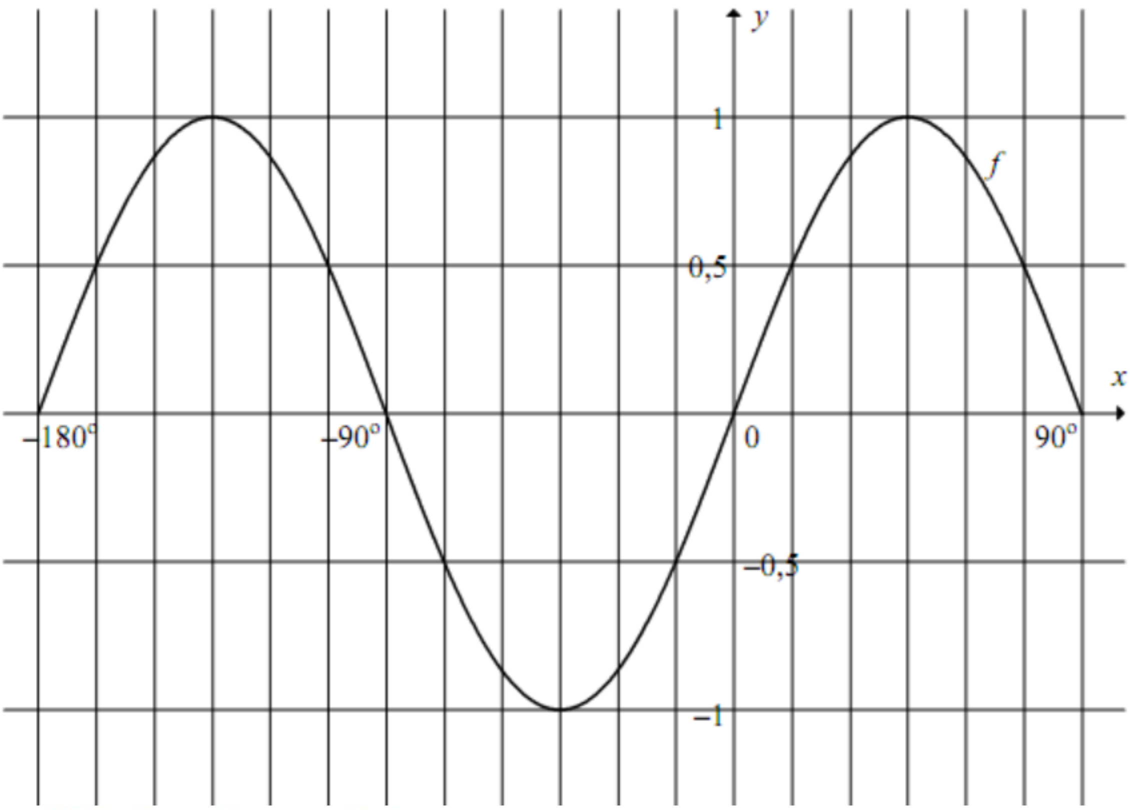
DIAGRAM SHEET 7

QUESTION 15.4



QUESTION 16

The graph of $f(x) = \sin 2x$ for $-180° \leq x \leq 90°$ is shown in the sketch below.



16.1 Write down the range of f.

16.2 Determine the period of $f\left(\frac{3}{2}x\right)$.

16.3 Draw the graph of $g(x) = \cos(x - 30°)$ for $-180° \leq x \leq 90°$ on the system of axes on DIAGRAM SHEET 8. Clearly label ALL x-intercepts and turning points.

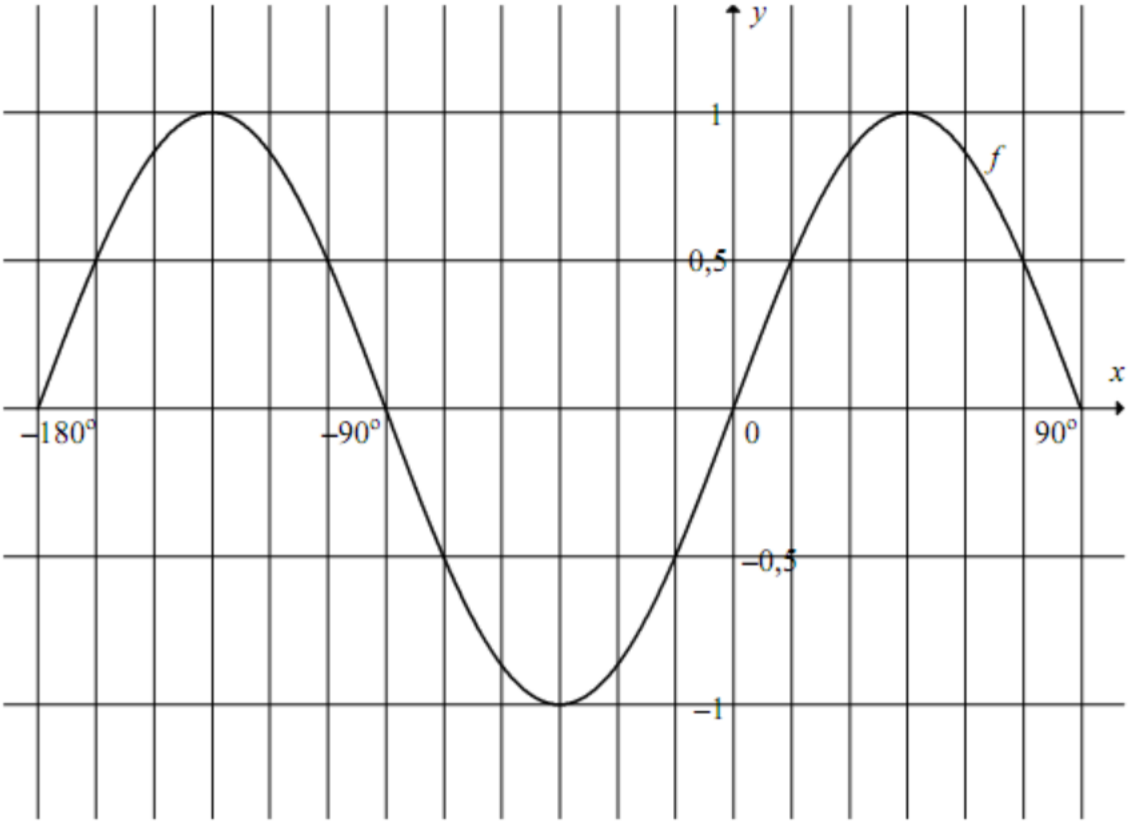
16.4 Hence, or otherwise, determine the values of x in the interval $-180° \leq x \leq 90°$ for which $f(x).g(x) < 0$.

16.5 Describe the transformation that graph f has to undergo to form $y = \sin(2x + 60°)$.

16.6 Determine the general solution of $\sin 2x = \cos(x - 30°)$.

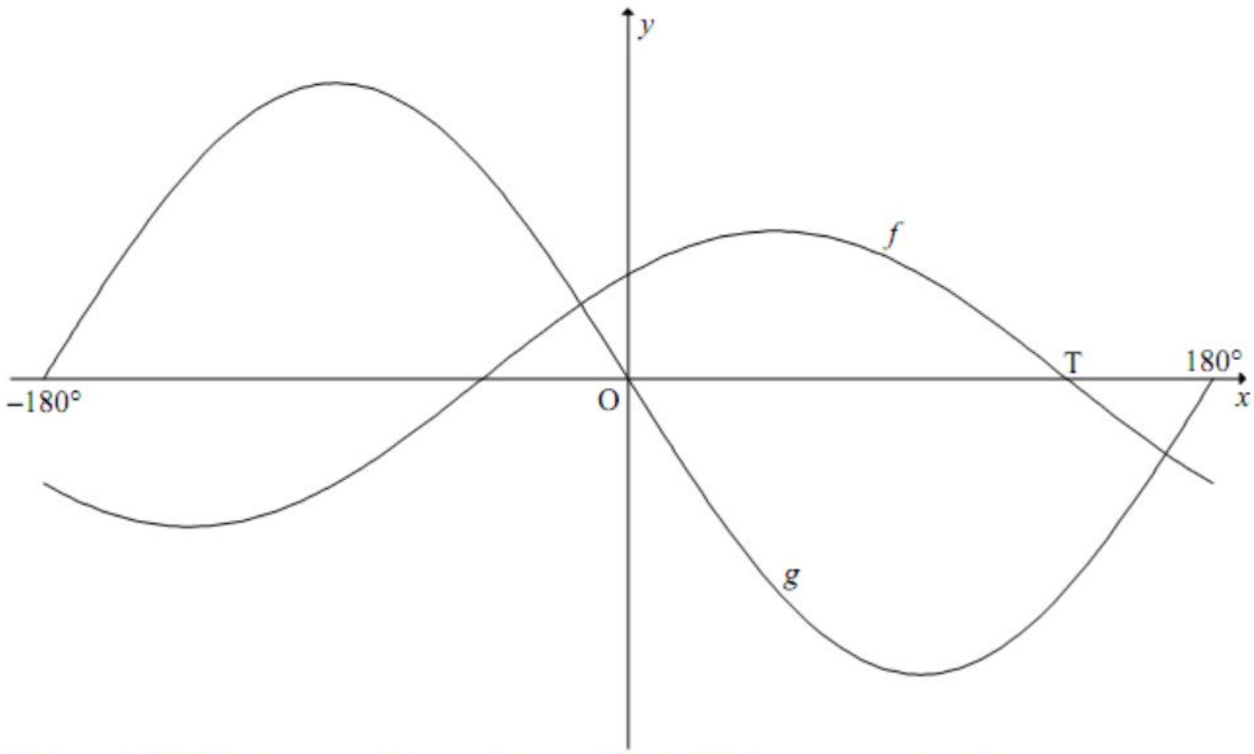
DIAGRAM SHEET 8

QUESTION 16.3



QUESTION 17

The graphs of $f(x) = \cos(x - 45°)$ and $g(x) = -2\sin x$ are drawn below for $x \in [-180°; 180°]$. The point T is an x-intercept of f as indicated on the diagram.



