JENN
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SUBJECT: MATHEMATICS

SOLUTIONS MANUAL

**GRADE 12**

2023

STATISTICS

JENN MAREMATLOU

A Division of Marematlou Group Holdings

QUESTION 1

1.1	$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{408}{19} = 21{,}47$$
1.2	Standard deviation $= 7{,}81$
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma \, ; \bar{x} + 1\sigma)$ $= (21{,}47 - 7{,}81; 21{,}47 + 7{,}81) = (13{,}66 \, ; 29{,}28)$ $\therefore$ 13 people lie within 1 standard deviation of the mean.
1.4	5 12 13 15 **18** 18 18 19 20 **21** 21 22 23 23 **26** 29 33 35 37 $\text{IQR} = 26 - 18$ $\phantom{\text{IQR}} = 8$
1.5	
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values. $\therefore$ 5 is an outlier

QUESTION 2

2.1

Class	Frequency	Cumulative frequency
$0 \leq m < 2$	7	7
$2 \leq m < 4$	15	22
$4 \leq m < 6$	26	48
$6 \leq m < 8$	29	77
$8 \leq m < 10$	36	113
$10 \leq m < 12$	31	144
$12 \leq m < 14$	14	158
$14 \leq m < 16$	2	160

2.2
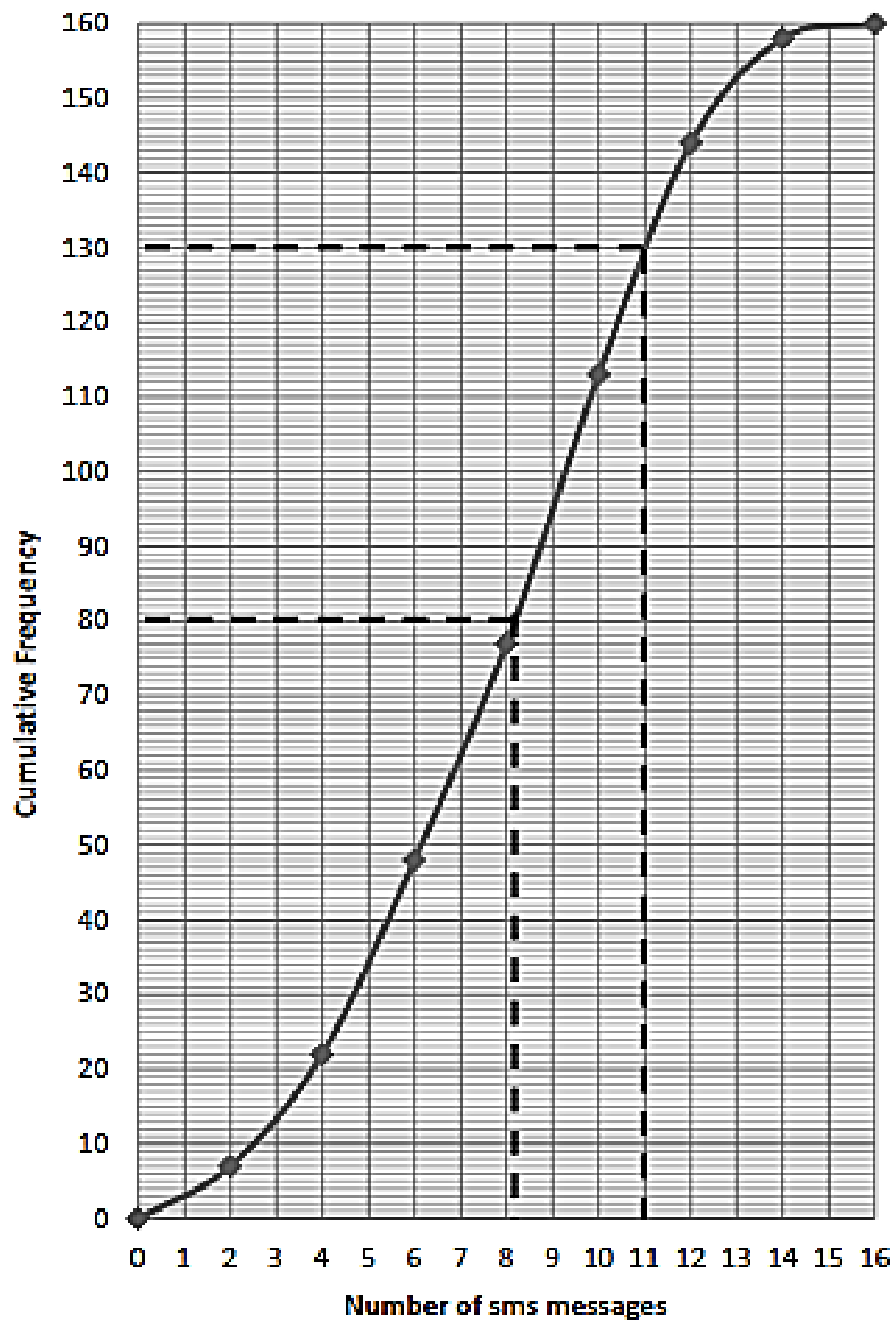
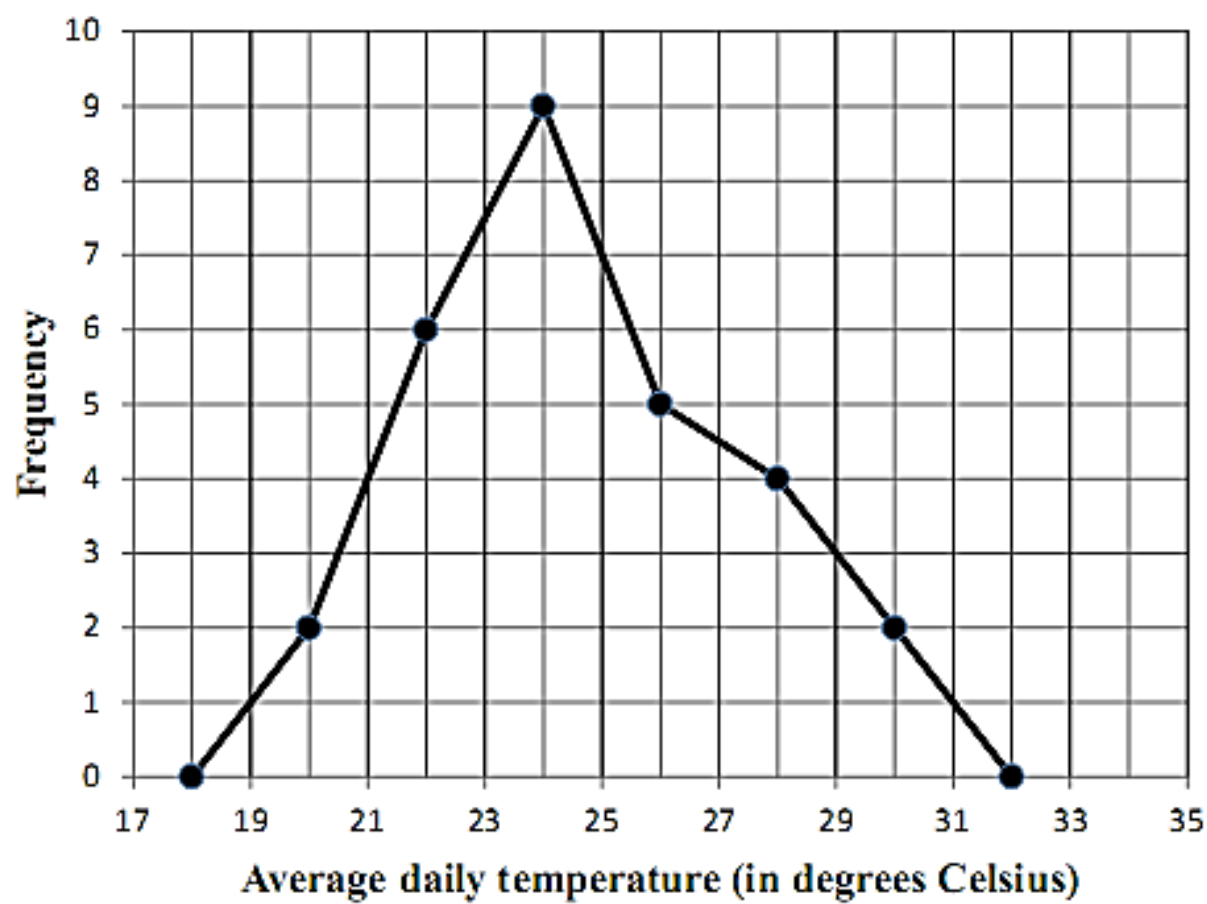| 2.3	The median for the data is approximately 8 messages.
2.4	Approximately 130 learners sent 11 or fewer messages. Therefore 30 learners sent more than 11 messages. $$\frac{30}{160} \times 100\% = 18{,}75\%$$
2.5	Skewed to the left or negatively skewed

