JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ACTIVITY MANUAL

GRADE 12

2023

LAST PUSH
PAPER 1 AND 2

JENN MAREMATLOU TRAINING INSTITUTE

A Division of Marematlou Group Holdings

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a(r^n-1)}{r-1}$; $\quad r\neq 1$ $\quad S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $\quad P=\frac{[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad a^2=b^2+c^2-2bc.\cos A$ $\quad area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad P(A\text{ or }B)=P(A)+P(B)-P(A\text{ and }B)$

$\hat{y}=a+bx$ $\quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

PAPER 1

TOPIC | **PAGE**

PAPER 2

ANNEXURES

PAPER 1

ALGEBRA, EQUATIONS AND INEQUALITIES

PAPER A

QUESTION 1

(a) Solve for x:

(1) $x^2 - 5x = -6$ (3)

(2) $(3x+1)(x-4) < 0$ (3)

(3) $\log_2(x+6) = 1$ (2)

(4) $2x + \sqrt{x+1} = 1$ (6)

(5) $12^{5+3x} = 1$ (2)

(b) Solve for x and y:

$2x - y = 8$ and $x^2 - xy + y^2 = 19$ (7)

(c) The polynomial $x^{10} - 2x^5 + c$ is divisible by $x+1$. Calculate the value of c. (3)

(d) Determine the slope of the tangent to the graph of $y = x^2$ at the point (–1; 1). (2)

PAPER B

QUESTION 1

(a) Solve for x:

(1) $\frac{4x}{2} - \frac{2x+1}{3} = 5$ (2)

(2) $(x-5)(x-6) \leq 56$ (5)

(d) Given the equation $x^2 + c = 0$, where $-2 < c < 5$.

Give two values of c for which the roots of the equation are unequal and rational. (2)

(e) The roots of a quadratic equation are given by $x = \frac{-1 \pm \sqrt{3-k}}{2}$.

Determine the value(s) of k for which the roots will be non-real. (2)

PAPER C

(a) Solve for x:

(1) $(x-1)^2 = 2(1-x)$ (4)

(2) $5^{-x}.5^{x-2} = \frac{25^{2x}}{5}$ (4)

Paper D

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x-3)^2 = 1$ (3)

1.1.2 $2x^2 + 4x - 7 = 0$ (Leave the answer in simplified surd form.) (4)

1.1.3 $x - \sqrt{2x-1} = 2$ (6)

1.2 Consider the equation: $x^2 + 5xy + 6y^2 = 0$

1.2.1 Calculate the values of the ratio $\frac{x}{y}$. (3)

1.2.2 Hence, calculate the values of x and y if $x + y = 8$. (5)

1.3 The solutions of a quadratic equation are given by $x = \frac{-2 \pm \sqrt{2p+5}}{7}$

For which value(s) of p will this equation have:

1.3.1 Two equal solutions (2)

1.3.2 No real solutions (1)

PAPER E

QUESTION 1

1.1 Solve for x:

1.1.4 $(x+1)(4-x) > 0$ (3)

1.2 Given: $2^x + 2^{x+2} = -5y + 20$

1.2.1 Express 2^x in terms of y. (2)

1.2.2 How many solutions for x will the equation have if $y = -4$? (2)

1.2.3 Solve for x if y is the largest possible integer value for which $2^x + 2^{x+2} = -5y + 20$ will have solutions. (3)

PAPER F

QUESTION 1

1.1 Solve for x correct to 2 decimal places where necessary:

1.1.1 $(x-3)(x+4)=18$ (4)

1.1.2 $x^2=6(x+2)$ (5)

1.1.3 $2-\frac{1}{x}=\frac{3}{x+2}$ (6)

1.2 Solve for y if:

$y^2+2y-\frac{8}{y^2+2y}=7$ (8)

1.3 Given: $3x^2-6px-9p^2=0$

Solve for x in terms of p (6)

1.4 Solve for x if $2x^2-5x\geq 7$ (4)

QUESTION 2

2.1 Solve for x and y:

$(2y+3)(x^2+4)=0$ (3)

2.2 Solve simultaneously for a and b:

$2a-b=7$ and

$a^2+ab+b^2=7$ (8)

2.3 The area of a room is 20 m^2. If the length is increased by 3 m and the width is increased by 1 m, the room will double in area. Determine the original dimensions of the room. (5)

PATTERNS, SEQUENCES AND SERIES

PAPER A

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to -6 and a fifth term equal to -14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)

QUESTION 7

The number pattern 1, 5, 11, 19, … is such that the second difference is constant.

7.1 Determine the 5^{th} number in the pattern. (1)

7.2 Derive a formula for the n^{th} number in the pattern. (7)

7.3 What is the 100^{th} number in the pattern? (3)

PAPER B

QUESTION 4

Consider the following quadratic number pattern: $-7 ; 0 ; 9 ; 20 ; \ldots$

4.1 Show that the general term of the quadratic number pattern is given by $T_n = n^2 + 4n - 12$. (4)

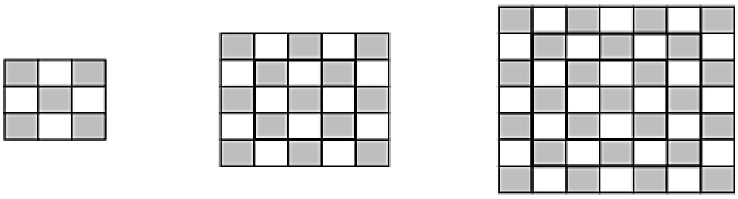
4.2 Which term of the quadratic pattern is equal to 128? (4)

4.3 Determine the general term of the first differences. (3)

4.4 Between which TWO terms of the quadratic pattern will the first difference be 599? (3)

QUESTION 5

Grey and white squares are arranged into patterns as indicated below.



Pattern 1 Pattern 2 Pattern 3

	Pattern 1	Pattern 2	Pattern 3
Number of grey squares	5	13	25

The number of grey squares in the n^{th} pattern is given by $T_n = 2n^2 + 2n + 1$.

5.1 How many white squares will be in the FOURTH pattern? (2)

5.2 Determine the number of white squares in the 157^{th} pattern. (3)

5.3 Calculate the largest value of n for which the pattern will have less than 613 grey squares. (4)

5.4 Show that the TOTAL number of squares in the n^{th} pattern is always an odd number. (3)

PAPER C

QUESTION 3

The first three terms of an arithmetic sequence are $2p - 3$; $p + 5$; $2p + 7$.

3.1 Determine the value(s) of p. (3)

3.2 Calculate the sum of the first 120 terms. (3)

3.3 The following pattern is true for the arithmetic sequence above:

$T_1 + T_4 = T_2 + T_3$

$T_5 + T_8 = T_6 + T_7$

$T_9 + T_{12} = T_{10} + T_{11}$

$\therefore T_k + T_{k+3} = T_x + T_y$

3.3.1 Write down the values of x and y in terms of k. (2)

3.3.2 Hence, calculate the value of $T_x + T_y$ in terms of k in simplest form. (4)

QUESTION 4

4.1 Given: $\sum_{k=1}^{\infty} 5(3^{2-k})$

4.1.1 Write down the value of the first TWO terms of the infinite geometric series. (2)

4.1.2 Calculate the sum to infinity of the series. (2)

4.2 Consider the following geometric sequence:

$\sin 30°; \cos 30°; \frac{3}{2}; \ldots; \frac{81\sqrt{3}}{2}$

Determine the number of terms in the sequence. (5)

PAPER D

QUESTION 3

3.1 The following geometric series is given:

$2(3x-1)+2(3x-1)^2+2(3x-1)^3 \ \ldots$

Determine the value(s) of x for which the series converges. (3)

3.2 The first two terms of a convergent geometric series are k and 6 respectively where $k \neq 0$.

The sum of the infinite series is 25.

Calculate the value(s) of k. (5)

3.3 Given the series:

$(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+\ldots+(81\times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series). (4)

PAPER E

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \ldots$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n = \frac{n}{2}(2a + (n-1)d)$. (4)

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \ldots$

3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

PAPER F

QUESTION 2

A cyclist, preparing for an ultra cycling race, cycled 20 km on the first day of training. She increases her distance by 4 km every day.

2.1 On which day does she cycle 100 km? (3)

2.2 Determine the total distance she would have cycled from day 1 to day 14. (3)

2.3 Would she be able to keep up this daily rate of increase in distance covered indefinitely? Substantiate your answer. (2)

QUESTION 3

Consider the following sequence: 5; 12; 21; 32; ...

3.1 Write down the next term of the sequence. (1)

3.2 Determine a formula for the n^{th} term of this sequence. (5)

QUESTION 4

4.1 Prove that $a + ar + ar^2 + \ldots \text{ (to } n \text{ terms)} = \dfrac{a(1-r^n)}{1-r}$ for $r \neq 1$ (6)

4.2 Given the geometric series $15 + 5 + \dfrac{5}{3} + \ldots$

4.2.1 Explain why the series converges. (2)

4.2.2 Evaluate $\sum_{n=1}^{\infty} 5(3^{2-n})$ (3)

4.3 The sum of the first n terms of a sequence is given by $S_n = 2^{n+2} - 4$

4.3.1 Determine the sum of the first 24 terms. (1)

4.3.2 Determine the 24^{th} term. (2)

4.3.3 Prove that the n^{th} term of the sequence is 2^{n+1}. (3)

FUNCTIONS AND GRAPHS

PAPER A

QUESTION 5

Given: $f(x) = -2x^2 + x + 6$

5.1 Calculate the coordinates of the turning point of f. (4)

5.2 Determine the y-intercept of f. (1)

5.3 Determine the x-intercepts of f. (4)

5.4 Sketch the graph of f showing clearly all intercepts with the axes and turning point. (3)

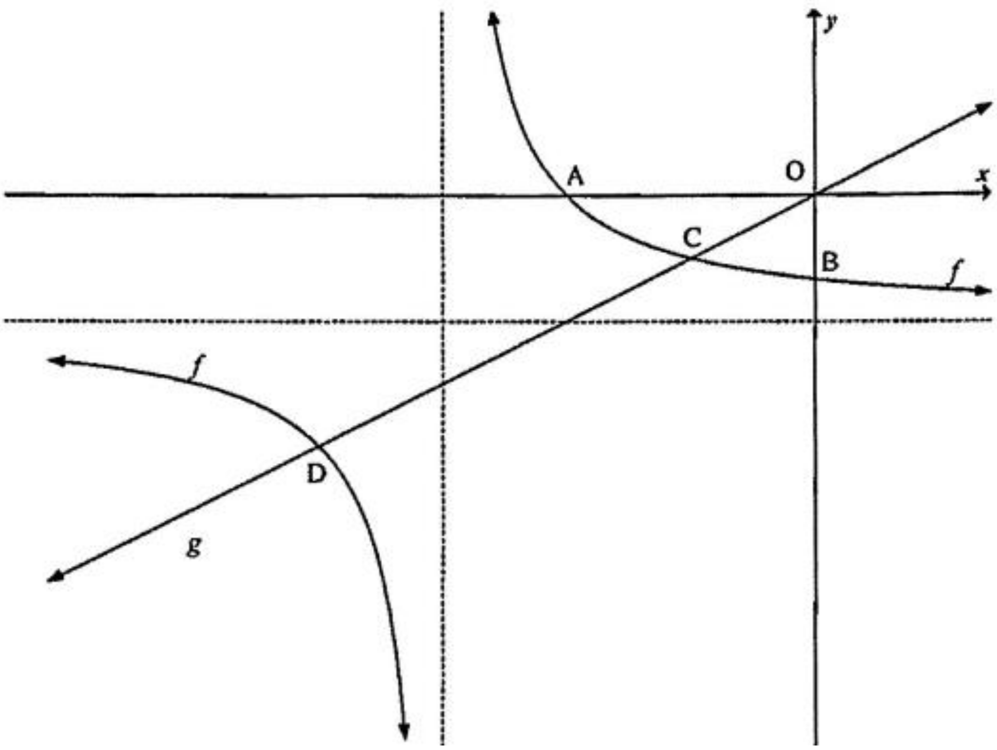
5.5 Determine the values of k such that $f(x) = k$ has equal roots. (2)

5.6 If the graph of f is shifted two units to the right and one unit upwards to form h, determine the equation h in the form $y = a(x+p)^2 + q$. (3)

QUESTION 6

The diagram below shows the graph of $f(x) = \dfrac{1}{x+3} - 1$ and $g(x) = \dfrac{1}{2}x$.

The graph of f intersects the x-axis at A and the y-axis at B.
The graph of f and g intersect at points C and D.



6.1 Write down the equations of the asymptotes of f. (2)

6.2 Determine the domain of f. (2)

6.3 Calculate the length of:

6.3.1 OB (2)

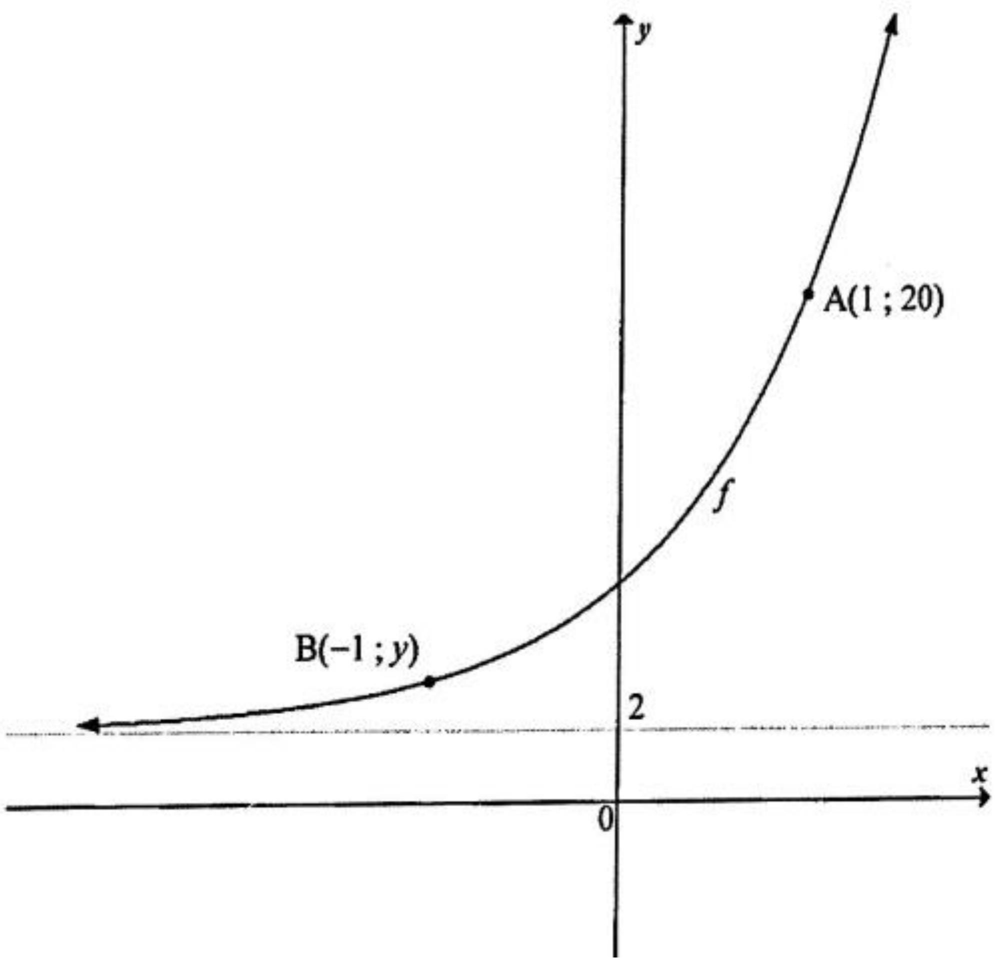
6.3.2 OA (3)

6.4 Determine the coordinates of C and D. (6)

6.5 Use the graphs to obtain the solution to: $\frac{1}{x+3} \geq \frac{x+2}{2}$ (4)

QUESTION 7

The sketch below is the graph of $f(x) = 2.b^{x+1} + q$.
The graph of f passes through the points A(1 ; 20) and B (–1 ; y).
The line $y = 2$ is an asymptote of f.



7.1 Show that the equation of f is $f(x)=2(3)^{x+1}+2$ (3)

7.2 Calculate the y-coordinate of the point B. (1)

7.3 Determine the average gradient of the curve between the points A and B. (2)

7.4 A new function h is obtained when f is reflected about its asymptote. Determine the equation of h. (2)

7.5 Write down the range of h. (1)

PAPER B

QUESTION 5

Given $f(x)=\dfrac{-4}{2-x}-1$

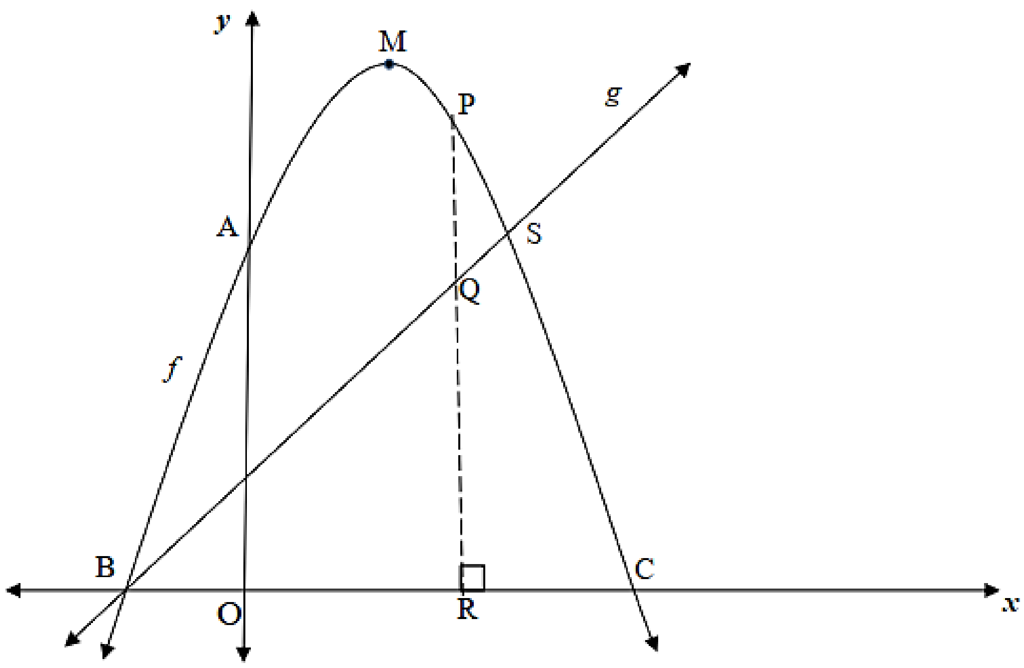
5.1 Write down the equations of the vertical and horizontal asymptotes of f. (2)

5.2 Determine the intercepts of the graph of f with the axes. (3)

5.3 Draw the graph of f. Show all intercepts with the axes as well as the asymptotes of the graph. (4)

QUESTION 6

In the diagram, the graphs of $f(x)=-x^2+5x+6$ and $g(x)=x+1$ are drawn below.
The graph of f intersects the $x-$axis at B and C and the $y-$axis at A.
The graph of g intersects the graph of f at B and S. PQR is perpendicular to the $x-$axis with points P and Q on f and g respectively. M is the turning point of f.



6.1 Write down the co-ordinates of A. (1)

6.2 S is the reflection of A about the axis of symmetry of *f*. Calculate the coordinates of S. (2)

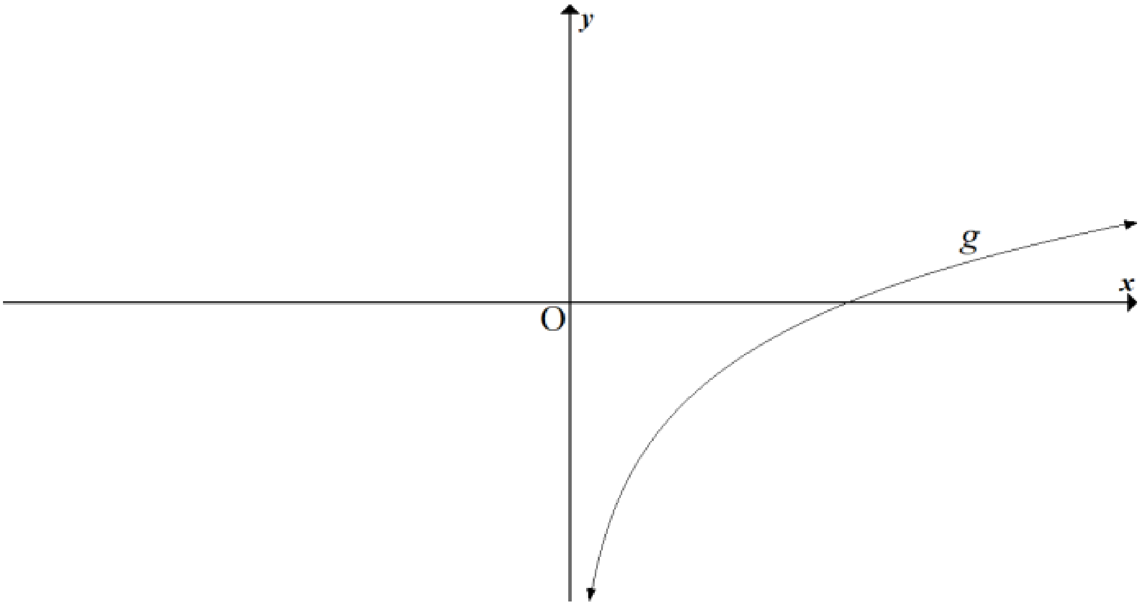
6.3 Calculate the coordinates of B and C. (3)

6.4 If PQ = 5 units, calculate the length of OR. (4)

6.5 Calculate the:
6.5.1 Coordinates of M. (4)
6.5.2 Maximum length of PQ between B and S. (4)

QUESTION 7

In the diagram, the graph of $g(x) = \log_5 x$ is drawn.



