JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ACTIVITY MANUAL SOLUTIONS

GRADE 12

2022

Trigonometry

JENN MAREMATLOU TRAINING INSTITUTE

A Division of Marematlou Group Holdings

CONTENTS **PAGE**

PART 1: REDUCTION AND OR / IDENTITIES

QUESTION 1

1.1.1

$$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$$

$$=\frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$$

$$=\frac{(-\tan 60°).(-\sin 60°).\cos 14°.(-\sin 45°)}{\cos 14°.(-\cos 45°)}$$

$$=\frac{\left(-\sqrt{3}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)}$$

$$=\frac{3}{2}$$

OR

$$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$$

$$=\frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$$

$$=\frac{(-\tan 60°).(-\sin 60°).\sin 76°.\tan 45°}{\sin 76°}$$

$$=\left(-\sqrt{3}\right)\left(-\frac{\sqrt{3}}{2}\right).1$$

$$=\frac{3}{2}$$

1.1.2

$$\cos 75°$$

$$=\cos(45°+30°)$$

$$=\cos 45°.\cos 30°-\sin 45°.\sin 30°$$

$$=\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}$$

$$=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

OR

$$\cos 75°$$

$$=\cos(45°+30°)$$

$$=\cos 45°.\cos 30°-\sin 45°.\sin 30°$$

$$=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}.\frac{1}{2}$$

$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$

$$=\frac{\sqrt{2}.\sqrt{3}-\sqrt{2}}{4}$$

$$=\frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

1.2	$\cos(90°-2x).\tan(180°+x)+\sin^2(360°-x)$ $=\sin 2x.\tan x+\sin^2 x$ $=2\sin x.\cos x.\dfrac{\sin x}{\cos x}+\sin^2 x$ $=2\sin^2 x+\sin^2 x$ $=3\sin^2 x$
1.3	$(\tan x-1)(\sin 2x-2\cos^2 x)$ $=\left(\dfrac{\sin x}{\cos x}-1\right)\left(2\sin x.\cos x-2\cos^2 x\right)$ $=\left(\dfrac{\sin x}{\cos x}-1\right)2\cos x(\sin x-\cos x)$ $=2(\sin x-\cos x)^2$ $=2(\sin^2 x-2\sin x.\cos x+\cos^2 x)$ $=2(1-2\sin x.\cos x)$ **OR** $(\tan x-1)(\sin 2x-2\cos^2 x)$ $=\left(\dfrac{\sin x}{\cos x}-1\right)\left(2\sin x.\cos x-2\cos^2 x\right)$ $=2\sin^2 x-2\sin x.\cos x-2\sin x.\cos x+2\cos^2 x$ $=2(\sin^2 x-2\sin x\cos x+\cos^2 x)$ $=2(1-2\sin x.\cos x)$ **OR** $2(1-2\sin x\cos x)$ $=2(\sin^2 x+\cos^2 x-2\sin x\cos x)$ $=2(\sin x-\cos x)^2$ $=2\cos^2 x(\dfrac{\sin x}{\cos x}-1)^2$ $=2\cos^2 x(\tan x-1)(\tan x-1)$ $=(2\cos^2 x.\tan x-2\cos^2 x)(\tan x-1)$ $=(2\sin x\cos x-2\cos^2 x)(\tan x-1)$ $=(\sin 2x-2\cos^2 x)(\tan x-1)$ **OR** $LHS=(\tan x-1)(\sin 2x-\cos^2 x)$ $\quad=\dfrac{\sin x-\cos x}{\cos x}\left(2\sin x.\cos x-\cos^2 x\right)$ $\quad=2(\sin x-\cos x)^2$ $RHS=2\left(\sin^2 x+\cos^2 x-2\sin x\cos x\right)$ $\quad=2(\sin x-\cos x)^2$ $\quad=LHS$
1.4	$\sin(90°-x).\cos(180°-x)+\tan x.\cos(-x).\sin(180°+x)$ $=\cos x(-\cos x)+\tan x(\cos x)(-\sin x)$ $=-\cos^2 x-\dfrac{\sin x}{\cos x}\cos x\sin x$ $=-\cos^2 x-\sin^2 x$ $=-(\cos^2 x+\sin^2 x)$ $=-1$

1.5

$$\frac{\sin 190^\circ \cos 225^\circ \tan 390^\circ}{\cos 100^\circ \sin 135^\circ}$$

$$= \frac{-\sin 10^\circ(-\cos 45^\circ)\tan 30^\circ}{-\sin 10^\circ \sin 45^\circ}$$

$$= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}} \qquad \text{or} \qquad = -\tan 30^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

If using $-\cos 80^\circ$: no penalty

If the candidate stop at

$$= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}} \qquad 6/7$$

QUESTION 2

2.1

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B - \sin A.\sin B}$$

$$= \frac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B - \sin A.\sin B} \times \frac{\frac{1}{\cos A.\cos B}}{\frac{1}{\cos A.\cos B}}$$

$$= \frac{\frac{\sin A.\cos B}{\cos A.\cos B} + \frac{\cos A.\sin B}{\cos A.\cos B}}{\frac{\cos A.\cos B}{\cos A.\cos B} - \frac{\sin A.\sin B}{\cos A.\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

OR

$$RHS \quad = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A}{\cos A}\frac{\sin B}{\cos B}} \times \frac{\cos A.\cos B}{\cos A.\cos B}$$

$$= \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B}$$

sin

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \tan(A+B)$$

$$= \text{LHS}$$

2.2

$$\tan C = \tan(180^\circ - (A+B))$$

$$\tan C = -\tan(A+B)$$

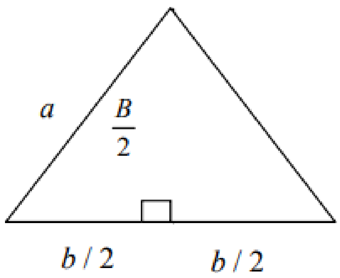
$$\tan C = -\left(\frac{\tan A + \tan B}{1 - \tan A.\tan B}\right)$$

$$\tan C(1 - \tan A.\tan B) = -(\tan A + \tan B)$$

$$\tan C - \tan A.\tan B.\tan C = -\tan A - \tan B$$

$$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$$

OR

	$\hat{C} = 180° - (\hat{A} + \hat{B})$ (angles in a triangle) $\tan C = \tan(180° - (A + B))$ $\tan C = \tan((180° - A) + (-B))$ $\tan C = \dfrac{\tan(180° - A) + \tan(-B)}{1 - \tan(180° - A).\tan(-B)}$ $\tan C(1 - \tan(180° - A).\tan(-B)) = \tan(180° - A) + \tan(-B)$ $\tan C - \tan C \tan A \tan B = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$
2.3	$\mathrm{AB} = \mathrm{BC} = a = c$ $b^2 = a^2 + c^2 - 2ac \times \cos \mathrm{B}$ $b^2 = a^2 + a^2 - 2a \times a \times \cos \mathrm{B}$ $b^2 = 2a^2 - 2a^2 \cos B$ $b^2 = 2a^2(1 - \cos B)$ $\dfrac{b^2}{2a^2} = 1 - \cos B$ $\cos \mathrm{B} = 1 - \dfrac{b^2}{2a^2}$ **OR** $\sin \dfrac{B}{2} = \dfrac{b}{2a}$ $\cos B = 1 - 2\sin^2 \dfrac{B}{2}$ $= 1 - 2\left(\dfrac{b}{2a}\right)^2$ $= 1 - \dfrac{b^2}{2a^2}$  **OR** $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ $but\ a = c$ $\cos B = \dfrac{a^2 + a^2 - b^2}{2a.a}$ $= \dfrac{2a^2 - b^2}{2a^2}$ $= 1 - \dfrac{b^2}{2a^2}$
2.4	$\dfrac{\sin(90° - x)\tan(360° - x)}{\cos(180° - x)}$ $= \dfrac{(\cos x)(-\tan x)}{-\cos x}$ $= \tan x$

2.5	$\frac{\cos(-60°)+\tan 135°}{\tan 315°+\cos 660°}$ $=\frac{\cos 60°-\tan 45°}{-\tan 45°+\cos 60°}$ $=\frac{\frac{1}{2}-1}{-1+\frac{1}{2}}$ $=1$
2.6	$\frac{\sin(90°+\theta)+\cos(180°+\theta)\sin(-\theta)}{\sin 180°-\tan 135°}$ $=\frac{\cos\theta+(-\cos\theta)(-\sin\theta)}{0+1}$ $=\cos\theta+\cos\theta.\sin\theta$ $=\cos\theta(1+\sin\theta)$

2.7	$\frac{4\sin A\cos A\cos 2A.\sin 15°}{\sin 2A(1-2\sin^2 A)}$ $=\frac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$ $=\frac{2\cos 2A.\sin 15°}{\cos 2A}$ $=2\sin 15°$ $=2\sin(45°-30°)$ $=2[\sin 45°\cos 30°-\cos 45°\sin 30°]$ $=2\left[\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}.\frac{1}{2}\right]$ $=2\left[\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right]$ $=\frac{\sqrt{6}-\sqrt{2}}{2}$ **OR** Left Hand Side $=\frac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$ $=\frac{2\cos 2A.\sin 15°}{\cos 2A}$ $=2\sin 15°$ $=2\sin(60°-45°)$ $=2[\sin 60°\cos 45°-\cos 60°\sin 45°]$ $=2\left[\frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}-\frac{1}{2}.\frac{\sqrt{2}}{2}\right]$ $=2\left[\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right]$ $=\frac{\sqrt{6}-\sqrt{2}}{2}=RHS$

	OR $\text{Left Hand Side} = \dfrac{4\sin A\cos A\cos 2A.\sin 15^\circ}{2\sin A\cos A(1-2\sin^2 A)}$ $= \dfrac{2\sin 2A\cos 2A.\sin 15^\circ}{\sin 2A\cos 2A}$ $= 2\sin 15^\circ$ $= 2\sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ\cos 30^\circ - \cos 45^\circ\sin 30^\circ]$ $= 2\left[\dfrac{\sqrt{2}}{2}.\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2}.\dfrac{1}{2}\right]$ $= 2\left[\dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4}\right]$ $= \dfrac{\sqrt{6}-\sqrt{2}}{2} = RHS$
2.8	$LHS = \dfrac{\sin(360^\circ + 90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \dfrac{\sin(90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \dfrac{\cos(x-\alpha)}{\cos(\alpha - x)}$ $= \dfrac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ **OR** $LHS = \dfrac{\sin[90^\circ - (\alpha - x)]}{\cos(\alpha - x)}$ $= \dfrac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ $= \text{RHS}$
2.9	
2.9.1	LHS: $\dfrac{\sin A\cos B - \cos A\sin B}{\sin B\cos B}$ $= \dfrac{\sin(A-B)}{\sin B\cos B}$ $\text{RHS} = \dfrac{2\sin(A-B)}{2\sin B\cos B}$ $= \dfrac{\sin(A-B)}{\sin B\cos B}$ $= \text{LHS}$

	OR LHS: $\frac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ $=\frac{\sin(A-B)}{\sin B\cos B}$ $= \; =\frac{2\sin(A-B)}{2\sin B\cos B}$ $=\frac{2\sin(A-B)}{\sin 2B}$ $=RHS$ **OR** $RHS=\frac{2\sin(A-B)}{\sin 2B}$ $=\frac{2(\sin A\cos B-\cos A\sin B)}{2\sin B\cos B}$ $=\frac{\sin A\cos B-\cos A\sin B}{\sin B\cos B}$ $=\frac{\sin A\cos B}{\sin B\cos B}-\frac{\cos A\sin B}{\sin B\cos B}$ $=\frac{\sin A}{\sin B}-\frac{\cos A}{\cos B}$ $=LHS$
a.	A = 5B $\frac{\sin 5B}{\sin B}-\frac{\cos 5B}{\cos B}=\frac{2\sin(5B-B)}{\sin 2B}$ $=\frac{2\sin 4B}{\sin 2B}$ $=\frac{4\sin 2B\cos 2B}{\sin 2B}$ $=4\cos 2B$ **OR** $\frac{\sin 5B}{\sin B}-\frac{\cos 5B}{\cos B}$ $=\frac{\sin 5B\cos B-\cos 5B\sin B}{\sin B\cos B}$ $=\frac{\sin(5B-B)}{\sin B\cos B}$ $= \; =\frac{\sin 4B}{\frac{1}{2}(2)\sin B\cos B}$ $=\frac{2\sin 2B\cos 2B}{\frac{1}{2}\sin 2B}$ $=4\cos 2B$

b.	Let $\sin 18° = a$ $\frac{1}{\sin 18°} = 4\cos 36°$ $\frac{1}{\sin 18°} = 4(1 - 2\sin^2 18°)$ $\therefore \frac{1}{a} = 4(1 - 2a^2)$ $\therefore 1 = 4a - 8a^3$ $\therefore 8a^3 - 4a + 1 = 0$ Hence $\sin 18°$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$ **OR** $\frac{1}{\sin 18°} = 4\cos 36°$ $\frac{1}{\sin 18°} = 4(1 - 2\sin^2 18°)$ $\frac{1}{\sin 18°} = 4 - 8\sin^2 18°$ $8(\sin 18°)^3 - 4(\sin 18) + 1 = 0$ Hence $\sin 18°$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$
2.10	$LHS = \frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x \cos x - \cos x}$ $= \frac{2\sin^2 x - \sin x}{2\sin x \cos x - \cos x}$ $= \frac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= RHS$ **OR**

	$LHS=\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\dfrac{1-(2\cos^2 x-1)-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{2-\cos^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{2(1-\cos^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\dfrac{\sin x}{\cos x}$ $=\tan x$ $=RHS$ **OR** $LHS=\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\dfrac{1-(\cos^2 x-\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{1-\cos^2 x+\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{\sin^2 x+\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\dfrac{\sin x}{\cos x}$ $=\tan x$ $=RHS$
2.11	
2.11.1	$\dfrac{\tan(180°+x)\cos(360°-x)}{\sin(180°-x)\cos(90°+x)+\cos(540°+x)\cos(-x)}$ $=\dfrac{\tan x.(\cos x)}{(\sin x).(-\sin x)-\cos x.\cos x}$ $=\dfrac{\frac{\sin x}{\cos x}\cos x}{-\sin^2 x-\cos^2 x}$ $=\dfrac{\sin x}{-\left(\sin^2 x+\cos^2 x\right)}$ $=-\sin x$

2.11.2	$\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\dfrac{1-(1-2\sin^2 x)-\sin x}{2\sin x.\cos x-\cos x}$ $=\dfrac{2\sin^2 x-\sin x}{2\sin x.\cos x-\cos x}$ $=\dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\dfrac{\sin x}{\cos x}$ $=\tan x$
	QUESTION 3
3.1.1	$\dfrac{\cos(360°-x).\tan^2 x}{\sin(x-180°).\cos(90°+x)}$ $=\dfrac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $=(\cos x)\left(\dfrac{\sin^2 x}{\cos^2 x}\right)\left(\dfrac{1}{\sin^2 x}\right)$ $=\dfrac{1}{\cos x}$
3.1.2	$x=30°$ $\dfrac{1}{\cos 30°}=\dfrac{1}{\frac{\sqrt{3}}{2}}=\dfrac{2}{\sqrt{3}}$
3.2	$b\sqrt{1-a^2}-a\sqrt{1-b^2}$ $=\cos 32°.\sqrt{1-\sin^2 28°}-\sin 28°\sqrt{1-\cos^2 32°}$ $=\cos 32°.\cos 28°-\sin 28°.\sin 32°$ $=\cos(32°+28°)$ $=\cos 60°$ $=\dfrac{1}{2}$
3.3.1	$\dfrac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ $=\dfrac{\sin 50°\times\tan 60°}{\cos 180°\times\tan 50°\times\sin 40°}$ $=\dfrac{\sin 50°\times\sqrt{3}}{-1\times\frac{\sin 50°}{\cos 50°}\times\cos 50°}$ $=-\dfrac{\sqrt{3}\cos 50°}{\cos 50°}$ $=-\sqrt{3}$

3.3.2	$(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=1-2\sin^2 75°$ $=\cos 150°$ $=\frac{-\sqrt{3}}{2}$ **OR** $\sin 75°$ $=\sin(45°+30°)$ $=\sin 45°.\cos 30°+\cos 45°.\sin 30°$ $=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2}$ $\sqrt{2}\sin 75°=\frac{\sqrt{3}}{2}+\frac{1}{2}=a$ $(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=(1-a)(1+a)$ $=1-a^2$ $=1-\left(\frac{3}{4}+\frac{1}{4}+2.\frac{\sqrt{3}}{2}.\frac{1}{2}\right)$ $=-\frac{\sqrt{3}}{2}$
3.4	$\text{LHS}=\frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ $=\frac{\cos^2 x+2\cos x\sin x+\sin^2 x-(\cos^2 x-2\sin x\cos x+\sin^2 x)}{(\cos x-\sin x)(\cos x+\sin x)}$ $=\frac{4\cos x\sin x}{\cos^2 x-\sin^2 x}$ $=\frac{2\sin 2x}{\cos 2x}$ $=2\tan x$ $=\text{RHS}$

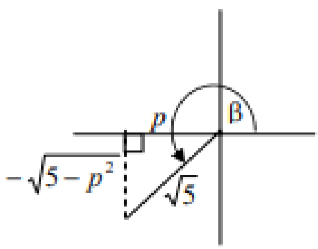
3.5	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times ....\times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $=\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)......\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)......\left(\frac{\sin 88^\circ}{\cos 88^\circ}\right)\left(\frac{\sin 89^\circ}{\cos 89^\circ}\right)$ $=\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)......\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)......\left(\frac{\sin(90^\circ-2^\circ)}{\cos(90^\circ-2^\circ)}\right)\left(\frac{\sin(90^\circ-1^\circ)}{\cos(90^\circ-1^\circ)}\right)$ $=\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)......\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)......\left(\frac{\cos 2^\circ}{\sin 2^\circ}\right)\left(\frac{\cos 1^\circ}{\sin 1^\circ}\right)$ $=\tan 45^\circ$ $=1$ **OR** $\tan 89^\circ = \cot 1^\circ \quad \tan 88^\circ = \cot 2^\circ \ ...$ $\therefore$ product is $(\tan 1^\circ.\cot 1^\circ)(\tan 2^\circ.\cot 2^\circ)...(\tan 44^\circ.\cot 44^\circ).\tan 45^\circ$ $= 1\times 1\times 1\times ...\times 1 = 1$
3.6	$1-\sin^2\theta + 3 - \cos^2\theta$ $=4-(\sin^2\theta+\cos^2\theta)$ $=3$ **OR** $\cos^2\theta + 3 - \cos^2\theta$ $=3$
3.7	$\sqrt{4^{\sin 150^\circ}.2^{3\tan 225^\circ}}$ $=\sqrt{4^{\sin 30^\circ}.2^{3\tan 45^\circ}}$ $=\sqrt{(2^2)^{\frac{1}{2}}.2^3}$ $=\sqrt{16}$ $=4$ **OR** $\sin 150^\circ = \frac{1}{2}$ $\tan 225^\circ = 1$ $\sqrt{4^{\sin 150^\circ}2^{3\tan 225^\circ}}$ $=\sqrt{4^{\frac{1}{2}}2^3}$ $=\sqrt{2.2^3}$ $=\sqrt{16}$ $=4$