QUESTION 3

3.1.1	IQR (A) $= 30 - 20$ $ = 10$
3.1.2	Data of Supermarket A is skewed to the left/*Data van Supermark A is skeef na links.* **OR** Negatively skewed/*negatief skeef*
3.1.3	Range/*Omvang* (B) $= 35 - 6$ $ = 29$
3.1.4	Supermarket A • Supermarket A received 25 or more deliveries on more than 7 days whilst Supermarket B received 25 or more deliveries on less than 7 days/ *Supermark A het op meer as 7 dae 25 of meer aflewerings ontvang terwyl Supermark B op minder as 7 dae soveel aflewerings ontvang het.*
3.2	$\bar{x} = 24{,}5$ $$\frac{2x + 293}{14} = 24{,}5$$ $2x + 293 = 343$ $2x = 50$ $x = 25$

QUESTION 4

4.1	28 days/*dae*
4.2	12 days $\therefore \dfrac{12}{28} \times 100$ $= 42,86\%$ Accept/*Aanvaar* 12,5 days which is/*dae, wat gelyk is aan* 44,64% OR Accept/*Aanvaar* 13 days which is/*dae, wat gelyk is aan* 46,43%
4.3	

Temperature, T, in degrees Celsius	Frequency
$19 \leq T < 21$	2
$21 \leq T < 23$	6
$23 \leq T < 25$	9
$25 \leq T < 27$	5
$27 \leq T < 29$	4
$29 \leq T < 31$	2

4.4

FREQUENCY POLYGON

Average daily temperature (in degrees Celsius)

QUESTION 5

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

5.1	Range/*Omvang* = 156 − 56 = 100 kg
5.2	Mode/*Modus* = 71 kg
5.3	Median/*Mediaan* = T_{14} = 93 kg
5.4	$Q_1 = T_7 = 82$ $Q_3 = T_{21} = 127$ IQR = $Q_3 - Q_1$ $\quad = 127 - 82$ $\quad = 45$ kg
5.5	
5.6	SD = 25,838 ≈ 25,84 kg
5.7	$\bar{x}$ = 98,59 $\bar{x} + 1\sigma = 98,59 + 25,84$ $\quad\quad\quad = 124,43$ kg 127 > 124,43 ∴ I agree with this person/*Ek stem met die persoon saam.*
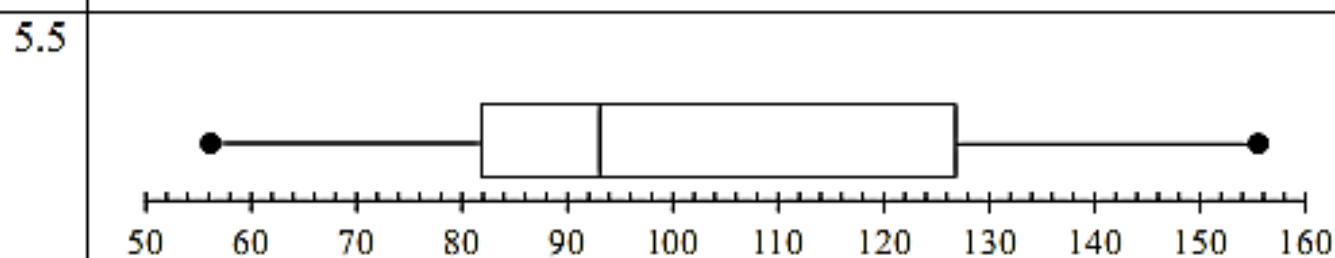
9