7.1 Write down the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

7.2 Write down the range of g^{-1}. (1)

7.3 Calculate the value(s) of x for which $g(x) \leq -4$. (4)

PAPER C

5.2 Given: $h(x) = 4\left(2^{-x}\right) + 1$

5.2.1 Determine the coordinates of the y-intercept of h. (2)

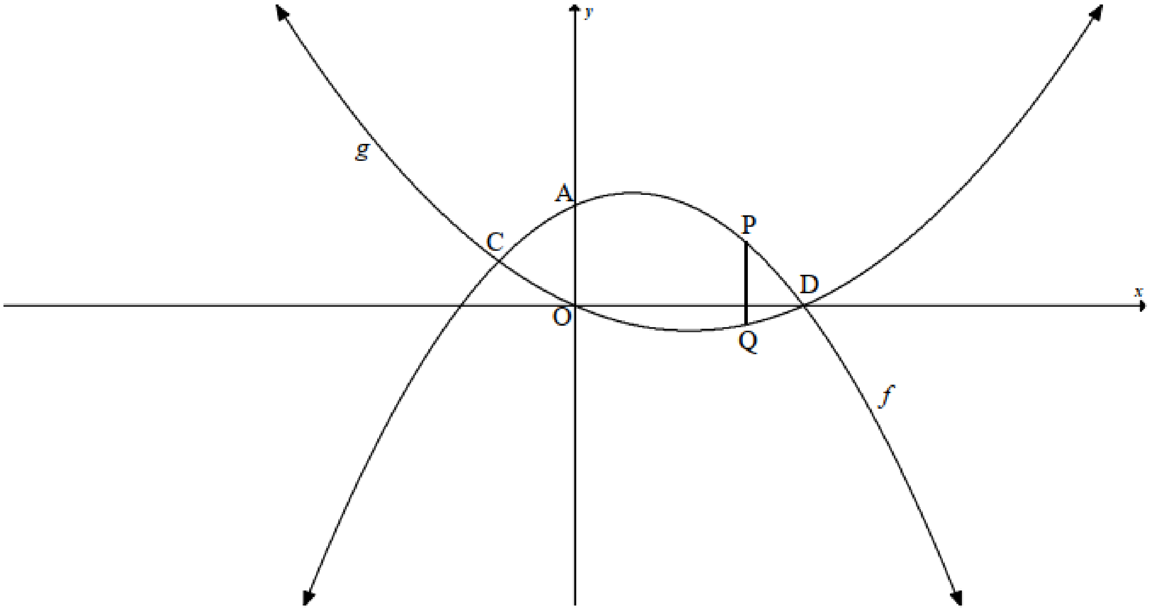
5.2.2 Explain why h does not have an x-intercept. (2)

5.2.3 Draw a sketch graph of h, clearly showing all asymptotes, intercepts with the axes and at least one other point on h. (3)

5.2.4 Describe the transformation from h to g if $g(x) = 4\left(2^{-x} + 2\right)$. (2)

QUESTION 6

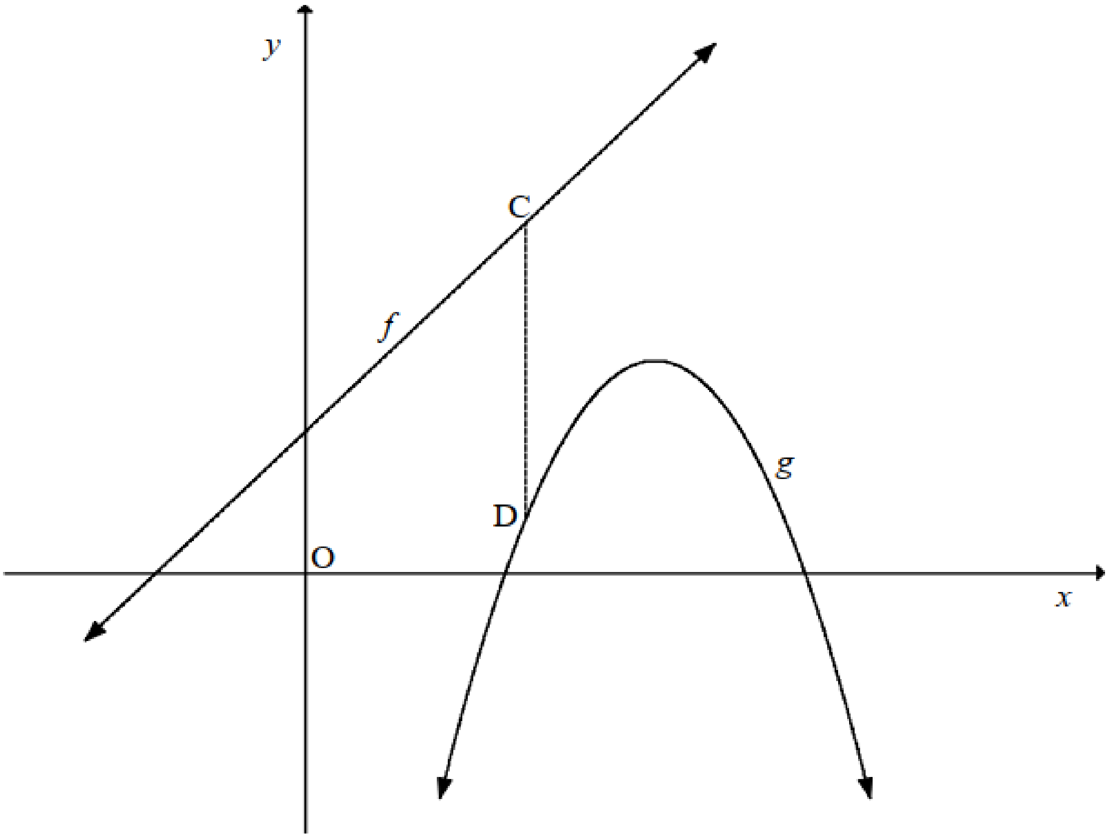
In the diagram, the graphs of $f(x) = -x^2 + x + 2$ and $g(x) = \frac{1}{2}x^2 - x$ are drawn below. f and g intersect at C and D. A is the y – intercept of f. P and Q are any points on f and g respectively. PQ is parallel to the y – axis.



6.1	Write down the co-ordinates of A.	(1)
6.2	Calculate the coordinates of C and D.	(5)
6.3	Determine the values of x for which $f(x) \leq g(x)$.	(2)
6.4	Calculate the maximum length of PQ where line PQ is between C and D.	(4)
6.5	Calculate the value of x where the gradient of f is equal to 3.	(3)
6.6	Determine the values of k for which $f(x) = k$ has two positive unequal roots.	(4)

QUESTION 7

The sketch below shows the graphs of $f(x) = 2x + 3$ and $g(x) = -2x^2 + 14x + k$.
C is any point on f and D any point on g, such that CD is parallel to the y-axis.
k is a value such that C lies above D.



7.1 Write down a simplified expression for the length of CD in terms of x and k. (3)

7.2 If the minimum length of CD is 5, calculate the value of k. (4)

PAPER D

QUESTION 5

5.1 Consider the function $f(x) = \left(\frac{5}{6}\right)^x$

5.1.1 Write down the equation of h, the reflection of f in the y-axis. (1)

5.1.2 Write down the equation of $f^{-1}(x)$ in the form $y = \ldots$ (2)

5.1.3 For which value(s) of x will $f^{-1}(x) \geq 0$? (2)

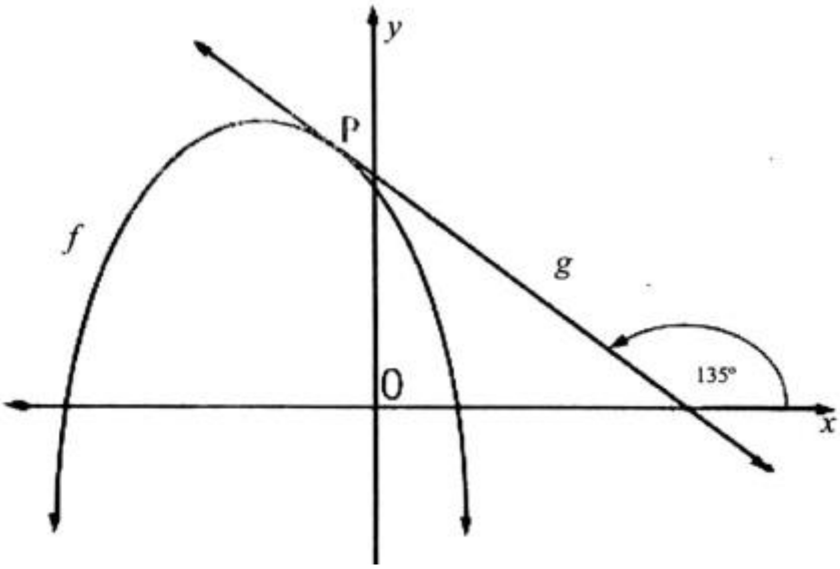
5.2 The function defined as $f(x) = ax^2 + bx + c$ has the following properties:

- $f'(-2,5) = 0$
- $f(1) = 0$
- $b^2 - 4ac > 0$
- $f(-2,5) = 6$

Draw a neat sketch graph of f. Clearly show all x-intercepts and turning point. (4)

QUESTION 6

The graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$ are sketched below.
The angle of inclination of g is 135°. Graph g is a tangent to f at point P.



6.1 Calculate the coordinates of the turning point of f. (3)

6.2 Write down the range of f. (1)

6.3 Calculate the coordinates of point P, the point of contact of f and g. (4)

6.4 Determine the value(s) of k for which the straight line $y = k$ is NOT a tangent to $y = 2x^2 + 5x - 3$. (2)

QUESTION 7

Given $f(x) = a^x$, where $a > 0$, passing through the point $(2\ ;\ \frac{1}{4})$ and $g(x) = 4x^2$.

7.1 Prove that $a = \frac{1}{2}$. (2)

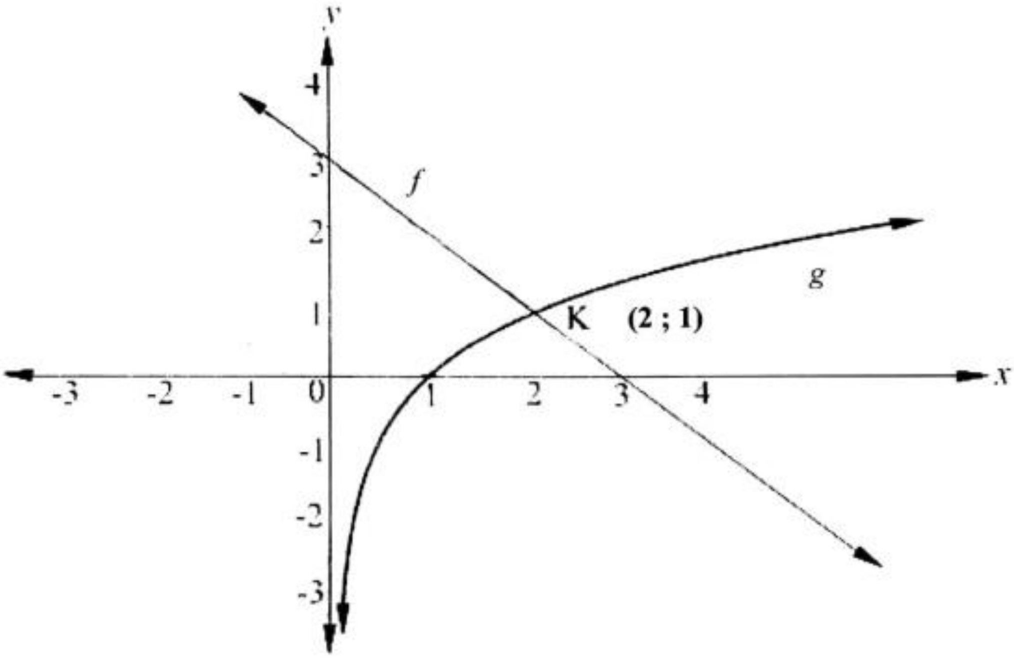
7.2 Determine the equation of $y = f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Determine the equation of $y = h(x)$ where h is the reflection of f in the x-axis. (1)

7.4 How must the domain of $g(x)$ be restricted so that $g^{-1}(x)$ will be a function? (2)

QUESTION 8

The graphs of $f(x) = -x + 3$ and $g(x) = \log_2 x$ are drawn below.
Graphs f and g intersect at point K(2 ; 1).



8.1 Write down value(s) of x for which:

8.1.1 $f(x) - g(x) > 0$ (2)

8.1.2 $g(x) \cdot g^{-1}(x) \leq 0$ (2)

8.2 8.2.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

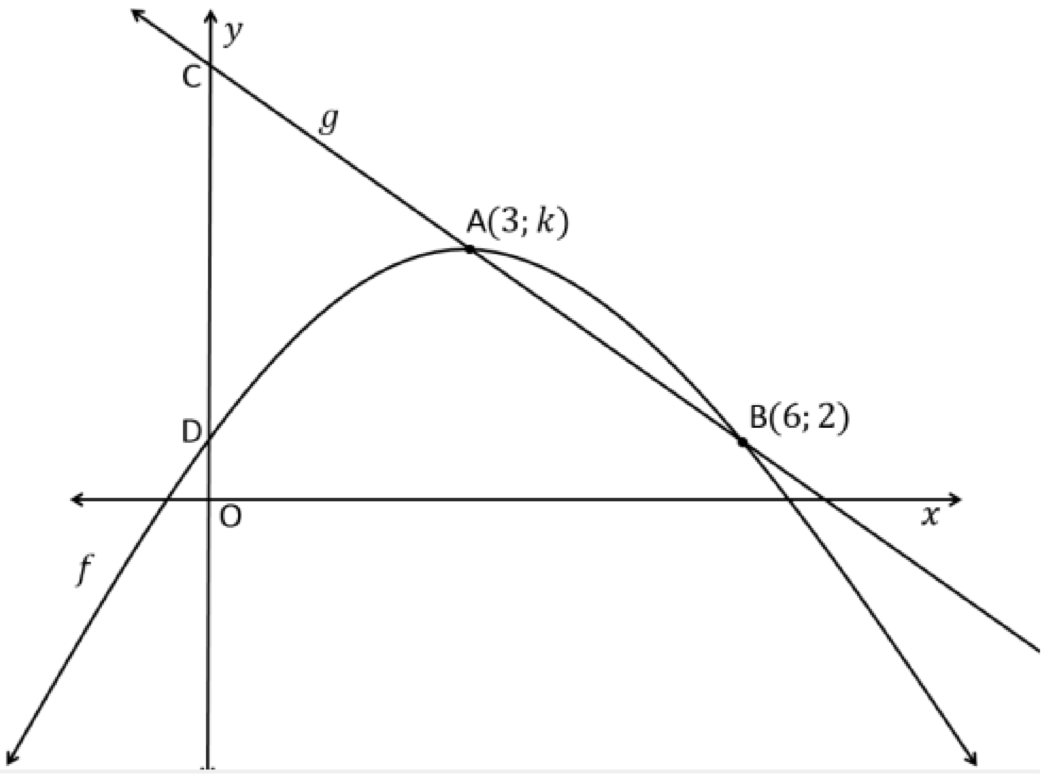
8.2.2 Explain how you could use the given sketch to solve the equation $\log_2(3 - x) = x$. (2)

8.2.3 Write down the solution to $\log_2(3 - x) = x$. (1)

PAPER E

QUESTION 5

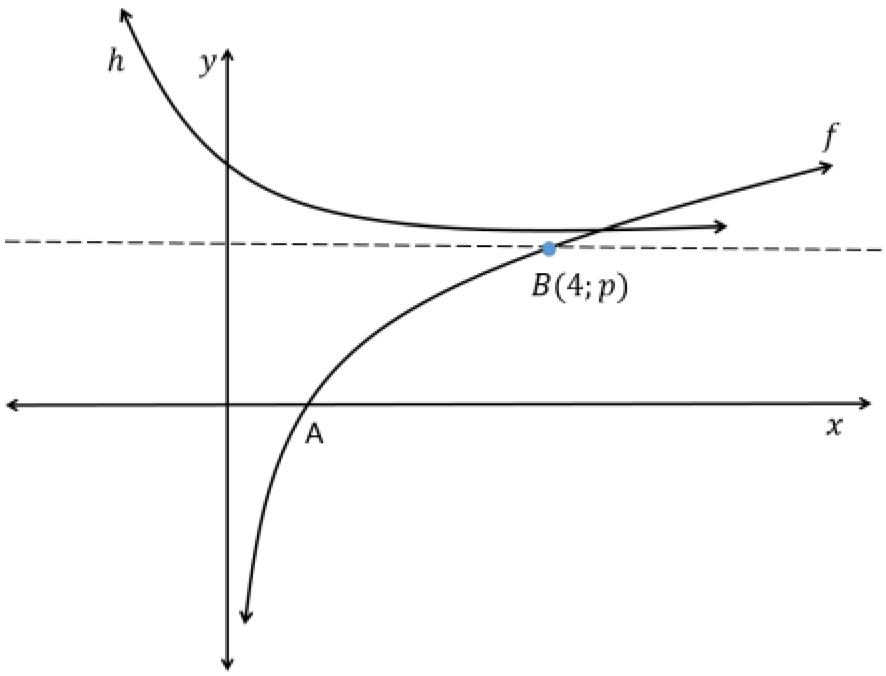
Sketched below are the graphs of $g(x) = -3x + 20$ and $f(x) = ax^2 + bx + c$.
Graph f has a turning point at A$(3; k)$. Graph f and g intersects at A and B$(6; 2)$.



5.1	Calculate the numerical value of k, the y-coordinate of A.	(2)
5.2	Determine the range of $y = -f(x)$.	(2)
5.3	Calculate the numerical values of a, b and c.	(6)
5.4	Determine the value(s) of x for which $f(x) > g(x)$.	(2)
5.5	Describe the nature of the roots for $f(x) - 11$.	(2)
5.6	Determine the value(s) of x for which $f'(x) \,.\, g'(x) > 0$.	(2)

QUESTION 6

Sketched below are the graphs of $h(x) = \left(\frac{1}{2}\right)^x + q$ and $f(x) = \log_2 x$.
Graph f and the asymptote of h intersect at $B(4; p)$.