17.1 Show that $\cos(x - 45°) = -2\sin x$ can be written as $\tan x = -0{,}2612$.

17.2 Solve the equation: $\cos(x - 45°) = -2\sin x$ for $x \in [-180°; 180°]$.

17.3 Write down the coordinates of point T.

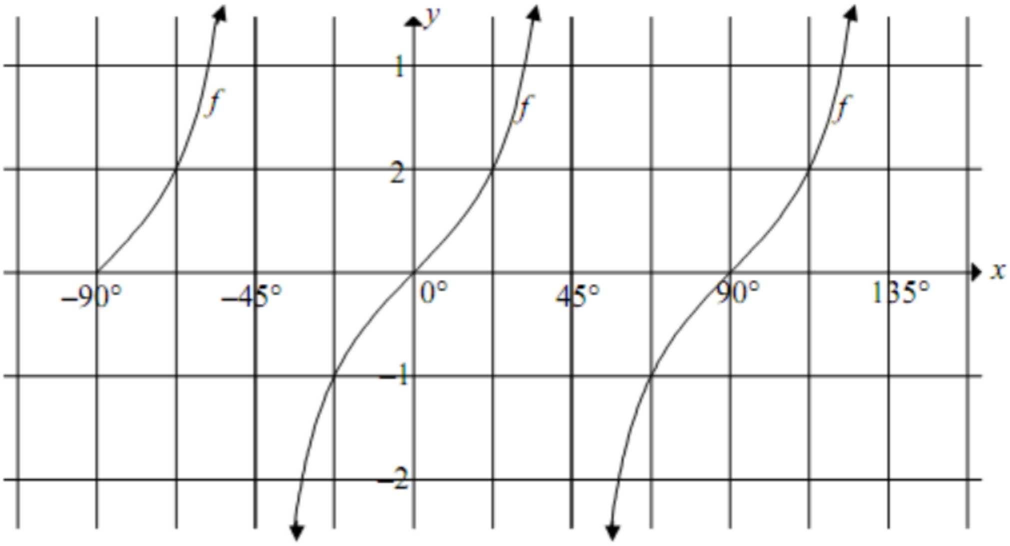
17.4 Write down the interval for which $f(x) \geq g(x)$.

17.5 Write down the interval for which both f and g are strictly increasing.

17.6 The graph h is obtained when the graph f is shifted 45° to the right. Write down the equation of h in its simplest form.

QUESTION 18

18.1 In the diagram, the graph of $f(x) = \tan bx$ is drawn for the interval $-90° \leq x \leq 135°$.

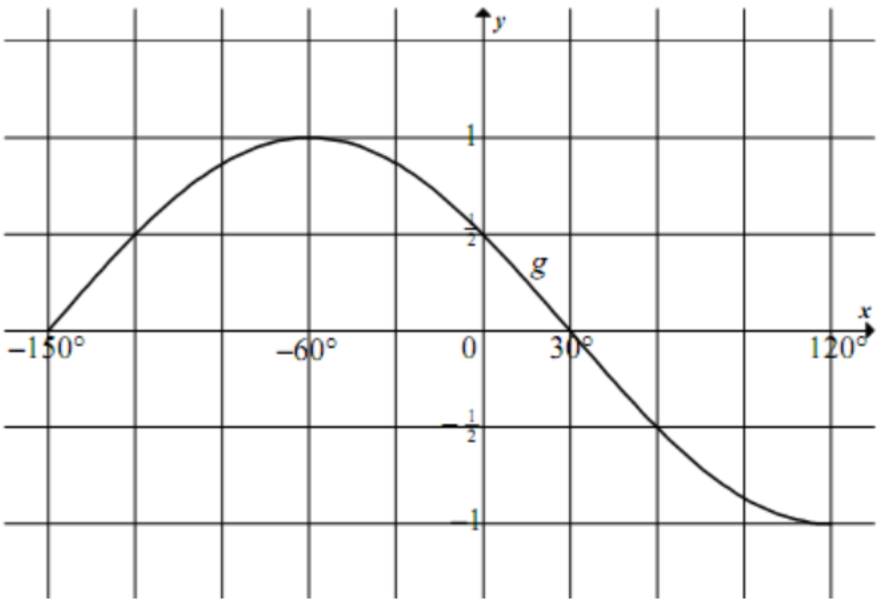


18.1.1 Determine the value of b.

18.1.2 Determine the values of x in the interval $0° \leq x \leq 135°$ for which $f(x) \leq -1$.

18.1.3 Graph h is defined as $h(x) = \tan b(x + 55°)$. Write down the equations of the asymptotes of h in the interval $-90° \leq x \leq 135°$.

18.2 In the diagram, the graph of $g(x) = \cos(x + 60°)$ is drawn for the interval $-150° \leq x \leq 120°$.



18.2.1 On the same system of axes, DIAGRAM SHEET 9, draw the graph of $k(x) = -\sin x$ for the interval $-150° \le x \le 120°$. Show ALL the intercepts with the axes as well as the coordinates of the turning points and end points of the graph.

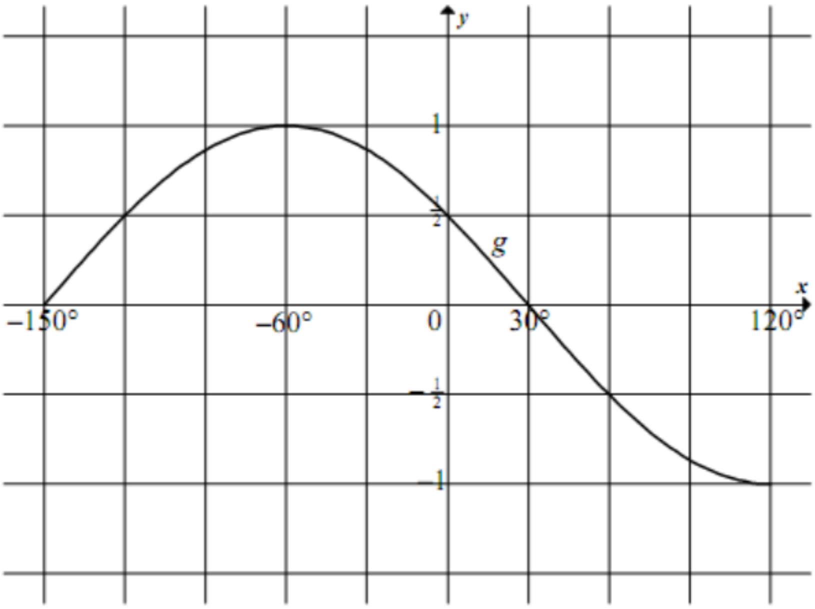
18.2.2 Determine the minimum value of $h(x) = \cos(x + 60°) - 3$.

18.2.3 Solve the equation $\cos(x + 60°) + \sin x = 0$ for the interval $-150° \le x \le 120°$.

18.2.4 Determine the values of x for the interval $-150° \le x \le 120°$, for which $\cos(x + 60°) + \sin x > 0$.

18.2.5 The function g can also be defined as $y = -\sin(x - \theta)$, where θ is an acute angle. Determine the value of θ.

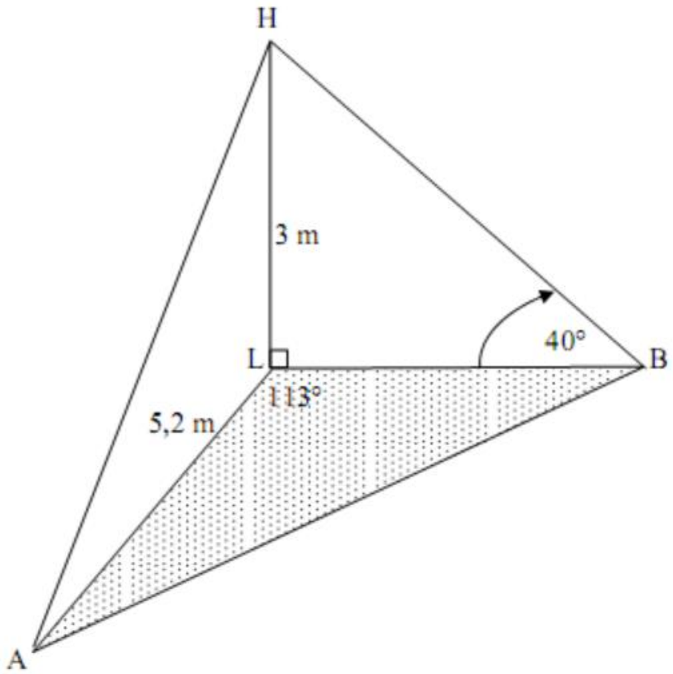
DIAGRAM SHEET 9



Part 5

QUESTION 1

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.



1.1 Calculate the length of LB.

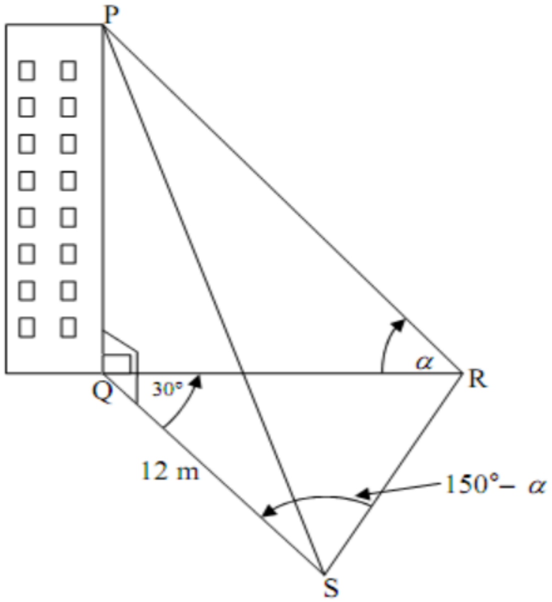
1.2 Hence, or otherwise, calculate the length of AB.

1.3 Determine the area of ΔABL.

QUESTION 2

2.1 In the sketch below, PQ is a vertical building. Q, R and S are points on the same horizontal plane. The angle of elevation of P, the top of the building, from R is α.