QUESTION 6

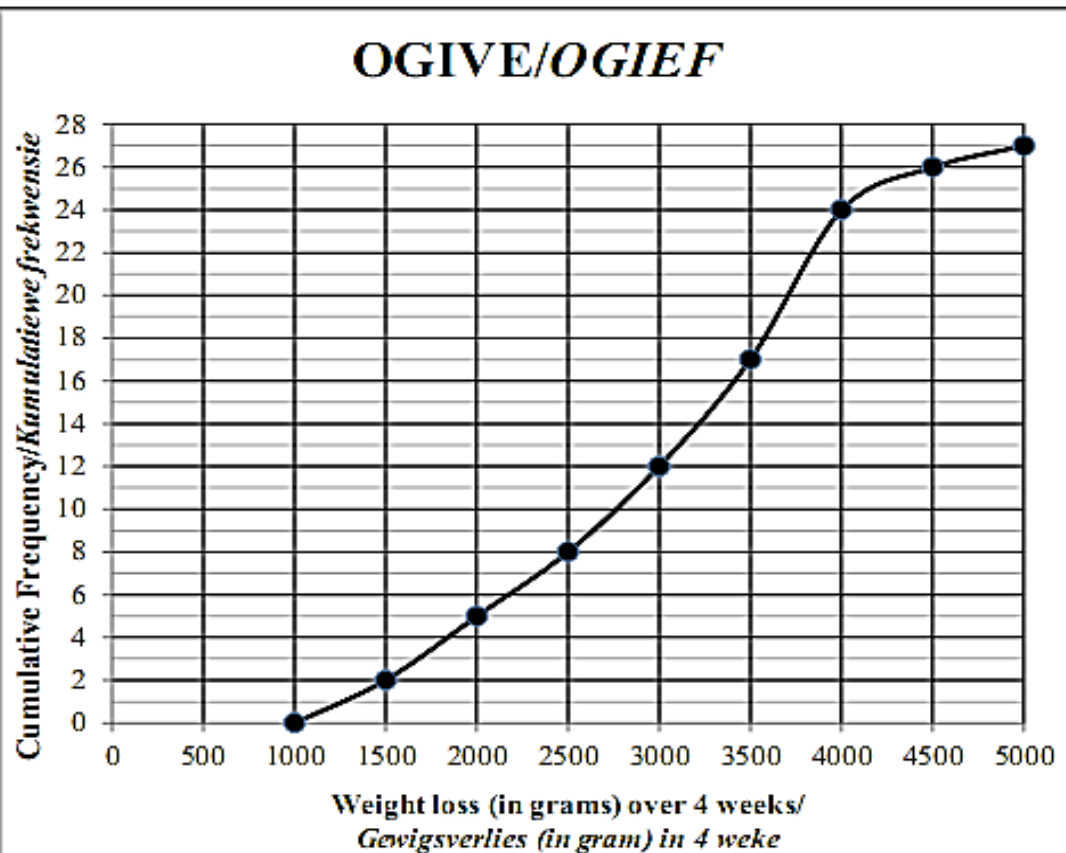| WEIGHT LOSS OVER 4 WEEKS GEWIGSVERLIES IN 4 WEKE (IN GRAMS/GRAM)	FREQUENCY FREKWENSIE f	CUMULATIVE KUMULATIEWE f
$1\,000 < x \leq 1\,500$	2	2
$1\,500 < x \leq 2\,000$	3	5
$2\,000 < x \leq 2\,500$	3	8
$2\,500 < x \leq 3\,000$	4	12
$3\,000 < x \leq 3\,500$	5	17
$3\,500 < x \leq 4\,000$	7	24
$4\,000 < x \leq 4\,500$	2	26
$4\,500 < x \leq 5\,000$	1	27

6.1	$\text{Average/}Gemiddelde = \dfrac{1250 \times 2 + 1750 \times 3 + \ldots + 4750 \times 1}{27}$
	$= \dfrac{81250}{27}$
	$= 3009{,}259 \approx 3009{,}26 \text{ g}$

6.2	

OGIVE/OGIEF

Cumulative Frequency/Kumulatiewe frekwensie (y-axis: 0 to 28)

Weight loss (in grams) over 4 weeks/
Gewigsverlies (in gram) in 4 weke (x-axis: 0 to 5000)

6.3	(3200 ; 14)
	27 − 14 = 13 participants/*deelnemers* (accept/*aanvaar*: 12 − 14)

QUESTION 7

7.1	Mean $= \dfrac{220}{10} = 22$ minutes
7.2	$\sigma = \sqrt{\dfrac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\dfrac{156}{10}} = 3{,}95$
7.3	One standard deviation of the mean is in the interval (22 - 3,95 ; 22 + 3,95) which is (18,05 ; 25,95) ∴ 6 runners completed the race within one standard deviation of the mean. (List of times: 21, 24, 20, 22, 19, 23)

QUESTION 8

8.1	The median, the maximum scores, IQR Note: Any two statements that are valid in the context of the problem apply.
8.2	IQR $= 90 - 72 = 18$.
8.3	No. In the calculation of the median only the value in the middle of an ordered data set is of importance. The extreme values are not taken into account. In this case, 25% of the learners in Class A had a score of less than 66 marks. The minimum mark in Class B is 66 marks. Hence the performance of the two classes differ significantly. OR No. The one is skewed to the left and the other is skewed to the right. The extreme values are not taken into account.

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OR

No.
The lower quartile of Class A is below the minimum of Class B. The extreme values are not taken into account.

OR

No.
The left whisker of Class A is much longer than the left whisker of Class B. The extreme values are not taken into account.

QUESTION 9

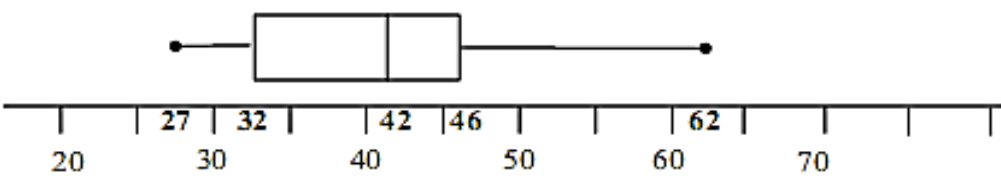| 9.1 | $\bar{x} = 26$ |
	(Calculator used)
9.2	$\sigma = 8,336666...$
	$\sigma = 8,34$
9.3	The times are more closely spread around the mean because 13 out of 20 travelling times from the data set falls within one standard deviation from the mean. So the teacher's observation is acceptable.