6.1 Write down the coordinates of A, the x –intercept of f. (1)

6.2 Determine the domain of f. (1)

6.3 Determine the equation of f^{-1} in the form $y = \ldots$ (2)

6.4 Sketch the graph of f^{-1}. Clearly labelling the intercept(s) with the axes as well as the coordinates of any one other point on the graph. (3)

6.5 Determine the equation of the asymptote of h. (2)

6.6 Describe, in words, the transformation of h to f^{-1}. (2)

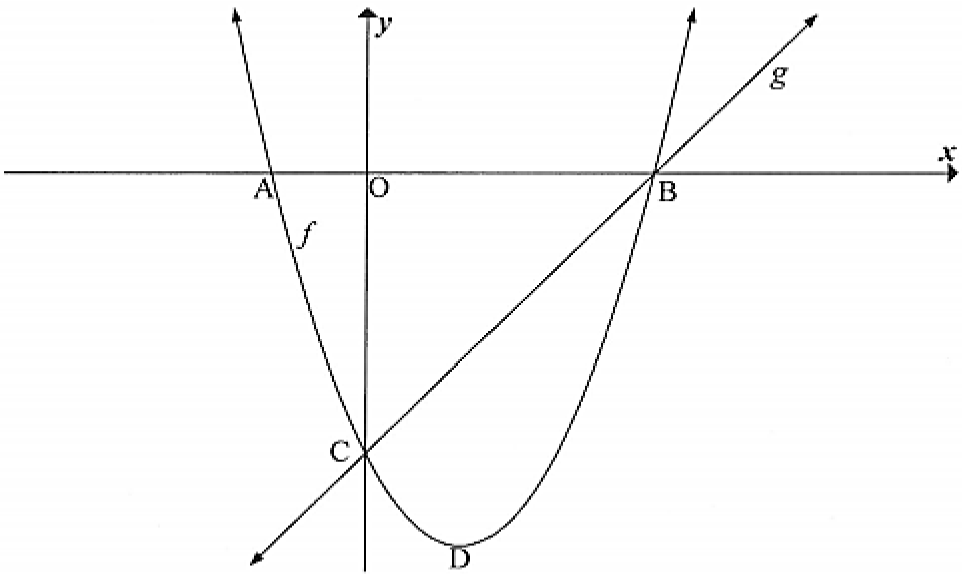
PAPER F

QUESTION 5

5.1 The sketch below shows the graphs of $f(x)=x^2-2x-3$ and $g(x)=x-3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1	Determine the coordinates of C.	(1)
5.1.2	Calculate the length of AB.	(4)
5.1.3	Determine the coordinates of D.	(2)
5.1.4	Calculate the average gradient of f between C and D.	(2)
5.1.5	Calculate the size of $O\hat{C}B$	(2)
5.1.6	Determine the values of k for which $f(x)=k$ will have two unequal positive real roots.	(3)
5.1.7	For which values of x will $f'(x)\,.\,f''(x)>0$?	(3)

5.2 The graph of a parabola f has x-intercepts at $x=1$ and $x=5$. $g(x)=4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

QUESTION 6

Given: $f(x)=\frac{1}{4}x^2,\ x\leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x)=\ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)

FINANCE, GROWTH AND DECAY

PAPER A

QUESTION 7

7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)

7.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

7.2.2
- One month after purchasing his present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a., compounded monthly.
- He continued to deposit the same amount at the end of each month for a total of 60 months.
- At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.

Calculate the value of x. (6)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be? (4)

PAPER B

QUESTION 7

7.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

7.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)

7.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012). (6)

PAPER C

QUESTION 8

8.1 A new cellphone was purchased for R7 200. Determine the depreciation value after 3 years if the cellphone depreciates at 25% p.a. on the reducing-balance method. (3)

8.2 Jill negotiates a loan of R300 000 with a bank which has to be repaid by means of monthly payments of R5 000 and a final payment which is less than R5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% per annum, compounded monthly.

8.2.1 Determine the number of payments required to settle the loan. (6)

8.2.2 Calculate the balance outstanding after Jill has paid the last R5 000. (5)

8.2.3 Calculate the value of the final payment made by Jill to settle the loan. (2)

8.2.4 Calculate the total amount that Jill repaid to the bank. (1)

PAPER D

QUESTION 8

8.1 A car depreciated at the rate of 13,5 % p.a. to R250 000 over 5 years according to the reducing balance method. Determine the original price of the car, to the nearest rand. (3)

8.2 Melissa takes a loan of R950 000 to buy a house. The interest is 14,25 % p.a. compounded monthly. His first instalment will commence one month after taking out the loan.

8.2.1 Calculate the monthly repayments over a period of 20 years. (4)

8.2.2 Determine the balance on the loan after the 100th instalment. (4)

8.2.3 If Melissa failed to pay the 101st, 102nd, 103rd and 104th instalments, calculate the value of the new instalment that will settle the loan in the same time period. (4)

PAPER E

QUESTION 8

8.1 An investor indicates that he will be able to treble the value of the investment at the end of 6 years. The interest rate is fixed and compounded monthly. Calculate the annual interest rate that the investor has on offer. (4)

8.2 Samuel decided to buy a car costing R192 000. He takes out a loan for 5 years at an interest rate charged at 12 % p.a. compounded monthly. Payments are made at the end of each month.

8.2.1 Calculate the monthly repayments over a period of 5 years. (4)

8.2.2 After Samuel had made 45 payments, he decides to settle the balance on the loan. Calculate the lump sum that he will need to pay off the loan after he has made the 45^{th} payment. (4)

PAPER F

QUESTION 4

4.1 How many years will it take for an investment of R3 000 to accumulate to R4 500, if it is invested at 8% p.a. compounded monthly? (4)

4.2 Bongani paid off a 20-year loan of R40 000. During the period of the loan the interest rate changed from 24% p.a. compounded monthly for the first five years to 18% p.a. compounded monthly for the remaining years.

4.2.1 Calculate the initial monthly payment before the interest rate changed. (4)

4.2.2 What is the outstanding balance of the loan after the FIRST five years? (4)

4.2.3 Determine the monthly payment after the interest rate changed. (4)

QUESTION 7

7.1 Sarah's investment earns interest at 11% p.a. compounded semi-annually.
Mary's investment earns an effective interest of 11,42% *p.a.*
Whose investment, Sarah's or Mary's, earns a higher rate of interest per annum. (3)

7.2 Buhle decided to start saving before retirement. She makes payments of R10 000 monthly into an account yielding 7,72% *p.a.* compounded monthly, starting on 1 November 2016 with a final payment on 1 April 2026.

7.2.1 Calculate how much will be in the savings account immediately after the last deposit is made. (4)

7.2.2 At the end of the investment period Buhle re-invested the full amount in order for her to be able to draw a monthly pension from the fund.

She re-invested the money at an interest rate of 10% p.a. compounded monthly. If she draws an amount of R30 000 per month from this investment, for how many full months will she be able to receive R30 000? (4)

7.2.3 After withdrawing R30 000 for 20 months Buhle requires R1 500 000. Determine whether she can access this amount of money from this annuity. (4)

DIFFERENTIAL CALCULUS

PAPER A

QUESTION 9

9.1 Determine $f'(x)$ from first principles given $f(x)=x^2-\frac{1}{2}x$. (5)

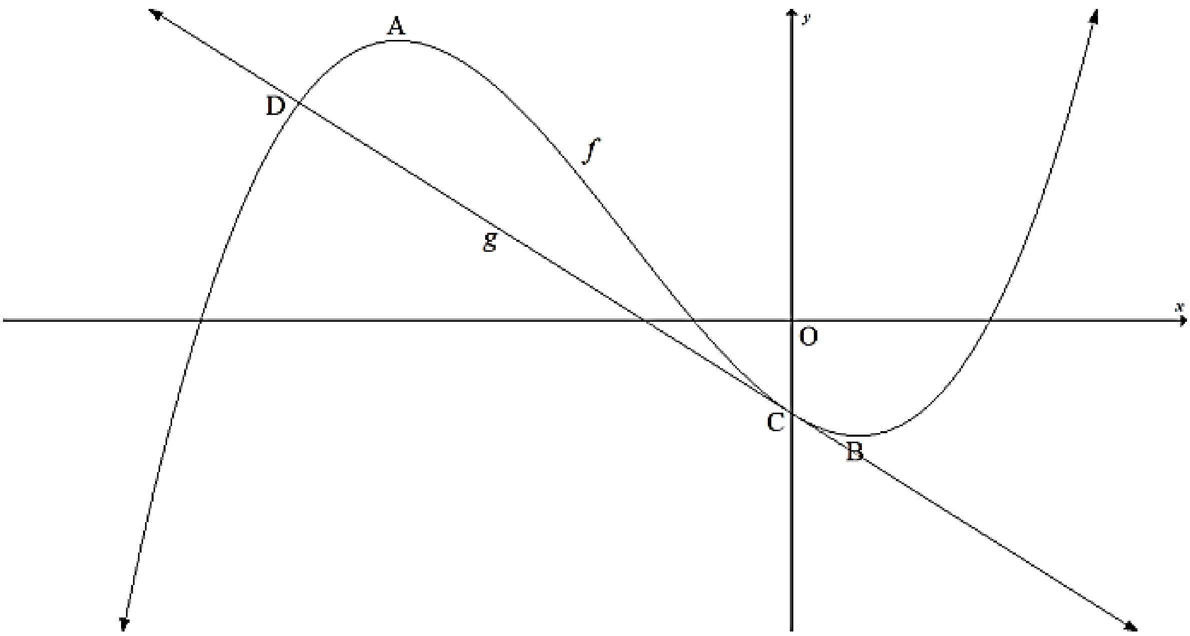
9.2 Determine:

9.2.1 $\frac{d}{dx}[3x^4+\sqrt[5]{x}+a^2]$ (3)

9.2.2 $\frac{dy}{dx}$, if $xy=x+x^2-1$. (4)

QUESTION 10

In the diagram, the graph of $f(x)=x^3+5x^2-8x-12$ is drawn. A and B are the turning points and C the y - intercept of f. $g(x)=mx+c$ is a tangent to the graph of f at C. D is the intersection of f and g.



10.1 Calculate the:

10.1.1 co-ordinates of the x-intercepts of f. (6)
10.1.2 co-ordinates of B. (4)
10.1.3 x – coordinate of the point of inflection of f. (2)

10.2 Determine the:

10.2.1 equation of the g. (2)
10.2.2 values of x for which $f'(x).\ g'(x) > 0$. (3)

PAPER B

QUESTION 9

9.1 Determine $f'(x)$ from first principles given $f(x) = x^2 - bx$, where b is a constant. (5)

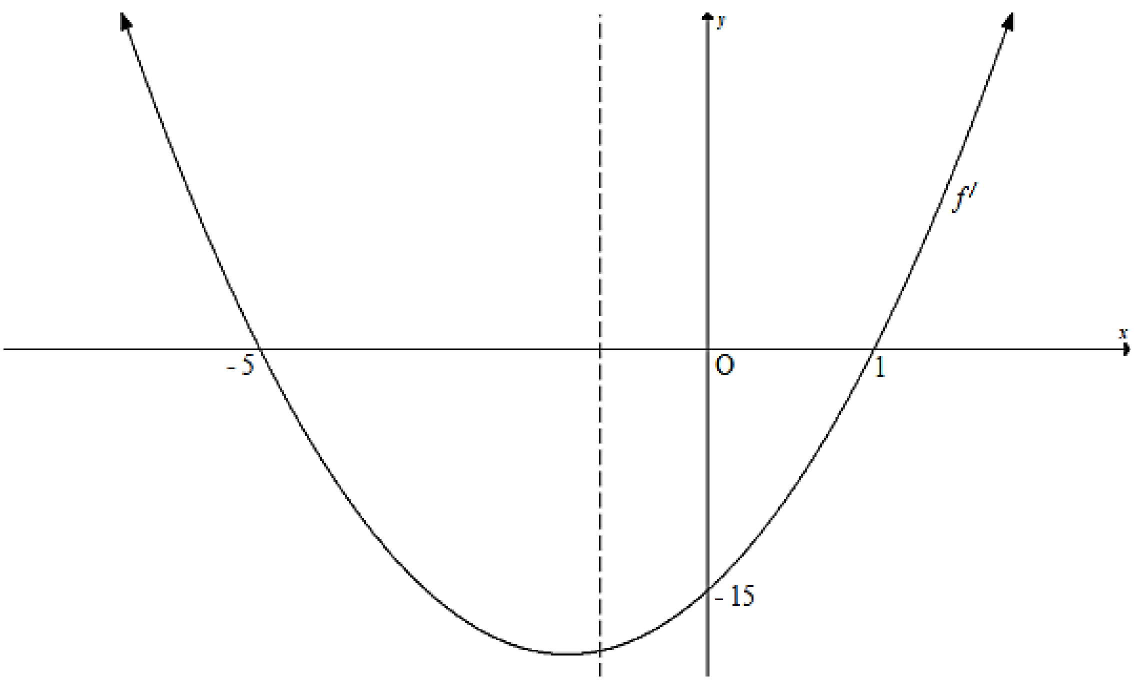
9.2 Determine:

9.2.1 $\frac{d}{dx}\left[\frac{x^4}{4} - 3.\sqrt[3]{x} + 7\right]$ (3)

9.2.2 $\frac{dy}{dx}$ if $y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ (4)

QUESTION 10

The graph of f', the derivative of f, is drawn below. $f(x) = ax^3 + bx^2 + cx + d\ ; a \neq 0$.
f' intersects the x – axis at -5 and 1 and the y – axis at -15.



10.1 Write down the:

10.1.1 x – values of the turning points of f. (2)

10.1.2 x – value(s) where the gradient of f is equal to -15. (2)

10.2 Show that the equation of $f^{/}$ is given by $y = 3x^2 + 12x - 15$. (3)

10.3 If $f(-3) = 0$, calculate the value of d. (4)

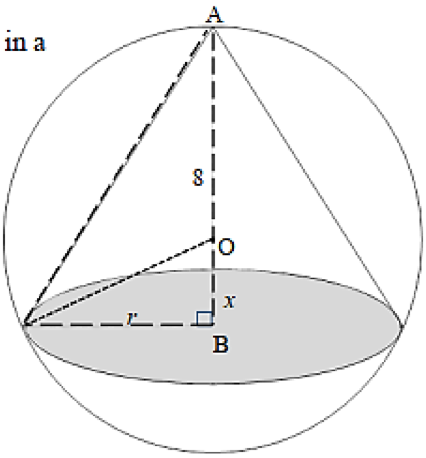
10.4 Determine the coordinates of the turning points of the graph of f and state whether they are maximum or minimum turning points. (4)

10.5 $y = tx + 4$ is a tangent to f. Calculate the value of t. (5)

PAPER C

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB $= x$.



Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$. (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

QUESTION 10

A cubic function has the following essential properties:

- $f(0) = 8$
- $f(4) = f(1) = 0$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

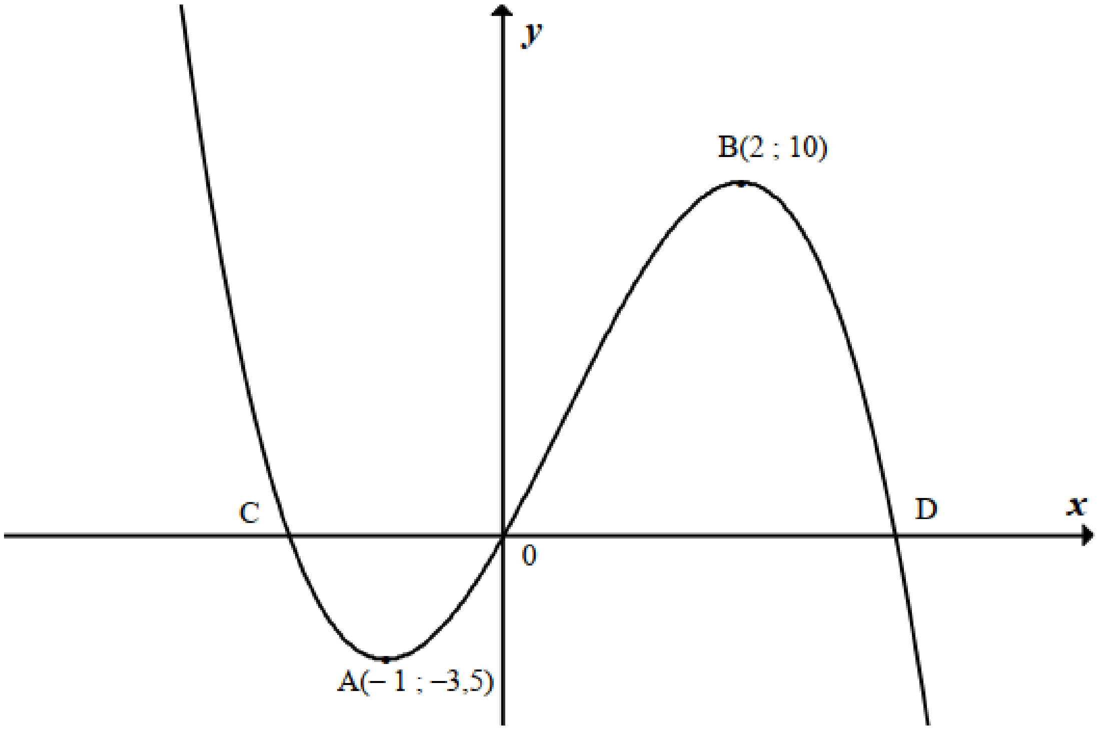
10.1 Sketch the graph of f in your ANSWER BOOK clearly indicating the turning point(s) and the points of intersection of the graph with the axes. (3)

10.2 Show that the defining equation of f is $f(x) = -2x^3 + 12x^2 - 18x + 8$. (4)

10.3 Determine the value(s) of x for which graph of f is concave down. (3)

QUESTION 12

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. A$(-1\ ;\ 3{,}5)$ and B$(2\ ;\ 10)$ are the turning points of h. The graph passes through the origin and further cuts the x-axis at C and D.



12.1	Show that $a = \frac{3}{2}$ and $b = 6$.	(6)
12.2	Calculate the average gradient between A and B.	(2)
12.3	Determine the equation of the tangent to h at $x = -2$.	(5)
12.4	Determine the x-value of the point of inflection of h.	(3)
12.5	Use the graph to determine the values of p for which the equation $-x^3 + \frac{3}{2}x^2 + 6x + p = 0$ will have ONE real root.	(2)

PAPER D

QUESTION 8

8.1	If $f(x) = 2x^2 - 5x + 3$, determine $f'(x)$ from first principles.	(5)
8.2	Determine $\frac{dy}{dx}$ if $y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$	(5)

QUESTION 9

9.1 Given: $f(x) = -2x^3 + 5x^2 + 4x - 3$

9.1.1 Calculate the coordinates of the x-intercepts of f if $f(3) = 0$. Show all calculations. (4)

9.1.2 Calculate the x-values of the stationary points of f. (4)

9.1.3 For which values of x is f concave up? (2)

9.2 The function g, defined by $g(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $g(-2) = g(4) = 0$
- The graph of $g'(x)$ is concave up.
- The graph of $g'(x)$ has x-intercepts at $x = 0$ and $x = 4$ and a turning point at $x = 2$.

9.2.1 Use this information to draw a neat sketch graph of g without actually solving for a, b, c and d. Clearly show all x-intercepts, x-values of the turning points and x-value of the point of inflection on your sketch. (4)

9.2.2 For which values of x will $g(x) \cdot g''(x) > 0$? (3)

QUESTION 10

A shopkeeper finds that the number of people visiting his shop at any moment during the 10 hours that the shop is open, is represented by:

$$N(t) = t^3 - 12t^2 + 36t + 8\ ,$$

where $N(t)$ is the number of people in the shop, t hours after the shop opened.

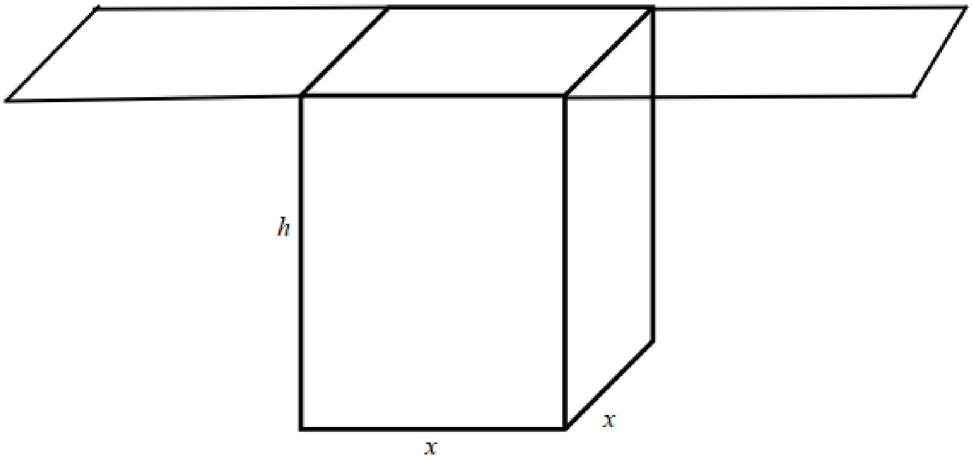
10.1 How many people are in the shop when the shop opens? (1)

10.2 At what stage is the number of people in the shop increasing? (5)

10.3 At which stage is it the best time for the shopkeeper to take a break and leave his assistant alone in the shop? (1)

QUESTION 11

The rectangular milk carton has a square base which holds 1 litre of milk. It has a specially designed fold-in top. The area of the cardboard used for the top is three times the area of the base.