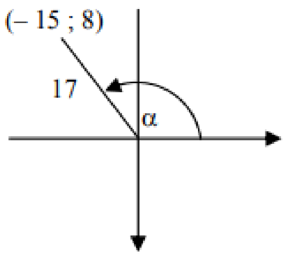
3.8	$LHS=\dfrac{\cos^2 x(\sin^2 x+\cos^2 x)}{1-\sin x}$ $=\dfrac{\cos^2 x.(1)}{1-\sin x}$ $=\dfrac{(1-\sin^2 x)}{1-\sin x}$ $=\dfrac{(1+\sin x)(1-\sin x)}{1-\sin x}$ $=1+\sin x$ $=RHS$
3.9	$\cos 3\theta$ $=\cos(2\theta+\theta)$ $=\cos 2\theta.\cos\theta-\sin 2\theta.\sin\theta$ $=(2\cos^2\theta-1).\cos\theta-2\sin\theta.\cos\theta.\sin\theta$ $=2\cos^3\theta-\cos\theta-2\sin^2\theta.\cos\theta$ $=2\cos^3\theta-\cos\theta-2(1-\cos^2\theta).\cos\theta$ $=2\cos^3\theta-\cos\theta-2\cos\theta+2\cos^3\theta$ $=4\cos^3\theta-3\cos\theta$
3.10	$\cos 3\theta=4\cos^3\theta-3\cos\theta$ $\cos 3(20°)=4\cos^3(20°)-3\cos(20°)$ $\dfrac{1}{2}=4x^3-3x$ $8x^3-6x-1=0$
3.11	$\dfrac{\cos 160°.\tan 200°}{2\sin(-10°)}$ $=\dfrac{(-\cos 20°)(\tan 20°)}{2(-\sin 10°)}$ $=\dfrac{(-\cos 20°)\left(\dfrac{\sin 20°}{\cos 20°}\right)}{-2\sin 10°}$ $=\dfrac{2\sin 10°\cos 10°}{2\sin 10°}$ $=\cos 10°$

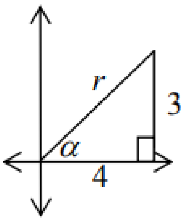
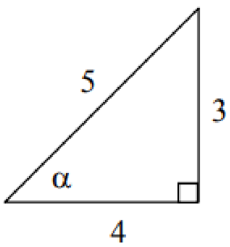
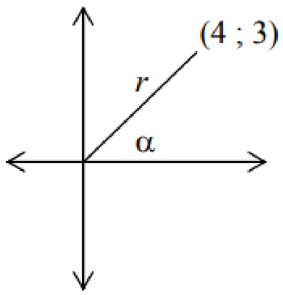
3.12	$LHS = \cos(x+45°).\cos(x-45°)$ $= (\cos x.\cos 45° - \sin x \sin 45°)(\cos x \cos 45° + \sin x \sin 45°)$ $= \cos^2 x.\cos^2 45° - \sin^2 x.\sin^2 45°$ $= \cos^2 x\left(\frac{\sqrt{2}}{2}\right)^2 - \sin^2\left(\frac{\sqrt{2}}{2}\right)^2$ or $\left[\cos^2 x\left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 x\left(\frac{1}{\sqrt{2}}\right)^2\right]$ $= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$ $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ $= \frac{1}{2}\cos 2x$ **OR** $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$ Let $\alpha = x + 45°$ and $\beta = x - 45°$ $\therefore \cos(x+45°)\cos(x-45°)$ $= \frac{1}{2}(\cos((x+45°+x-45°) + \cos(x+45°-x+45°))$ $= \frac{1}{2}(\cos 2x + \cos 90°)$ $= \frac{1}{2}\cos 2x$
3.13	$\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \sin 40° - \sin 10° \cos 40°$ $= \sin 40° \cos 10° - \cos 40° \sin 10°$ $= \sin(40° - 10°)$ $= \sin 30°$ $= \frac{1}{2}$ **OR** $\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \cos 50° - \sin 10° \sin 50°$ $= \cos(10° + 50°)$ $= \cos 60°$ $= \frac{1}{2}$ **NOTE:** There are many solutions.

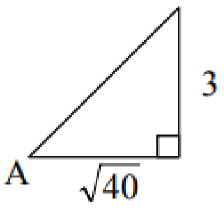
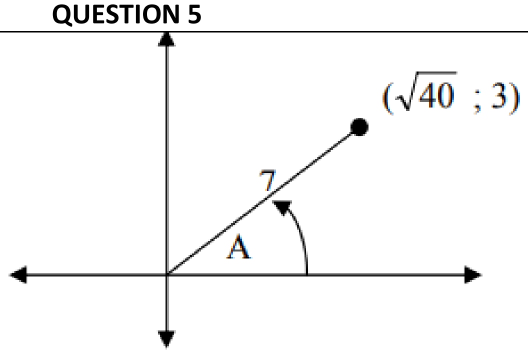
3.14.1	$LHS = 1 - \cos 2Q$ $= 1 - (1 - 2\sin^2 Q)$ $= 2\sin^2 Q$ $= RHS$
3.14.2.a	$LHS = \sin 2R$ $= \sin 2[180° - (P + Q)]$ $= \sin[360° - 2(P + Q)]$ $= -\sin 2(P + Q)$ $= -\sin(2P + 2Q)$ $= RHS$
b.	$LHS = \sin 2P + \sin 2Q + \sin 2R$ $= \sin 2P + \sin 2Q - \sin(2P + 2Q)$ $= \sin 2P + \sin 2Q - [\sin 2P \cos 2Q + \cos 2P \sin 2Q]$ $= \sin 2P + \sin 2Q - \sin 2P \cos 2Q - \cos 2P \sin 2Q$ $= \sin 2P(1 - \cos 2Q) + \sin 2Q(1 - \cos 2P)$ $= \sin 2P(2\sin^2 Q) + \sin 2Q(2\sin^2 P)$ $= 2\sin P \cos P . 2\sin^2 Q + 2\sin Q \cos Q . 2\sin^2 P$ $= 4\sin P \sin Q(\sin Q \cos P + \cos Q \sin P)$ $= 4\sin P \sin Q(\sin(Q + P))$ $= 4\sin P \sin Q(\sin[180° - (Q + P)])$ $= 4\sin P \sin Q \sin R$ $= RHS$
3.15	
3.16	
3.17	
3.18	
3.19	a.
	b.

PART 2

	Question 1
1.1.1	$\cos\beta = \frac{p}{\sqrt{5}}$ $x = p$ $r = \sqrt{5}$ $y = -\sqrt{5 - p^2}$ $\therefore \tan\beta = \frac{-\sqrt{5 - p^2}}{p}$ 

1.1.2	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ $= \frac{2p^2}{5} - 1$ **OR** $\cos 2\beta = 1 - 2\sin^2\beta$ $= 1 - 2\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= 1 - \frac{2(5-p^2)}{5}$ $= \frac{2p^2-5}{5}$ **OR** $\cos 2\beta = \cos^2\beta - \sin^2\beta$ $= \left(\frac{p}{\sqrt{5}}\right)^2 - \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= \frac{p^2}{5} - \frac{5-p^2}{5}$ $= \frac{2p^2-5}{5}$
	QUESTION 2
2.1	$\sin\alpha = \frac{8}{17}$ $\sin\alpha > 0 \therefore$ in second quadrant $y_\alpha = 8 \quad r_\alpha = 17$ $x_\alpha = -15 \quad$ (Pythagoras) $\tan\alpha = -\frac{8}{15}$ 
2.2	$\sin(90° + \alpha) = \cos\alpha$ $= -\frac{15}{17}$

2.3	$\cos 2\alpha = 1 - 2\sin^2 \alpha$ $= 1 - 2\left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$ **OR** $\cos 2\alpha = 2\cos^2 \alpha - 1$ $= 2\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$ **OR** $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$
	QUESTION 3
3.1.1	$\tan\theta = -\frac{5}{12}$
3.1.2	$r^2 = (-12)^2 + (5)^2 = 169$ $\therefore r = 13$ $\cos\theta \sin\theta = \frac{-12}{13} \times \frac{5}{13} = -\frac{60}{169}$
	QUESTIN 4
4.1	   $r = \sqrt{16+9} = 5$ (Pyth) $\sin\alpha = \frac{3}{5}$ Accept : 0,6

4.2	$\cos^2(90° - \alpha) - 1$ $= \sin^2\alpha - 1$ $= \left(\frac{3}{5}\right)^2 - \frac{25}{25}$ $= \frac{-16}{25}$ $= -0{,}64$ **OR** $\cos^2(90° - \alpha) - 1$ $= \sin^2\alpha - (\sin^2\alpha + \cos^2\alpha)$ $= -\cos^2\alpha$ $= -\left(\frac{4}{5}\right)^2$ $= \frac{-16}{25}$ $= -0{,}64$
4.3	$1 - \sin 2\alpha$ $= 1 - 2\sin\alpha\cos\alpha$ $= 1 - 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= 1 - \frac{24}{25}$ $= \frac{1}{25}$ **OR** $1 - \sin 2\alpha$ $= \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha$ $= (\sin\alpha - \cos\alpha)^2$ $= \left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2$ $= \left(-\frac{1}{5}\right)^2$ $= \frac{1}{25}$
	QUESTION 5
5.1.1	 or  $r^2 = 40 + 9$ $r = 7$ $\cos A = \frac{\sqrt{40}}{7}$

5.1.2	$\sin(180° + A)$ $= -\sin A$ $= -\frac{3}{7}$ **OR** $\sin(180° + A) = \sin 180°.\cos A + \cos 180°.\sin A$ $= 0.\cos A - 1.\sin A$ $= -\sin A$ $= -\frac{3}{7}$
5.2	
5.2.1	$r = 5$ $\sin R\hat{O}P = \frac{3}{5} = 0{,}6$
5.2.2	$R\hat{O}P = 36{,}87°$ $Q\hat{O}P = 180° - 36{,}869....°$ $Q\hat{O}P = 143{,}13°$ Answer only: Full Marks

5.2.3

$x_m = x\cos\theta + y\sin\theta$

$a = 4\cos 115° + 3\sin 115°$

$a = 1{,}03$

> **Note**: Penalise 1 mark for rounding incorrectly
> **Note**: If incorrect angle is used in the x- formula: 1 mark

OR

Rotation of 115° clockwise = 245° anticlockwise

$x_m = x\cos\theta - y\sin\theta$

$a = 4\cos 245° - 3\sin 245°$

$a = 1{,}03$

OR

$\tan P\hat{O}R = \dfrac{3}{4}$

$P\hat{O}R = 36{,}86...°$

$M\hat{O}R = 78{,}13....°$

$\cos M\hat{O}R = \dfrac{a}{5}$

$a = 5\ \cos 78{,}13°$

$a = 1{,}03$

QUESTION 6

6.1

6.1.1

OT $= k$, PT $= 8$ and OP $= 17$

$k^2 + 8^2 = 17^2$

$k^2 = 289 - 64$

$k^2 = 225$

$k = \pm 15$

$k > 0$

$k = 15$

OR

$k^2 = 17^2 - 8^2$

$k^2 = (17-8)(17+8)$

$= 25 \times 9$

$= 225$

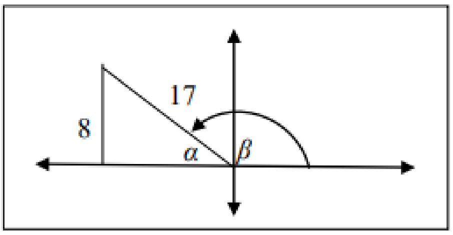
$k = \pm 15$

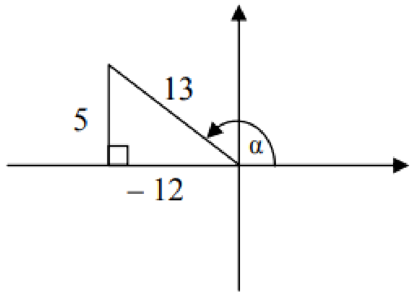
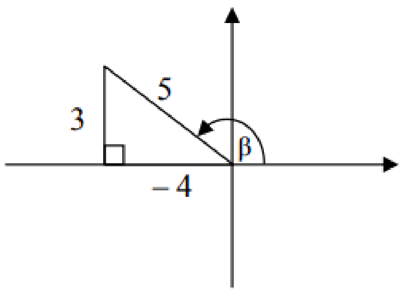
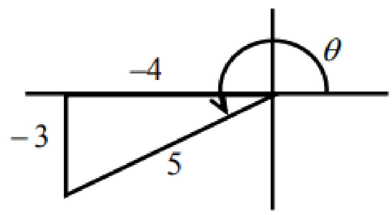
$k > 0$

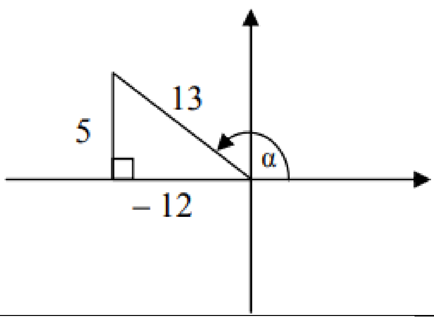
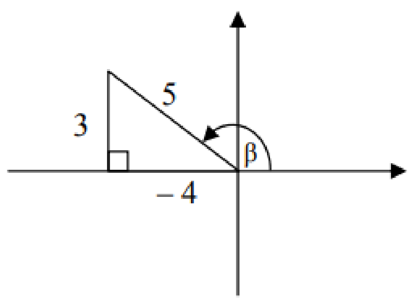
$k = 15$

6.1.2

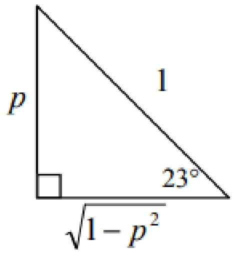
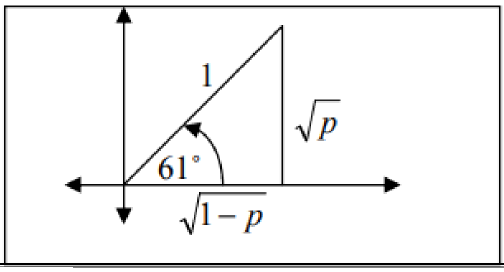
$\cos\alpha = \dfrac{15}{17}$

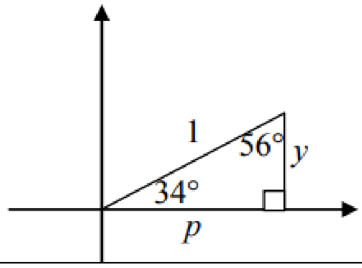
6.1.3	$\alpha+\beta=180°$ $\beta=180°-\alpha$ $\therefore \cos\beta=\cos(180°-\alpha)$ $=-\cos\alpha$ $=-\frac{15}{17}$ **OR**  $\therefore \cos\beta=\cos(180°-\alpha)$ $=-\cos\alpha$ $=-\frac{15}{17}$
6.1.4	$\sin(\beta-\alpha)$ $=\sin\beta\cos\alpha-\cos\beta\sin\alpha$ $=\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)-\left(-\frac{15}{17}\right)\left(\frac{8}{17}\right)$ $=\frac{120}{289}+\frac{120}{289}$ $=\frac{240}{289}$ **OR** $\beta-\alpha=(180°-\alpha)-\alpha$ $=180°-2\alpha$ $\sin(\beta-\alpha)=\sin(180°-2\alpha)$ $=\sin 2\alpha$ $=2\sin\alpha.\cos\alpha$ $=2\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)$ $=\frac{240}{289}$
6.2	

6.2.1	$\sin\alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos\alpha = -\frac{12}{13}$	
6.2.2	$\tan\beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$ $\cos(\alpha+\beta)$ $= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48-15}{65}$ $= \frac{33}{65}$	
	QUESTION 7	
7.1.1	$\sin\theta = -\frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$ $\sin\theta + \cos\theta = -\frac{7}{5}$	

7.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$ **OR** $\tan 2\theta$ $= \frac{2\tan\theta}{1-\tan^2\theta}$ $= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$
	QUESTION 8
8.1	
8.1.1	
8.1.2	
8.2	
8.2.1	$\sin\alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos\alpha = -\frac{12}{13}$ 
8.2.2	$\tan\beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$  $\cos(\alpha+\beta)$ $= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $= \left(-\frac{12}{13}\right).\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right).\left(\frac{3}{5}\right)$ $= \frac{48-15}{65}$ $= \frac{33}{65}$

PART 3

	QUESTION 1
1.1.1	
1.1.2	
	QUESTION 2
2.1.1	$\cos 113°$ $= \cos(90° + 23°)$ $= -\sin 23°$ $= -p$
2.1.2	$\cos 23°$ $= \sqrt{1-p^2}$  **OR** $\cos^2 23° + \sin^2 23° = 1$ $\cos^2 23° = 1 - p^2$ $\cos 23° = \sqrt{1-p^2}$
2.1.3	$\sin 46°$ $= 2\sin 23°.\cos 23°$ $= 2p\sqrt{1-p^2}$
	QUESTION 3
3.1	
3.1.1	$\sin 61° = \sqrt{p}$ $\sin 241° = \sin(180° + 61°)$ $= -\sin 61°$ $= -\sqrt{p}$ 
3.1.2	$\cos 61° = \sqrt{1-\sin^2 61°}$ $= \sqrt{1-p}$
3.1.3	$\cos 122° = \cos 2(61°)$ $= 2\cos^2 61° - 1$ $= 2\left(\sqrt{1-p}\right)^2 - 1$ $= 2(1-p) - 1$ $= 2 - 2p - 1$ $= 1 - 2p$

3.1.4	$\cos 73°\cos 15° + \sin 73°.\sin 15°$ $= \cos(73° - 15°)$ $= \cos 58° = (\cos 180° - 122°)$ $= -(\cos 122°)$ $= -(1 - 2p)$ $= 2p - 1$
3.2	
3.2.1	$\cos\ 214°$ $= \quad \cos(180° + 34°)$ $= -\cos 34°$ $= -p$
3.2.2	$\cos 68°$ $= \cos[2(34°)]$ $= 2\cos^2 34° - 1$ $= 2p^2 - 1$
3.2.3	$\tan 56° = \dfrac{\sin 56°}{\cos 56°}$ $= \dfrac{\cos 34°}{\sin 34°}$ $= \dfrac{\cos 34°}{\sqrt{1-\cos^2 34°}}$ $= \dfrac{p}{\sqrt{1-p^2}}$ **OR** $y^2 = 1 - p^2$ $y = \sqrt{1-p^2}$ $\therefore \tan 56° = \dfrac{p}{\sqrt{1-p^2}}$ 
	QUESTION 4
4.1.1	$\cos 28° = \sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$
4.1.2	$\cos 64°$ $= \cos 2(32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$