QUESTION 10

10.1	575 598 599 667 691 701 701 701
	704 716 724 747 747 764 825 891
	946 996 1 070
	Median = 716
	Lower quartile = 691
	Upper quartile = 825
10.2	**Petrol**
10.3	There are 19 data points. The lower quartile (600) is at position 5 and the upper quartile (800) is at position 15. There are 11 data points from 600 to 800.
	Therefore there are 9 data points strictly between 600 and 800.
	OR
	$19 - (4 + 1 + 4 + 1) = 9$

QUESTION 11

11.1.1	500
11.1.2	About 2 050
11.1.3	$IQR = Q_3 - Q_1$ $\quad\quad\quad = 2\,300 - 1\,625$ $\quad\quad\quad = 675$
11.1.4	$240 - 160 = 80$ light bulbs
11.2	$500 - 420 = 80$ light bulbs Therefore the cost will be: $R5,00 \times 80 = R400$

QUESTION 12

12.1	Median = 42	
12.2	Lower quartile = 32 Upper quartile = 46 Inter quartile range = 46 – 32 = 14	Answer only: FULL MARKS
12.3		
12.4	There is a **greater spread** of scores to the right of the median (42). OR There is a **greater spread** of scores in the top 50%. OR The spread of the scores on the left hand side of the median is closer to each other.	

QUESTION 13

13.1	$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{580}{8} = 72,5$$ **Note:** If rounded off to 73: 1 mark
13.2	Standard deviation $(\sigma) = 2,78$ (2,783882181...) **Note:** If rounded off to 2,8: 1 mark
13.3	$\therefore$ 2 golfers' scores lie outside 1 standard deviation of the mean. The interval for 1 standard deviation of the mean is $(72,5 - 2,78 ; 72,5 + 2,78) = (69,72 ; 75,28)$

QUESTION 14

|	
14.1	Average number of runs $$\bar{x} = \frac{\sum x}{n} = \frac{128}{8} = 16$$
14.2	Standard deviation = 7,55
14.3	Standard deviation = 9,71 Standard deviation increases. **OR** 2 and 35 are far from the mean, namely 16. Since the standard deviation depends on how far data points are from the mean, the standard deviation would be expected to increase.
14.4	Total number of runs required is 20 x 16 = 320 Total number of runs to be scored in last five games = 320 – 59 – 128 = 133 Average number of runs for last five games is $$\frac{133}{5} = 26,6$$

QUESTION 15

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

1	School B, because the standard deviation of B is the largest.
2	There is no difference in the spread of the marks.
3	Add/increase each score in School A by 5 marks.
4	The mean will decrease (by 10%) The standard deviation will also decrease (by 10%)

QUESTION 16

16.1	40	
16.2	**Time, t, in minutes**	**Frequency**
	$0 \leq t < 5$	3
	$5 \leq t < 10$	5
	$10 \leq t < 15$	10
	$15 \leq t < 20$	15
	$20 \leq t < 25$	7