11.1 Show that the Total Surface Area of the carton is given by (3)

$$A(x) = 4x^2 + \frac{4000}{x}$$

11.2 Determine the dimensions of the carton so that minimum amount of cardboard is used. (6)

PAPER E

QUESTION 8

8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)

8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (7)

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

9.5 Write down the values of x for which $f'(x) < 0$. (2)

QUESTION 10

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

10.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

10.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

10.3 After how many seconds will the particle be closest to the fixed point? (2)

PAPER F

QUESTION 8

8.1 If $f(x) = \frac{4}{x}$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 5x^2 + 5x + 2$ (2)

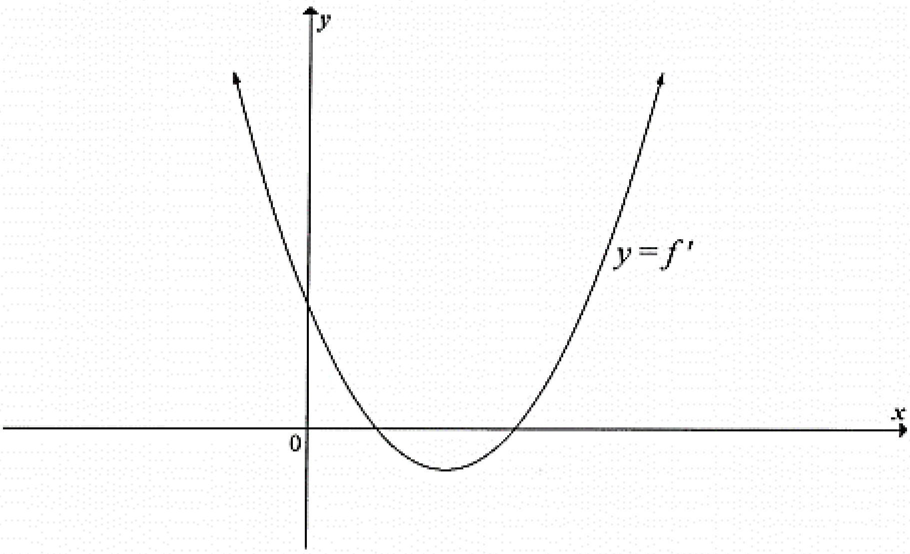
8.2.2 $D_x\left[\sqrt[3]{x^2} - \frac{1}{2}x\right]$ (3)

8.3 Given: $p(x) = x^3 + 2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of p to have a negative gradient. (3)

QUESTION 9

The graph of $y = ax^2 + bx + c$ below represents the derivative of f.
It is given that $f'(1) = 0$, $f'(3) = 0$ and $f'(0) = 6$.



9.1 Write down the x-coordinates of the stationary points of f. (2)

9.2 For which value(s) of x is f strictly decreasing? (2)

9.3 Explain at which value of x the stationary point of f will be a local minimum. (2)

9.4 Determine the x-coordinate of the point of inflection of f. (1)

9.5 For which value(s) of x is f concave up? (2)

QUESTION 10

The mass of a baby in the first 30 days of life is given by

$M(t) = t^3 - 9t^2 + 3\,000 \quad ; \quad 0 \leq t \leq 30$.

t is the time in days and M is the mass of the baby in grams.

10.1 Write down the mass of the baby at birth. (1)

10.2 A baby's mass usually decreases in the first few days after birth.
On which day will the baby's mass return to its birth mass? (4)

10.3 On which day will this baby have a minimum mass? (4)

10.4 On which day will the baby's mass be decreasing the fastest? (2)

PROBABILITY AND COUNTING

BASIC PROBABILITY QUESTIONS AND/OR VENN DIAGRAMS

PAPER A

QUESTION 4

P(A) = 0,3 and P(B) = 0,5.

Calculate P(A or B) if:

4.1 A and B are mutually exclusive events (2)

4.2 A and B are independent events (3)

PAPER B

QUESTION 4

The events A, B and C are such: A and B are independent, B and C are independent and A and C are mutually exclusive. Their probabilities are P(A) = 0,3 , P(B) = 0,4 and P(C) = 0,2.

Calculate the probability of the following events occurring:

4.1 Both A and C occur. (2)

4.2 Both B and C occur. (2)

4.3 At least one of A or B occur. (4)

PAPER C

QUESTION 12

12.1 It is given that A and B are independent events. P(A) = 0,4 and P(B) = 0,5.

Use a Venn diagram and calculate:

12.1.1 P(A or B) (4)

12.1.2 P(neither A or B) (1)

PAPER D

QUESTION 4

4.1 A survey of 80 students at a local library indicated the reading preferences below:

44 read the *National Geographic* magazine

33 read the *Getaway* magazine

39 read the *Leadership* magazine

23 read both *National Geographic* and *Leadership* magazines

19 read both *Getaway* and *Leadership* magazines

9 read all three magazines

69 read at least one magazine

4.1.1 How many students did not read any magazine? (1)

4.1.2 Let the number of students who read *National Geographic* and *Getaway*, but not *Leadership*, be represented by x. Draw a Venn diagram to represent reading preferences. (5)

4.1.3 Hence show that $x = 5$. (3)

4.1.4 What is the probability, correct to THREE decimal places, that a student selected at random will read at least two of the three magazines? (3)

4.2 A smoke detector system in a large warehouse uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0,95. The probability that it will be detected by device B is 0,98 and the probability that it will be detected by both devices simultaneously is 0,94.

4.2.1 If smoke is present, what is the probability that it will be detected by device A or device B or both devices? (3)

4.2.2 What is the probability that the smoke will not be detected? (1)

PAPER E

QUESTION 3

The probability that it will rain on a given day is 63%. A child has a 12% chance of falling in dry weather and is three times as likely to fall in wet weather.

3.1 Draw a tree diagram to represent all outcomes of the above information. (6)

3.2 What is the probability that a child will not fall on any given day? (3)

3.3 What is the probability that a child will fall in dry weather? (2)

4.2 There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

4.2.1 Calculate the probability that the first learner chosen is a boy. (1)

4.2.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate on your diagram ALL possible outcomes. (4)

4.2.3 Calculate the probability that a boy, then a girl and then another boy is chosen in that order. (3)

4.2.4 Calculate the probability that all three learners chosen are girls. (2)

4.2.5 Calculate the probability that at least one of the learners chosen is a boy. (3)

QUESTION 5

Alfred and Barry have an equal chance of winning a point in a game.

5.1 Draw a tree diagram to represent the situation after a total of 3 points have been contested. Indicate on your diagram the probabilities and all the outcomes associated with each branch. (5)

5.2 Calculate the probability that Barry would have won all 3 points. (2)

5.3 Calculate the probability that Alfred would have won 2 points and Barry would have won 1 point of the 3 points contested. (2)

5.4 Barry and Alfred play a fourth point. Calculate the probability that Alfred will win 3 of the 4 points contested. (4)

CONTINGENCY TABLES

PAPER A

QUESTION 5

The sports director at a school analysed data to determine how many learners play sport and what the gender of each learner is. The data is presented in the table below.

	DO NOT PLAY SPORT	PLAY SPORT	TOTAL
Male	51	69	120
Female	49	67	116
Total	100	136	236

5.1 Determine the probability that a learner, selected at random, is:

5.1.1 Male (2)

5.1.2 Female and plays sport (2)

5.2 Are the events 'male' and 'do not play sport' mutually exclusive? Use the values in the table to justify your answer. (2)

5.3 Are the events 'male' and 'do not play sport' independent? Show ALL calculations to support your answer. (4)

PAPER B

QUESTION 5

In a survey 1 530 skydivers were asked if they had broken a limb. The results of the survey were as follows:

	Broken a limb	Not broken a limb	TOTAL
Male	463	b	782
Female	a	c	d
TOTAL	913	617	1 530

5.1 Calculate the values of a, b, c and d. (4)

5.2 Calculate the probability of choosing at random in the survey, a female skydiver who has not broken a limb. (2)

5.3 Is being a female skydiver and having broken a limb independent? Use calculations, correct to TWO decimal places, to motivate your answer. (4)

PROBABILITY AND COUNTING

PAPER A

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

10.2.2 Repetition of digits IS allowed in the code (1)

PAPER B

QUESTION 7

Consider the digits 1, 2, 3, 4, 5, 6, 7 and 8 and answer the following questions:

7.1 How many 2-digit numbers can be formed if repetition is allowed? (2)

7.2 How many 4-digit numbers can be formed if repetition is NOT allowed? (3)

7.3 How many numbers between 4 000 and 5 000 can be formed? (3)

PAPER C

QUESTION 12

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 All the letters of the alphabet can be used (2)

12.1.2 The password must start with a 'D' and end with an 'L' (2)

PAPER D

QUESTION 5

Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.

5.1 How many personal identity numbers (PINs) can be made if:

5.1.1 Digits can be repeated (2)

5.1.2 Digits cannot be repeated (2)

5.2 Suppose that a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4)

PAPER E

5.2 A photographer has placed six chairs in the front row of a studio. Three boys and three girls are to be seated in these chairs.

In how many different ways can they be seated if:

5.2.1 Any learner may be seated in any chair (2)

5.2.2 Two particular learners wish to be seated next to each other (3)

PAPER F

QUESTION 6

There are 7 different shirts and 4 different pairs of trousers in a cupboard. The clothes have to be hung on the rail.

6.1 In how many different ways can the clothes be arranged on the rail? (2)

6.2 In how many different ways can the clothes be arranged if all the shirts are to be hung next to each another and the pairs of trousers are to be hung next to each another on the rail? (3)

6.3 What is the probability that a pair of trousers will hang at the beginning of the rail and a shirt will hang at the end of the rail? (4)

PAPER G

QUESTION 5

Consider the word: PRODUCT.

5.1 How many different arrangements are possible if all the letters are used? (2)

5.2 How many different arrangements can be made if the first letter is T and the fifth letter is C? (2)

5.3 How many different arrangements can be made if the letters R, O and D must follow each other, in any order? (3)

PAPER H

QUESTION 11

11.1 The letters of the word EQUATION are randomly used to form a new word consisting of five letters. How many of these words are possible if letters may not be repeated? (2)

PAPER I

Question 6

Consider the letters of the word MAREMATLOU

6.1 What is the probability that the word arrangement formed with the letters, MAREMATLOU, will start and end with the same letter?
Repeated letters are treated as identical. (5)

PAPER 2

STATISTICS

PAPER A

QUESTION 1

The total number of red cards issued per country to players during a soccer competition are given in the table below:

NUMBER OF RED CARDS	NUMBER OF COUNTRIES (f)	MIDPOINT OF INTERVAL (x)	$f.x$
$0 < x \leq 2$	27		
$2 < x \leq 4$	15		
$4 < x \leq 6$	5		
$6 < x \leq 8$	5		
$8 < x \leq 10$	3		
TOTAL			

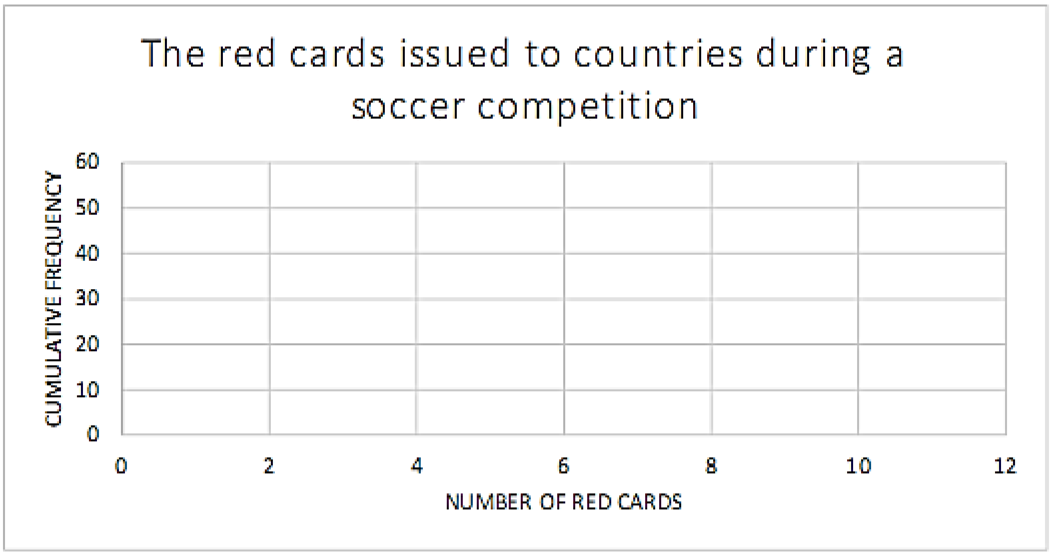
1.1 Calculate the estimated mean of the number of red cards per country. (3)

1.2 Draw an ogive curve to represent the above data. (3)

1.3 Calculate the interquartile range of the number of red cards issued per country in the competition. (2)

OGIVE CURVE GRID

1.2



QUESTION 2

The table below shows a relationship between the monthly rent (x) a person pays for an apartment and the person's monthly income (y). Both are given in thousands of rands.

YEAR	2003	2004	2005	2006	2007	2008
Rent (x)	2	3	3,5	5,2	5,6	6
Income (y)	9	13,5	15	16,5	17	20

2.1 Determine the equation of the regression line. (4)

2.2 Determine the estimated monthly income if the rent per month is R9000. (2)

2.3 Calculate the value of the correlation coefficient. (2)

2.4 Describe the relationship between the monthly rent and the monthly income. (2)

PAPER B

QUESTION 1

1. The following information represents the amount of maize exported to other countries over 11 years in 1000 tons.

39	42	48	54	62	68	78	78	82	91	93

1.1 Calculate the mean amount of maize exported over the 11 years. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Calculate the number of years that are within one standard deviation of the mean. (2)

1.4 Draw a box and whisker diagram to represent the data. (4)

1.5 Comment on the skewness of the data. (1)

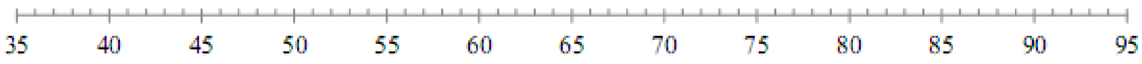
1.6 There was an error in the data. The mean amount of maize exported over the 11 years should increase by 1,25 thousands of tons. What impact will this error have on the:

1.6.1 yearly data provided in the above table? (1)

1.6.2 on the interquartile range of the given data above? (1)

BOX AND WHISKER NUMBER LINE

1.4



QUESTION 2

The following information (in %) represent contributions made by the Agricultural and Mining industries in order to evaluate the GDP(GROSS DOMESTIC PRODUCT) of a certain country.

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1 Determine the equation of the least squares regression line. (3)

2.2 Estimate the percentage that the mining industry will contribute if the agriculture industry dropped to 1,2 %. (2)

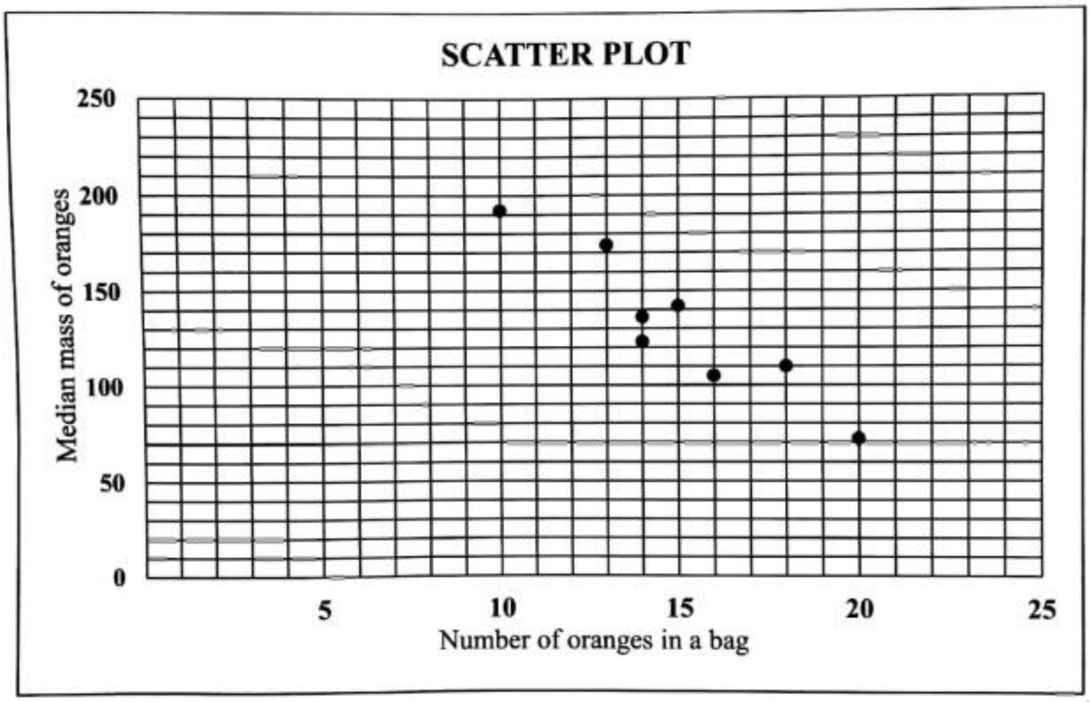
2.3 Comment on the strength of the correlation between the contributions made by these two industries. Motivate your answer. (2)

PAPER C

Question 1

A student is investigating the number of oranges in a bag in relation to the median mass of the oranges filled in the same bag. The findings are recorded in the table below.

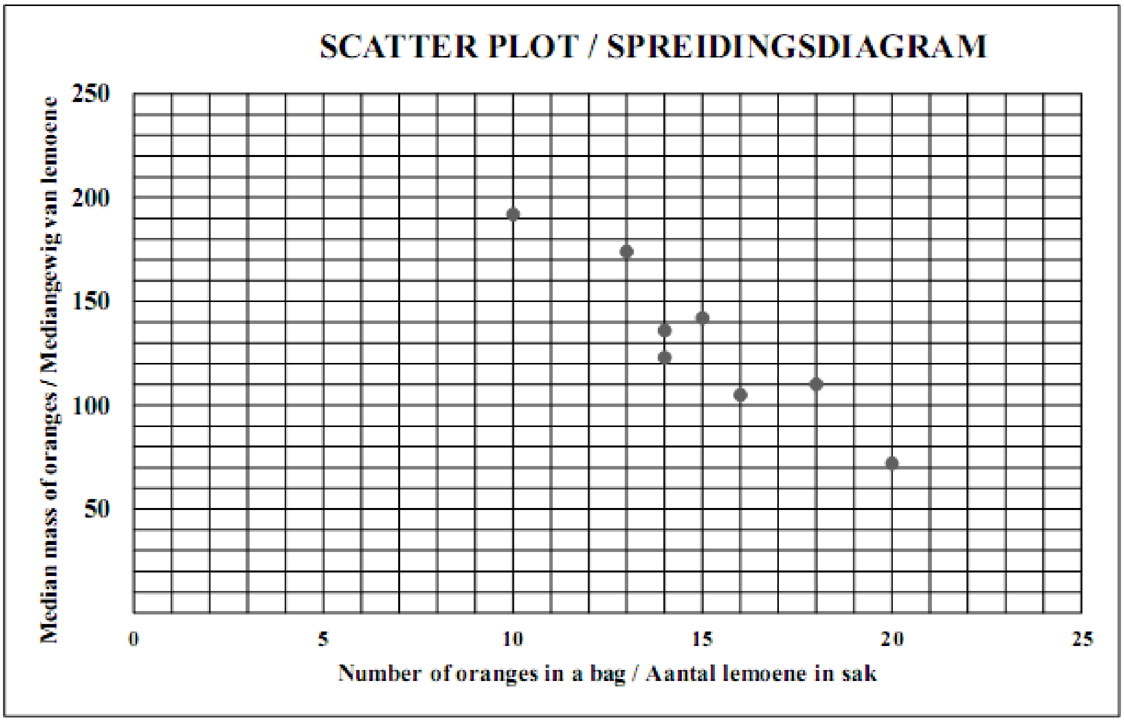
Number of oranges in a bag	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram)	110	105	72	142	123	174	136	192



1.1	Determine the equation of the least squares regression line for the data.	(3)
1.2	Write down the correlation coefficient of the data.	(1)
1.3	Draw the least squares regression line on the scatter plot given in your ANSWER BOOK.	(2)
1.4	Comment on the strength of the relationship between the number of oranges in the bag and the median mass of the oranges.	(1)
1.5	Determine the possible median mass of oranges in a bag, if there are 12 oranges in that bag.	(2)

SCATTER PLOT

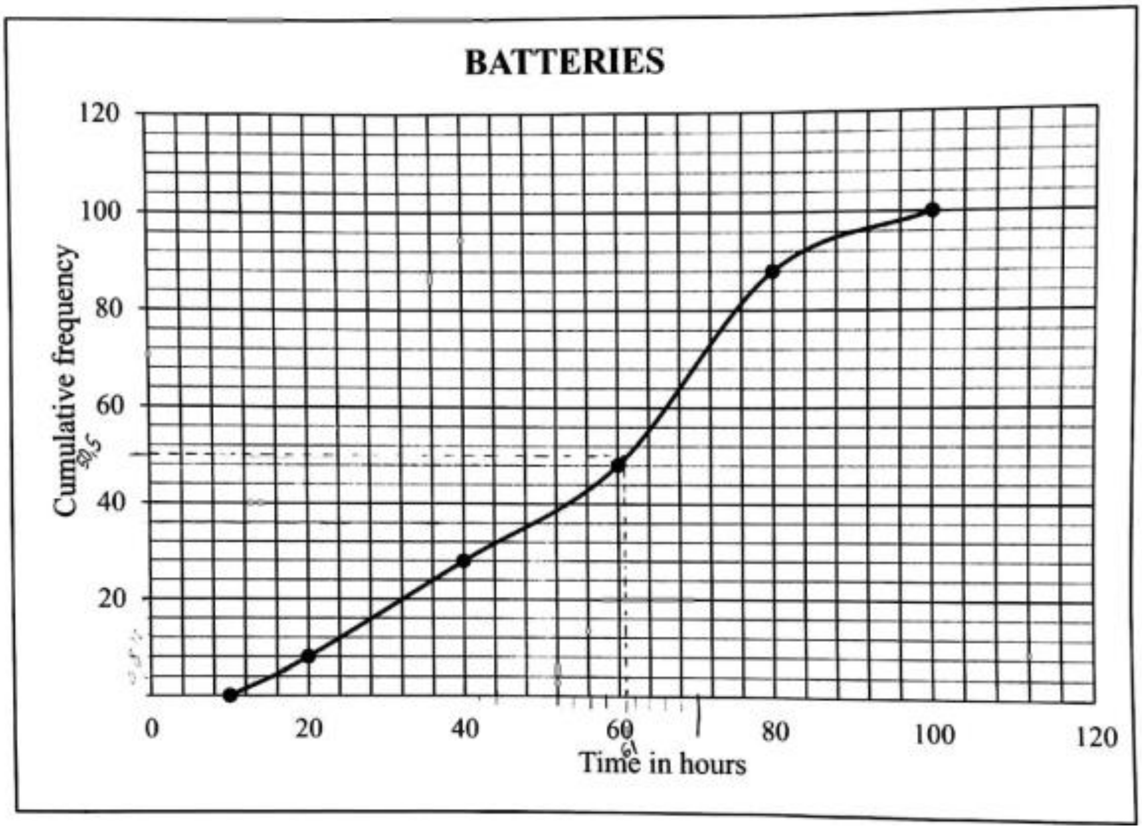
1.3



QUESTION 2

2.1 Batteries are used in everyday life. The Grade 12 Physical Sciences learners investigated the life span of batteries under constant test conditions.