$R\hat{Q}S = 30°$ and $Q\hat{S}R = 150° - \alpha$.
$QS = 12\text{ m}$

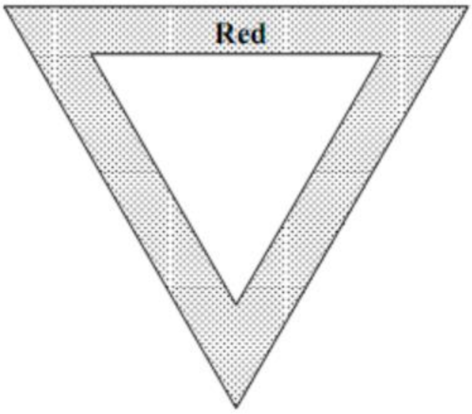


2.1.1 Determine QR in terms of $\sin\alpha$ and $\cos\alpha$. (Hint: Use the sine rule in ΔQRS.)

2.1.2 Hence, or otherwise, show that the height PQ of the building is $PQ = 6 + 6\sqrt{3}\tan\alpha$.

2.1.3 Hence, or otherwise, calculate α, the angle of elevation of P from R, if PQ = 23 m.

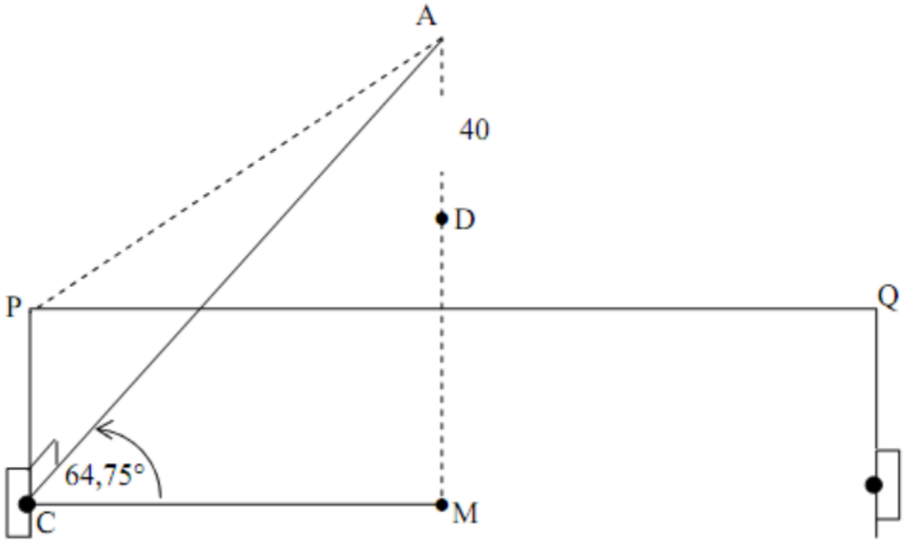
2.2 A yield sign consists of two equilateral triangles. The length of the side of the inner triangle is 50 cm and the length of the side of the outer triangle is 80 cm.



Calculate the area of the red part of the yield sign. (Indicated as the shaded region on the diagram).

QUESTION 3

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is 64,75°. The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also $AC \perp PC$. In the figure below PQ = 100 metres and PC = 32 metres.



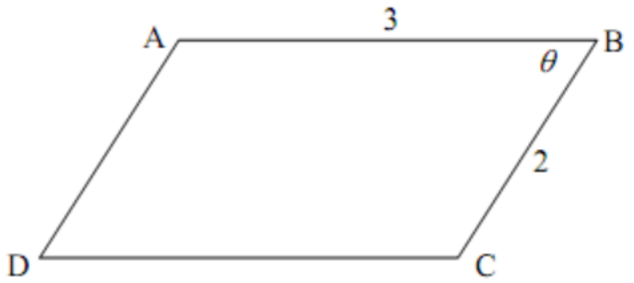
3.1 Determine AC.

3.2 Calculate $P\hat{A}C$.

3.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C.

QUESTION 4

ABCD is a parallelogram with AB = 3 units, BC = 2 units and $A\hat{B}C = \theta$ for $0° < \theta \le 90°$.



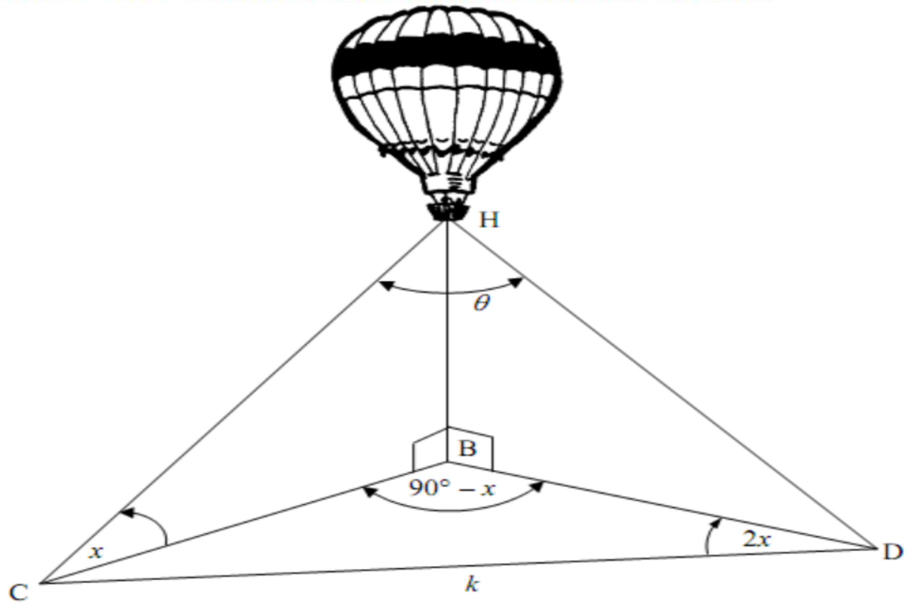
4.1 Prove that the area of parallelogram ABCD is $6\sin\theta$.

4.2 Calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units.

4.3 Determine the value of θ for which the parallelogram has the maximum area.

QUESTION 5

A hot-air balloon H is directly above point B on the ground. Two ropes are used to keep the hot-air balloon in position. The ropes are held by two people on the ground at point C and point D. B, C and D are in the same horizontal plane. The angle of elevation from C to H is x. $C\hat{D}B = 2x$ and $C\hat{B}D = 90° - x$. The distance between C and D is k metres.



5.1 Show that CB = $2k \sin x$.

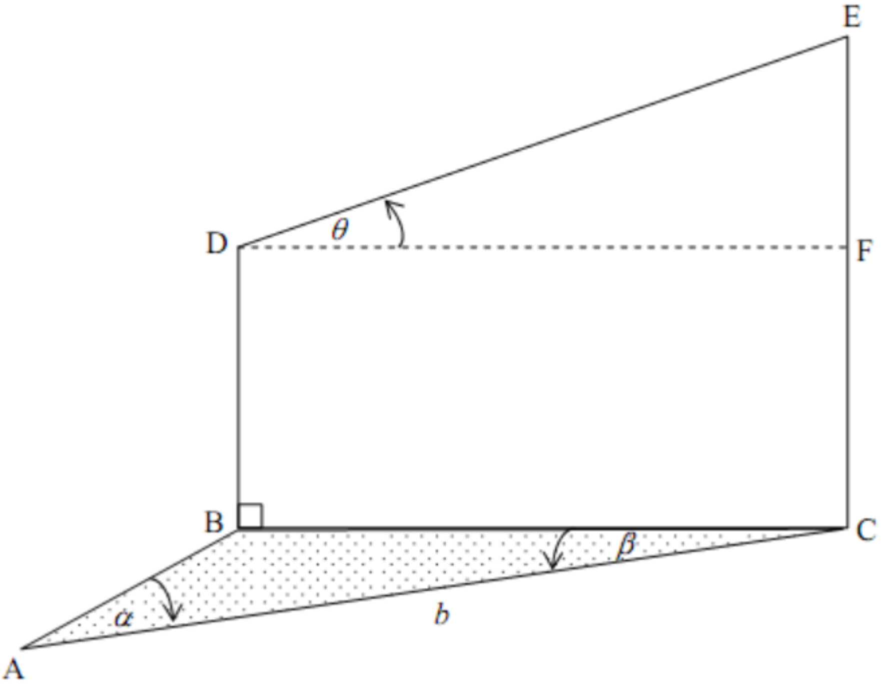
5.2 Hence, show that the length of rope HC is $2k \tan x$.

5.3 If k = 40 m, x = 23° and HD = 31,8 m, calculate θ, the angle between the two ropes.

QUESTION 6

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is $\theta°$. F is a point on EC such that DF || BC.

$B\hat{A}C = \alpha$, $A\hat{C}B = \beta$ and AC = b metres.

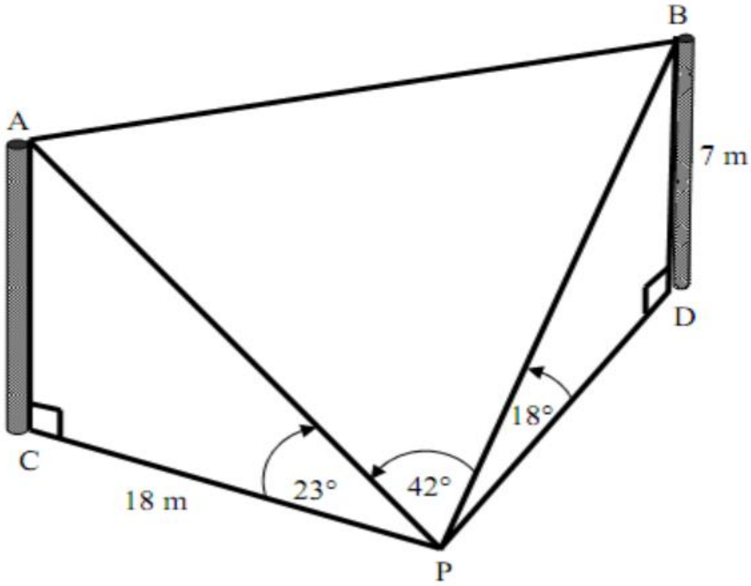


6.1 Prove that $DE = \dfrac{b\sin\alpha}{\sin(\alpha+\beta)\cos\theta}$

6.2 Calculate DE if $b = 2\ 000$ metres, $\alpha = 43°$, $\beta = 36°$ and $\theta = 27°$.

QUESTION 7

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

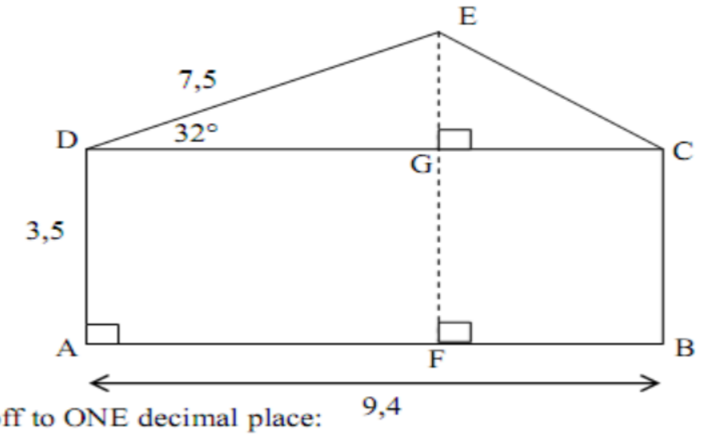
7.1 The distance from Thandi to the top of pole BD

7.2 The distance from Thandi to the top of pole AC

7.3 The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$

QUESTION 8

The sketch below shows one side of the elevation of a house. Some dimensions (in metres) are indicated on the figure.