16.3
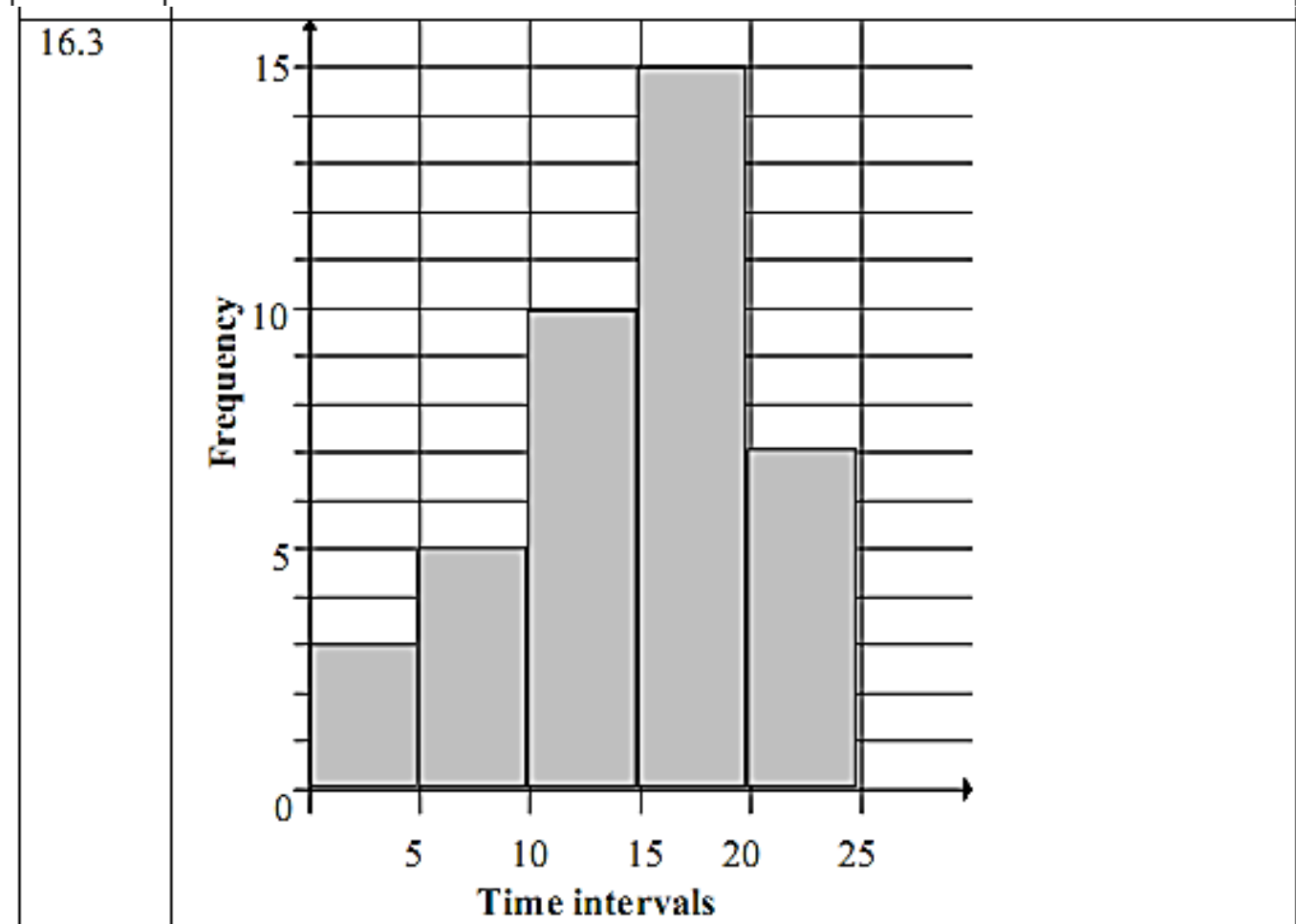
QUESTION 17

$a = 7$	$b = 15$	$c = 17$	$d = 23$	$e = 34$	$f = 37$	$g = 42$

OR

$g = 42$; $a = 7$; $d = 23$; $f = 37$; $b = 15$
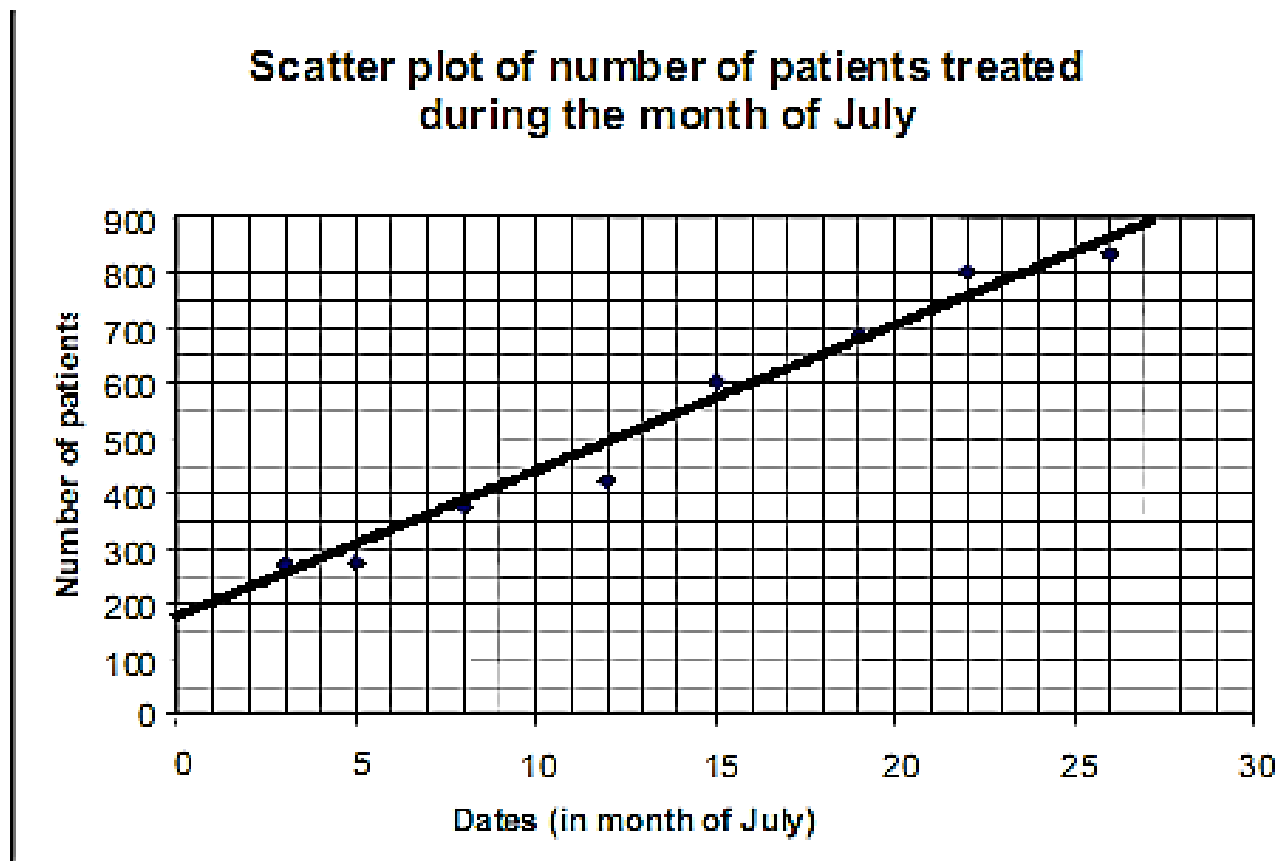
$$\frac{42 + 7 + 23 + 37 + 15 + 3c}{7} = 25$$
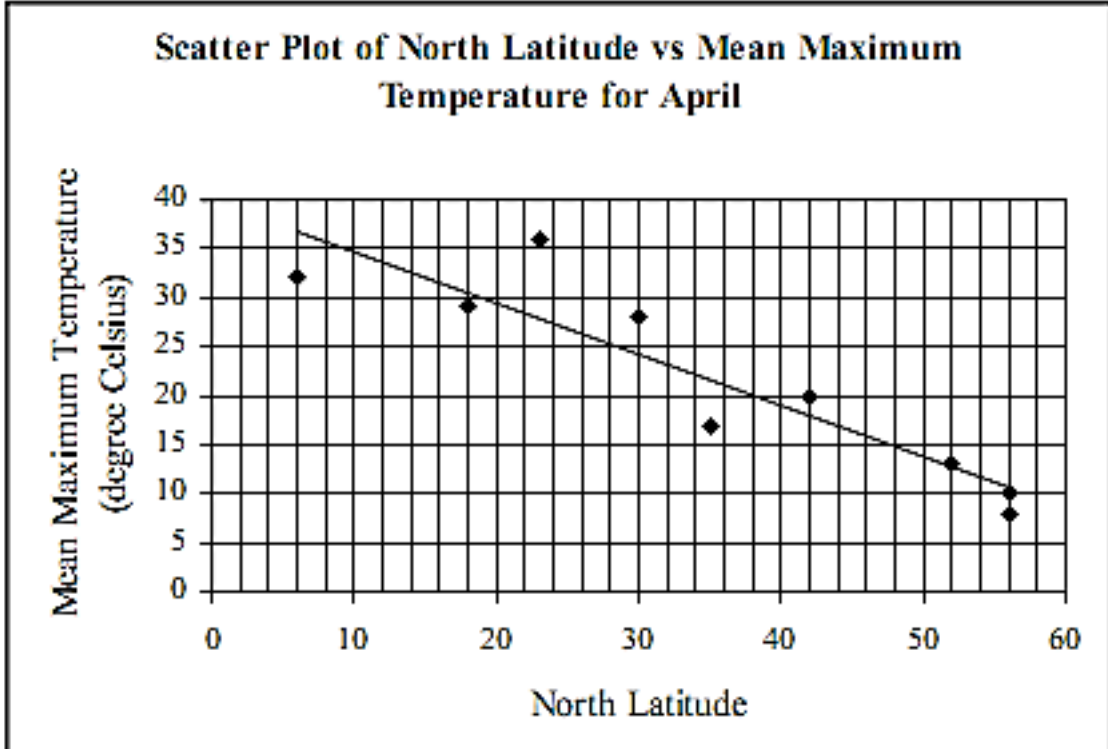
$$3c = 51$$
$$c = 17$$
$$e = 34$$