The ogive (cumulative frequency graph) below shows the lifespan (in hours) of the batteries.



2.1.1 How many batteries were tested for this investigation?

2.1.2 Use the graph to estimate the median time for the life span (in hours) of the batteries.

2.1.3 The minimum lifespan of batteries is 10 hours and the maximum lifespan is 100 hours. Use the cumulative frequency graph to draw a box and whisker diagram in your ANSWER BOOK.

2.1.4 Comment on the skewness of the distribution of the lifespan of the batteries.

BOX AND WHISKER NUMBER LINE

2.1.3

2.2 The table below represents values in a data set written in increasing order. None of the values in the data set are repeated.

5	a	19	b	c	d	35

Determine the values of a, b, c, and d if:

- The median is 20.
- The semi interquartile range is 8.
- The upper quartile is twice the lower quartile.
- The mean is 22.

PAPER D

QUESTION 1

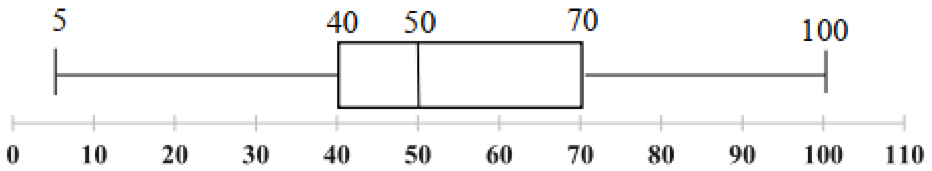
The A-Rithmetic High School decided to compare the results of 31 Grade 12 learners in Mathematics and Physical Sciences in the 2019 Preparatory Examination.

- The Mathematics results are recorded in the table below.
- The box and whisker plot below illustrates the results of Physical Sciences.
- Marks are recorded as percentages.

Mathematics Results

7	11	15	19	19	23	28	28	31	38	39
40	41	48	48	52	53	55	57	59	59	64
67	72	76	83	85	87	89	92	96	-	-

Physical Sciences Results



1.1 Calculate the mean mark of the Mathematics learners. (2)

1.2 Comment on the skewness of the Mathematics data. (1)

1.3 Determine which subject performed better in the 2019 Preparatory Examination. Give a reason for your conclusion. (2)

1.4 Write down a possible mark for a learner who achieved the tenth lowest mark in Physical Sciences. (2)

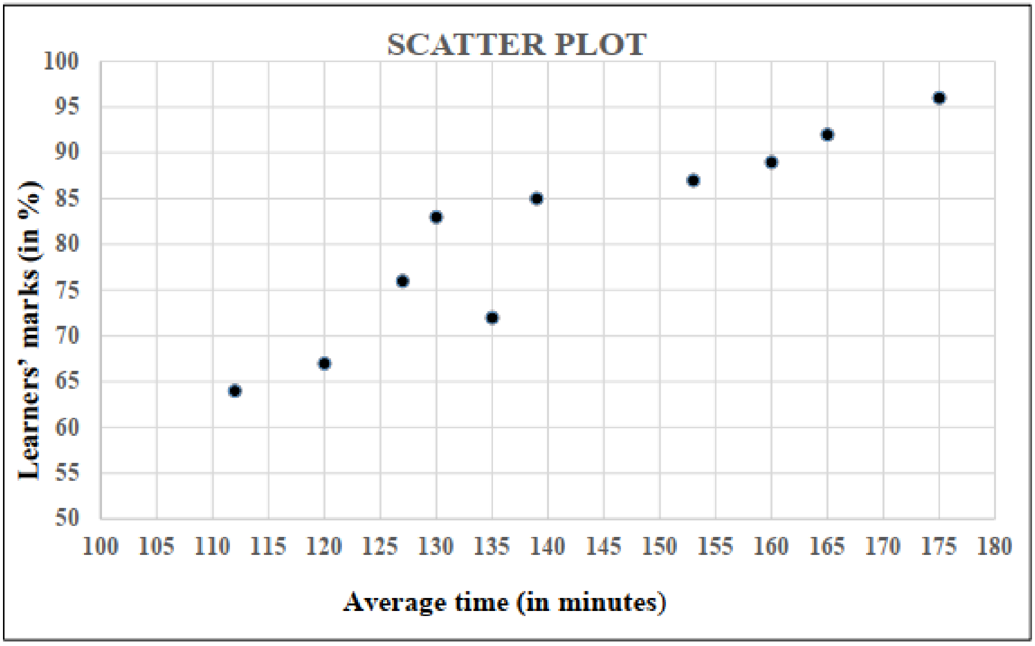
1.5 A learner scored the fourth highest in both subjects. The learner obtained the GREATEST possible difference between both subjects. Calculate the learner's mark in Physical Sciences. (2)

QUESTION 2

A question raised by many educators is whether the results that a learner achieves in an examination is dependent on the time that the learner takes to complete the examination.

The average time taken by each of the top 10 Mathematics learners was recorded. The data is represented in the table and scatter plot below.

Average time (in minutes)	175	165	160	153	139	130	127	135	120	112
Learners' marks (in %)	96	92	89	87	85	83	76	72	67	64



2.1 Calculate the equation of the least squares regression line for the data. (3)

2.2 A learner completed the exam in 2,5 hours. Predict the mark that the learner achieved. (2)

2.3 Explain within the context why the regression line is not reliable. (1)

2.4 Calculate the standard deviation of the top 10 Mathematics learners. (2)

2.5 It is further given that $(p\ ;\ 103{,}59)$ is the interval of 15 random learners' marks within ONE standard deviation of the mean. If $\bar{x} = 63{,}96$, calculate the value of p. (3)

PAPER E

QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)

1.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)

1.3 Identify an outlier in the data set. (1)

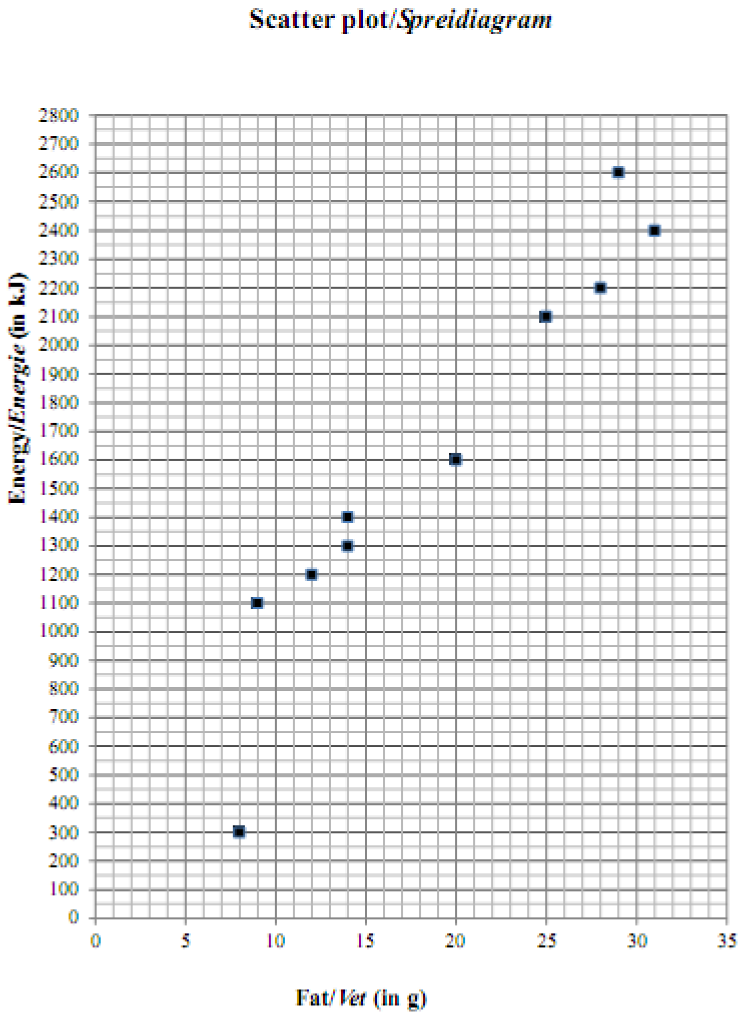
1.4 Calculate the value of the correlation coefficient. (2)

1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)

1.1 AND 1.2.2 GRID

1.1

1.2.2



QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

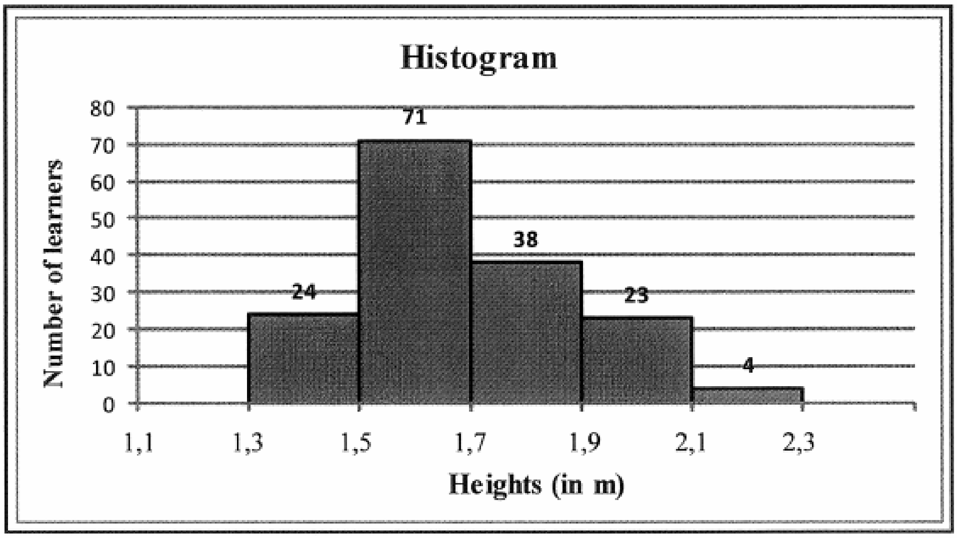
2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)

PAPER F

QUESTION 2

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

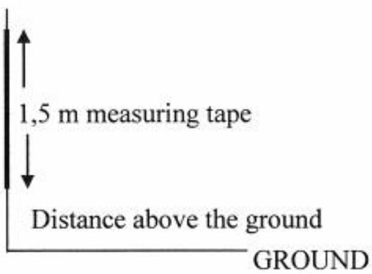
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

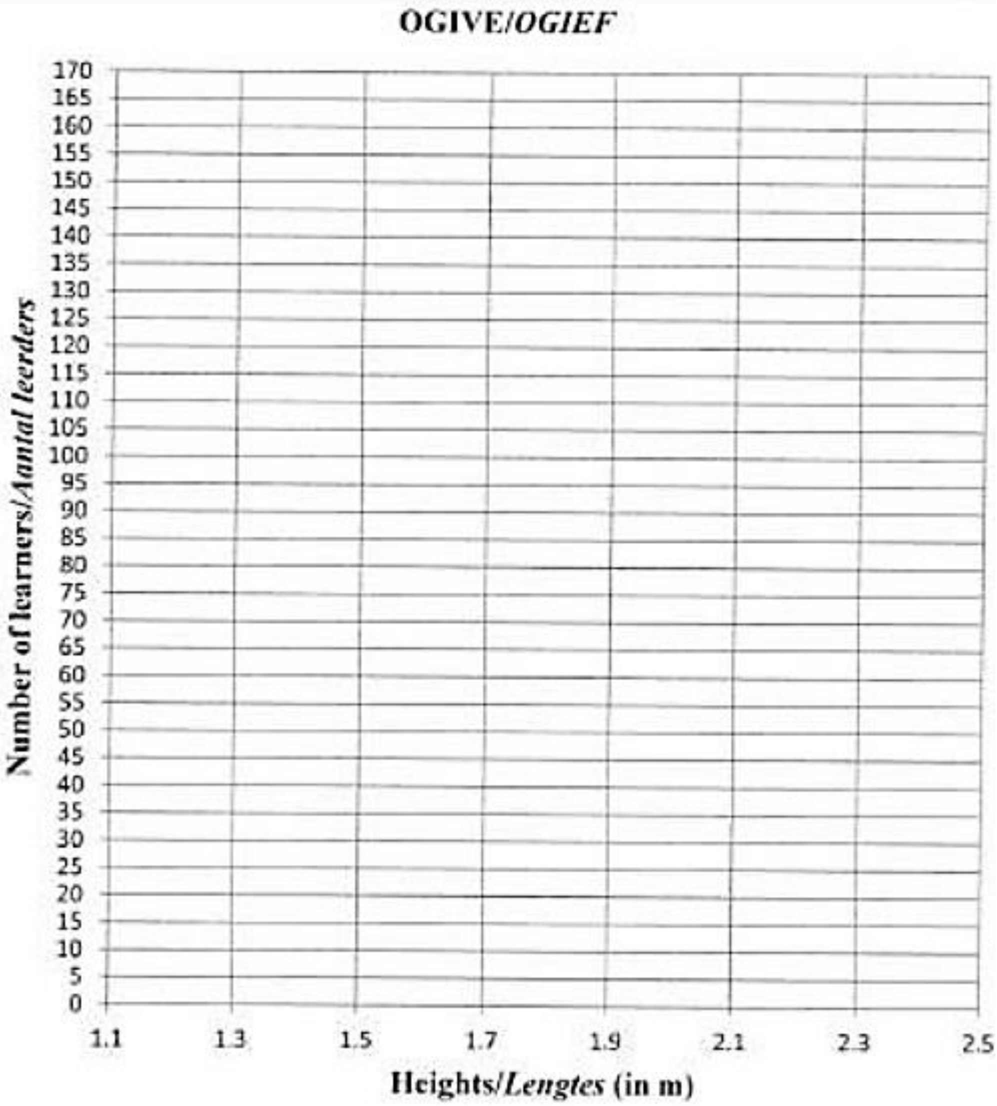
2.6.1 Mean of the data set (1)

2.6.2 Standard deviation of the data set (1)

2.3 CUMULATIVE FREQUENCY COLUMN

Intervals *Klasse*	Cumulative frequency *Kumulatiewe frekwensie*
$1{,}3 \leq x < 1{,}5$	
$1{,}5 \leq x < 1{,}7$	
$1{,}7 \leq x < 1{,}9$	
$1{,}9 \leq x < 2{,}1$	
$2{,}1 \leq x < 2{,}3$	

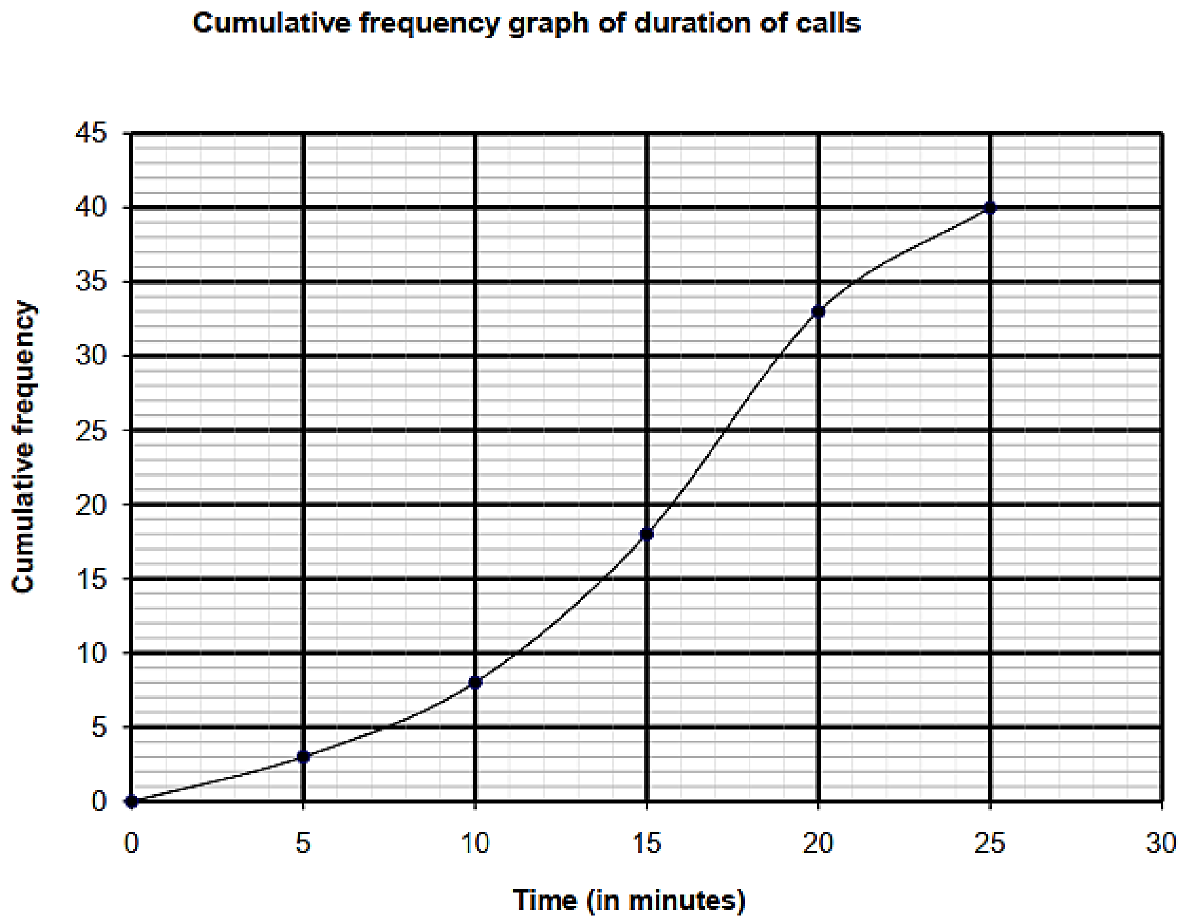
2.4 OGIVE GRID



PAPER G

QUESTION 3

The length of time, in minutes, of a certain number of telephone calls was recorded. No call lasted 25 minutes or longer. A cumulative frequency diagram of this data is shown below.