Calculate, rounded off to ONE decimal place:

8.1 EC

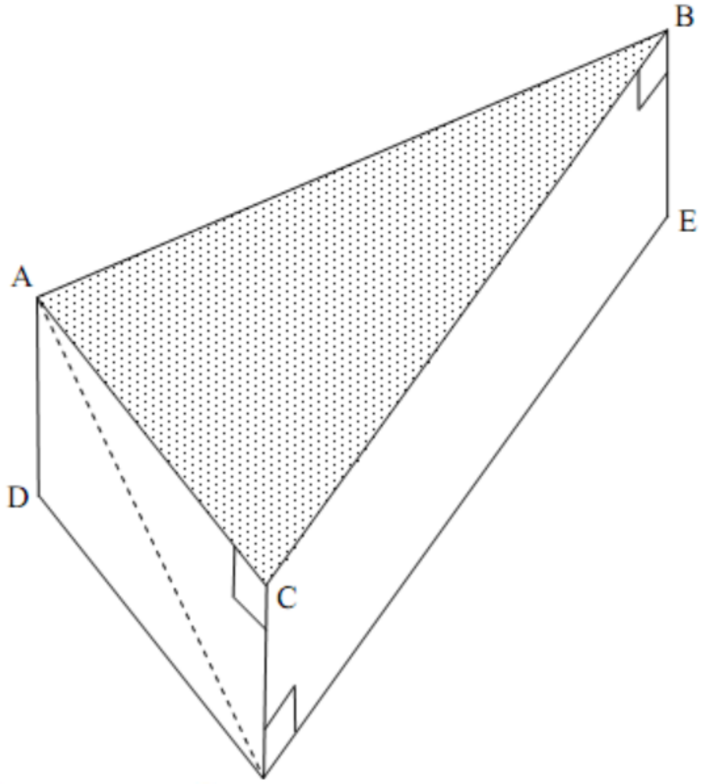
8.2 $D\hat{C}E$

8.3 Area of ΔDEC

8.4 The height EF

QUESTION 9

The figure below represents a triangular right prism with BA = BC = 5 units, $A\hat{B}C = 50°$ and $F\hat{A}C = 25°$.



9.1 Determine the area of ΔABC.

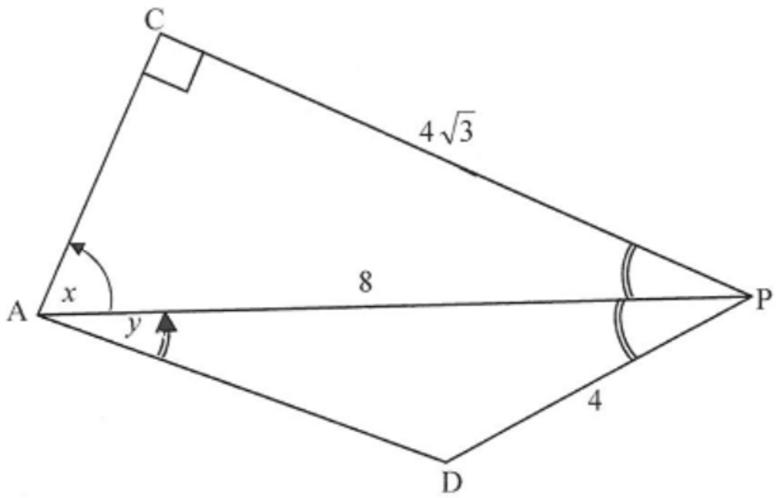
9.2 Calculate the length of AC.

9.3 Hence, determine the height FC of the prism.

Part 6

QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



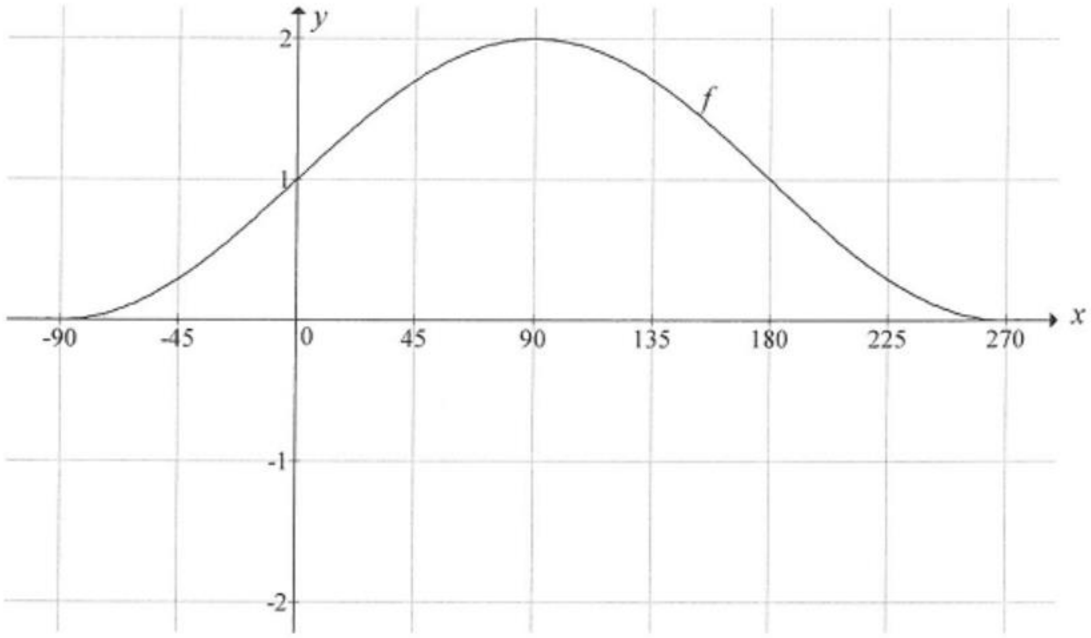
5.1	Show, by calculation, that $x = 60°$.	(2)
5.2	Calculate the length of AD.	(4)
5.3	Determine y.	(3)
		[9]

QUESTION 6

6.1	Prove the identity: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$	(5)
6.2	Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$.	(3)
6.3	If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$.	(4)
		[12]

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$$1 + 2\cos 2x + 4\cos^2 2x + \ldots$$

Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)

[19]

QUESTION 5

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90°+x)}{\sin(30°-x).\cos x+\cos(30°-x).\sin x}$$ (6)

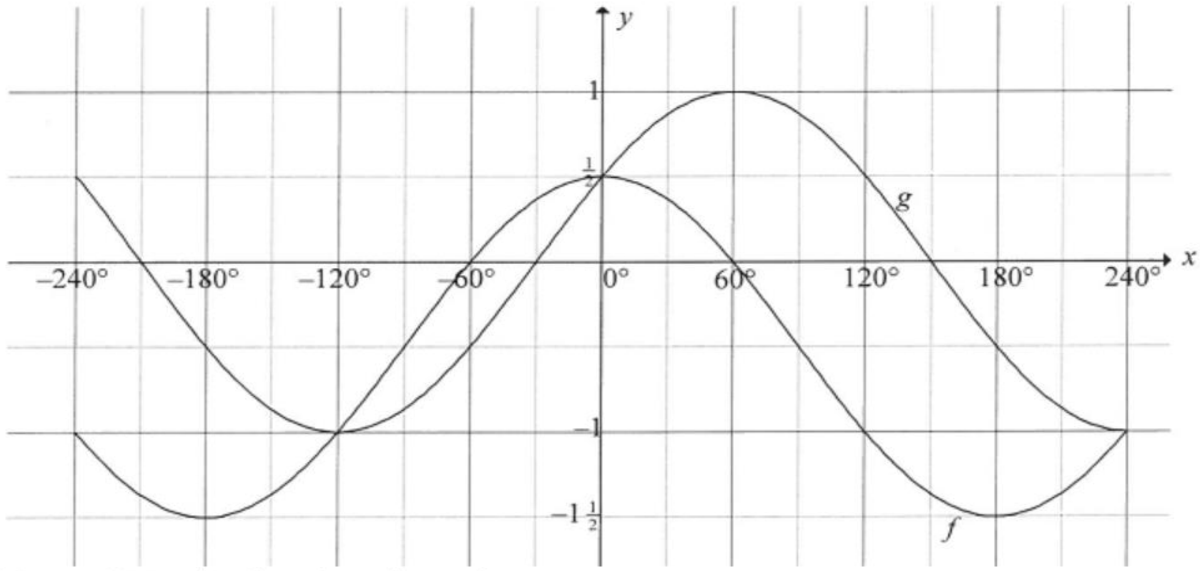
5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \le x \le 240°$. The graphs intersect at $\left(0°\,;\frac{1}{2}\right)$, $(-120°\,;-1)$ and $(240°\,;-1)$.



6.1 Determine the values of p and q. (4)

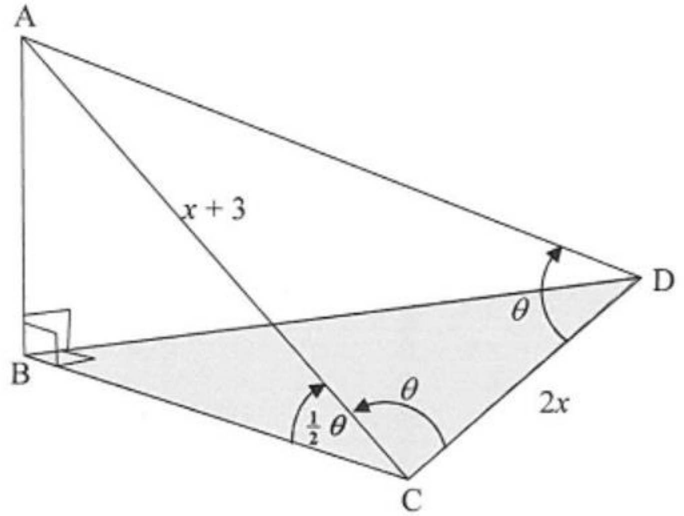
6.2 Determine the values of x in the interval $-240° \le x \le 240°$ for which $f(x) > g(x)$. (2)

6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2)

[8]

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below.
The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.
Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

7.2 Prove that $\cos\theta = \dfrac{x}{x+3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)
[10]

QUESTION 5

5.1 Given: $\sin 16° = p$
Determine the following in terms of p, **without using a calculator.**

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.3 Simplify $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90°+A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

5.4.1 $\cos B$ (3)