QUESTION 1

1.1 & 1.3

Scatter plot of number of patients treated during the month of July

1.2   By using a calculator : $a = 161{,}24$ $(161{,}2371188...)$
                                     $b = 26{,}88$ $(26{,}88275499...)$
    $\therefore$ equation of line of least squares is $y = 161{,}24 + 26{,}88x$

1.4   On 30 June, $x = 0$.
       Therefore approximately 161 patients were treated on 30 June.

1.5   On 24 July, $x = 24$.
       $\hat{y} = 161{,}24 + 26{,}88(24) = 806{,}36$
       Approximately 806 patients were treated as at 24 July.

1.6   By using a calculator, $r = 0,98$ $(0,9847864966...)$
There is a very strong positive correlation between the number of days elapsed in July and the number of patients that were treated. This would suggest that there was a rapid spread of the influenza virus in the community.

QUESTION 2

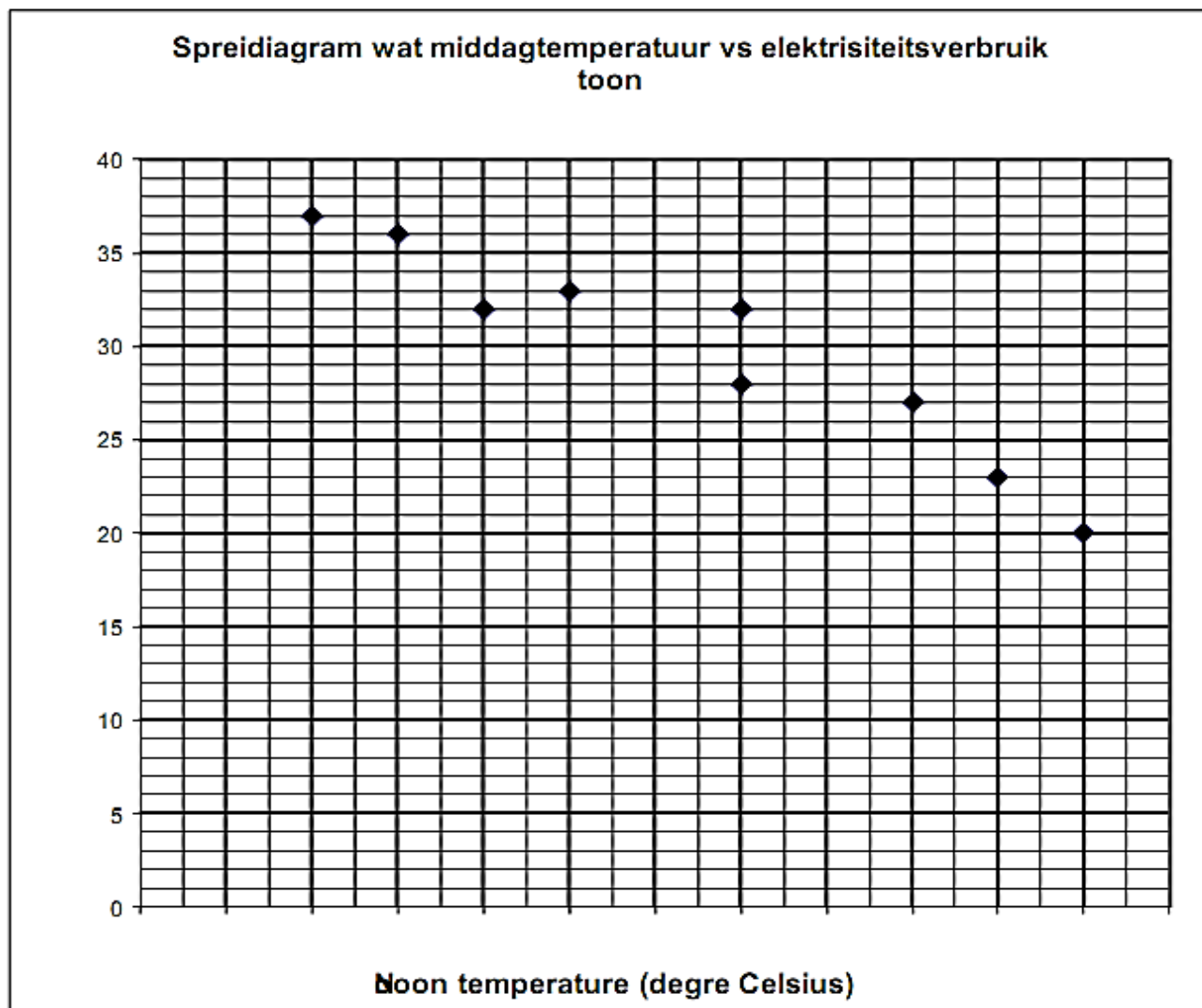|	
2.1 & 2.3	Scatter Plot of North Latitude vs Mean Maximum Temperature for April
2.2	$a = 39,94$ $(39,94369425...)$ $b = -0,52$ $(-0,5235636749...)$ Equation of regression line $\hat{y} = 39,94 - 0,52x$
2.4	The y-intercept represents the mean maximum temperature for April at the equator.

2.5	Mean maximum temperature for April in Madrid $= 39,94 - 0,52(40)$ $= 19,14 \, °C$
2.6	$r = -0,91$ $(-0,9129015212...)$
2.7	The value of r is close to -1 and suggests that there is a very strong relationship between distance from the equator and the mean maximum temperature for April. The further one moves away from the equator, the colder it gets.