4.1.3	$\sin 4°$ $= \sin(32° - 28°)$ $= \sin 32° \cos 28° - \cos 32° \sin 28°$ $= \sqrt{1-b^2}.\sqrt{1-a^2} - ab$ **OR** $\sin 4°$ $= \sin(60° - 2 \times 28°)$ $= \sin 60° \cos(2 \times 28°) - \cos 60° \sin(2 \times 28°)$ $= \frac{\sqrt{3}}{2}\left(1 - 2a^2\right) - \frac{1}{2}(2a)\sqrt{1-a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1-a^2}$ **OR** $\sin 4°$ $= \sin(2 \times 32° - 60°)$ $= \sin(2 \times 32°) \cos 60° - \cos(2 \times 32°).\sin 60°$ $= 2.b\sqrt{1-b^2}.\frac{1}{2} - \frac{\sqrt{3}}{2}\left(2b^2 - 1\right)$ $= b\sqrt{1-b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ **OR** Using sin(A+B) + sin(A – B) = 2.sinA.cosB With A = 28° and B = 32° $\sin 60° + \sin(-4°) = 2ab$ $\sin 4° = \frac{\sqrt{3}}{2} - 2ab$

	Using sin(A+B) + sin(A – B) = 2.sinA.cosB With A = 32° and B = 28° $\sin 60° + \sin(4°) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4° = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ **OR** Using $\sin 4° = 2\sin 2°.\cos 2°$ and $\sin 2° = \sin(30° - 28°) = \frac{1}{2}\left(\sqrt{1-a^2} - \sqrt{3}a\right)$ and $\sin 2° = \sin(32° - 30°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-b^2} - b\right)$ and $\cos 2° = \cos(30° - 28°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-a^2} + a\right)$ and $\cos 2° = \cos(32° - 30°) = \frac{1}{2}\left(\sqrt{3}b + \sqrt{1-b^2}\right)$ then $\sin 4° = \frac{1}{2}\left\{\sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2}\right\}$ **OR** $\sin 4° = \frac{1}{2}\left\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\right\}$

PART4

	QUESTION 1

a.	$\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=-78{,}7°+k.180°$ $k\in Z$ **OR** $\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=101{,}3°+k.180°$ $k\in Z$ **OR** $\frac{\tan x-1}{2}=-3$ $\tan x-1=-6$ $\tan x=-5$ $x=101{,}3°+k.360°$ *or* $x=281{,}3°+k.360°$ $k\in Z$ **OR** If the candidate has used $\tan(x-1)=-6$ max of 2 / 5
b.	$\sin x+2\cos^2 x=1$ $\sin x+2(1-\sin^2 x)=1$ $-2\sin^2 x+\sin x+1=0$ $2\sin^2 x-\sin x-1=0$ $(2\sin x+1)(\sin x-1)=0$ $\sin x=1$ $x=90°+k.360°;k\in Z$ Or

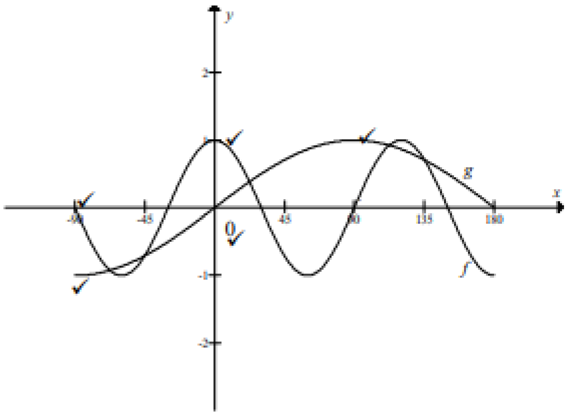
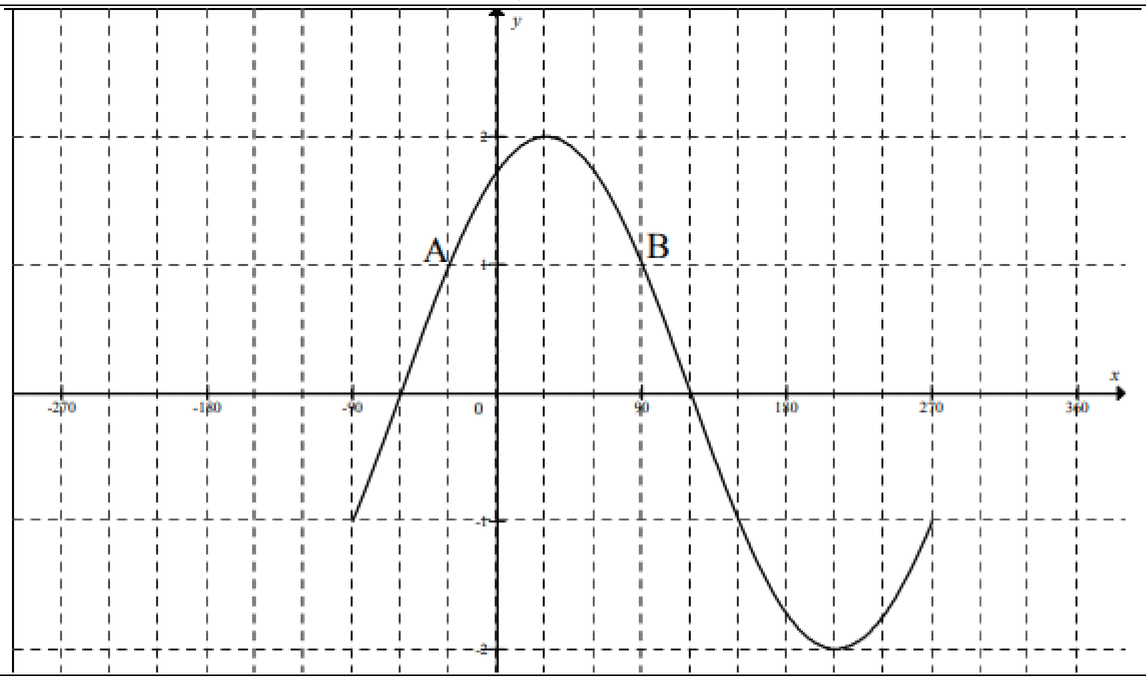
	$\sin x = -\frac{1}{2}$ $x = 210° + k.360°; k \in Z$ *OR* $x = 210° + k.360°$ or $x = 330° + k.360°; k \in Z$ *or* $x = -30° + k.360°$ OR $x = -150° + k.360°; k \in Z$ *OR* $x = -150° + k.360°; k \in Z$ *or* $x = 330° + k.360°$ *or* $x = -30° + k.360°$ **OR** $\sin x + 2\cos^2 x = 1$ $\sin x = 1 - 2\cos^2 x$ $\sin x = -\cos 2x$ $\sin x = -[\sin(90° - 2x)]$ $x = 180° + (90° - 2x) + k360°$ $3x = 270° + k360°$ $x = 90° + k120°$ $k \in Z$ or $x = 360° - (90° - 2x) + k360°$ $x = -270° - k360°$ **OR** $\sin x + 2\cos^2 x = 1$ $\sin x = 1 - 2\cos^2 x$ $\sin x = -\cos 2x$ $-\cos(90° - x) = \cos 2x$ $2x = 180° - (90° - x) + k360°$ $x = 90° + k360°$ $k \in Z$ or $2x = 180° + (90° - x) + k360°$ $3x = 270° + k360°$ $x = 30° + k120°$
c.	$\frac{1}{2}\sin x = -0{,}243$ $\sin x = -0{,}486$ Ref angle $= 29{,}078...°$ $\therefore x = 180° + 29{,}078...° + k.360°$ or $x = 360° - 29{,}078...° + k.360°$ $x = 209{,}08° + k.360°, k \in Z$ or $x = 330{,}92° + k.360°, k \in Z$ **OR** $x = k.360° - 150{,}92°$ or $x = k.360° - 29{,}08°$

d.	$\frac{\sin x}{\cos x} = \frac{\sqrt{3}\sin x}{\sin x}$ $\tan x = \sqrt{3}$ $x = 60°$ or $x = 240°$
e.	$6\cos x - 5 = \frac{4}{\cos x}$ $6\cos^2 x - 5\cos x = 4$ $6\cos^2 x - 5\cos x - 4 = 0$ $(3\cos x - 4)(2\cos x + 1) = 0$ $\cos x = \frac{4}{3}$ *or* $\cos x = \frac{-1}{2}$ no solution or $x = 120° + k.360°, k \in Z$ or $x = 240° + k.360°, k \in Z$ Alternative solution for $\cos x = \frac{-1}{2}$ $x = k.360° \pm 120°\ k \in Z$ **Note:** If candidate puts $\pm k.360$ then $k \in N_0$
f.	$\cos 2x = 1 - 3\cos x$ $2\cos^2 x - 1 = 1 - 3\cos x$ $2\cos^2 x + 3\cos x - 2 = 0$ $(2\cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{2}$ or $\cos x = -2$ n/a $x = 60° + k.360°$; $k \in Z$ or $x = 300° + k.360°$; $k \in Z$ **OR** $x = \pm 60° + k.360°$; $k \in Z$

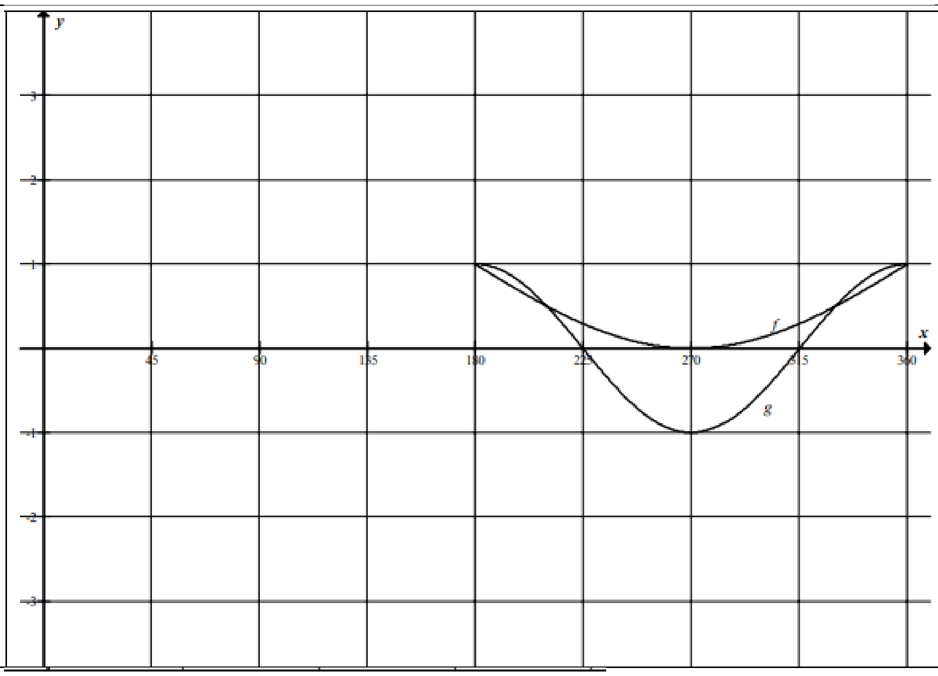
g.	$\sin 2x - \cos x = 0$ $2\sin x\cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0$ $x = 90° + 360°k$ or $x = 270° + 360°k$ $k \in Z$ or $\sin x = \frac{1}{2}$ $x = 30° + 360°k$ or $x = 150° + 360°k$ $x = 90°$ or $x = 270°$ or $x = 30°$ or $x = 150°$ **OR** $\sin 2x = \cos x$ $\sin 2x = \sin(90° - x)$ $2x = 90° - x + 360°k; k \in Z$ or $2x = 180° - (90° - x) + 360°k$ $3x = 90° + 360°k$ $2x = 90° + x + 360°k$ $x = 30° + 120°k$ $x = 90° + 360°k$ $x = 30°$ or $x = 150°$ or $x = 270°$ or $x = 90°$
h.	$\sin^2 x + \cos 2x - \cos x = 0$ $\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$ $\cos^2 x - \cos x = 0$ $\cos x(\cos x - 1) = 0$ $\cos x = 0$ *or* $\cos x = 1$ $x = \pm 90° + k.360°$ *or* $x = 0° + k.360°$ $k \in Z$ $= k.360°$ (i.e. $x = 90° + k.180°$ or $x = k.360° \pm 90°, k \in Z$)
i.	1. $x = 0°; 90°; 180°$
	2.

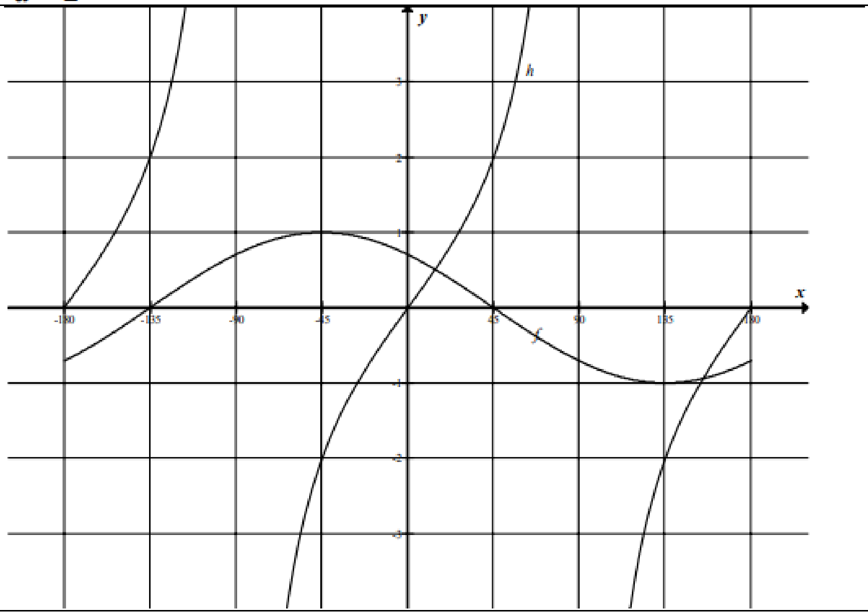
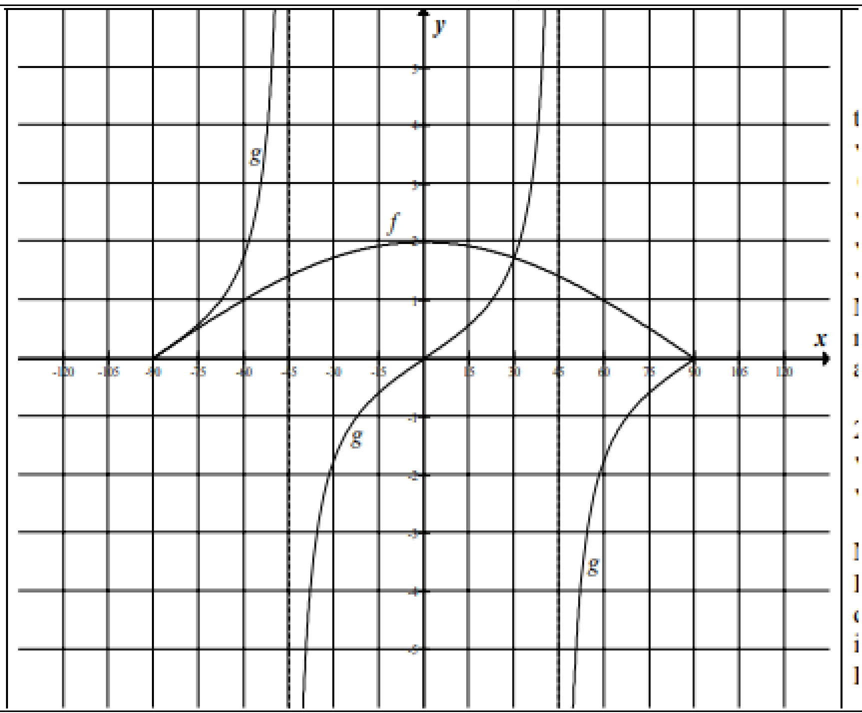
	$\frac{\cos 2x.\tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x).\frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x.\sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$
j.	1. $LHS = \cos(x+45°).\cos(x-45°)$ $= (\cos x.\cos 45° - \sin x \sin 45°)(\cos x \cos 45° + \sin x \sin 45°)$ $= \cos^2 x.\cos^2 45° - \sin^2 x.\sin^2 45°$ $= \cos^2 x\left(\frac{\sqrt{2}}{2}\right)^2 - \sin^2\left(\frac{\sqrt{2}}{2}\right)^2$ or $\left[\cos^2 x\left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 x\left(\frac{1}{\sqrt{2}}\right)^2\right]$ $= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$ $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ $= \frac{1}{2}\cos 2x$ **OR** $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$ Let $\alpha = x + 45°$ and $\beta = x - 45°$ $\therefore \cos(x+45°)\cos(x-45°)$ $= \frac{1}{2}(\cos((x+45°+x-45°) + \cos(x+45°-x+45°))$ $= \frac{1}{2}(\cos 2x + \cos 90°)$ $= \frac{1}{2}\cos 2x$
	2.