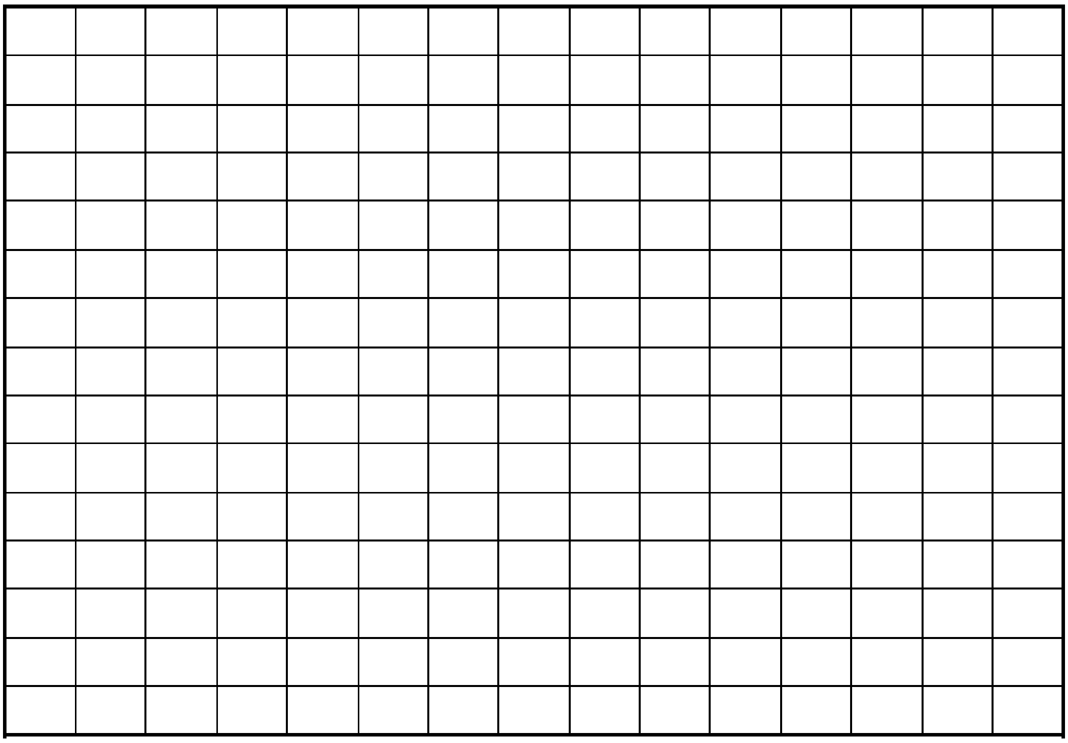
3.1	Determine the total number of calls recorded.	(1)
3.2	Complete the frequency table for the data on DIAGRAM SHEET 3.	(3)
3.3	Hence, draw a histogram on the grid on DIAGRAM SHEET 3.	(3)

DIAGRAM SHEET 3

QUESTION 3.2

Time, t, in minutes	Frequency
$0 \leq t < 5$	
$5 \leq t < 10$	
$10 \leq t < 15$	
$15 \leq t < 20$	
$20 \leq t < 25$	

QUESTION 3.3

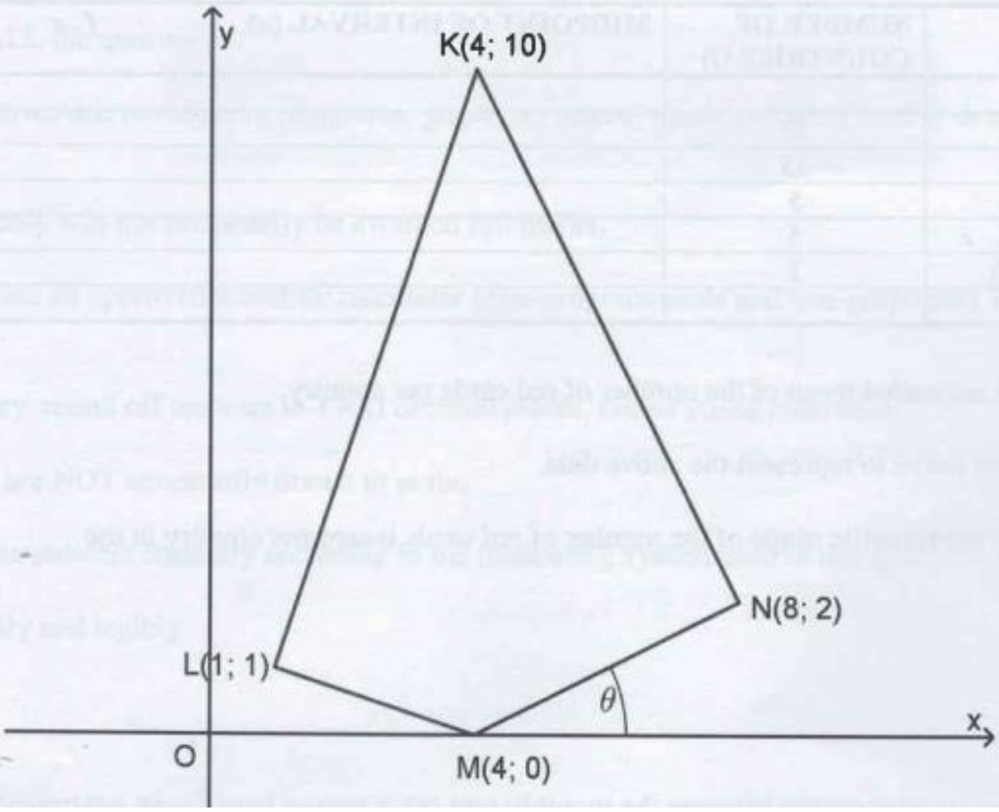


ANALYTICAL GEOMETRY

PAPER A

QUESTION 3

In the diagram KLMN is a quadrilateral with K(4; 10), L(1; 1), M(4; 0) and N(8; 2).



3.1 Determine the:

3.1.1 gradient of LM and MN (4)

3.1.2 length of KM. (2)

3.1.3 value of θ (2)

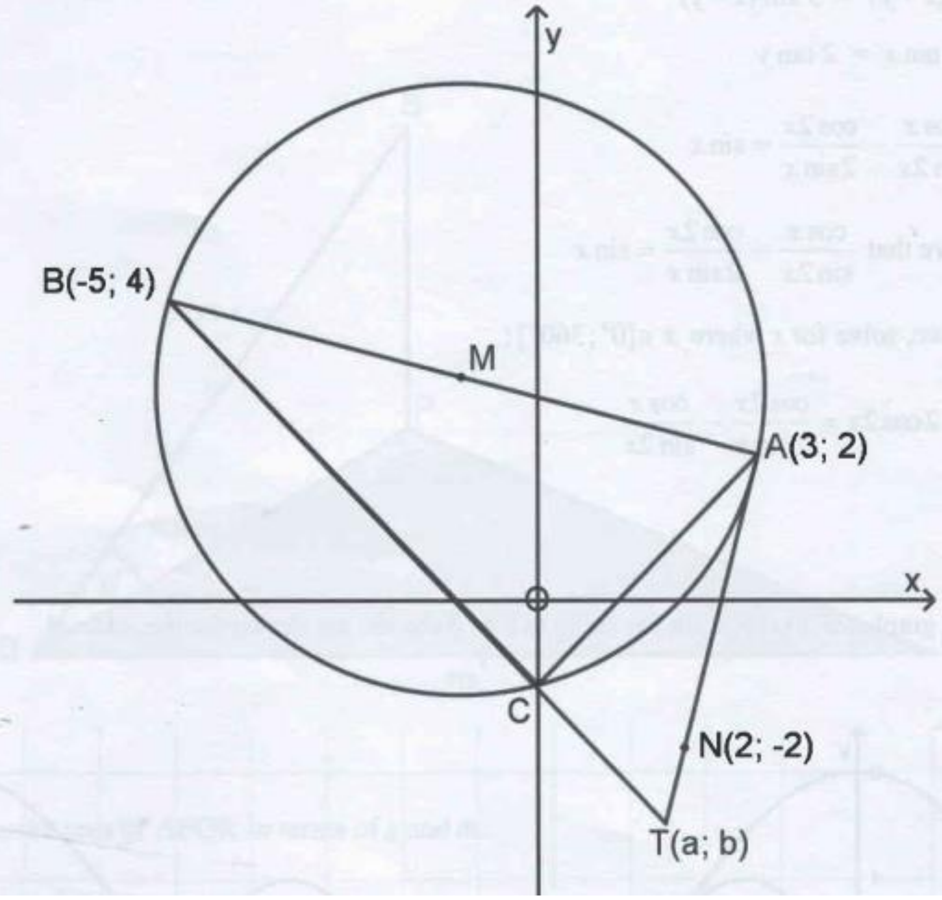
3.1.4 midpoint of LN (2)

3.2 Show that KL ⊥ LM (3)

3.3 Prove that KLMN is a cyclic quadrilateral. (4)

QUESTION 4

In the sketch below, AB is a diameter with coordinates A(3; 2) and B(–5; 4) of circle ABC. M is the centre of the circle. BC produced meets AT in T. N(2; –2) is a point on the line TA. C is the y – intercept of the circle.

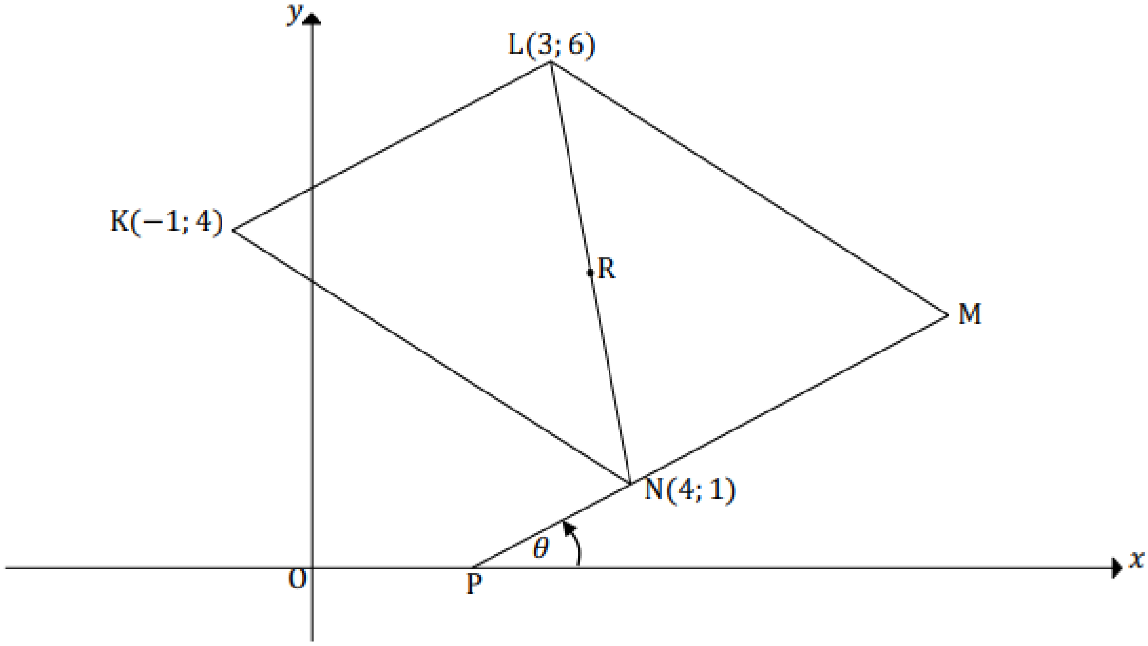


4.1	Determine the co-ordinates of M the centre of the circle	(2)
4.2	Write down the equation of the circle in the form $(x-p)^2 + (y-q)^2 = r^2$	(3)
4.3	Prove that TA is a tangent to the circle at A.	(5)
4.4	Determine the equations of the lines	
	4.4.1 TA and	(4)
	4.4.2 BT	(6)
4.5	If the coordinates of T are $(a; b)$, calculate the values of a and b.	(3)

PAPER B

QUESTION 3

In the diagram, K (–1 ; 4) ; L(3 ; 6); M and N(4 ; 1) are vertices of a parallelogram. R is the midpoint of LN. P is the x-intercept of the line MN produced.



3.1 Calculate the:

3.1.1 gradient of KL. (2)

3.1.2 coordinates of R. (3)

3.1.3 coordinates of M. (4)

3.2 Determine the equation of NM in the form $y = mx + c$. (3)

3.3 Calculate the:

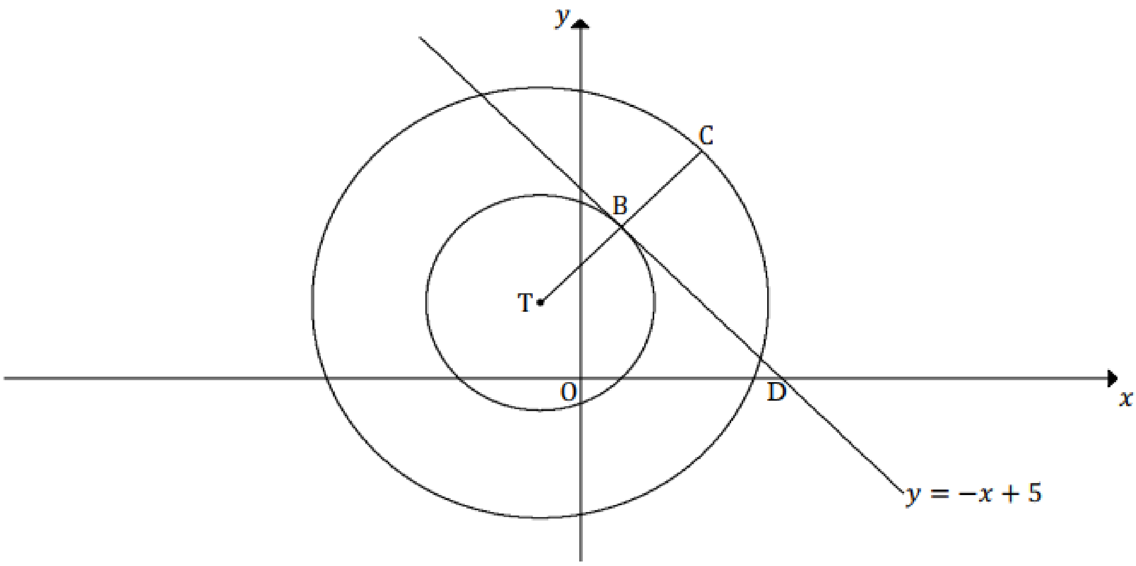
3.3.1 coordinates of P. (2)

3.3.2 size of θ, the inclination of PM. (2)

3.3.3 size of $K\hat{P}N$. (4)

QUESTION 4

In the diagram, T is the centre of two concentric circles. The larger circle has equation $x^2 + y^2 - 4y + 2x - 27 = 0$. The smaller circle touches the straight line $y = -x + 5$ at point B. BD is a tangent to smaller circle T. D is the x-intercept of the straight line. C is a point on the larger circle such that TBC is a straight line.



4.1 Calculate the coordinates of T. (4)

4.2 Show that equation of TB is given by $y = x + 3$. (3)

4.3 Calculate the coordinates of B (3)

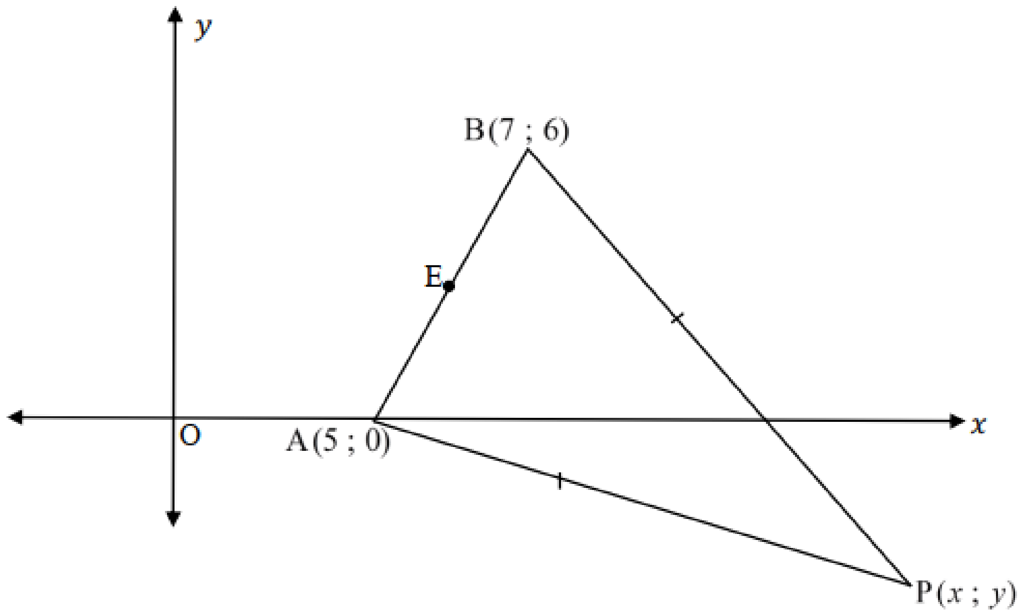
4.4 Determine the equation of the smaller circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.5 Calculate the area of quadrilateral OTBD. (7)

PAPER C

QUESTION 3

In the diagram below, points A$(5\ ;\ 0)$, B$(7\ ;\ 6)$ and P$(x\ ;\ y)$ form a triangle.
BP = AP and E is the midpoint of AB.



3.1 Determine the coordinates of E. (2)

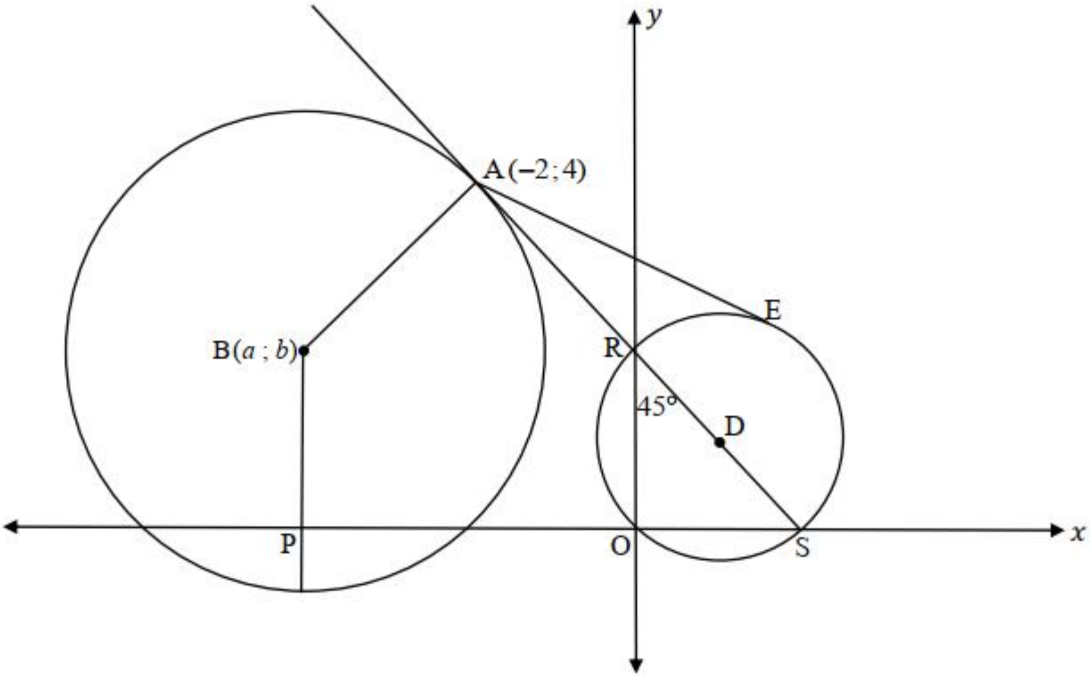
3.2 Determine the equation of line BA. (3)

3.3 Line BA is parallel to the straight line with equation $rx - 3y + 5 = 0$.
Calculate the value of r. (3)

3.4 If the area of ΔAOP = 10 units^2 and $y < 0$, calculate the coordinates of P. (7)

QUESTION 4

The diagram below shows a circle with centre $B(a\,;\,b)$. BP is parallel to the y-axis with P on the x-axis. AS is a tangent to circle B at $A(-2\,;\,4)$ and intersects the x-axis at S and the y-axis at R. AE is a tangent to the smaller circle with centre D and touches the circle at E. $O\hat{R}S = 45°$.