QUESTION 3

Noon temperature (in °C)	2	3	4	5	7	7	9	10	11
Units of eletriity used	37	36	32	33	32	28	27	23	20

3.1

Spreidiagram wat middagtemperatuur vs elektrisiteitsverbruik toon

Noon temperature (degre Celsius)

3.2	$a = 40,97 \qquad (40,97108844\ldots)$ $b = -1,74 \qquad (-1,736394558\ldots)$ $\hat{y} = 40,97 - 1,74x$
3.3	$r = -0,97 \qquad (-0,9699269087\ldots)$
3.4	Strong negative
3.5	$\hat{y} \approx 40,97 - 1,74(8)$ $\quad \approx 27,05$

QUESTION 4

4.1	$a = 91,27 \qquad (91,26785714\ldots)$ $b = -4,91 \qquad (-4,910714286\ldots)$ $\hat{y} = 91,27 - 4,91x$
4.2	 SCATTER PLOT SHOWING THE NUMBER OF SATURDAYS ABSENT AND THE FINAL MARK ACHIEVED

4.3	$r = -0,87$ $\qquad(-0,8748915491...)$
4.4	The greater the number of Saturdays absent, the lower the mark.
4.5	$\hat{y} = 91,27 - 4,91(4)$ $\qquad \approx 71,63\%$ $\qquad \approx 72\%$ **NOTE:** Allow for the range 70%–74% for a student who reads off the graph.

November 2015

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1

1.2.2

Scatter plot/*Spreidiagram*

Energy/*Energie* (in kJ)

Fat/*Vet* (in g)
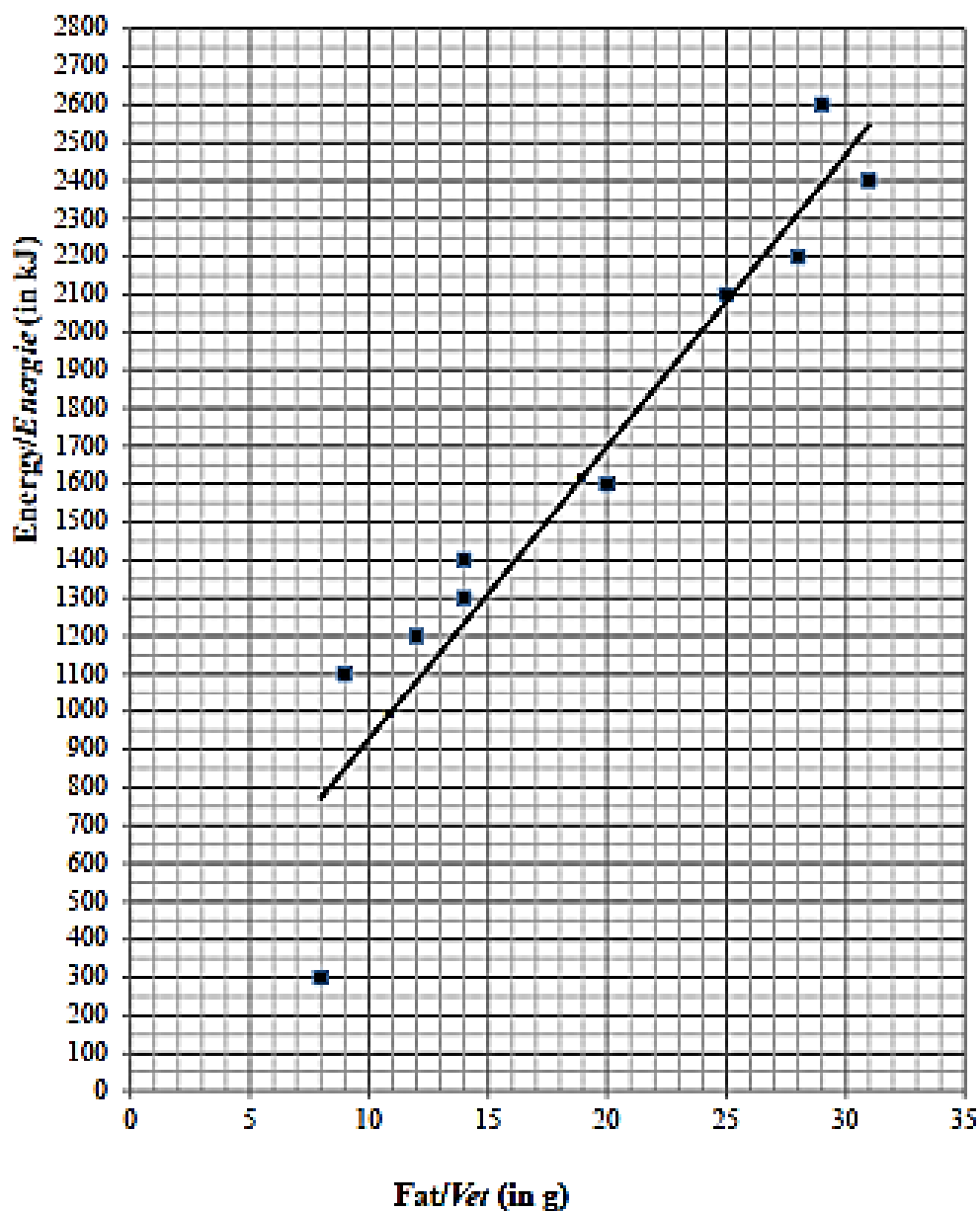
1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $\quad = 1\ 542{,}94 \approx 1\ 500\ kJ$
1.3	$(8\ ;\ 300)$
1.4	$r = 0{,}9520... \approx 0{,}95$
1.5	very strong positive relationship/ *baie sterk positiewe verband*

November 2018

QUESTION 1

1.1.1
140 food items

1.1.2
20 – 30 minutes

1.1.3
20 minutes

1.1.4
140 – 126 = 14 food items

1.1.5
37 minutes

1.1.6
$Q_1 = 21{,}5$
$Q_3 = 37$
$\therefore IQR = 15{,}5\ minutes$

1.2.1

(a)
$$\bar{x} = \frac{1420}{15} = R94{,}67$$

(b)
$$\sigma = R22{,}69$$

1.2.2

(a)
They each collected the same in total over the 15-day period.