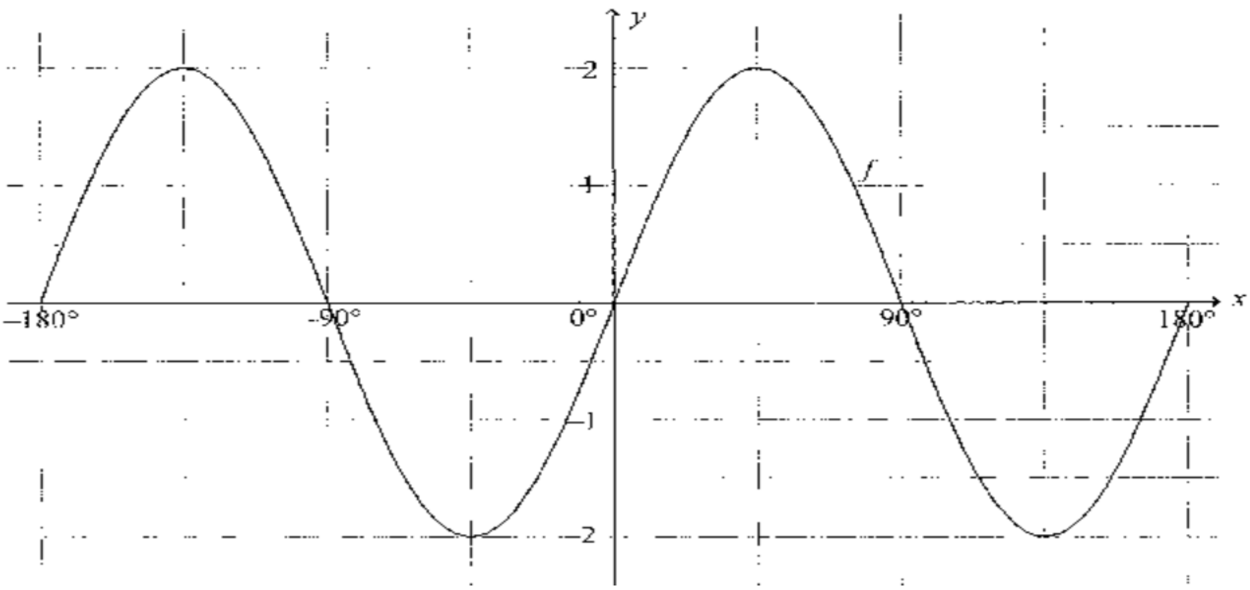
5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45°)$ (4)

[21]

QUESTION 6

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

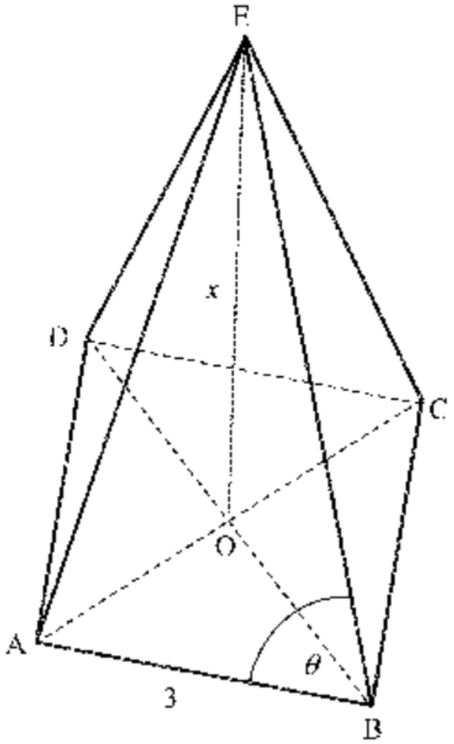
6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180° ; 0°]$. (3)

[12]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, $AB = 3$ m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}$(area of base)$\times$(⊥ height)



7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \dfrac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is $15\ m^3$, calculate the value of θ. (4)

[12]

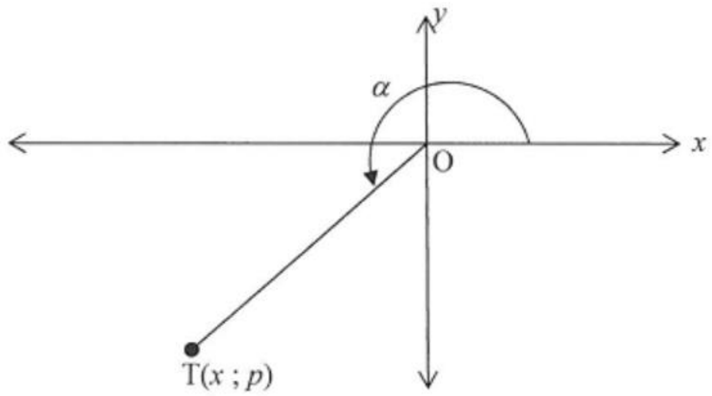
QUESTION 5

5.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$\sin(540^\circ - x).\sin(-x) - \cos(180^\circ - x).\sin(90^\circ + x)$ (6)

5.3 In the diagram below, $T(x\,;\,p)$ is a point in the third quadrant and it is given that $\sin\alpha = \frac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180^\circ + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$. (3)

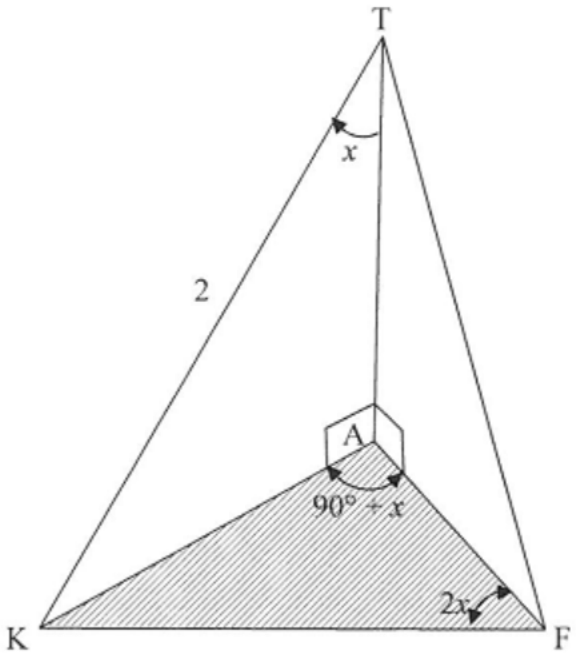
5.4 5.4.1 For which value(s) of x will $\frac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0^\circ \leq x \leq 180^\circ$? (3)

5.4.2 Prove the identity: $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

QUESTION 6

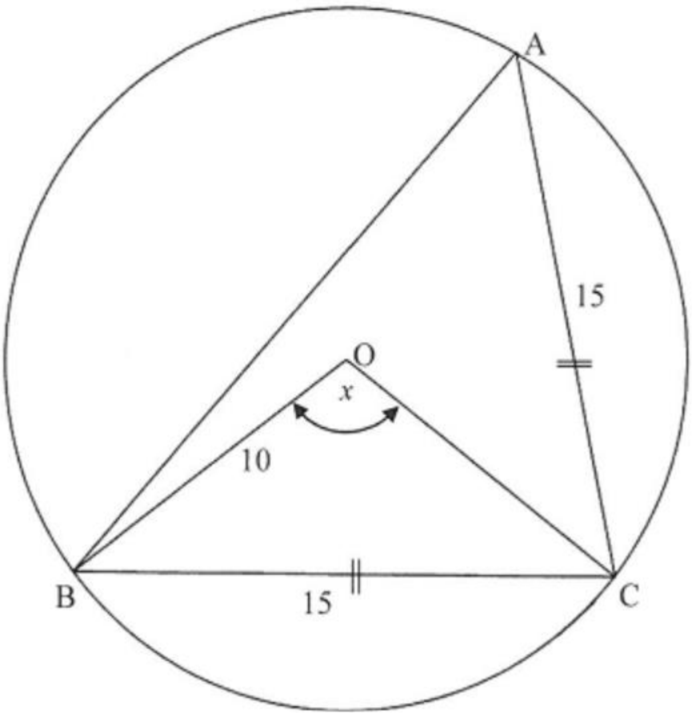
6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. $TK = 2$ units.



6.1.1 Express AK in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of KF. (5)

6.2 In the diagram below, a circle with centre O passes through A, B and C. $BC = AC = 15$ units. BO and OC are joined. $OB = 10$ units and $B\hat{O}C = x$.



Calculate:

6.2.1 The size of x (4)

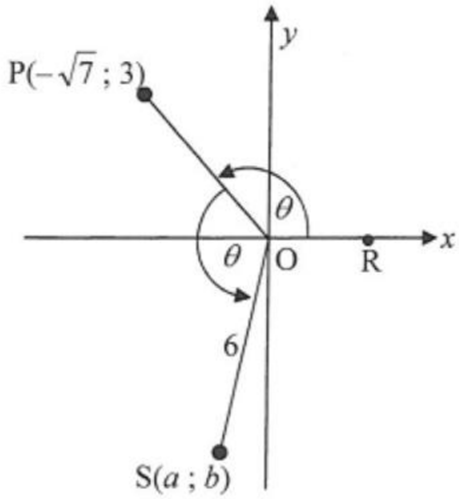
6.2.2 The size of $A\hat{C}B$ (3)

6.2.3 The area of ΔABC (2)

[16]

QUESTION 5

5.1 $P(-\sqrt{7}\,;3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and $OS = 6$.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