	$\cos(x+45°)\cos(x-45°)$ has a minimum when $\frac{1}{2}\cos 2x$ has a minimum. The minimum value of $\cos 2x$ is -1 $\cos 2x = -1$ $2x = 180°$ $x = 90°$
	QUESTION 2
2.1	
2.2	
2.3	**QUESTION 3**
3.1	$\cos 3x = \sin x$ $\sin(90° - 3x) = \sin x$ $90° - 3x = x + k.360°$ or $90° - 3x = 180° - x + k.360°\ k \in Z$ $-4x = -90° + k.360°$ or $-2x = 90° + k.360°$ $x = 22{,}5° - k.90°\ k \in Z$; $x = -45° - k.180°\ k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$; $x = -45°; 135°$ **OR** $\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ or $3x = 360° - (90° - x) + k.360°$ $4x = 90° + k.360°$ or $2x = 270° + k.360°$ $x = 22{,}5° + k.90°\quad k \in Z$; $x = 135° + k.180°\quad k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$; $x = -45°; 135°$ **OR** $\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ or $3x = -90° + x + k.360°$ $4x = 90° + k.360°$ or $2x = -90° + k.360°$ $x = 22{,}5° + k.90°\quad k \in Z$; $x = -45° - k.180°\quad k \in Z$ $x = -67{,}5°; 22{,}5°; 112{,}5°$; $x = -45°; 135°$

3.2	
3.3	$-67{,}5° \leq x \leq -45°$ **OR** $x \in [-67{,}5°; -45°]$ **OR** From $-67{,}5°$ up to and including $-45°$
	QUESTION 4
4.1	

4.2	$\cos(x-30°)=\frac{1}{2}$ $2\cos(x-30°)=1$ See points A and B on the graph **Note:** If drawn the line y = $\frac{1}{2}$ and put A and B on the graph: 0/2 If A and B on the x-axis: 1/2 If A$=-30°$ and B$=90°$: 1/2
4.3	$\cos(x-30°)=0{,}5$ $x-30°=60°$ OR $x-30°=-60°$ $x=90°$ $x=-30°$
4.4	$g'(x)=0$ is at maximum and minimum values of graph $x=30°;\ 210°$
4.5	$x\in[-90°\,;-60°)\cup(120°\,;270°]$ **OR** $-90°\le x<-60°$ or $120°<x\le 270°$ **OR** *If* $x<-60°$ *or* $x>120°$ 2/3
	QUESTION 5
5.1	
5.2	
5.3	
5.4	
	QUESTION 6
6.1	
6.2	
6.3	
6.3.1	
6.3.2	
	QUESTION 7

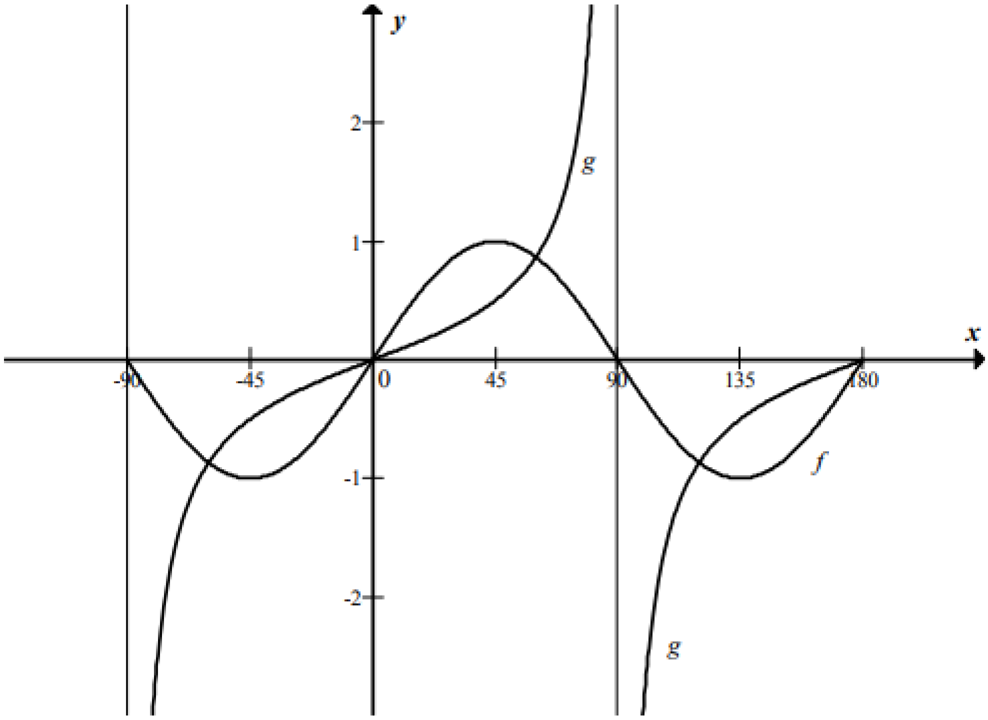
7.1	$1+\sin x=\cos 2x$ $1+\sin x=1-2\sin^2 x$ $\sin x+2\sin^2 x=0$ $\sin x(1+2\sin x)=0$ $\sin x=0$ or $\sin x=-\frac{1}{2}$ $x=k.180$ or $x=-30°+k.360$ $k\in Z$ $x=210°+k.360$ $x\in\{180°;210;330°;360°\}$ **OR** $1+\sin x=\cos 2x$ $1+\sin x=\cos^2 x-\sin^2 x$ $1+\sin x=1-\sin^2 x-\sin^2 x$ $\sin x+2\sin^2 x=0$ $\sin x(1+2\sin x)=0$ $\sin x=0$ or $\sin x=-\frac{1}{2}$ $x=k.180$ or $x=-30°+k.360$ $k\in Z$ $x=210°+k.360$ $x\in\{180°;210;330°;360°\}$
7.2	
7.3	$180°\le x\le 210°$ or $330°\le x\le 360°$
	QUESTION 8

8.1	$a \tan 45° = 2$ $a = 2$
8.2	
8.3	2
8.4	$f(14{,}5°) = \cos 59{,}5° = 0{,}5075$ $h(14{,}5°) = 2\tan 14{,}5° = 0{,}5172 \ > \ f(14{,}5°)$ From the graph, 14,5° lies to the RIGHT of the point of intersection. $\theta < 14{,}5°$
	QUESTION 9
9.1	

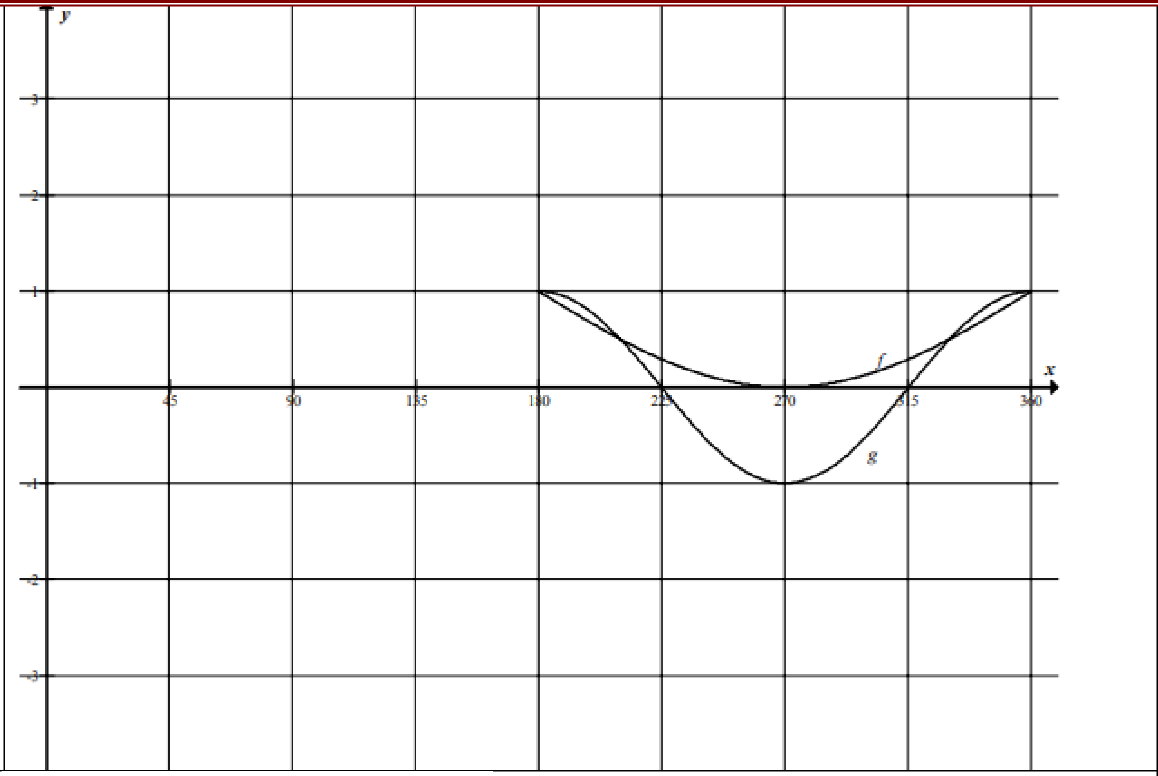
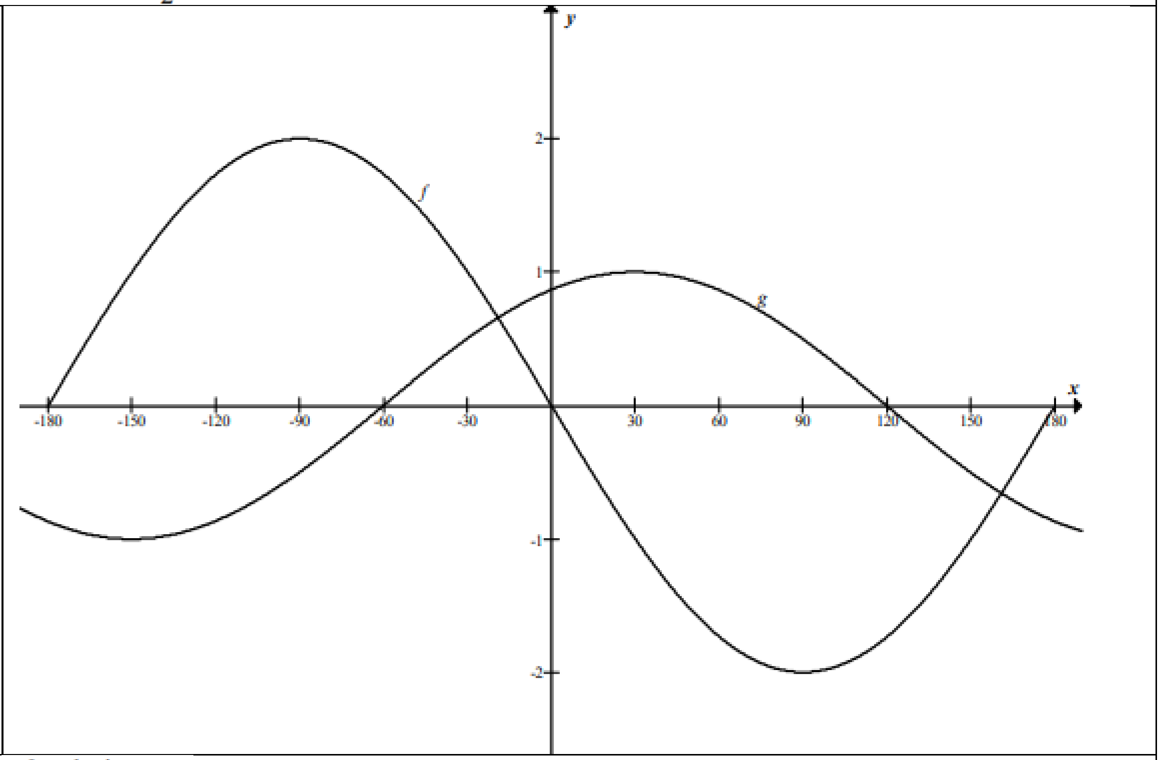
9.2	For these values of x, $\cos 2x \neq 0$ $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $= \dfrac{2\sin x\cos x}{1-2\sin^2 x}$ $1-2\sin^2 x = \sin x$ or $\cos x = 0$ $2\sin^2 x + \sin x - 1 = 0$ or $\cos x = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$ **OR** For these values of x, $\cos 2x \neq 0$ $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $= \dfrac{2\sin x\cos x}{2\cos^2 x - 1}$ $2\cos x(2\cos^2 x - 1) = 2\sin x\cos x$ $2\cos x(2(1-\sin^2 x)-1) = 2\sin x\cos x$ $2\cos x(1-2\sin^2 x) - 2\sin x\cos x = 0$ $2\cos x(2\sin^2 x + \sin x - 1) = 0$ $2\sin^2 x + \sin x - 1 = 0$ or $\cos x = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$ **OR** $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $2\cos x.\cos 2x = \sin 2x$ $2\cos x.\cos 2x - 2\sin x.\cos x = 0$ $2\cos x(\cos 2x - \sin x) = 0$ $\cos 2x = \sin x$ or $2\cos x = 0$ $1-2\sin^2 x = \sin x$ $2\sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$
9.3	$0° < x < 45°$ Or $-90° < x < -45°$
9.4	Period $= 2(360°) = 720°$

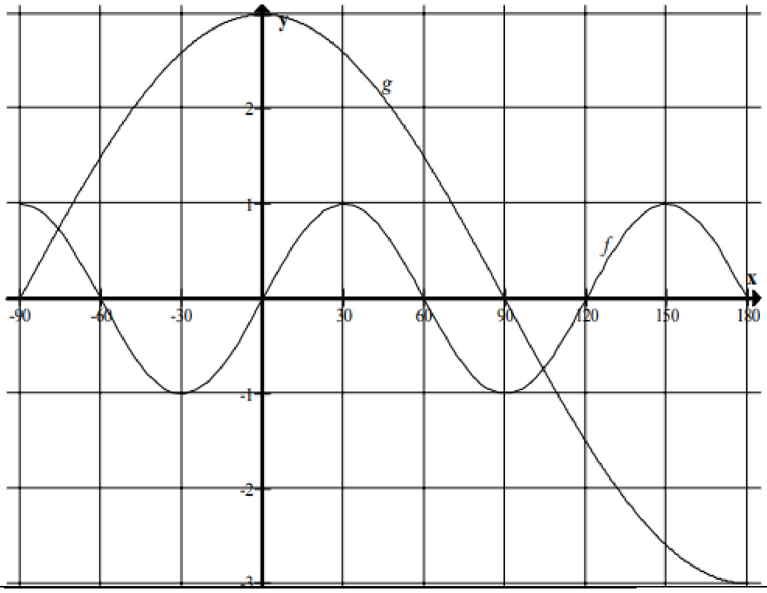
9.5	$x = -45° + 25° = -20°$ $x = 45° + 25° = 70°$ **Note:** Answer only: **full marks** **OR** $2(x-25°) = -90°$ and $2(x-25°) = 90°$ $2x - 50° = -90°$ and $2x - 50° = 90°$ $2x = -40°$ and $2x = 140°$ $x = -20°$ and $x = 70°$
	QUESTION 10
10.1	$f(225°) = 2$ $\therefore a \tan 225° = 2 \quad \therefore a = 2$ $g(0) = 4$ $\therefore b \cos 0° = 4 \quad \therefore b = 4$ Answer only: Full marks
10.2	Minimum value of $g(x) + 2 = -4 + 2 = -2$ Answer only: Full marks
10.3	$\text{Period} = \dfrac{180°}{\frac{1}{2}} = 360°$ Answer only: Full marks
10.4	At P $f(\theta) = g(\theta)$ $2\tan\theta = 4\cos\theta$ for $180° - \theta$: $2\tan(180° - \theta) = -2\tan\theta$ and $4\cos(180° - \theta) = -4\cos\theta$ $2\tan\theta = 4\cos\theta$ at P $\therefore -2\tan\theta = -4\cos\theta$ $\therefore 2\tan(180° - \theta) = 4\cos(180° - \theta)$ at Q **OR** $2\tan\theta = 4\cos\theta$ $\dfrac{\sin\theta}{\cos\theta} = 2\cos\theta$ $\sin\theta = 2\cos^2\theta$ $= 2(1-\sin^2\theta)$ $2\sin^2\theta + \sin\theta - 2 = 0$ $\sin\theta = \dfrac{-1 \pm \sqrt{1-4(2)(-2)}}{4}$ $\sin\theta = 0{,}78077...$ $\theta = 51{,}33°$ or $128{,}67°$ $\therefore$ the x-coordinate of Q is $180° - x_p$
	QUESTION 11
11.1	$f(0) - g(0) = 0{,}5 - (-2) = 2{,}5$

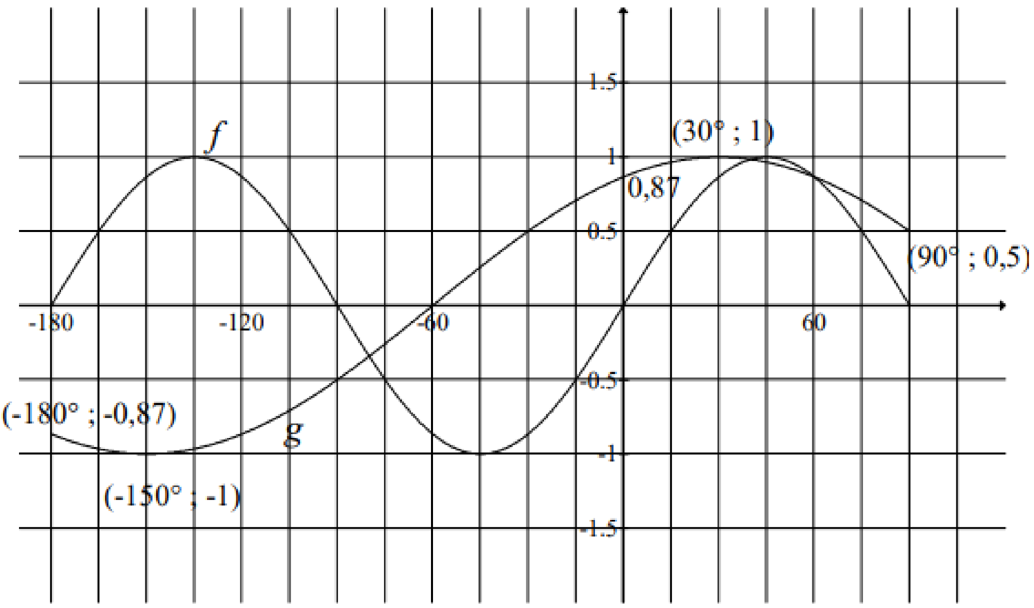
11.2	$\sin(x+30°)=-2\cos x$ $\sin x.\cos 30° + \cos x.\sin 30° = -2\cos x$ $\left(\frac{\sqrt{3}}{2}\right)\sin x + \left(\frac{1}{2}\right)\cos x = -2\cos x$ $\sqrt{3}\sin x + \cos x = -4\cos x$ $\sqrt{3}\sin x = -5\cos x$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 109{,}11° + 180°k \quad ; \quad k \in Z$ $x_P = -70{,}89°$ *and* $x_Q = 109{,}11°$ **OR** $\sin(x+30°) = -2\cos x$ $\cos(90° - x - 30°) = -2\cos x$ $\cos(60° - x) = -2\cos x$ $\cos 60° \cos x + \sin 60° \sin x = -2\cos x$ $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = -2\cos x$ $\cos x + \sqrt{3}\sin x = -4\cos x$ $\sqrt{3}\sin x = -5\cos x$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 109{,}11° + 180°.k \,;\, k \in Z$ $x_P = -70{,}89°$ *and* $x_Q = 109{,}11°$
11.3	$-70{,}89° \leq x \leq 109{,}11°$ **OR** $[-70{,}89° \,;\, 109{,}11°]$ **OR** $x_P \leq x \leq x_Q$
11.4	$h(x) = 2\sin(x + 60° + 30°) = 2\sin(x + 90°) = 2\cos x = -g(x)$ h is the reflection of g about the x-axis. **OR** f is shifted to the left through 60° and then doubled. $\therefore$ h is the reflection of g about the x-axis.
	QUESTION 12