4.1 Determine the equation of tangent AS. (4)

4.2 If OP = 4 units, determine the values of a and b, the centre of the larger circle. (4)

4.3 Determine the equation of the circle with centre B. (3)

4.4 The equation of the smaller circle with centre D is $x^2 - 2x + y^2 - 2y = 0$.
Write this equation in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

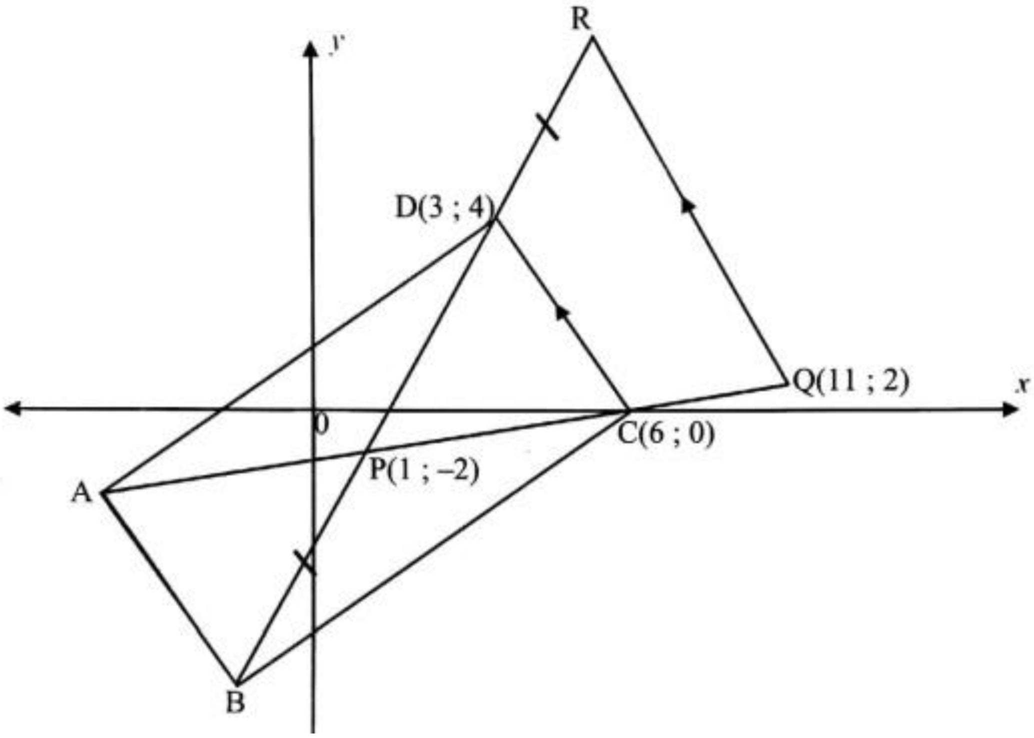
4.5 Write down the coordinates of D, the centre of the smaller circle. (1)

4.6 Calculate the length of AE, the tangent to circle D at E. (6)

PAPER D

QUESTION 3

3.1 In the diagram below, A, B, C(6 ; 0) and D(3 ; 4) are the vertices of rectangle ABCD. Diagonals AC and BD bisect each other at P(1 ; – 2). AC is produced to Q(11 ; 2) and BD is produced to R such that BP = DR and CD ∥ QR.



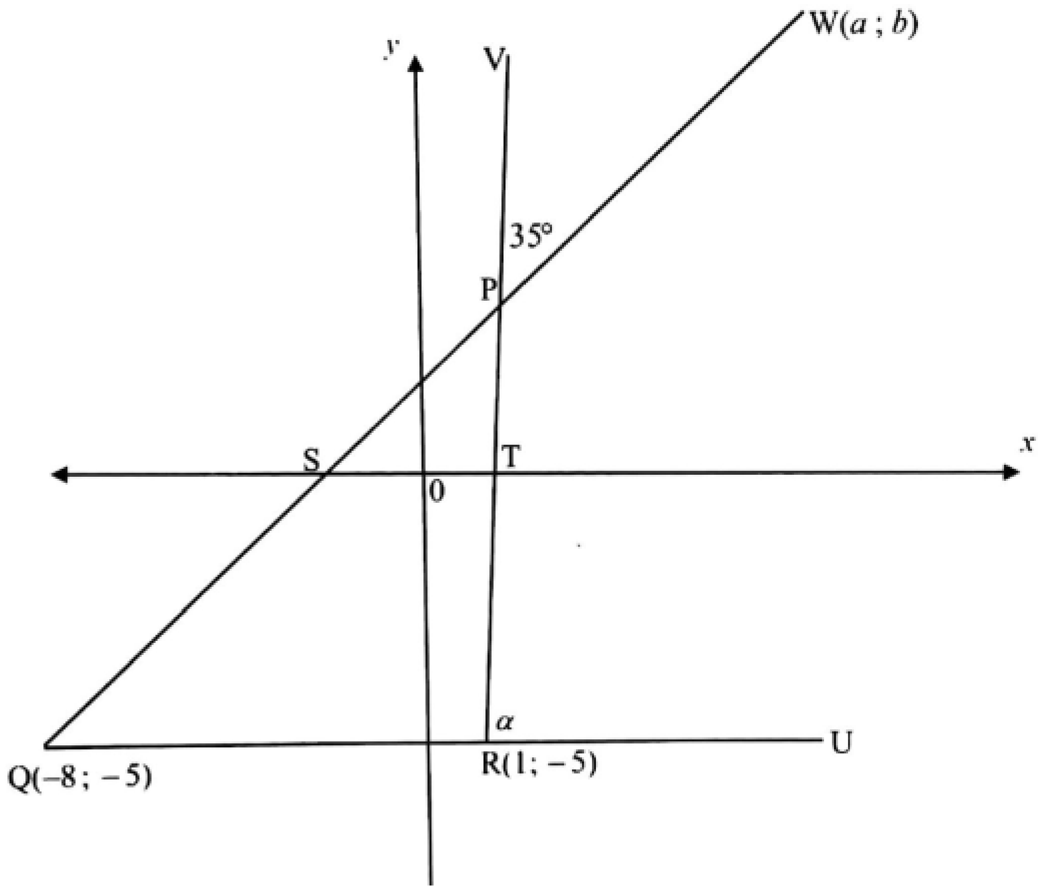
3.1.1 Calculate the coordinates of B. (3)

3.1.2 Determine the gradient of CD. (2)

3.1.3 Show that the equation of QR is $y = -\frac{4}{3}x + \frac{50}{3}$. (2)

3.1.4 If K(4 ; y) is a point in the 4th quadrant such that PK = RQ, calculate the value of y. (6)

3.2 In the diagram below, P, Q(−8 ; −5) and R(1 ; −5) are the vertices of ΔPQR. RP is produced to V and QP is produced to W(a ; b) such that $V\hat{P}W = 35°$. The equation of QW is $y = x + \frac{2}{3}$. QR is produced to U and $U\hat{R}V = \alpha$. QW and RV intersect the x-axis at S and T respectively.



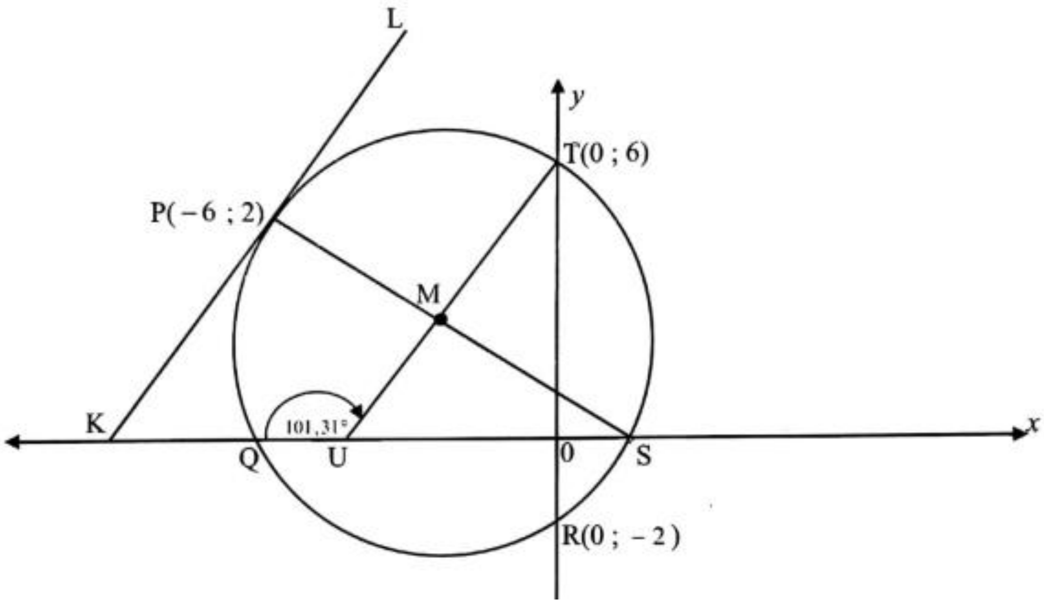
3.2.1 Calculate the size of α. (5)

3.2.2 It is further given that QU ⊥ WU and R is the midpoint of QU. Calculate the area of ΔQWU. (6)

QUESTION 4

In the diagram below, a circle with centre M, cuts the x-axis at Q and S and the y-axis at T(0 ; 6) and R(0 ; –2). The equation of diameter SMP is $y = \frac{-1}{5}x + \frac{4}{5}$.

KPL is a tangent to the circle at P(–6 ; 2). TM produced cuts the x-axis at U. $Q\hat{U}T = 101{,}31°$.

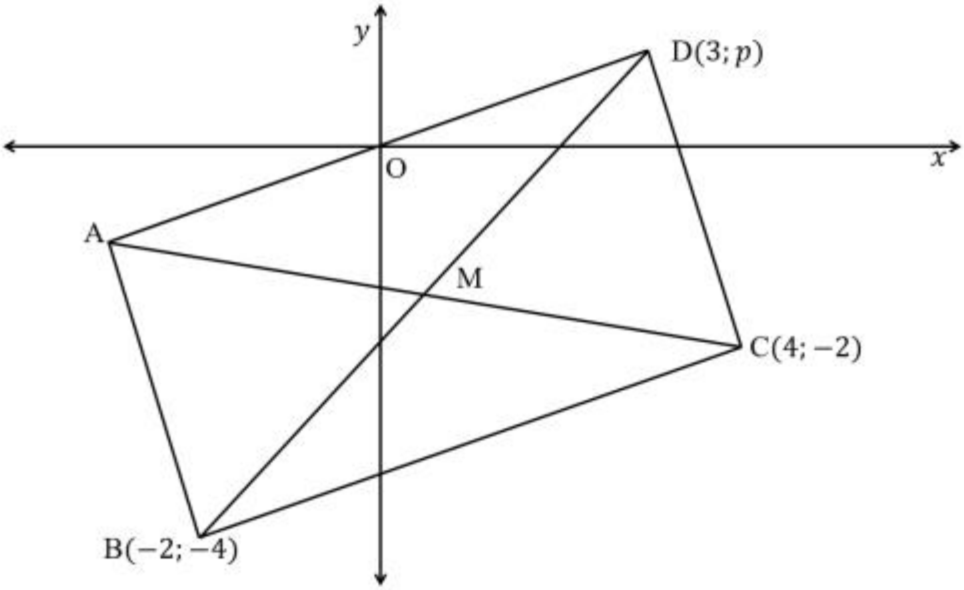


4.1	Determine the equation of TU.	(3)
4.2	Calculate the coordinates of M.	(3)
4.3	If the coordinates of M are (–1 ; 1), determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.	(3)
4.4	Prove that KL is parallel to TU.	(3)
4.5	Is the point $V\left(-\frac{1}{2}; 7\right)$ inside the circle? Support your answer with calculations.	(3)

PAPER E

QUESTION 3

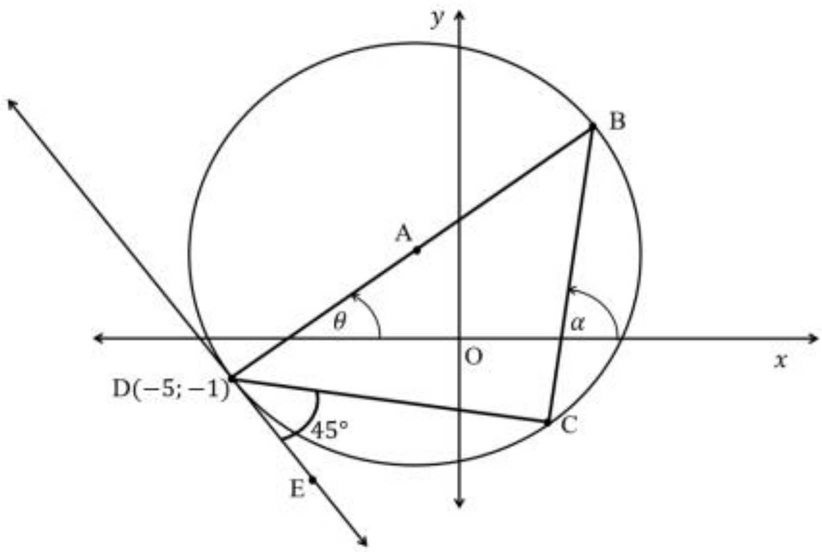
In the diagram A, B$(-2;-4)$, C$(4;-2)$ and D$(3;\ p\ )$ are the vertices of a rectangle. The diagonals AC and BD intersect at M.



3.1	Given that the length of AC is $\sqrt{50}$ units, show that $p = 1$.	(4)
3.2	Determine the coordinates of M.	(3)
3.3	Calculate the gradient of DC	(2)
3.4	Determine the equation of line AB in the form $y = mx + c$.	(2)

QUESTION 4

In the diagram is the circle with equation $(x+1)^2+(y-2)^2=25$.
DB is the diameter of the circle and A the centre of the circle. DE is a tangent to the circle at $D(-5;-1)$. The angle $E\hat{D}C = 45°$. The inclination angles of AD and BC is θ and α respectively. B and C are points on the circumference of the circle.



4.1 Determine:

4.1.1 The coordinates of A, the centre of the circle. (2)

4.1.2 The coordinates of B (3)

4.1.3 The gradient of AD. (2)

4.1.4 The value of θ, the inclination angle of AD. (2)

4.1.5 The equation of the tangent DE. (3)

4.2 Calculate the gradient of BC. (4)

4.3 Another circle with equation $x^2+y^2-6x+2y=8$ is given.

Show that:

4.3.1 The coordinates of the centre of the circle is $M(3;-1)$. (4)

4.3.2 The two circles will intersect each other. Show all calculations. (4)

PAPER F

Question 2

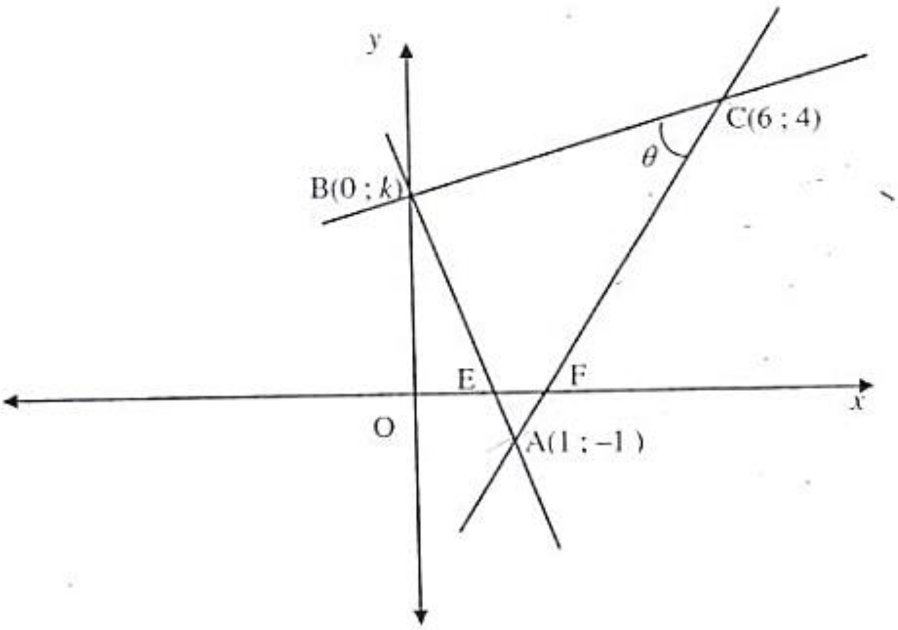
2.2 A circle is represented by the equation $x^2 + 2x + y^2 - 4y - 5 = 0$.

2.2.1 A transformation moves every point 2 units to the left and 4 units up. Determine the equation of the new circle after the transformation. (3)

2.2.2 Does the origin lie within the new circle? Give a reason for your answer. (2)

QUESTION 3

In the diagram, A(1 ; −1), B(0 ; k) and C(6 ; 4) are the vertices of ΔABC. The equations of the sides AB and AC are $y + 3x - 2 = 0$ and $y = x - 2$ respectively. AB cuts the x-axis at E and AC cuts the x-axis at F.



3.1 Write down the value of k. (2)

3.2 Calculate the length of AC and leave your answer in simplest surd form. (2)

3.3 Prove that $A\hat{B}C = 90°$. (3)

3.4 Calculate the size of θ. (5)

3.5 Determine the equation of the circle passing through A, B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

3.6 If D is a point in the first quadrant, calculate the coordinates of D such that ABCD in that order, forms a parallelogram. (4)

QUESTION 5

5.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

5.1.1 Prove that the point (2 ; – 9) is on the circumference of the circle. (2)

5.1.2 Determine an equation of the tangent to the circle at the point (2 ; – 9). (7)

5.2 Calculate the length of the tangent AB drawn from the point A(6 ; 4) to the circle with equation $(x-3)^2 + (y+1)^2 = 10$. (5)

QUESTION 6

6.1 Determine the centre and radius of the circle with the equation $x^2 + y^2 + 8x + 4y - 38 = 0$. (4)

6.2 A second circle has the equation $(x-4)^2 + (y-6)^2 = 26$. Calculate the distance between the centres of the two circles. (2)

6.3 Hence, show that the circles described in QUESTION 6.1 and QUESTION 6.2 intersect each other. (3)

6.4 Show that the two circles intersect along the line $y = -x + 4$. (4)

TRIGONOMETRY

PAPER A

QUESTION 5

5.1 Without using a calculator, evaluate

Cos $79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$ (4)

5.2 Given: $\sin(x+y) = 3\sin(x-y)$

Prove that: $\tan x = 2\tan y$ (4)

5.3 Given: $\dfrac{\cos x}{\sin 2x} - \dfrac{\cos 2x}{2\sin x} = \sin x$

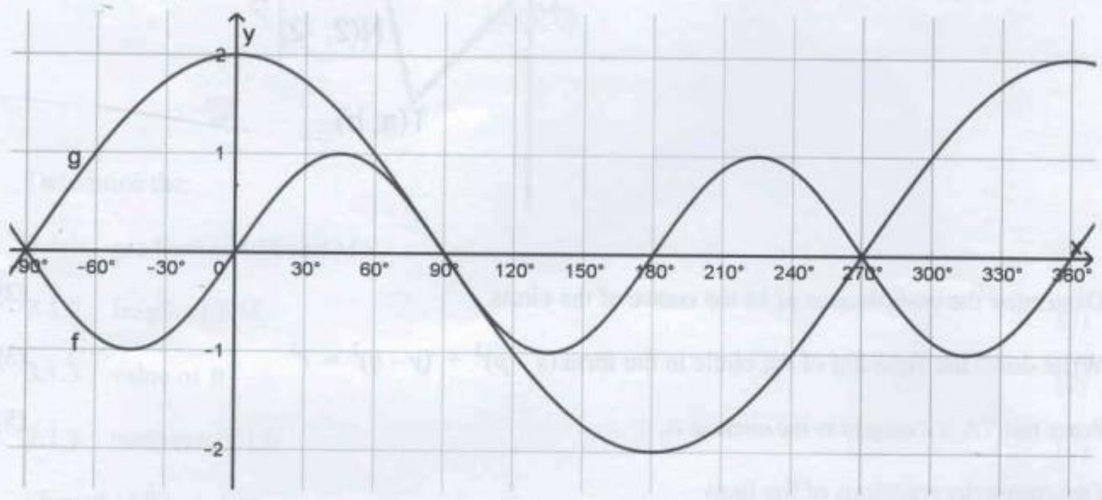
5.3.1 Prove that $\dfrac{\cos x}{\sin 2x} - \dfrac{\cos 2x}{2\sin x} = \sin x$ (4)

5.3.2 Hence, solve for x where $x \in [0^\circ ; 360^\circ]$:

$1 + 2\cos 2x = \dfrac{\cos 2x}{2\sin x} - \dfrac{\cos x}{\sin 2x}$ (6)

QUESTION 6

In the diagram, the graphs of $f(x) = a \sin bx$ and $g(x) = c \cos dx$ are drawn for the interval $x \in [-90^\circ ; 360^\circ]$



6.1 Determine the values of a, b, c and d. (4)

6.2 Write down the period of g. (1)

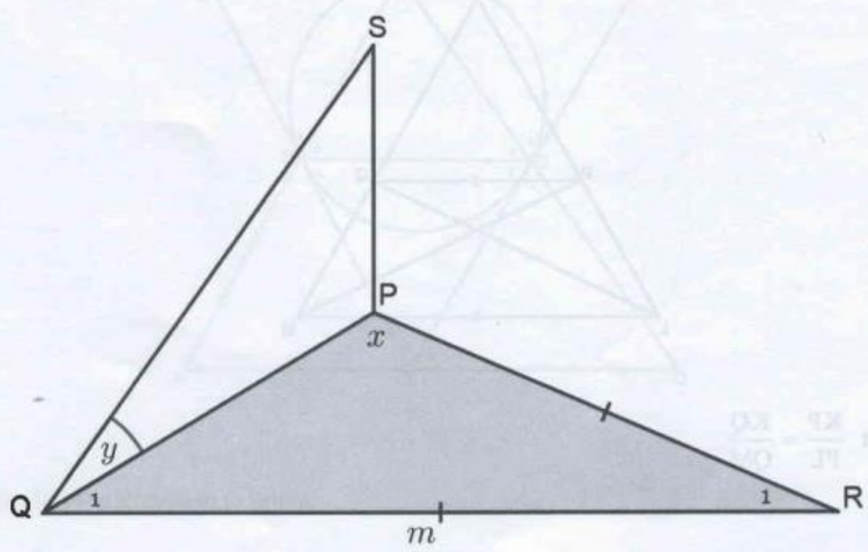
6.3 Determine the value(s) of x in the interval $x \in [-90^\circ ; 360^\circ]$, for which

6.3.1 $f(x) \leq g(x)$ (2)

6.3.2 $f'(x) \times g'(x) > 0$ where $g(x) > 0$ (3)

QUESTION 7

In the diagram P, Q and R are three points in the same horizontal plane. PR = QR = m, $Q\hat{P}R = x$. SP is perpendicular to PQ. The angle of elevation of S from Q is y.