(b)
Mary's tips were more consistent or less varied than Reggie's.

QUESTION 2

2.1

251 km/h

2.2.1

0,52 – a weak positive correlation

2.2.2

The least-squares regression line has a positive slope which suggests a direct relationship. However, the data points are not all that closely packed around the least-squares regression line, so the correlation is not as strong as 0,93.

2.3

Loosely speaking, the taller someone is the faster they can serve.

2.4

That would suggest that a player of zero height can still serve at 27,07 km/h. This is a case of extrapolation and makes no sense in this context.

November 2019

QUESTION/*VRAAG* 1

Monthly income (in rands) Maandelikse inkomste (in rand)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) Maandelikse paaiement (in rand)	2 000	3 000	3 500	5 200	5 500	6 000

1.1	$a = -1946,875... = -1946,88$
	$b = 0,41$
	$\hat{y} = -1946,88 + 0,41x$

Answer only: Full marks

1.2	Monthly repayment $\approx$ R3 727,16 (calculator)
	Maandelikse paaiement $\approx$ R3 727,16
	OR
	$\hat{y} = -1946,88 + 0,41(14000)$
	$\approx$ R3 793,12

1.3	$r = 0,946 .... \approx 0,95$

1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR D**

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19
	Aantal mense wat R200 of minder betaal het = 19
2.2	$7 + 12 + a + 35 + b + 6 = 100$
	$a = 40 - b$
	$309 = \dfrac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$
	$309 = \dfrac{(50 \times 7) + (150 \times 12) + (250 \times (40 - b)) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$
	$350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$
	$200b = 3200$
	$b = 16$
	$a = 24$

OR/*OF*

$7 + 12 + a + 35 + b + 6 = 100$

$b = 40 - a$

$$309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$$

$$309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times (40 - a)) + (550 \times 6)}{100}$$

$350 + 1800 + 250a + 12250 + 1800 - 450a = 30900$

$200a = 4\ 800$

$a = 24$

$b = 16$

2.3	Modal class/*modale klas:* $300 < x \leq 400$

QUESTION/*VRAAG* 1

110	112	156	164	167	169
171	176	192	228	278	360

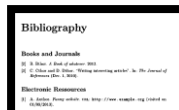| 1.1.1	Mean/*Gemiddelde* $= \dfrac{2283}{12}$ $= 190,25$ Mean profit/*Gemiddelde wins* $=$ R190 250,00 or 190,25 thousand rands	✓ sum/*som* ✓ answer ✓ answer in thousands of rands (3)
1.1.2	Median $= \dfrac{169+171}{2} = 170$ thousand rands $=$ R170 000	✓ answer (1)
1.2		✓ whiskers ✓ quartiles (2)
1.3	IQR $= Q_3 - Q_1$ $= 210 - 160$ thousand rands $=$ R50 000	✓ answer (1)
1.4	Skewed to the right or positively skewed.	✓ answer (1)
1.5.1	$\sigma = 67,04118759$ thousand rands $=$ R67 041,19	✓ answer (1)
1.5.2	$\bar{x} - \sigma = 123,21$ thousand rands For **2** months the profit was less than one standard deviation below the mean.	✓ lower limit ✓ answer (2)
		[11]

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QUESTION/*VRAAG* 2

CHIRPS/*TJIRPGELUIDE* PER MINUTE/ *PER MINUUT*	AIR TEMPERATURE/ *LUGTEMPERATUUR* IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1	SCATTER PLOT/*SPREIDIAGRAM*	3 marks: All points correct 2 marks: 6 – 9 points correct 1 mark: 3 – 5 points correct (3)
2.2	The points lie almost in a straight line. This suggests a very strong positive relationship between the number of chirps per minute and the temperature of the air. *Die punte lê amper in 'n reguitlyn, wat beteken dat daar 'n baie sterk positiewe verband tussen die aantal tjirpgeluide per minuut en die lugtemperatuur is.* **OR/*OF*** r = 0,99 so there is a very strong positive relationship between the number of chirps per minute and the temperature of the air. *r = 0,99, dus is daar 'n baie sterk positiewe verband tussen die aantal kriekgeluide per minuut en die lugtemperatuur.*	✓ justify with straight line / *Motivering mbv reguitlyn* (1) ✓ link with / *gebruik* r = 0,99 *om te motiveer* (1)

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2.3	$a = 3,97$ $b = 0,15$ $\hat{y} = 3,97 + 0,15x$	✓ $a = 3,97$ ✓ $b = 0,15$ ✓ equation (3)
2.4	Air temperature $\approx 15,67°C$ (calculator) **OR** $\hat{y} \approx 3,97 + 0,15(80)$ $\approx 15,97°C$ **OR** Air temperature $\approx 16°C$ (graph: Accept between 15°C and 17°C)	✓✓ answer (2) ✓ substitution ✓ answer (2) ✓✓ answer (2)
		[9]

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QUESTION/*VRAAG* 1

1.1	Modal class: $9 < m \leq 11$	✓ answer (1)

1.2				

Mass (in kg)	Frequency	Cumulative frequency	
$5 < m \leq 7$	6	6	
$7 < m \leq 9$	18	24	✓ adding
$9 < m \leq 11$	21	45	
$11 < m \leq 13$	19	64	
$13 < m \leq 15$	11	75	
$15 < m \leq 17$	4	79	
$17 < m \leq 19$	1	80	✓ 80 (2)

1.3		✓ grounding (5 ; 0) ✓ points ✓ shape (3)

1.4	Median mass: 10,5 kg	✓✓ answer (2)

1.5.1	$\bar{x} = \dfrac{(6 \times 6 + 18 \times 8 + 21 \times 10 + 19 \times 12 + 11 \times 14 + 4 \times 16 + 1 \times 18)}{80}$ $= \dfrac{854}{80}$ $= 10{,}68$ Answer only 2/2	✓ 854 ✓ answer (2)

1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer ✓ 8 kg (2)

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OR/ *OF* Learners' bags are heavier than the stipulated international guideline. Estimated mean $= \dfrac{10,68}{80} \times 100$ $\qquad\qquad\quad = 13,35\%$ $13,35\% > 10\%$	✓ answer ✓ 13,35% (2)
	[12]

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Bibliography

1. **MATHEMATICS NOVEMBER AND MARCH NATIONAL PAPERS GRADE 11 AND 12**