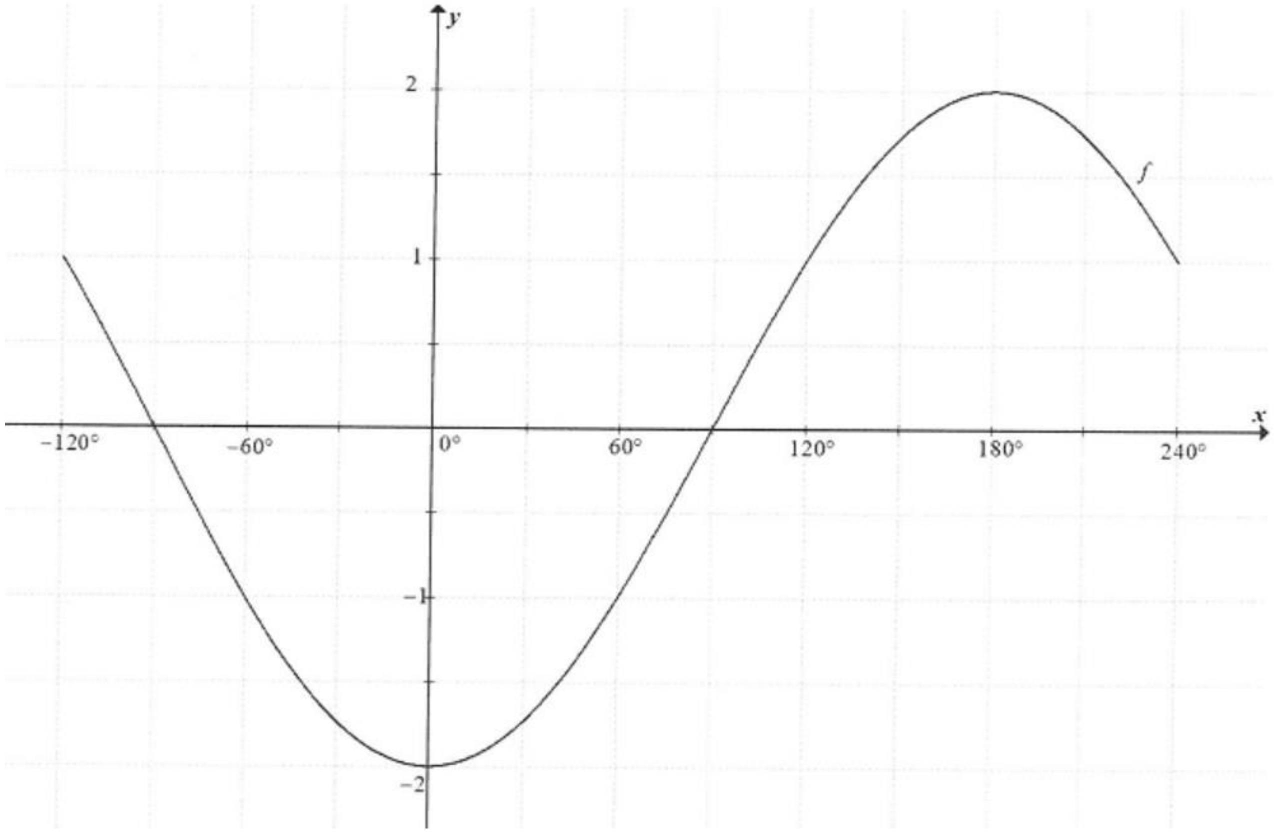
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



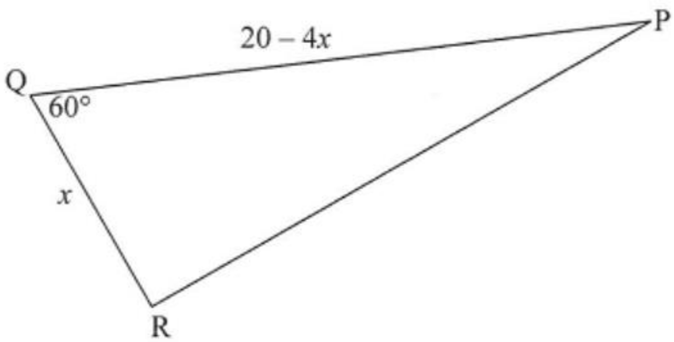
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

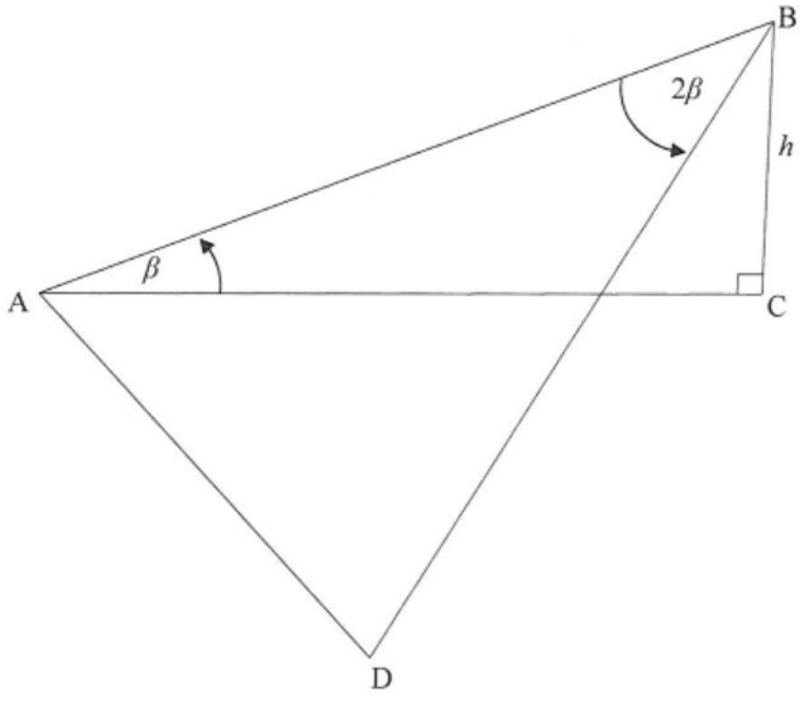


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and $BA = BD$.

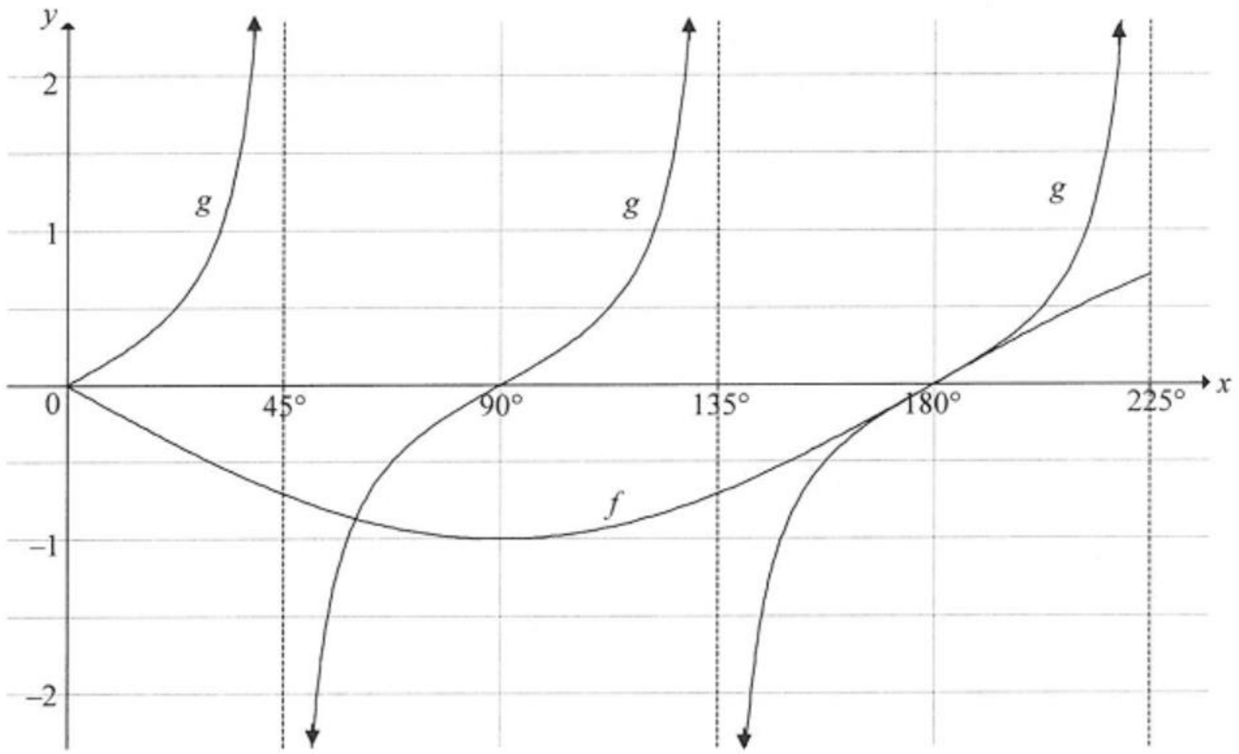


Determine the distance AD between the two anchors in terms of h. (7)

[15]

QUESTION 5

In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \leq x \leq 225°$.



5.1 Write down the values of a and b. (2)

5.2 Write down the period of $f(3x)$. (2)

5.3 Determine the values of x in the interval $90° \leq x \leq 225°$ for which $f(x).g(x) \leq 0$. (3)

[7]

QUESTION 6

6.1 **Without using a calculator**, determine the following in terms of $\sin 36°$:

6.1.1 $\sin 324°$ (1)

6.1.2 $\cos 72°$ (2)

6.2 Prove the identity: $1-\dfrac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$

(6)

6.4 Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Use the formula for $\cos(A-B)$ to derive a formula for $\sin(A-B)$. (4)

6.4.2 **Without using a calculator**, show that

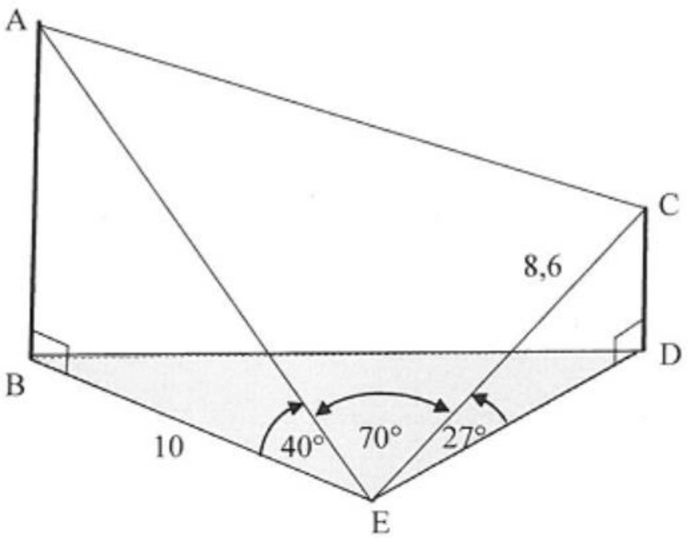
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

for all values of x. (6)

[23]

QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. CE = 8,6 m, BE = 10 m, $A\hat{E}B=40°$, $A\hat{E}C=70°$ and $C\hat{E}D=27°$.