12.1	
12.2	$\sin 2x = \frac{1}{2}\tan x$ $2\sin x.\cos x = \frac{\sin x}{2\cos x}$ $4\sin x.\cos^2 x - \sin x = 0$ $\sin x(4\cos^2 x - 1) = 0$ $\sin x = 0$ or $\cos^2 x = \frac{1}{4}$ $x = 0°$ or $180°$ or $\cos x = \pm\frac{1}{2}$ $x = 60°\ ;\ -60°$ or $120°$
12.3	$\{x \mid -60° < x < 0°\} \cup \{x \mid 60° < x < 90°\} \cup \{x \mid 120° < x < 180°\}$ **OR** $x \in (-60°\ ;\ 0°) \cup (60°\ ;\ 90°) \cup (120°\ ;\ 180°)$ **OR** $-60° < x < 0°$ or $60° < x < 90°$ or $120° < x < 180°$
	QUESTION 13

13.1	$1+\sin x=\cos 2x$ $1+\sin x=1-2\sin^2 x$ $\sin x+2\sin^2 x=0$ $\sin x(1+2\sin x)=0$ $\sin x=0$ or $\sin x=-\frac{1}{2}$ $x=k.180$ or $x=-30°+k.360$ $k\in Z$ $x=210°+k.360$ $x\in\{180°;210;330°;360°\}$ **OR** $1+\sin x=\cos 2x$ $1+\sin x=\cos^2 x-\sin^2 x$ $1+\sin x=1-\sin^2 x-\sin^2 x$ $\sin x+2\sin^2 x=0$ $\sin x(1+2\sin x)=0$ $\sin x=0$ or $\sin x=-\frac{1}{2}$ $x=k.180$ or $x=-30°+k.360$ $k\in Z$ $x=210°+k.360$ $x\in\{180°;210;330°;360°\}$

13.2	
13.3	$180° \le x \le 210°$ or $330° \le x \le 360°$
	QUESTION 14
14.1	Period = $360°$
14.2	Amplitude = $\frac{1}{2}$
14.3	
14.4	2 solutions
14.5	$-60° \le x \le 120°$ or $x \in [-60°;120°]$
14.6	$-90° < x < 30°$ or $x \in (-90°;30°)$

	QUESTION 15
15.1	Period = 120°
15.2	$\sin 3x = -1$ $x = -30^{\circ}$ or $x = 90^{\circ}$
15.3	Maximum value of $f(x)$ is 1 $\therefore$ Maximum value of $h(x)$ is 0
15.4	
15.5	$\frac{\sin 3x}{3} - \cos x = 0$ $\sin 3x - 3\cos x = 0$ $\therefore \sin 3x = 3\cos x$ There are 2 solutions where graphs *f* and *g* are equal
15.6	$f(x).g(x) < 0$ $x \in (-60^{\circ} ; 0^{\circ})$ or $(60^{\circ} ; 90^{\circ})$ or $(120^{\circ} ; 180^{\circ})$ **OR** $-60^{\circ} < x < 0^{\circ}$ or $60^{\circ} < x < 90^{\circ}$ or $120^{\circ} < x < 180^{\circ}$
	QUESTION 16
16.1	Range = $[-1 ; 1]$

16.2	$f\left(\frac{3}{2}x\right)=\sin 2\left(\frac{3}{2}x\right)$ $=\sin 3x$ $\therefore\ Period=\frac{360°}{3}=120°$ **OR** $f\left(\frac{3}{2}x\right)=\sin 2\left(\frac{3}{2}x\right)$ $=\sin 3x$ $=\sin(3x+360°)$ $=\sin 3(x+120°)$ $\therefore\ Period=120°$
16.3	
16.4	$(-180°;-90°)$ *or* $(-60°\ ;\ 0°)$ **OR** $-180°<x<-90°$ *or* $-60°<x<0°$
16.5	$y=\sin 2(x+30°)$ $\therefore$ translation of 30° to the left

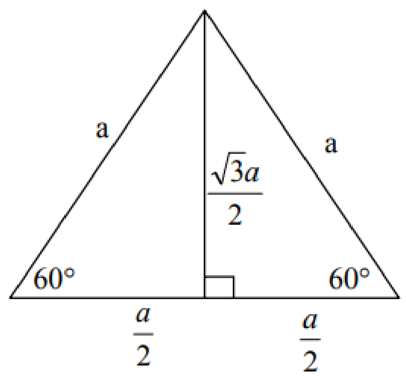
16.6	$\sin 2x = \cos(x-30°)$ $\sin 2x = \sin[90° - (x-30°)]$ $= \sin(120° - x)$ $2x = 120° - x + 360°k; k \in Z$ $2x = 180° - (120° - x) + 360°k$ $3x = 120° + 360°k$ **or** $2x - x = 60° + 360°k$ $x = 40° + 120°k; k \in Z$ $x = 60° + 360°k; k \in Z$ **OR** $\sin 2x = \cos(x-30°)$ $\cos(90° - 2x) = \cos(x-30°)$ $90° - 2x = x - 30° + 360°k$ or $90° - 2x = 360° - (x-30°) + 360°k$ $-3x = -120° + 360°k$ $-x = 300° + 360°k$ $x = 40° - 120°k; k \in Z$ $x = -300° - 360°k; k \in Z$ $\therefore x = 40° + 120°k$ or $x = 60° + 360°k$; $k \in Z$
	QUESTION 17
17.1	$\cos(x-45°) = -2\sin x$ $\cos x\cos 45° + \sin x \sin 45° = -2\sin x$ $\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = -2\sin x$ $\sqrt{2}\cos x = (-4-\sqrt{2})\sin x$ $\sqrt{2} = \frac{(-4-\sqrt{2})\sin x}{\cos x}$ $\tan x = \frac{\sqrt{2}}{-4-\sqrt{2}} = -0{,}2612$
17.2	$\tan x = -0{,}2612...$ $x = 165{,}36° + 180°k; k \in Z$ $x = -14{,}64°$ *or* $165{,}36°$
17.3	T (135° ; 0)
17.4	$f(x) \geq g(x)$ $-14{,}64° \leq x \leq 165{,}36°$ **OR** $x \in [-14{,}64° ; 165{,}36°]$
17.5	$-135° < x < -90°$ **OR** $x \in (-135° ; -90°)$
17.6	$h(x) = \cos(x - 45° - 45°)$ $= \cos(x - 90°)$ $= \sin x$ Answer only: full marks
	QUESTION 18
18.1.1	
18.1.2	

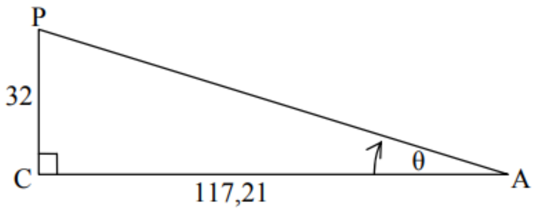
18.1.3	
18.2.1	
18.2.2	
18.2.3	
18.2.4	
18.2.5	

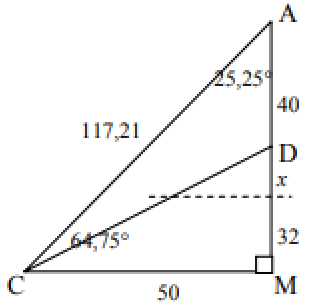
PART 5

	QUESTION 1
1.1.	$\frac{3}{LB} = \tan 40°$ $LB = \frac{3}{\tan 40°}$ $LB = 3{,}58 \text{ m}$ (3,5752......) (3,5 m ; 3,57 m ; 3,6 m) **OR** $\frac{LB}{\sin 50°} = \frac{3}{\sin 40°}$ $LB = \frac{3\sin 50°}{\sin 40°}$ $LB = 3{,}58 \text{ m}$ (3,5752......)
1.2	$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113°$ $AB^2 = (5.2)^2 + (3.58)^2 - 2(5.2)(3.58)\cos 113°$ $AB^2 = 54{,}40410138 \text{ m}^2$ $AB = 7{,}38 \text{ m}$ (7,37591..........) Note: AB = 7,3 m or 7,4 m: accept
1.3	$Area\ of\ \Delta ABL = \frac{1}{2} AL.BL.\sin A\hat{L}B$ $= \frac{1}{2}(5.2)(3.58)\sin 113°$ $= 8.568059176$ $= 8{,}57 \text{ m}$ Note: Area = 8,5 or 8,6 : accept
	QUESTION 2
2.1	
2.1.1	

	$Q\hat{R}S = 180° - (30° + 150° - \alpha)$ (3 angles of triangle) $= \alpha$ In triangle QRS: $\frac{QR}{\sin(150° - \alpha)} = \frac{12}{\sin\alpha}$ $QR = \frac{12\sin(150° - \alpha)}{\sin\alpha}$ $= \frac{12(\sin 150° \cos\alpha - \cos 150° \sin\alpha)}{\sin\alpha}$ $= \frac{12\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)}{\sin\alpha}$ $= \frac{6(\cos\alpha + \sqrt{3}\sin\alpha)}{\sin\alpha}$
2.1.2	$\frac{PQ}{QR} = \tan\alpha$ $PQ = \left(\frac{6(\cos\alpha + \sqrt{3}\sin\alpha)}{\sin\alpha}\right)\tan\alpha$ $PQ = \left(\frac{6\cos\alpha + 6\sqrt{3}\sin\alpha}{\sin\alpha}\right)\frac{\sin\alpha}{\cos\alpha}$ $PQ = \frac{6\cos\alpha + 6\sqrt{3}\sin\alpha}{\cos\alpha}$ $PQ = \frac{6\cos\alpha}{\cos\alpha} + \frac{6\sqrt{3}\sin\alpha}{\cos\alpha}$ $PQ = 6 + 6\sqrt{3}.\tan\alpha$ **OR** In triangle QPR: $Q\hat{P}R = 180° - (90° + \alpha) = 90° - \alpha$ (3 angles triangle) $\frac{PQ}{\sin\alpha} = \frac{QR}{\sin(90° - \alpha)}$ PQ $= \frac{QR\sin\alpha}{\cos\alpha}$ $= QR\tan\alpha$ $= \frac{12\sin(150° - \alpha)\tan\alpha}{\sin\alpha}$ $= \frac{12(\sin 150° \cos\alpha - \cos 150° \sin\alpha)\tan\alpha}{\sin\alpha}$ $= \frac{12\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)}{\sin\alpha} \times \frac{\sin\alpha}{\cos\alpha}$ $= 12\left(\frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sin\alpha}{\cos\alpha}\right)$ $= 6 + 6\sqrt{3}\tan\alpha$ $= 6(1 + \sqrt{3}\tan\alpha)$

2.1.3	$6+6\sqrt{3}.\tan\alpha) = 23$ $6\sqrt{3}.\tan\alpha = 17$ $\tan\alpha = \frac{17}{6\sqrt{3}}$ $\tan\alpha = 1{,}635825763...$ $\alpha = 58{,}56°$
2.2	Area of red part $= \text{Area of bigger triangle} - \text{area smaller triangle}$ $= \frac{1}{2}.(80)(80).\sin 60° - \frac{1}{2}(50)(50).\sin 60°$ $= 1\ 688{,}75\ \text{cm}^2$ **OR**  Area of equilateral triangle of side $a = \frac{1}{2}.a.\frac{\sqrt{3}q}{2} = \frac{\sqrt{3}}{4}a^2$ $\therefore \frac{\sqrt{3}}{4}(80^2 - 50^2) = 975\sqrt{3}\ cm^2$ **OR** Area of red part $= \frac{\sqrt{3}}{4}(80^2 - 50^2)$ $= 975\sqrt{3}$ $= 1\ 688{,}75\ \text{cm}^2$
	QUESTION 3

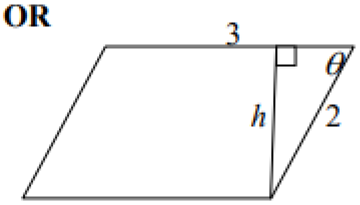
3.1	$\cos 64{,}75° = \frac{50}{AC}$ $AC = \frac{50}{\cos 64{,}75°}$ $= 117{,}21 \text{ m}$ **OR** $AC = \frac{50}{\cos 64{,}75°}$ $\therefore AC = 117{,}2144026 \text{ m}$ $\therefore AC = 117{,}21 \text{ m}$ **OR** $\frac{50}{\sin 25{,}25°} = \frac{AC}{\sin 90°}$ $\therefore AC = \frac{50 \sin 90°}{\sin 25{,}25°}$ $\therefore AC = 117{,}21 \text{ m}$	
3.2	PC is given to be $\frac{1}{2}(64) = 32$ m  $\tan P\hat{A}C = \frac{32}{117{,}21}$ $\theta = 15{,}27°$ (15,27042173...) **Note:** If the candidate takes the unrounded answer for AC, then the answer is 15,27° (15,26987495...)	

3.3	$CD^2 = 117{,}21^2 + 40^2 - 2(117{,}21)(40)\cos 25{,}25$ $= 6857{,}289092$ $\therefore CD = 82{,}81 \quad m$ **Note:** If don't use the rounded off then CD = 82,81 m. Accept this answer. **OR**  $AM = AC\sin 64{,}75°$ **OR** $AM = CM\tan 64{,}75°$ **OR** $AM = AC\cos 25{,}25°$ $= 106{,}0111876$ $= 50\tan 64{,}75°$ $= 117{,}21.\cos 25{,}25°$ $= 106{,}01$ $= 106{,}01$ $= 106{,}01$ $DM = 106{,}01 - 40$ $= 66{,}01$ $CD^2 = CM^2 + DM^2$ $= (50)^2 + (66{,}01)^2$ $= 6857{,}3201$ $CD = 82{,}81$ metres **OR** $AM = AC\sin 64{,}75°$ **OR** $AM = CM\tan 64{,}75°$ **OR** $AM = AC\cos 25{,}25°$ $= 106{,}0111876$ $= 50\tan 64{,}75°$ $= 117{,}21.\cos 25{,}25°$ $= 106{,}01$ $= 106{,}01$ $= 106{,}01$ $DM = 106{,}01 - 40$ $= 66{,}01$ $DC^2 = (50)^2 + (66{,}01)^2 - 2(50)(66{,}01).\cos 90°$ $= 6857{,}3201$ $CD = 82{,}81$ metres **OR** $\sin 64{,}75° = \dfrac{40 + x + 32}{117{,}21}$ $x = 34{,}01$ $CD^2 = CM^2 + DM^2$ $= (50)^2 + (32 + 34{,}01)^2$ $= 6857{,}3201$ $CD = 82{,}81$ metres
	QUESTION 4

4.1

$$\text{Area parallelogram ABCD} = 2 \times \text{Area } \Delta ABC$$
$$= 2\left[\left(\frac{1}{2}\right)(3)(2)\sin\theta\right]$$
$$= 6\sin\theta$$

OR



NOTE:
If no working is shown, then 0/3

$$\frac{h}{2} = \sin\theta$$
$$h = 2\sin\theta$$
$$\therefore \text{Area } ABCD = \text{base} \times \text{height} = 3h = 3.2\sin\theta = 6\sin\theta$$

OR

$$\text{Area of parallelogram ABCD} = \text{area of } \Delta ABC + \text{area of } \Delta ADC$$
$$= \left(\frac{1}{2}\right)(3)(2)\sin\theta + \left(\frac{1}{2}\right)(3)(2)\sin\theta$$
$$= 6\sin\theta$$

OR

$$\text{Area} = \frac{1}{2}(\text{sum of // sides}) \times h$$
$$= \frac{1}{2}(3+3) \times 2\sin\theta$$
$$= 6\sin\theta$$

4.2

$$\text{Area of parallelogram ABCD} = 3\sqrt{3}$$
$$6\sin\theta = 3\sqrt{3}$$
$$\sin\theta = \frac{\sqrt{3}}{2}$$
$$\theta = 60^\circ$$

NOTE:
Deduct 1 mark if both 60° and 120° are given as answers

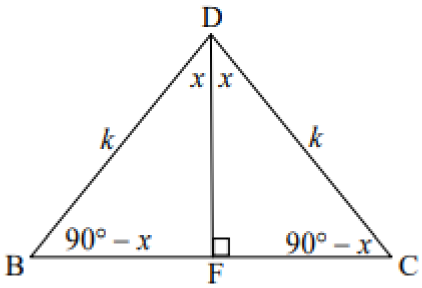
OR

$$6\sin 60^\circ = 3\sqrt{3}$$
$$\therefore \theta = 60^\circ$$

4.3

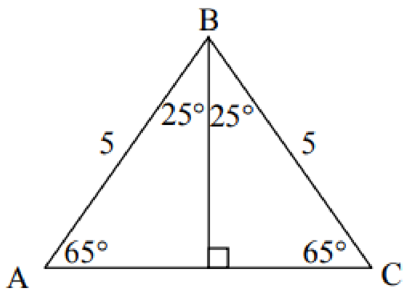
Maximum area of parallelogram occurs when $\sin\theta = 1$, that is when $\theta = 90^\circ$

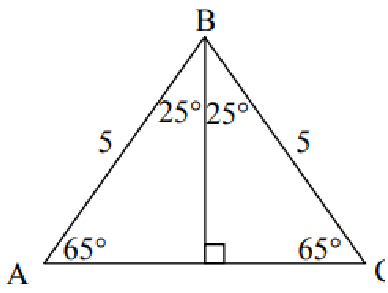
QUESTION 5

5.1	$\frac{CB}{\sin B\hat{D}C} = \frac{CD}{\sin C\hat{B}D}$ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90° - x)}$ $CB = \frac{k.\sin 2x}{\sin(90° - x)}$ $CB = \frac{k.2\sin x \cos x}{\cos x}$ $= 2k \sin x$ **OR** $D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$ $\therefore DC = DB = k$  Draw DF $\perp$ BC $\frac{CF}{CD} = \sin x$ $CF = k \sin x$ $CB = 2CF$ $CB = 2k \sin x$ **OR** $D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$ $\therefore DC = DB = k$ $CB^2 = CD^2 + BD^2 - 2.CD.BD.\cos 2x$ $CB^2 = k^2 + k^2 - 2k^2 \cos 2x$ $= 2k^2(1 - \cos 2x)$ $= 2k^2(1 - (1 - 2\sin^2 x))$ $= 2k^2(2\sin^2 x)$ $= 4k^2 \sin^2 x$ $= (2k \sin x)^2$ $CB = 2k \sin x$

5.2	$\cos x = \frac{BC}{HC}$ $HC = \frac{BC}{\cos x}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$ **OR** $\frac{HC}{\sin 90°} = \frac{BC}{\sin(90° - x)}$ $HC = \frac{BC}{\sin(90° - x)}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$
5.3	$HC = 2k \tan x = 2(40).\tan(23°) = 33{,}9579...$ In ΔHCD: $CD^2 = HC^2 + HD^2 - 2HC.HD.\cos\theta$ $\cos\theta = \frac{HC^2 + HD^2 - CD^2}{2HC.HD}$ $= \frac{(33{,}9579...)^2 + 31{,}8^2 - 40^2}{2(33{,}9579...)(31{,}8)}$ $\cos\theta = 0{,}2613...$ $\therefore \theta = 74{,}85°$
	QUESTION 6
6.1	$\frac{b}{\sin[180° - (\alpha + \beta)} = \frac{BC}{\sin\alpha}$ $BC \sin(\alpha + \beta) = b \sin\alpha$ $BC = \frac{b\sin\alpha}{\sin(\alpha + \beta)}$ but $BC = DF$ $\therefore DF = \frac{b\sin\alpha}{\sin(\alpha + \beta)}$ $\cos\theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos\theta}$ $\therefore DE = \frac{b\sin\alpha}{\sin(\alpha + \beta)\cos\theta}$