Calculate the:

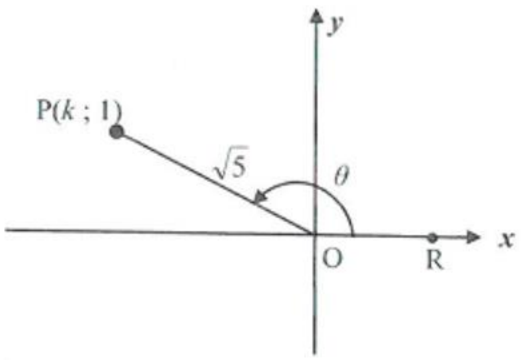
7.1 Height of pole CD (2)

7.2 Length of cable AE (2)

7.3 Length of cable AC (4)
[8]

QUESTION 5

5.1 In the diagram, P(k ; 1) is a point in the 2^{nd} quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $\hat{ROP} = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\dfrac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)
[23]

QUESTION 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

6.1 Write down the period of f. (1)

6.2 The point A(t ; 2) lies on the graph. Determine the general solution of t. (3)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120° ; 180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

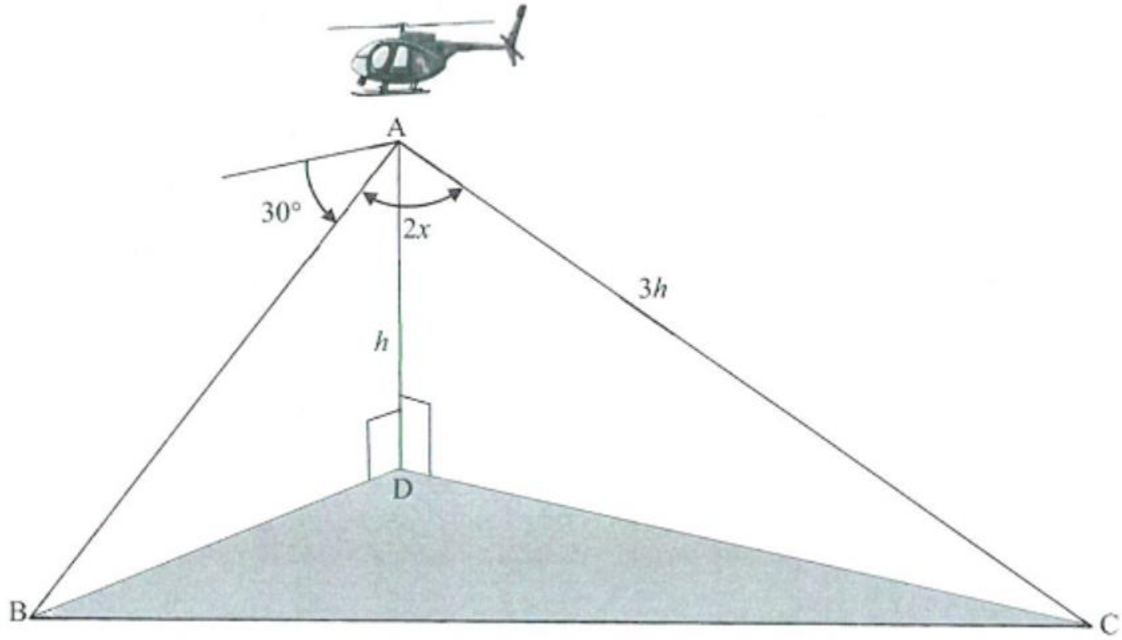
6.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120° ; 180°]$. (3)

6.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is $30°$. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)

[6]

QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$ (5)

5.2 **Without using a calculator**, determine the value of: $\dfrac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ (4)

5.3 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m. (4)

5.4 Consider: $f(x)=\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°$

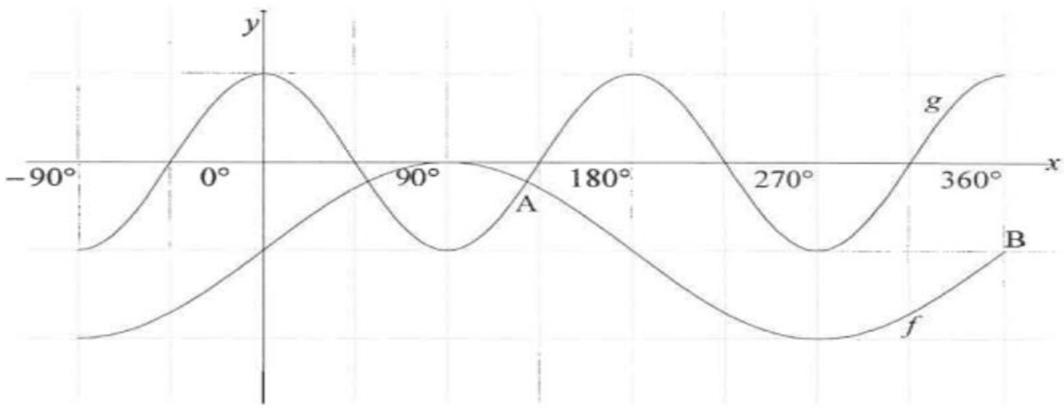
5.4.1 Determine the general solution of $f(x)=\tan 165°$ (6)

5.4.2 Determine the value(s) of x in the interval $x\in[0°;360°]$ for which $f(x)$ will have a minimum value. (3)

[22]

QUESTION 6

In the diagram, the graphs of $f(x)=\sin x-1$ and $g(x)=\cos 2x$ are drawn for the interval $x\in[-90°;360°]$. Graphs f and g intersect at A. B(360° ; −1) is a point on f.



6.1 Write down the range of f. (2)

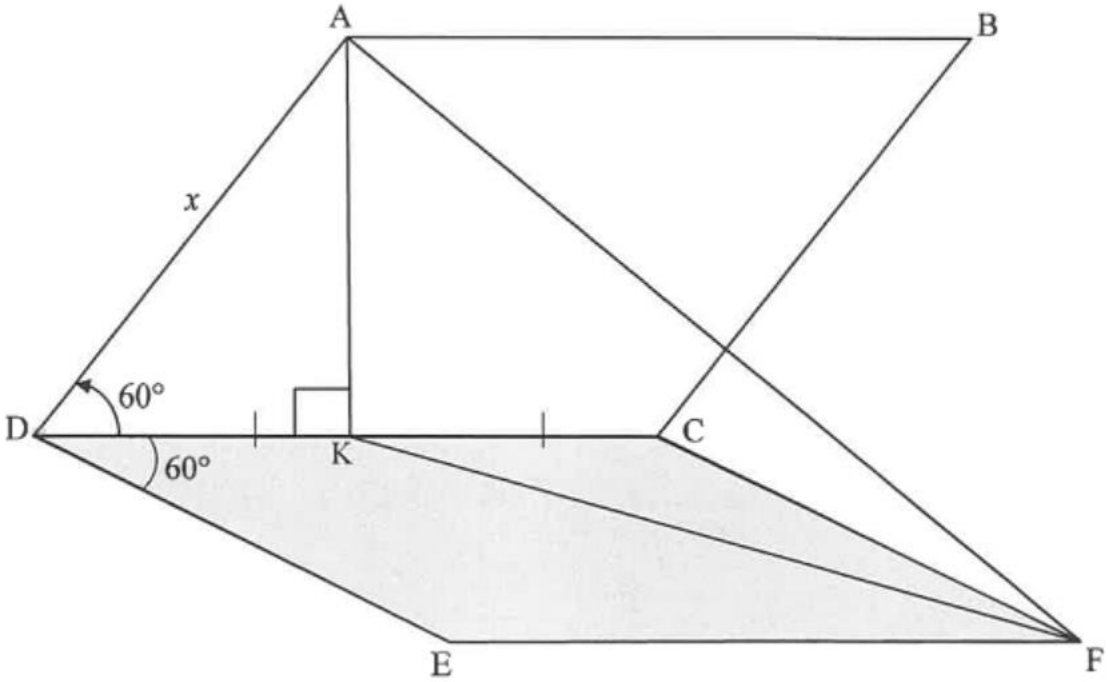
6.2 Write down the values of x in the interval $x\in[-90°;360°]$ for which graph f is decreasing. (2)

6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)

[10]

QUESTION 7

The diagram below shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K is a point on DC such that DK = KC and AK $\perp$ DC. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and AD = x units.



7.1 Determine AK in terms of x. (2)

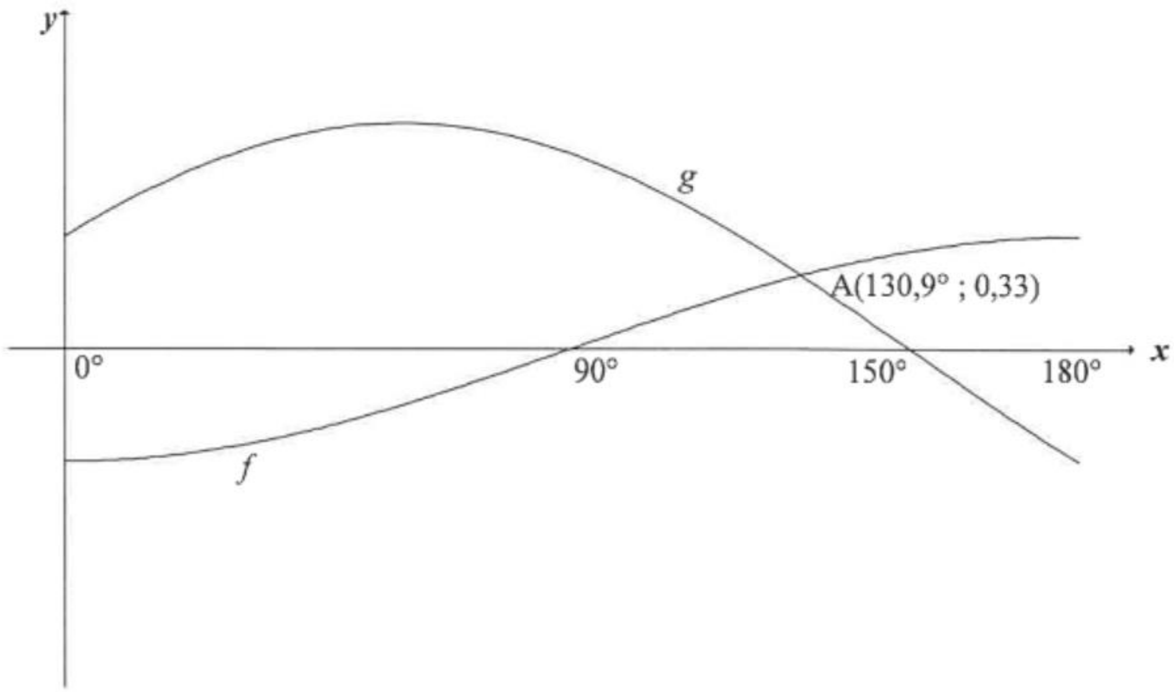
7.2 Write down the size of $K\hat{C}F$. (1)

7.3 It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y. (7)

[10]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}\cos x$ and $g(x) = \sin(x + 30°)$, for the interval $x \in [0°; 180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.



5.1 Write down the period of g. (1)

5.2 Write down the amplitude of f (1)

5.3 Determine the value of $f(180°) - g(180°)$ (1)

5.4 Use the graphs to determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

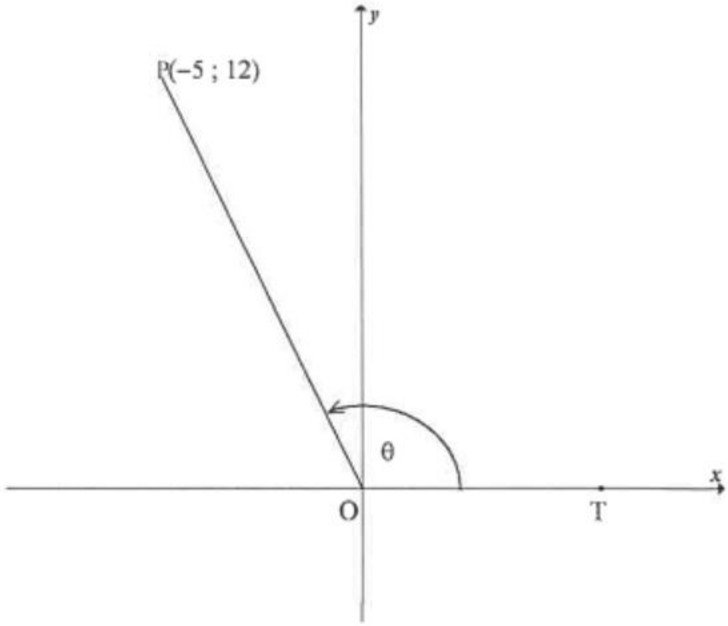
5.4.1 $f(x - 10°) = g(x - 10°)$ (1)

5.4.2 $\sqrt{3}\sin x + \cos x \geq 1$ (4)

[8]

QUESTION 6

6.1 In the diagram, $P(-5 ; 12)$ and T lies on the positive x-axis. $P\hat{O}T = \theta$



Answer the following **without using a calculator**:

6.1.1 Write down the value of $\tan\theta$ (1)

6.1.2 Calculate the value of $\cos\theta$ (3)

6.1.3 $S(a ; b)$ is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and $OS = 6{,}5$ units. Calculate the value of b. (4)

6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

6.3 Determine the general solution of the following equation:

$$6\sin^2 x + 7\cos x - 3 = 0$$ (6)

6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

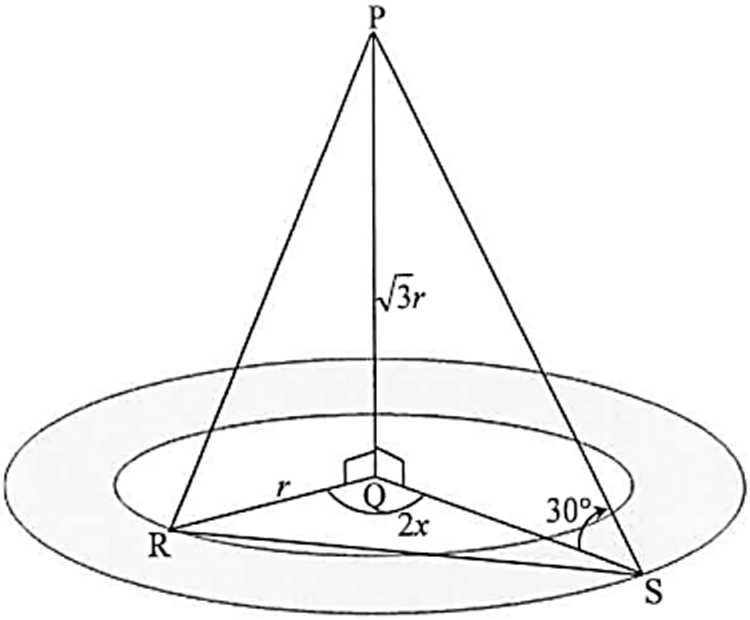
Determine the value of $\cos 2A$ **without using a calculator**. (5)

[24]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is $30°$.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



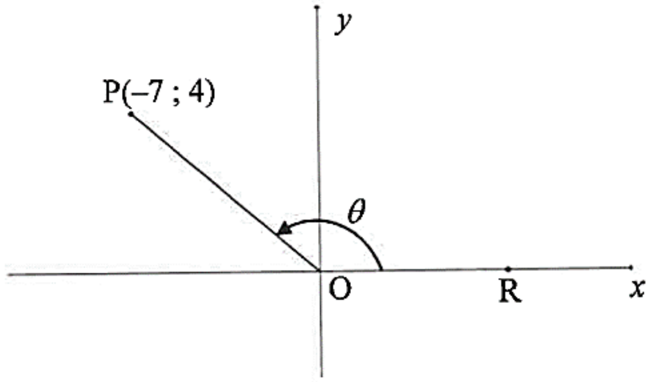
7.1	Show that $QS = 3r$	(3)
7.2	Determine, in terms of r, the area of the flower garden.	(2)
7.3	Show that $RS = r\sqrt{10 - 6\cos 2x}$	(3)
7.4	If $r = 10$ metres and $x = 56°$, calculate RS.	(2)
		[10]

EXTRACTS FROM PREVIUOS QUESTION PAPERS

ACTIVITY 1

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $\hat{POR} = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP

5.1.2 Value of:

(a) $\tan\theta$

(b) $\cos(\theta - 180°)$

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above.

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined.

ACTIVITY 2

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta).\sin 2\theta$$

6.2 Given: $\sin(60^\circ + 2x) + \sin(60^\circ - 2x)$

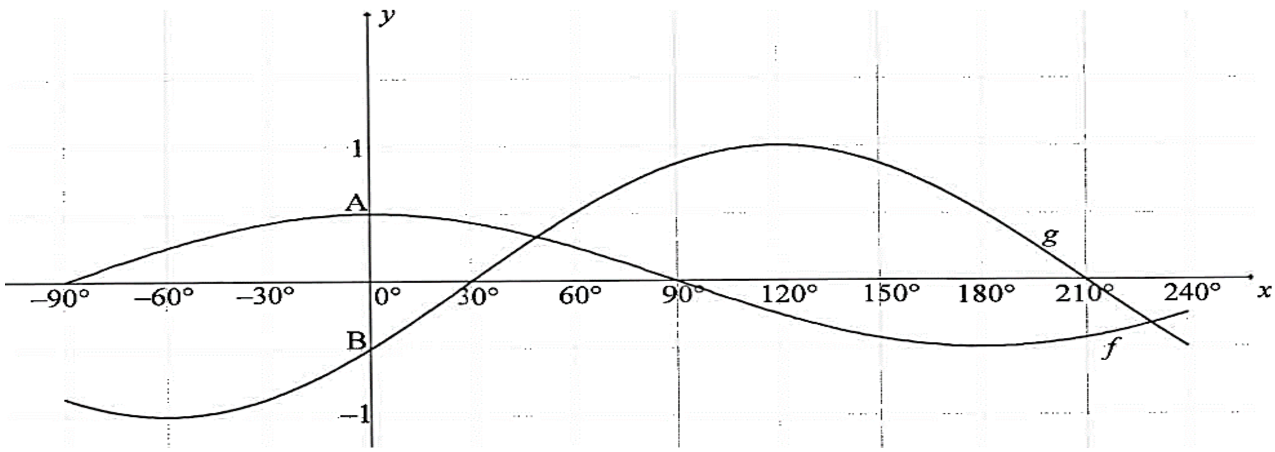
6.2.1 Calculate the value of k if $\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k\cos 2x$.

6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60^\circ[\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$ in terms of t.

ACTIVITY 3

QUESTION 7

In the diagram below, the graphs of $f(x) = \frac{1}{2}\cos x$ and $g(x) = \sin(x - 30^\circ)$ are drawn for the interval $x \in [-90^\circ ; 240^\circ]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB.

7.2 Write down the range of $3f(x) + 2$.

7.3 Read off from the graphs a value of x for which $g(x) - f(x) = \frac{\sqrt{3}}{2}$.

7.4 For which values of x, in the interval $x \in [-90^\circ ; 240^\circ]$, will:

7.4.1 $f(x).g(x) > 0$

7.4.2 $g'(x - 5^\circ) > 0$

ACTIVITY 4

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

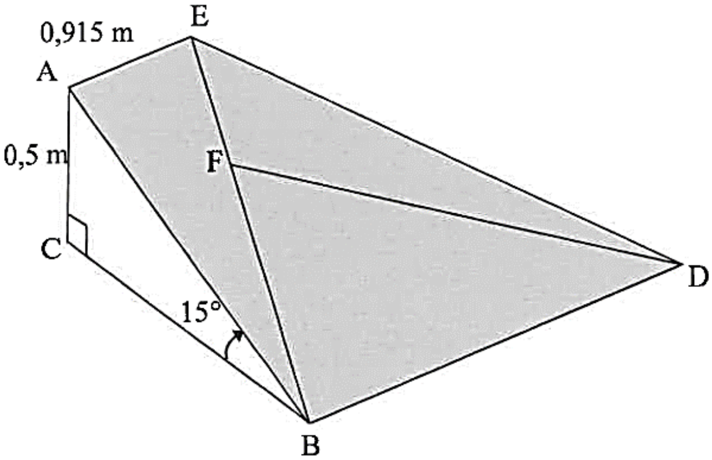


FIGURE I

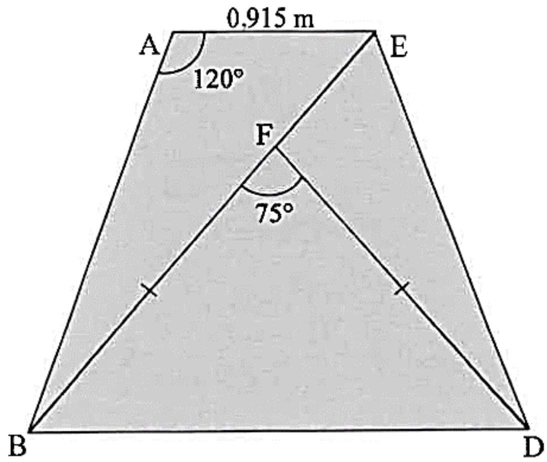


FIGURE II (top view)

8.1 Calculate the length of AB.

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

If $B\hat{A}E = 120°$, calculate the length of BE.

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$.

EXAMINATION GUIDELINES

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $\; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

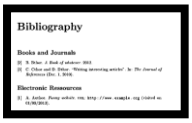
$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Bibliography



1. MATHEMATICS EXAMINATION GUIDELINES
2. MATHEMATICS QUESTION PAPERS GRADE 12 (2008 – 2020) NATIONAL