6.2	$DE = \frac{2000\sin 43°}{\sin 79°.\cos 27°}$ $= 1559,50\text{ m}$
	QUESTION 7
7.1	$\frac{7}{PB} = \sin 18°$ $PB = \frac{7}{\sin 18°}$ $PB = 22,65\text{ m} \quad (22,65247584...)$
7.2	$\frac{18}{PA} = \cos 23°$ $PA = \frac{18}{\cos 23°}$ $PA = 19,55\text{ m} \quad (19,55448679..)$
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55).\cos 42°$ $= 237,0847954...$ $AB = 15,40\text{ m} \quad (15,3975581...)$
	QUESTION 8
8.1	$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$ $= (7,5)^2 + (9,4)^2 - 2.(7,5)(9,4)\cos 32°$ $= 25,03521844...$ $EC = 5,0\text{ metres}$
8.2	$\frac{\sin D\hat{C}E}{7,5} = \frac{\sin 32°}{5,0}$ $\sin D\hat{C}E = \frac{7,5.\sin 32°}{5,0}$ $= 0,7948788963$ $D\hat{C}E = 52,6°$
8.3	Area of ΔDEC $= \frac{1}{2}DE.DC\sin\hat{D}$ $= \frac{1}{2}(7,5)(9,4)\sin 32°$ $= 18,7m^2$ OR Area of ΔDEC $= \frac{1}{2}CE.DC\sin 52,6°$ $= \frac{1}{2}(5,0)(9,4)\sin 52,6°$ $= 18,7m^2$

8.4	$\sin 32° = \dfrac{EG}{7,5}$ $EG = 7,5.\sin 32°$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5$ metres **OR** $EG = EC.\sin 52,6°$ $= (5,0).\sin 52,6°$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ **OR** $\frac{1}{2}.DC.EG = \text{area } \Delta DEC$ $\frac{1}{2}(9,4)EG = 18,7$ $\therefore\ EG = \dfrac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$
	QUESTION 9
9.1	Area $\Delta ABC = \frac{1}{2}.AB.BC.\sin 50°$ $= \frac{1}{2}(5)(5)\sin 50°$ $= 9,58\ units^2$ **OR** Area of ΔABC $= \frac{1}{2}(2)(5)\sin 25°)(5\cos 25°)$ $= 9,58\ units^2$  **OR** Area of ΔABC $= [\frac{1}{2}(5\cos 65°)(5\sin 65°)](2)$ $= 9,58\ units^2$

9.2	$AC^2 = 5^2 + 5^2 - 2(5)(5)\cos 50°$ $AC^2 = 17{,}86061952$ $AC = 4{,}23\ units$ **OR** $\hat{A} = \hat{C} = 65°$ (angles opposite equal sides) $\dfrac{\sin 65°}{5} = \dfrac{\sin 50°}{AC}$ $AC = \dfrac{5\sin 50°}{\sin 65°}$ $= 4{,}23\ units$ **OR** $\sin 25° = \dfrac{\frac{1}{2}(AC)}{5}$ $AC = 2(5)\sin 25°$ $= 4{,}23\ units$  **OR** $\cos 65° = \dfrac{\frac{1}{2}(AC)}{5}$ $AC = 2(5)\cos 65°$ $AC = 4{,}23\ units$
9.3	$\tan 25° = \dfrac{CF}{AC}$ $\therefore CF = 4{,}23 \times \tan 25°$ $\therefore CF = 1{,}97\ units$ **OR** $\dfrac{FC}{\sin 25°} = \dfrac{4{,}23}{\sin 65°}$ $FC = \dfrac{4{,}23\sin 25°}{\sin 65°}$ $= 1{,}97\ units$

PART 6

November 2014

5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$
5.2	$C\hat{P}A = D\hat{P}A = 30°$ (AP bisects $D\hat{P}C$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24{,}57...$ $AD = 4{,}96$

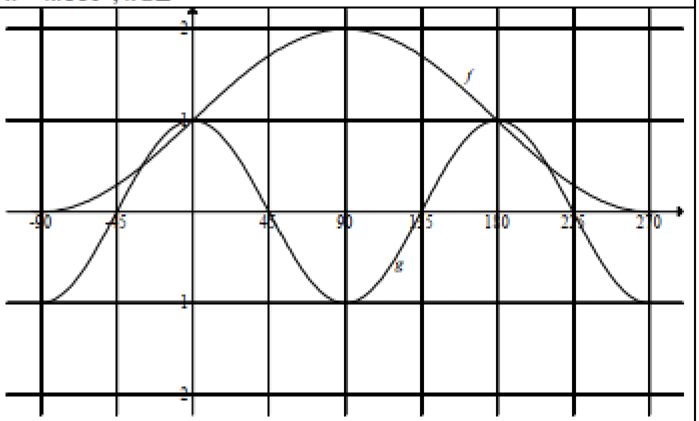
5.3	$\frac{\sin D\hat{A}P}{DP} = \frac{\sin A\hat{P}D}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30°}{4{,}96}$ $\sin y = \frac{4\sin 30°}{4{,}96}$ $= 0{,}403...$ $y = 23{,}78°$ **OR/*OF*** $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos D\hat{A}P$ $4^2 = 8^2 + (4{,}96)^2 - 2(8)(4{,}96).\cos y$ $\cos y = \frac{8^2 + (4{,}96)^2 - 4^2}{2(8)(4{,}96)}$ $\cos y = 0{,}9148...$ $y = 23{,}82°$

QUESTION/VRAAG 6

6.1	$\cos^2(180°+x)+\tan(x-180°)\sin(720°-x)\cos x$ $=(-\cos x)^2+[-(-\tan x)](-\sin x)(\cos x)$ $=\cos^2 x+\left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x)$ $=\cos^2 x-\sin^2 x$ $=\cos 2x$
6.2	$\sin(\alpha-\beta)$ $=\cos[90°-(\alpha-\beta)]$ $=\cos[(90°-\alpha)+\beta]$ $=\cos(90°-\alpha)\cos\beta-\sin(90°-\alpha)\sin\beta$ $=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ **OR/*OF*** $\sin(\alpha-\beta)$ $=\cos[90°-(\alpha-\beta)]$ $=\cos[(90°+\beta)+(-\alpha)]$ $=\cos(90°+\beta)\cos(-\alpha)-\sin(90°+\beta)\sin(-\alpha)$ $=(-\sin\beta)\cos\alpha-\cos\beta(-\sin\alpha)$ $=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

6.3	x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=-(\cos^2 76°-\sin^2 76°)$ $=-\cos 2(76°)$ $=-\cos 152°$ $=-(-\cos 28°)$ **OR/*OF*** $=-\cos(90°+62°)$ $=\cos 28°$ $=-(-\sin 62°)$ $=\cos(90°-62°)$ $=\sin 62°$ $=\sin 62°$ **OR/*OF*** x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=\sin 76°\sin 76°-\cos 76°\cos 76°$ $=\sin 76°\cos 14°-\cos 76°\sin 14°$ $=\sin(76°-14°)$ $=\sin 62°$ **OR/*OF*** x^2-y^2 $=\sin^2 76°-\cos^2 76°$ $=\cos^2 14°-\sin^2 14°$ $=\cos 2(14°)$ $=\cos 28°$ $=\sin 62°$

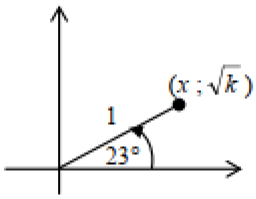
QUESTION/VRAAG 7

7.1	$0 \le y \le 2$ or $y \in [0\ ;2]$
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ $x = 0° + k.360°$ or $x = 210° + k.360°$ or $x = 180° + k.360°$ $x = 330° + k.360°,\ k \in Z$ **OR/*OF*** $x = k.180°,\ k \in Z$
7.4	

7.5	$f(x) = g(x)$ at/*by*: $x = -30°\ ;\ 0°\ ;\ 180°\ ;\ 210°$ $\therefore f(x + 30°) = g(x + 30°)$ at/*by*: $x = -60°\ ;\ -30°\ ;\ 150°\ ;\ 180°$
7.6	Series will converge if/*Reeks sal konvergeer as*: $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore\ 30° < x < 60°$ or $x \in (30°\ ;\ 60°)$

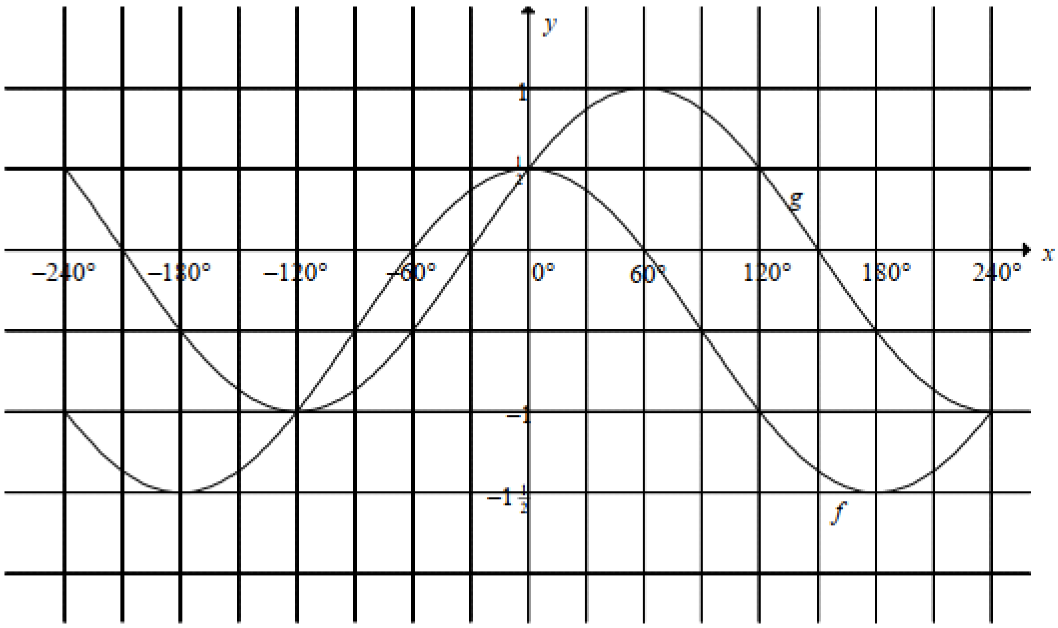
November 2015

QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}$ 
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\frac{\sin 23°}{\cos 23°}$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$
5.2	$\frac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \frac{-4\sin x.\cos x}{\sin 30°}$ $= \frac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$

5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$

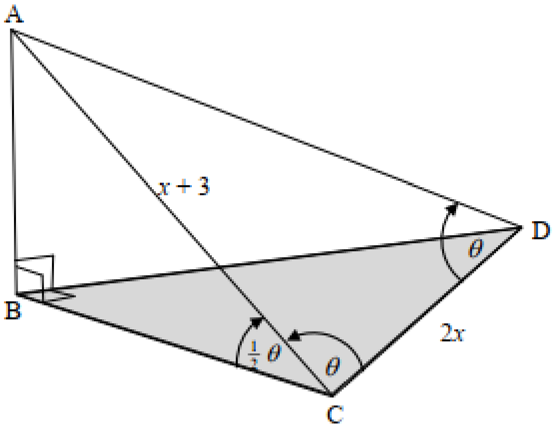
QUESTION/*VRAAG* 6

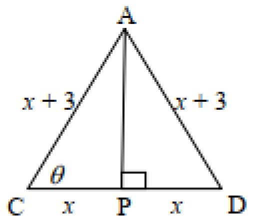


6.1	$f(x) = \cos x - \frac{1}{2}$ and/*en* $g(x) = \sin(x + 30°)$ $\therefore p = 30°$ and/*en* $q = -\frac{1}{2}$ **OR/*OF*** $\sin(60° + p) = 1$ and/*en* $\cos 0° + q = \frac{1}{2}$ $\therefore p = 30°$ $\therefore q = -\frac{1}{2}$	
6.2	$x \in (-120°\,;\,0°)$ **OR/*OF*** $-120° < x < 0°$	

6.3	The graph of g has to shift 60° to the left and then be reflected about the x-axis./*Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.* **OR/*OF*** The graph of g must be reflected about the x-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.* **OR/*OF*** The graph of g has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuif word.* **OR/*OF*** The graph of g has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuif word*	

QUESTION/*VRAAG* 7

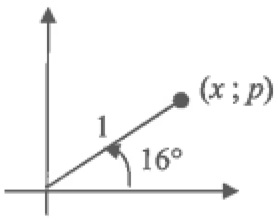


7.1	$\hat{CAD} = 180° - 2\theta$ [$\angle$s sum of Δ/ $\angle$*e som van* Δ]
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$ **OR/*OF*** $AD = x + 3$ [sides opp = $\angle$s/*sye to* = $\angle$*e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$ **OR/*OF*** Draw/*Trek* AP $\perp$ CD $\cos\theta = \frac{x}{x+3}$ 

7.3	$\cos\theta = \frac{2}{5}$ $\therefore\ \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore\ AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\frac{\theta}{2} = \frac{AB}{5}$ $\therefore\ AB = 5\sin\frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore\ AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$

November 2016

QUESTION/*VRAAG* 5

5.1.1	$\sin 196° = -\sin 16°$ $= -p$
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF*** $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore\ \cos 16° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$

5.3

$$\frac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90^\circ+A)}$$

$$=\frac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$$

$$=\frac{\sin 2A}{\cos A.(-\sin A)}$$

$$=\frac{2\sin A\cos A}{\cos A.(-\sin A)}$$

$$=-2$$

OR/OF

$$\frac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90^\circ+A)}=\frac{\sqrt{1-(2\cos^2 A-1)^2}}{\cos A.-\sin A}$$

$$=\frac{\sqrt{1-(4\cos^4 A-4\cos^2 A+1)}}{\cos A.-\sin A}=\frac{\sqrt{4\cos^2 A-4\cos^4 A}}{\cos A.-\sin A}$$

$$=\frac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A}=\frac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$$

$$=\frac{2\cos A.\sin A}{\cos A.-\sin A}$$

$$=-2$$

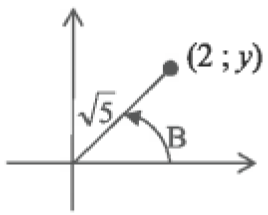
5.4.1

$$\cos 2B=\frac{3}{5}$$

$$2\cos^2 B-1=\frac{3}{5}$$

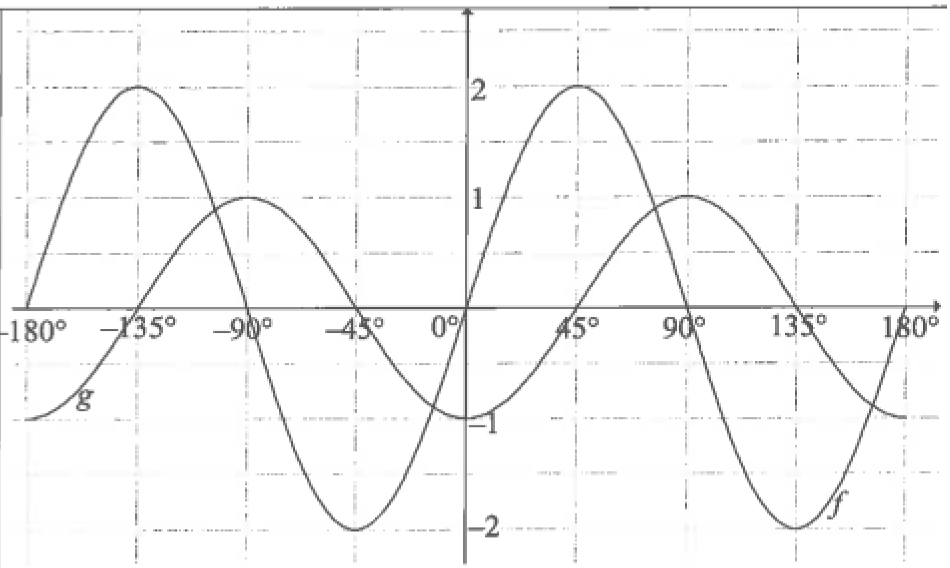
$$\cos^2 B=\frac{4}{5}$$

$$\therefore\ \cos B=\sqrt{\frac{4}{5}}\ or\ \frac{2}{\sqrt{5}}\ or\ \frac{2\sqrt{5}}{5}\quad [0^\circ\le B\le 90^\circ]$$

5.4.2	$\sin^2 B = 1 - \cos^2 B$ $= 1 - \left(\frac{2}{\sqrt{5}}\right)^2$ $= \frac{1}{5} \qquad \therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2 + y^2 = (\sqrt{5})^2$ $4 + y^2 = 5$ $y^2 = 1$ $y = 1$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$ 

5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ *or* $\frac{\sqrt{10}}{10}$

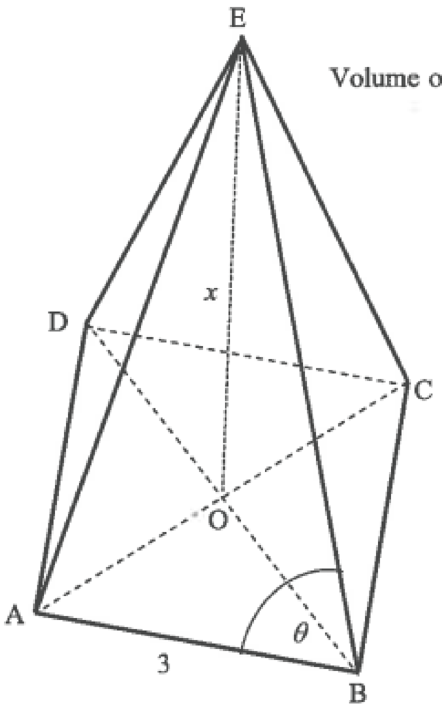
QUESTION/*VRAAG* 6

6.1	
6.2	$f(x) - 3 = 2\sin 2x - 3$ $\therefore$ maximum value $= 2 - 3 = -1$

6.3	$2\sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $ref\angle = 26{,}57°$ $2x = 153{,}43° + k.180°; k \in Z$ $x = 76{,}72° + k.90°; k \in Z \quad or \quad x = -13.28° + k.90°, k \in Z$ **OR/*OF*** $2\sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $ref\angle = 26{,}57°$ $2x = 153{,}43° + k.360° \quad or \quad 333{,}43° + k.360°, k \in Z$ $x = 76{,}72° + k.180° \qquad or \; 166{,}72° + k.180° \; ; k \in Z$

6.4	$x \in (-103{,}28°; -13{,}28°)$ **OR/*OF*** $-103{,}28° < x < -13{,}28°$

QUESTION/*VRAAG* 7



$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

7.1

$$DB^2 = 3^2 + 3^2 \qquad \text{[Theorem of Pyth]}$$
$$= 18$$
$$DB = \sqrt{18}$$
$$OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2} \ or \ \frac{3}{\sqrt{2}} \ or \ \frac{3\sqrt{2}}{2} \ or \ 2{,}12$$

OR/*OF*

$$\sin 45° = \frac{OB}{3}$$
$$OB = 3\sin 45°$$
$$OB = \frac{3\sqrt{2}}{2} \ or \ \frac{3}{\sqrt{2}} \ or \ 2{,}12$$

OF/OR

$$\cos 45° = \frac{OB}{3}$$
$$\frac{1}{\sqrt{2}} = \frac{OB}{3}$$
$$OB = \frac{3}{\sqrt{2}} \ or \ \frac{3\sqrt{2}}{2} \ or \ 2{,}12$$

7.2

$$BE^2 = EO^2 + OB^2 \qquad \text{(Pyth)}$$
$$BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$$
$$BE = \sqrt{x^2 + \frac{9}{2}}$$
$$AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$$
$$\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB} \qquad [EB = AE]$$
$$\cos\theta = \frac{AB}{2EB}$$
$$\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$$

7.3	$\text{Volume} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$ $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25+\frac{9}{2}}}$ $\therefore\ \theta = 73{,}97°$

Feb/Mar 2015

QUESTION/VRAAG 5

5.1	$x^2 + y^2$ $=(3\sin\theta)^2 + (3\cos\theta)^2$ $=9\sin^2\theta + 9\cos^2\theta$ $=9(\sin^2\theta + \cos^2\theta)$ $=9(1)$ $=9$
5.2	$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $\sin(180° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin\alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$

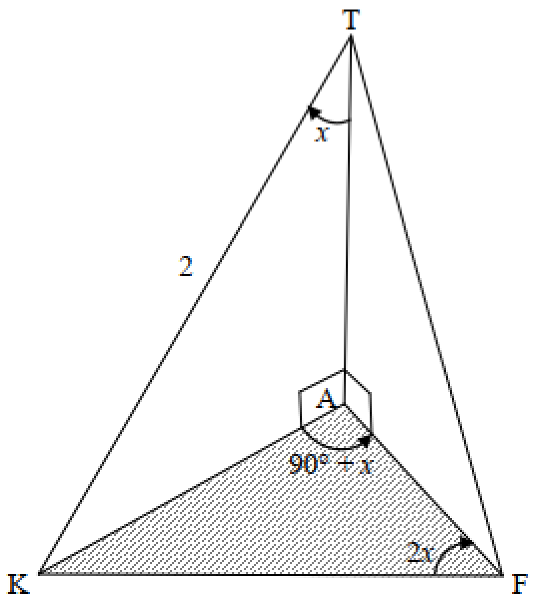
5.3.2	$\cos(180° + \alpha)$ $= -\cos\alpha$ $= -\left(\frac{-1}{\sqrt{1+p^2}}\right)$ $= \frac{1}{\sqrt{1+p^2}}$

5.3.3	$\cos 2\alpha$ $= \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - \left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= \frac{1}{1+p^2} - \frac{p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$

5.4.1	The identity is undefined for/*die identiteit is ongedefinieer* $2\sin^2 x = 0$ $\therefore$ $\underline{\sin x = 0:}$ $x = 0°\ ;\ 180°$ or/*of* $\underline{\tan x = \infty:}$ $x = 90°$ $\therefore\ x = 0°\ ;\ 90°\ ;\ 180°$
5.4.2	LHS/LK $= \frac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \frac{2\left(\frac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x}$ $= \left(\frac{2\sin x - 2\sin x\cos^2 x}{\cos x}\right) \times \frac{1}{2\sin^2 x}$ $= \frac{2\sin x(1-\cos^2 x)}{\cos x} \times \frac{1}{2\sin^2 x}$ $= \frac{2\sin x(\sin^2 x)}{\cos x} \times \frac{1}{2\sin^2 x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK

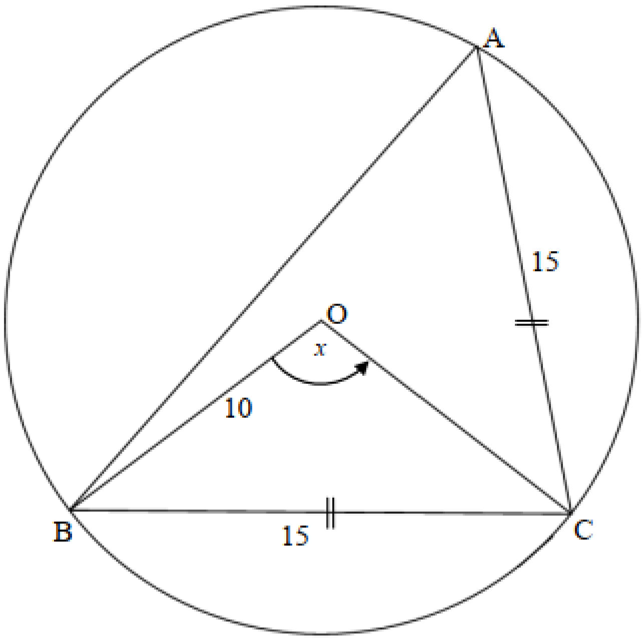
QUESTION/VRAAG 6

6.1



6.1.1	In ΔTAK: $\frac{AK}{KT} = \sin K\hat{T}A$ $AK = KT.\sin x$ $= 2\sin x$
6.1.2	In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{2\sin x.\cos x}{2\sin x.\cos x}$ $= 1$

6.2



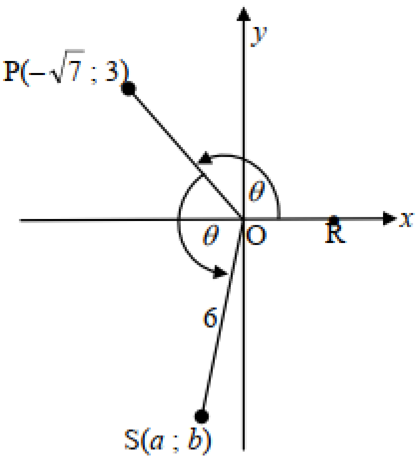
6.2.1	In ΔBOC: $BC^2 = BO^2 + CO^2 - 2.BO.\ CO\ .\cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10).\ \cos x$ $200\cos x = -25$ $\cos x = -0{,}125$ $x = 180° - 82{,}82°$ $\quad = 97{,}18°$

6.2.2	$B\hat{A}C = 48{,}59°$ ($\angle$ at centre=2×$\angle$at circ/$\angle$ *by midpt=2×$\angle$omt*) $A\hat{B}C = B\hat{A}C = 48{,}59°$ ($\angle$'s opp equal sides/$\angle$*e teenoor = sye*) $\therefore A\hat{C}B = 82{,}82°$ (sum of $\angle$s of Δ/*som van $\angle$e van Δ*) **OR/*OF*** $A\hat{C}B = \frac{1}{2}A\hat{O}B$ ($\angle$at centre = 2× $\angle$at circle) ($\angle$ *by midpt=2×$\angle$omt*) $= \frac{1}{2}[360° - 2(97{,}18°)]$ $= 82{,}82°$

6.2.3	Area/*Oppervlakte* ΔABC $= \frac{1}{2}(BC)(AC)\sin A\hat{C}B$ $= \frac{1}{2}(15)(15)(\sin 82{,}82°)$ $= 111{,}62\ \text{cm}^2$

Feb/Mar 2016

QUESTION/*VRAAG* 5

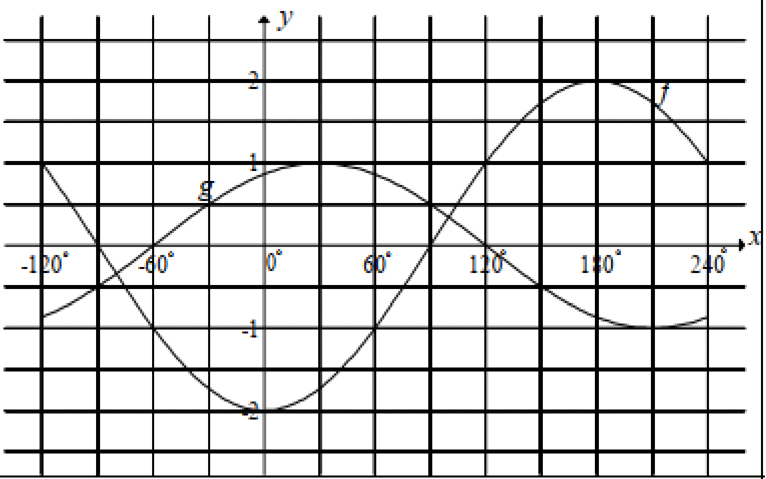


5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$

5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x-1}=\frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $=\frac{2\sin 2x}{-\cos 2x}$ $=-2\tan 2x$
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15°-1}=-2\tan 2(15°)$ $=-2\tan 30°$ $=-2\left(\frac{1}{\sqrt{3}}\right)$

QUESTION/*VRAAG* 6

6.1	$\sin(x+60°)+2\cos x=0$ $\sin x\cos 60°+\cos x\sin 60°+2\cos x=0$ $\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x+2\cos x=0$ $\frac{1}{2}\sin x=-2\cos x-\frac{\sqrt{3}}{2}\cos x$ $\sin x=-4\cos x-\sqrt{3}\cos x$ $\sin x=\cos x(-4-\sqrt{3})$ $\frac{\sin x}{\cos x}=\frac{\cos x(-4-\sqrt{3})}{\cos x}$ $\therefore \tan x=-4-\sqrt{3}$
6.2	$\tan x=-4-\sqrt{3}$ $\tan x=-(4+\sqrt{3})$ *ref* $\angle=80{,}10°$ $x=-80{,}1°$ or/*of* $99{,}9°$

6.3.1	
6.3.2	$\therefore \sin(x + 60°) > -2\cos x$ $x \in (-80{,}10° \ ; 99{,}90°)$ **OR/*OF*** $-80{,}10° < x < 99{,}90°$

QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2} PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore\ x_{max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* $2{,}5$
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore\ RP = 9{,}01$

7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: AB = BD and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$

Feb/Mar 2017

QUESTION/*VRAAG* 5

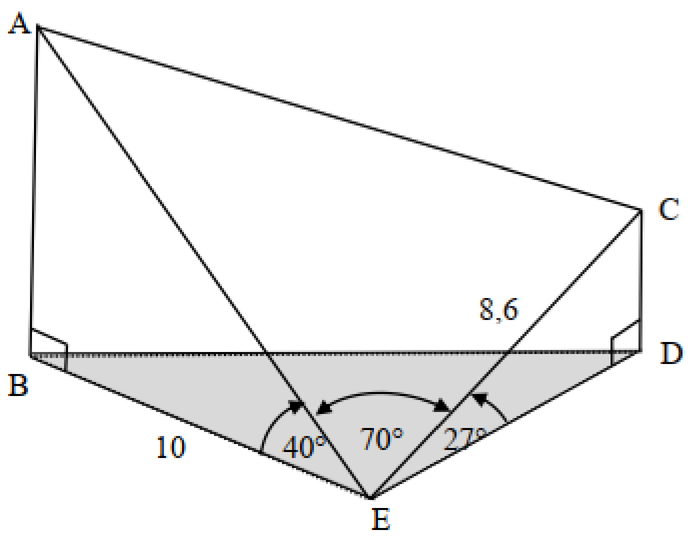
5.1	$a = -1$ $b = 2$
5.2	$f(3x) = -\sin 3x$ Period of $f(3x) = \frac{360°}{3}$ $= 120°$
5.3	$x \in [90° ; 135°) \cup \{180°\}$

QUESTION/*VRAAG* 6

6.1.1	$\sin\ (360° - 36°) = -\sin 36°$
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$
6.2	R.T.P.: $1 - \dfrac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $\text{LHS} = \dfrac{1+\tan^2\theta - \tan^2\theta}{1+\tan^2\theta}$ $= \dfrac{1}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$ $= \dfrac{1}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \dfrac{1}{\dfrac{1}{\cos^2\theta}}$ $= \cos^2\theta$ $= \text{RHS}$
6.3	$\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}\ \ or\ -\frac{1}{2}$ $\frac{1}{2}x = 60° + k.360°$ or $\frac{1}{2}x = 300° + k.360°$ or $\frac{1}{2}x = 120° + k.360°$ or $\frac{1}{2}x = 240° + k.360°$ $x = 120° + k.720°$ or $x = 600° + k.720°$ or $x = 240° + k.720°$ or $x = 480° + k.720°;\ \ k \in Z$
6.4.1	$\sin(\text{A}-\text{B}) = \cos[90° - (\text{A}-\text{B})]$ $= \cos[(90° - \text{A}) - (-\text{B})]$ $= \cos(90° - \text{A})\cos(-\text{B}) + \sin(90° - \text{A})\sin(-\text{B})$ $= \sin\text{A}\cos\text{B} + \cos\text{A}(-\sin\text{B})$ $= \sin\text{A}\cos\text{B} - \cos\text{A}\sin\text{B}$

6.4.2	$\sin(x+64^\circ)\cos(x+379^\circ)+\sin(x+19^\circ)\cos(x+244^\circ)$ $=\sin(x+64^\circ)\cos(x+19^\circ)+\sin(x+19^\circ)[-\cos(x+64^\circ)]$ $=\sin(x+64^\circ)\cos(x+19^\circ)-\cos(x+64^\circ)\sin(x+19^\circ)$ $=\sin[x+64^\circ-(x+19^\circ)]$ $=\sin 45^\circ$ $=\frac{1}{\sqrt{2}}$

QUESTION/*VRAAG* 7



7.1	$\sin 27^\circ=\frac{CD}{8{,}6}$ $CD=8{,}6\sin 27^\circ$ $CD=3{,}90\text{ m}$
7.2	$\cos 40^\circ=\frac{10}{AE}$ $AE=\frac{10}{\cos 40^\circ}$ $AE=13{,}05\text{ m}$
7.3	$AC^2=CE^2+AE^2-2\,CE.AE(\cos A\hat{E}C)$ $=(8{,}6)^2+(13{,}05)^2-2(8{,}6)(13{,}05)(\cos 70^\circ)$ $=167{,}49$ $AC=12{,}94\text{ m}$

November 2018

QUESTION 5

5.1.1

$$\therefore k = -\sqrt{\left(\sqrt{5}\right)^2 - 1^2} \;\; (Pythagoras)$$

$$\therefore k = -2$$

5.1.2

(a)

$$\tan\theta = -\frac{1}{2}$$

(b)

$$\cos(180° + \theta) = -\cos\theta = -\left(-\frac{2}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}} \;\; or \;\; \frac{2\sqrt{5}}{5}$$

(c)

$$\begin{aligned}\sin(\theta + 60°) &= \sin\theta\cos 60° + \cos\theta\sin 60° \\ &= \frac{1}{\sqrt{5}} \times \frac{1}{2} + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{1}{2\sqrt{5}} - \frac{2\sqrt{3}}{2\sqrt{5}} \\ &= \frac{1-2\sqrt{3}}{2\sqrt{5}} \\ &= \frac{1-2\sqrt{3}}{\sqrt{20}}\end{aligned}$$

5.1.3

$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$\therefore \theta = 180° - \sin^{-1}\frac{1}{\sqrt{5}} = 153{,}43$$

$$\therefore \tan(2 \times 153{,}43 - 40°) = 18{,}287 = 18.3\,(1\;d.p.)$$

5.2

$$LHS = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$$
$$= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$$
$$= \frac{\cos^2 x + 2\sin x \cos x + \sin^2 x - \cos^2 x + 2\sin x \cos x - \sin^2 x}{\cos^2 x - \sin^2 x}$$
$$= \frac{2(2\sin x \cos x)}{\cos^2 x - \sin^2 x}$$
$$= \frac{2(\sin 2x)}{\cos 2x}$$
$$= 2\tan 2x$$
$$= RHS$$

5.3

$$\sum_{A=38^\circ}^{52^\circ} \cos^2 A = \cos^2 38 + \cos^2 39 + \cos^2 40 + ... + \cos^2 51 + \cos^2 52$$
$$= \cos^2 38 + \cos^2 39 + \cos^2 40 + ... + \sin^2 39 + \sin^2 38$$
$$= 7 \times (\cos^2 \alpha + \sin^2 \alpha) + \cos^2 45^\circ \ \ (\cos^2 45 \textit{ is in the middle})$$
$$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$$
$$= 7\frac{1}{2} \textit{ or } \frac{15}{2}$$

QUESTION 6

6.1

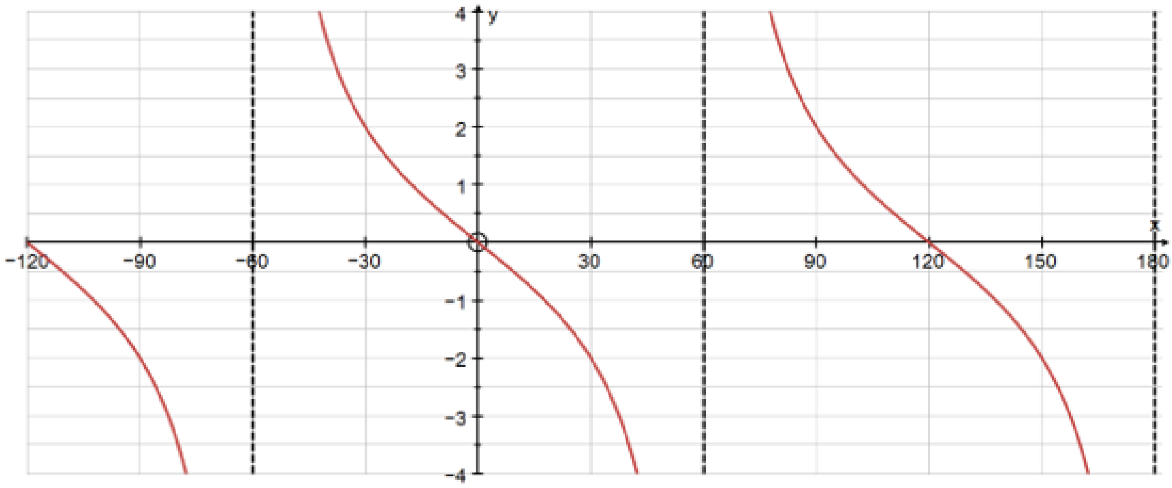
$$period = \frac{180^\circ}{\frac{3}{2}} = 120^\circ$$

6.2

sub in the point $(t;2)$,

$$\therefore 2 = -2\tan\frac{3}{2}t$$
$$\therefore \tan\frac{3}{2}t = -1$$
$$\therefore \frac{3}{2}t = -45^\circ + k.180^\circ \textit{ or } \frac{3}{2}t = 135^\circ + k.180^\circ$$
$$\therefore t = -30^\circ + k.120^\circ \ \textit{ or } \ t = 90^\circ + k.120^\circ \ \textit{ for } \ k \in \mathbb{Z}$$

6.3



6.4

$x \in (-60°; -30°]$ *or* $x \in (60°; 90°]$

6.5

$$g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$$

$$\therefore g(x) = -2\tan\left[\frac{3}{2}(x + 40°)\right]$$

$\therefore f(x)$ *has been shifted* $40°$ *to the left*

QUESTION 7

7.1

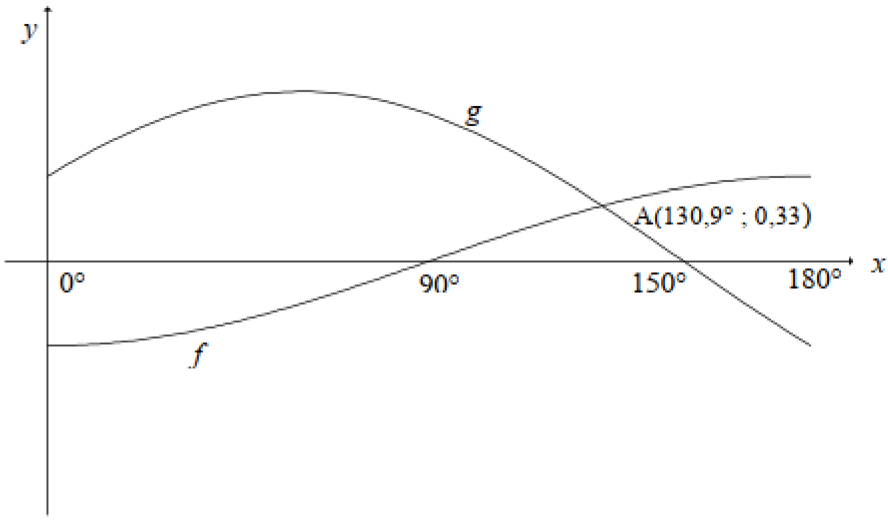
$$A\hat{B}D = 30^\circ \left(alt.\ \angle s \text{ on } \|\ lines\right)$$
$$\therefore \sin 30^\circ = \frac{h}{AB}$$
$$\therefore \frac{1}{2} = \frac{h}{AB}$$
$$\therefore AB = 2h$$

7.2

$$BC^2 = \sqrt{(2h)^2 + (3h)^2 - 2(2h)(3h)\cos(2x)}$$
$$= \sqrt{13h^2 - 12h^2\cos(2x)}$$
$$= \sqrt{13h^2 - 12h^2\left(2\cos^2 x - 1\right)}$$
$$= \sqrt{13h^2 - 24h^2\cos^2 x + 12h^2}$$
$$= \sqrt{25h^2 - 24h^2\cos^2 x}$$
$$= \sqrt{h^2\left(25 - 24\cos^2 x\right)}$$
$$= h\sqrt{25 - 24\cos^2 x} \quad as \quad required$$

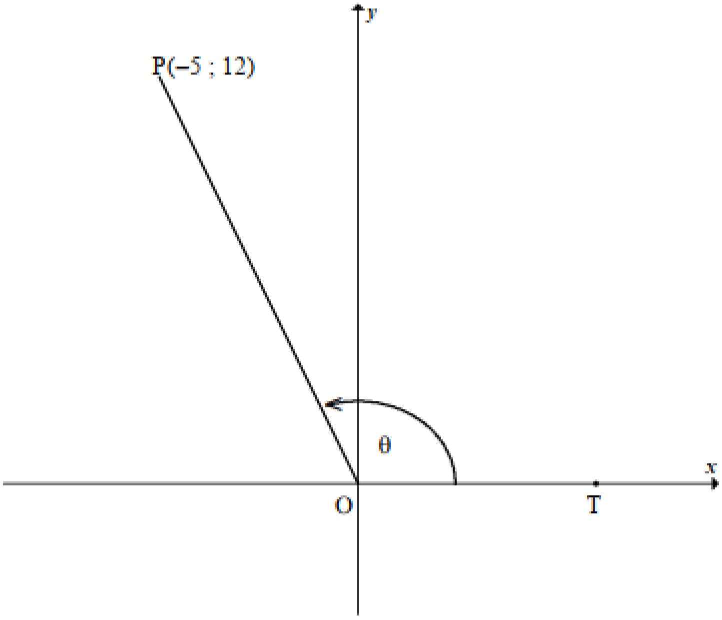
November 2020

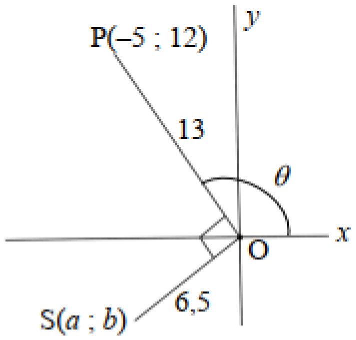
QUESTION/*VRAAG* 5



5.1	Period of $g = 360°$
5.2	Amplitude of $f = \frac{1}{2}$
5.3	$f(180°) - g(180°)$ $= \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1$
5.4.1	$x = 140{,}9°$
5.4.2	$\sqrt{3}\sin x + \cos x \geq 1$ $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x \geq \frac{1}{2}$ $\sin x \cos 30° + \cos x \sin 30° \geq \frac{1}{2}$ $\sin(x + 30°) \geq \frac{1}{2}$ $\sin(x + 30°) = \frac{1}{2}$ at $x = 0°$ or $x = 120°$ $\therefore x \in [0°;120°]$ **OR** $0° \leq x \leq 120°$

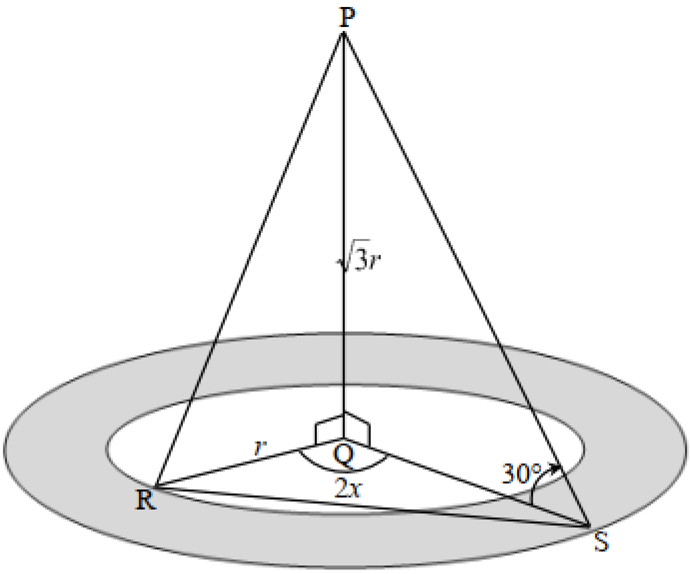
QUESTION/*VRAAG* 6



6.1.1	$\tan\theta = -\frac{12}{5}$ or $-2\frac{2}{5}$
6.1.2	$(OP)^2 = (-5)^2 + (12)^2$ $OP = 13$ $\cos\theta = -\frac{5}{13}$
6.1.3	$\sin(\theta + 90°) = \frac{b}{6{,}5}$ $\cos\theta = \frac{b}{6{,}5}$ $\frac{-5}{13} = \frac{b}{6{,}5}$ $b = -\frac{5}{2}$ **OR** $\cos(90° + \theta) = \frac{a}{6{,}5}$ $-\sin\theta = \frac{a}{6{,}5}$ $-\frac{12}{13} = \frac{a}{6{,}5} \quad \therefore a = -6$ $b = \sqrt{(6{,}5)^2 - (-6)^2} = -\frac{5}{2}$ 
6.2	$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$ $= \frac{\sin 2x \cos x + \cos 2x(-\sin x)}{-\sin x}$ $= \frac{\sin(2x - x)}{-\sin x}$ $= \frac{\sin x}{-\sin x}$ $= -1$
6.3	$6\sin^2 x + 7\cos x - 3 = 0$ $6(1 - \cos^2 x) + 7\cos x - 3 = 0$ $6 - 6\cos^2 x + 7\cos x - 3 = 0$ $6\cos^2 x - 7\cos x - 3 = 0$ $(3\cos x + 1)(2\cos x - 3) = 0$ $\cos x = -\frac{1}{3}$ or $\cos x = \frac{3}{2}$ (N/A) $\therefore x = 109{,}47° + k.360°;\ k \in Z$ or $x = 250{,}53° + k.360°;\ k \in Z$

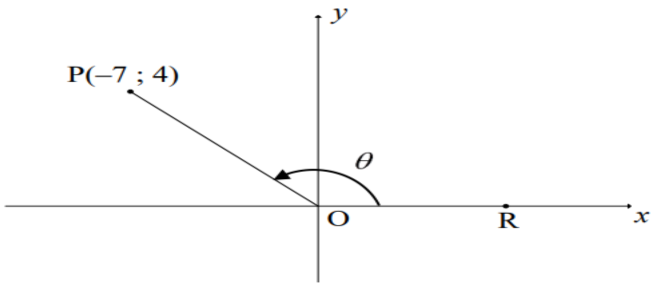
6.4	$x+\frac{1}{x}=3\cos A$ $(3\cos A)^2=\left(x+\frac{1}{x}\right)^2$ $9\cos^2 A=x^2+\frac{1}{x^2}+2$ $9\cos^2 A=2+2$ $\cos^2 A=\frac{4}{9}$ $\cos 2A=2\cos^2 A-1$ $=2\left(\frac{4}{9}\right)-1$ $=-\frac{1}{9}$

QUESTION/*VRAAG* 7



7.1	$\tan 30° = \frac{\sqrt{3}r}{QS}$ **OR** $\tan 60° = \frac{QS}{\sqrt{3}r}$ $QS = \frac{\sqrt{3}r}{\tan 30°}$ $\sqrt{3} = \frac{QS}{\sqrt{3}r}$ $= \frac{\sqrt{3}r}{\frac{1}{\sqrt{3}}}$ *or* $\frac{\sqrt{3}r}{\frac{\sqrt{3}}{3}}$ $QS = 3r$ $= 3r$
7.2	Area of flower garden $= \pi(3r)^2 - \pi r^2$ $= 9\pi r^2 - \pi r^2$ $= 8\pi r^2$
7.3	$RS^2 = r^2 + (3r)^2 - 2(r)(3r)\cos 2x$ $= r^2 + 9r^2 - 6r^2 \cos 2x$ $= 10r^2 - 6r^2 \cos 2x$ $= r^2(10 - 6\cos 2x)$ $RS = r\sqrt{10 - 6\cos 2x}$
7.4	$RS = 10\sqrt{10 - 6\cos 2(56)}$ $= 34{,}9966...$ ≈ 35 m

QUESTION/*VRAAG* 5



ACTIVITY 1

5.1.1	$\text{OP}=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$ Answer only 2/2	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan\theta=\dfrac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta-180°)=-\cos\theta$ $=\dfrac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.180°$; $k\in Z$ **OR/*OF*** $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.360°$ or $x=251{,}57°+k.360°$; $k\in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ ✓ $+k.180°;\ k\in Z$ (7) ✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ and $251{,}57°$ ✓ $+k.360°;\ k\in Z$ (7)

5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ $0°$ ✓ $60°$ (3)
		[18]

5.3.1	$\text{LHS} = \dfrac{\sin 3x}{1-\cos 3x} \times \dfrac{1+\cos 3x}{1+\cos 3x}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \dfrac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = \dfrac{\sin 3x}{1-\cos 3x} \times \dfrac{\sin 3x}{\sin 3x}$ $= \dfrac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \dfrac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \dfrac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \dfrac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$	✓ multiply by “1” ✓ $1-\cos^2 3x$ ✓ square identity (3) ✓ multiply by “1” ✓ square identity ✓ factors (3)

5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ $0°$ ✓ $60°$ (3)
		[18]

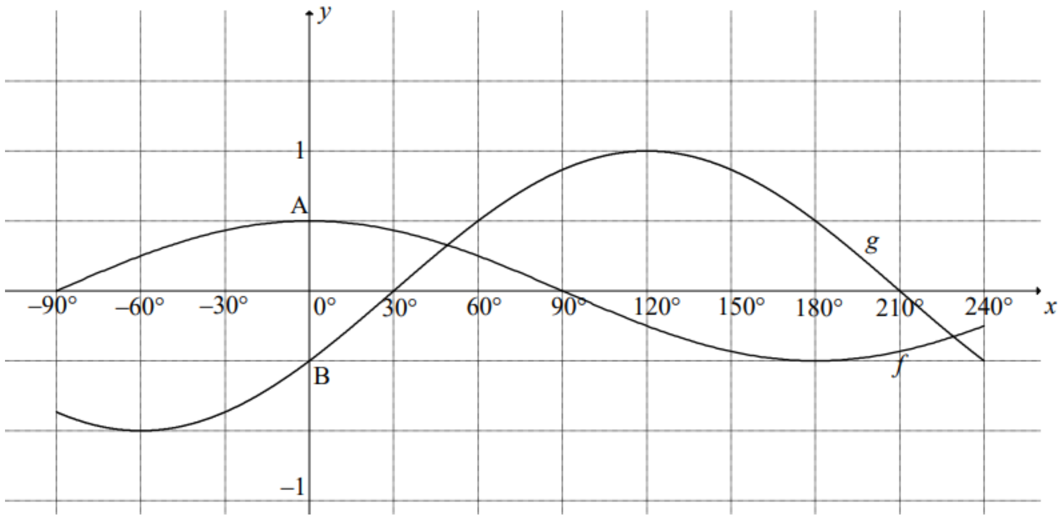
ACTIVITY 2

QUESTION/*VRAAG* 6

6.1	$\dfrac{\sin 10°}{\cos 440°}+\tan(360°-\theta).\sin 2\theta$ $=\dfrac{\cos 80°}{\cos 80°}-\tan\theta(2\sin\theta\cos\theta)$ $=1-\dfrac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$ $=1-2\sin^2\theta$ $=\cos 2\theta$	✓ $-\tan\theta$ ✓ cos 80° ✓ co-ratio ✓ double angle ✓ quotient identity ✓ answer (6)
6.2.1	$\sin(60°+2x)+\sin(60°-2x)=k\cos 2x$ $(\sin 60°\cos 2x+\cos 60°\sin 2x)+(\sin 60°\cos 2x-\cos 60°\sin 2x)=k\cos 2x$ $2\sin 60°\cos 2x=k\cos 2x$ $2\left(\dfrac{\sqrt{3}}{2}\right)\cos 2x=k\cos 2x$ $\therefore k=\sqrt{3}$	✓ both expansions correct ✓ special ∠s ✓ answer (3)
6.2.2	$\tan 60°[\sin(60°+2x)+\sin(60°-2x)]$ $=\tan 60°[k\cos 2x]$ $=\sqrt{3}(\sqrt{3}\cos 2x)$ $=3(2\cos^2 x-1)$ $=3\left(2(\sqrt{t})^2-1\right)$ $=6(\sqrt{t})^2-3$ $=6t-3$	✓ special ∠ ✓ double ∠s ✓ answer i.t.o t (3)
		[12]

ACTIVITY 3

QUESTION/*VRAAG* 7



7.1	$A\left(0;\frac{1}{2}\right)$ $B\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit Answer only 2/2	✓ y-values ✓ answer (2)
7.2	Range of $f:\ y\in\left[-\frac{1}{2};\frac{1}{2}\right]$ Range of $3f(x)+2:\ \ y\in\left[\frac{1}{2};3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\leq y\leq 3\frac{1}{2}$	✓ critical values ✓ answer (2)
7.3	$x=90°$	✓✓ $x=90°$ (2)
7.4.1	$x\in(30°;90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\leq 240°$	✓ $x\in(30°;90°)$ ✓ $(210°;240°]$ (2) ✓ $30°<x<90°$ ✓ $210°<x\leq 240°$ (2)
7.4.2	$x\in(-55°;125°)$ **OR/*OF*** $-55°<x<125°$	✓ critical values ✓ answer (2) ✓ critical values ✓ answer (2)
		[10]

ACTIVITY 4

QUESTION/*VRAAG* 8

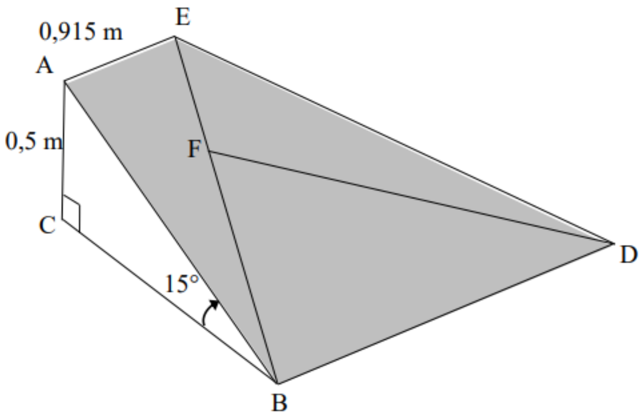


FIGURE I

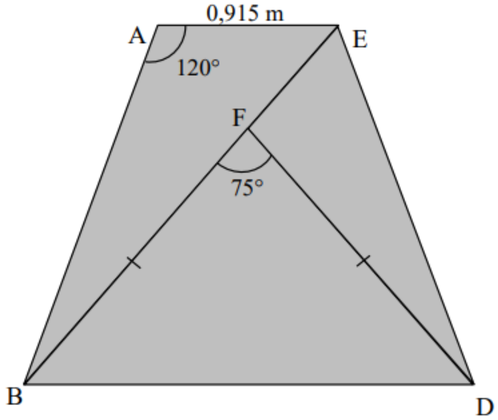


FIGURE II (top view)

8.1	$\frac{0,5}{AB}=\sin 15°$ $AB=\frac{0,5}{\sin 15°}$ $AB=1,93$ m Answer only 2/2	✓ trig ratio ✓ answer (2)
8.2	$BE^2=AB^2+AE^2-2(AB)(AE)\cos B\hat{A}E$ $BE^2=(1,93)^2+(0,915)^2-2(1,93)(0,915)(\cos 120°)$ $BE=2,52$ m	✓ correct use of cosine rule ✓ substitution ✓ answer (3)
8.3	$BF=FD=\frac{5}{7}(2,52)\ =1,80$ m Area $\Delta BFD=\frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $=\frac{1}{2}(1,8)(1,8)(\sin 75°)$ $=1,56\ m^2$	✓ BF ✓ correct substitution into the area rule ✓ answer (3)
		[8]