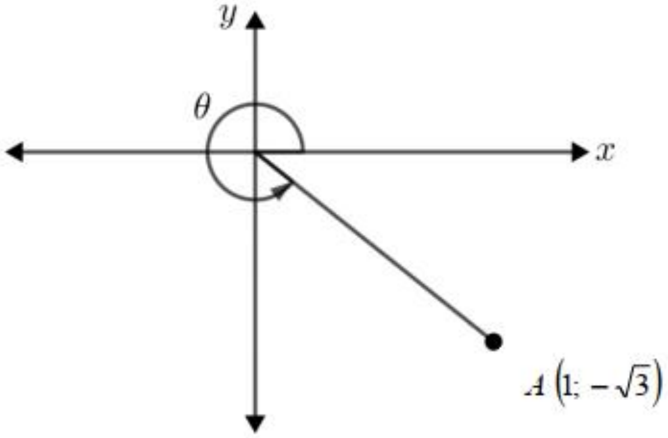


7.1	Express the area of ΔPQR in terms of x and m.	(5)
7.2	Show that $PQ = 2m\cos x$	(4)
7.3	Hence, prove that $SP = 2m\cos x\tan y$	(2)

PAPER B

QUESTION 5

5.1 Use the diagram below to calculate, **without the use of a calculator,** the following



5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 $\sin(\theta - 60°)$ (4)

5.2 Determine the value of the following trigonometric expression:

$$\frac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300° \sin(\theta - 360°)}$$ (6)

5.3 Consider: $\frac{\cos 2x - 1}{\sin 2x} = -\tan x$

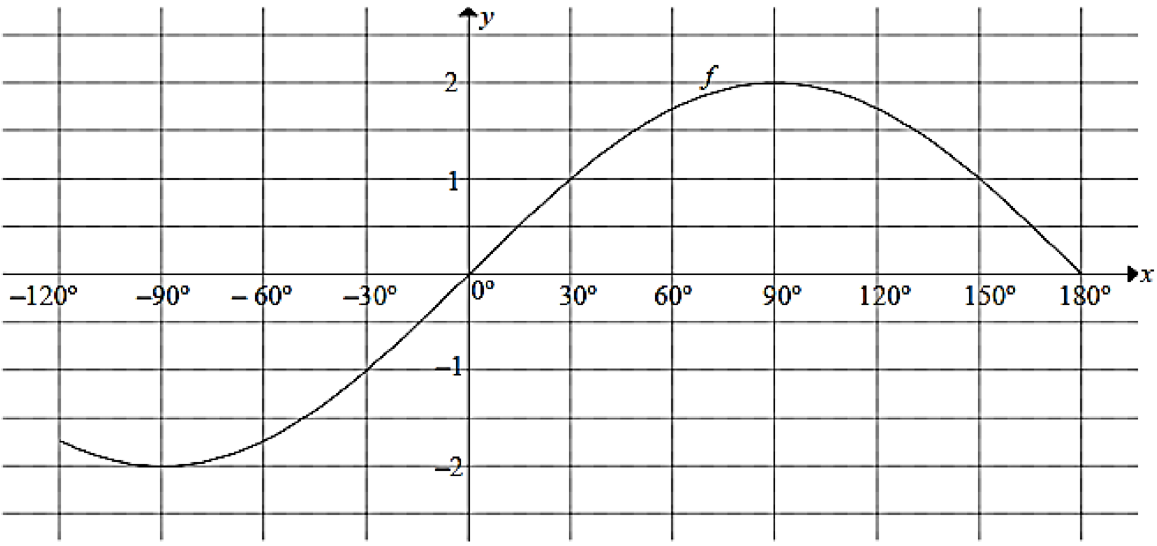
5.3.1 Prove the identity (3)

5.3.2 For which value(s) of x, $0° < x < 360°$, is this identity undefined? (3)

5.3.3 Hence or otherwise, find the general solution of $\frac{\sin 4x}{\cos 4x - 1} = 4$. (4)

QUESTION 6

In the diagram below, the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-120° ; 180°]$.



6.1 Draw on the same system of axes the graph of $g(x) = \cos(x + 30°)$, for the interval $x \in [-120° ; 180°]$. Show all intercepts with the axes as well as the turning and end Points of the graph. (4)

6.2 Write down the period of f. (1)

6.3 For which values of x in the interval $x \in [-120° ; 180°]$ is:

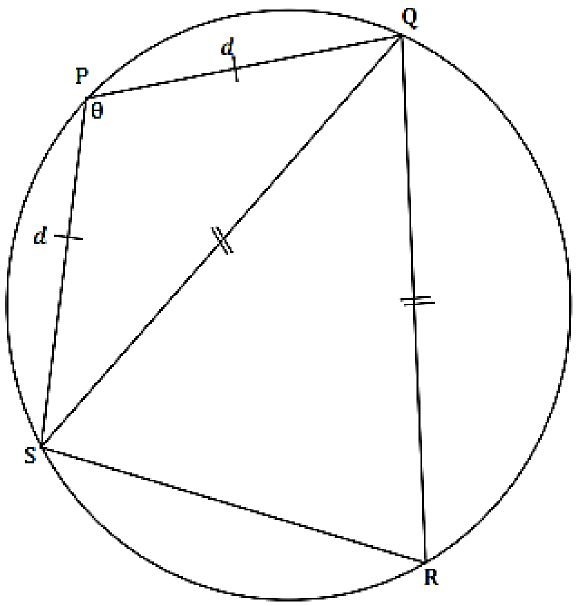
6.3.1 The graph of g decreasing? (2)

6.3.2 $f(x).\ g(x) > 0$? (2)

6.4 If the graph of g is moved 60° to the left, determine the equation of the new graph which is formed, in its simplest form. (2)

QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral with QS = QR and PQ = PS = d units. $Q\hat{P}S = \theta$.



Use the diagram to prove that:

7.1. $QS = d\sqrt{2(1 - \cos\theta)}$ (2)

7.2 The area of $\Delta QRS = -d^2 \sin 2\theta\ (1 - \cos\theta)$ (3)

PAPER C

QUESTION 5

5.1 Calculate the value of $1 - 4\sin^2 15°$ without the use of a calculator. (5)

5.2 Simplify without the use of a calculator:

$$\frac{\sqrt{3}\sin x.\sin^2 72° + \sin^2 198°.\sqrt{3}\cos(x - 90°)}{\tan 120°.\ \sin x}$$ (6)

5.3 Determine the general solution of the following:

$$6\sin x.\cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0$$ (7)

5.4 Prove that:

$$(1 - \tan A)\left(\frac{\cos A}{\cos 2A}\right) = \frac{1}{\cos A + \sin A}$$ (4)

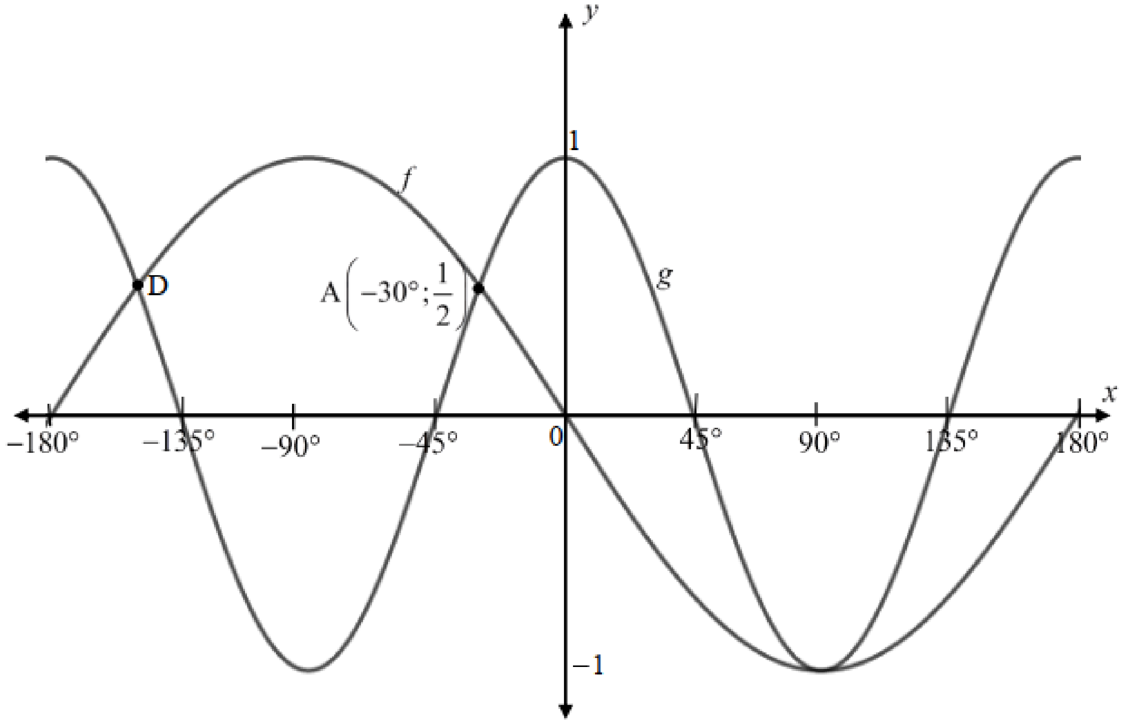
5.5 If $\sin 2\theta = k$ and $0° < 2\theta < 90°$, determine in terms of k:

5.5.1 $\cos 2\theta$ (2)

5.5.2 $\dfrac{\sin 2\theta}{\tan \theta}$ (5)

QUESTION 6

The sketch below shows the graphs of $f(x) = a\sin x$ and $g(x) = \cos dx$ for $x \in [-180° ; 180°]$. $A\left(-30° ; \frac{1}{2}\right)$ is a point of intersection of f and g.



6.1 Write down the values of a and d. (2)

6.2 Determine the coordinates of D. (1)

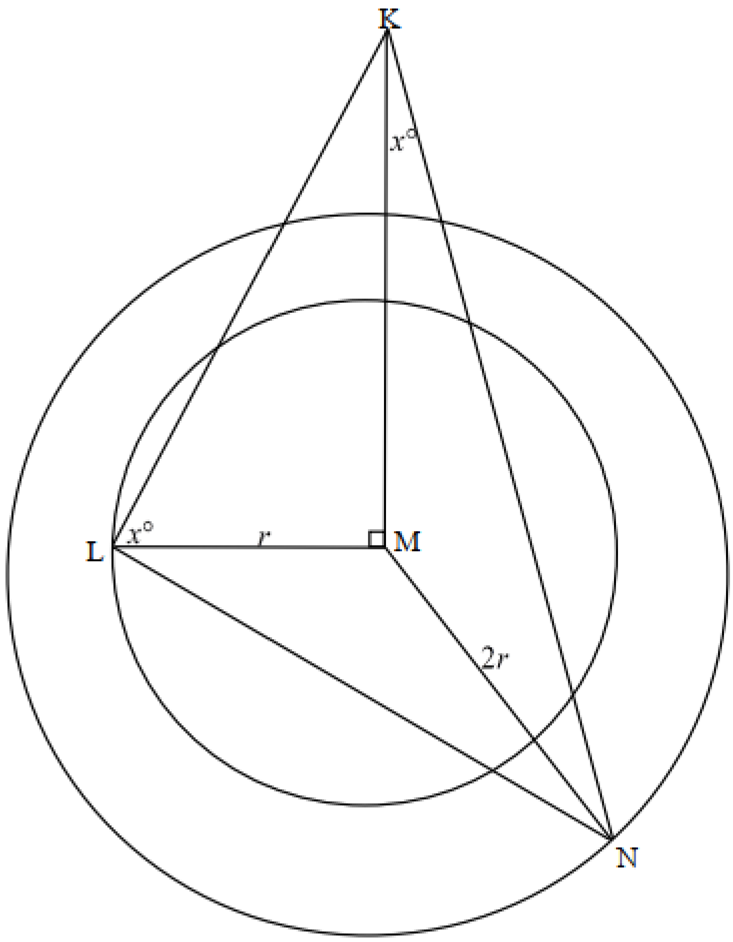
6.3 For which value(s) of x is:

6.3.1 f decreasing for $x \in [-180°\,;\,180°]$? (2)

6.3.2 $f(x)\,.\,g(x) < 0$ for $x \in [-180°\,;\,0°]$? (2)

QUESTION 7

In the figure below, KM is a vertical flag post set in the centre of two circles which lie on the same horizontal plane. $M\hat{K}N = M\hat{L}K = x°$. The radius of the inner circle $ML = r$ units and the radius of the outer circle $MN = 2r$ units.



7.1 Calculate the value of x. (6)

7.2 If $r = 5m$ and $L\hat{M}N = 110°$, calculate the length of LN. (2)

PAPER D

QUESTION 5

5.1 If $\sin 16^\circ = \dfrac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a calculator.**

5.1.1 $\tan 16^\circ$ (2)

5.1.2 $\cos 32^\circ$ (3)

5.2 Simplify the following expression.

$$\frac{\cos(90^\circ + x)\sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$$ (6)

5.3 Calculate the value of the following, **without the use of a calculator.**

$\cos 75^\circ.\cos 45^\circ - \cos 15^\circ.\cos 45^\circ$ (4)

5.4 Given: $\tan\theta\left(\sin 2\theta + \dfrac{3\cos^2\theta}{\sin\theta}\right) = -2\cos^2\theta + 3\cos\theta + 2$

5.4.1 Prove the identity. (3)

5.4.2 Determine the general solution of:

$$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right) = 0$$ (4)

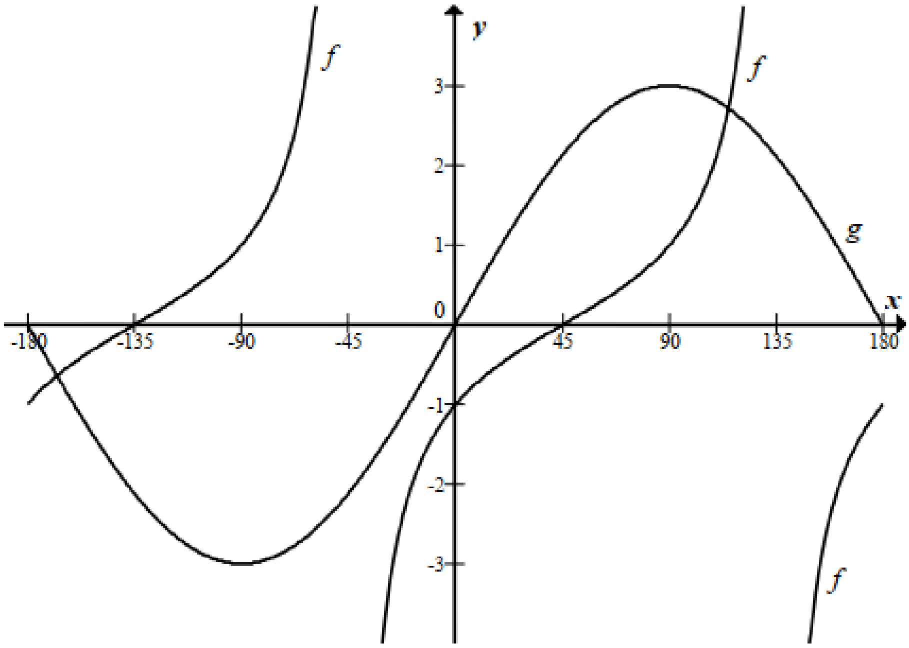
5.5 Solve for a and b:

$\cos(a+b) = -\dfrac{\sqrt{2}}{2}$ if $a+b \in [0^\circ ; 180^\circ]$

$\cos(a-2b) = \dfrac{1}{2}$ if $a-2b \in [0^\circ ; 180^\circ]$ (4)

QUESTION 6

Sketched below are the graphs of the functions $f(x)=\tan(x-45°)$ and $g(x)=3\sin x$ for $x\in[-180°\,;180°]$.



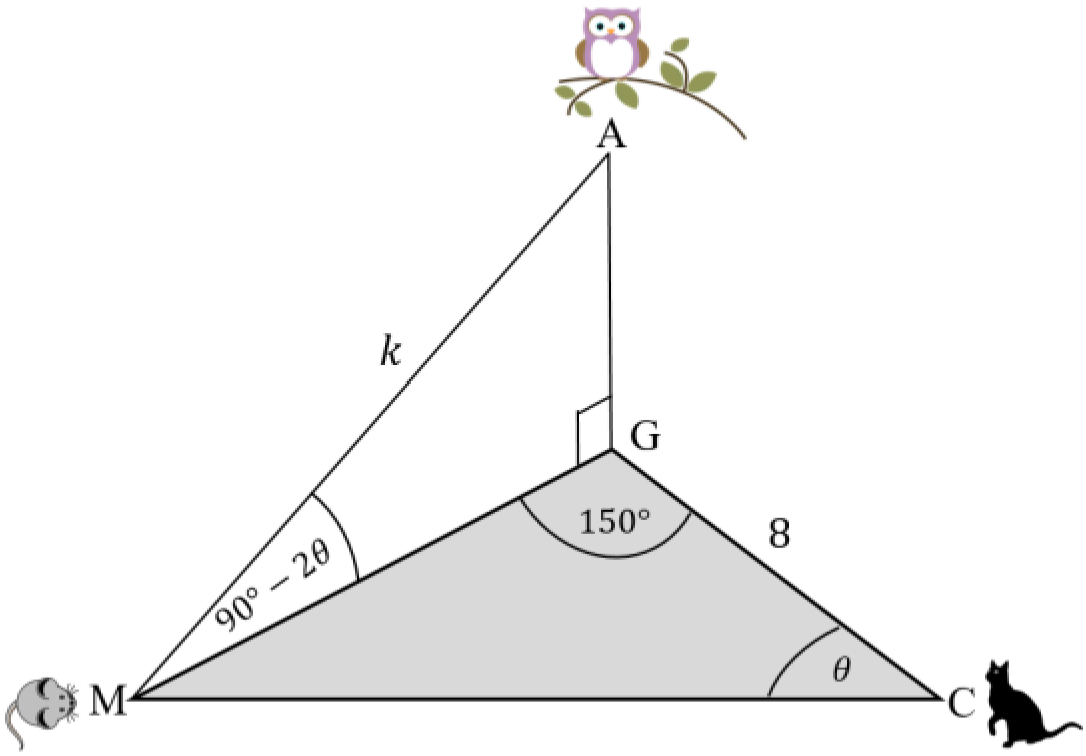
6.1 Write down the equations of the asymptotes of $y=f(x)$ for $x\in[-90°\,;180°]$. (2)

6.2 Describe the transformation of the graph of f to h if $h(x)=\tan(45°-x)$. (2)

6.3 The period of g is reduced to 180° and the amplitude and y-intercept remain the same. Write down the equation of the resulting function. (2)

QUESTION 7

A mouse on the ground is looking up to an owl in a tree and a cat to his right on the ground. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$.
AM $= k$ *units*, GC$=$ 8 *units* , M$\hat{G}$C$=$ 150° and M$\hat{C}$G $= \theta$



7.1	Give the size of M$\hat{A}$G in terms of θ.	(1)
7.2	Show that MG$=$ $k \sin 2\theta$	(2)
7.3	Show that MC$=$ $k \cos\theta$	(4)
7.4	Show that the area of ΔMGC$=$ $2k \sin 2\theta$	(2)

PAPER E

QUESTION 5

5.1 If $\sin 40°.\cos 22° + \cos 40°.\ \sin 22° = k$, determine without the use of a calculator, the value of the following in terms of k.

5.1.1 $\sin 62°$ (2)

5.1.2 $\tan 118°$ (4)

5.1.3 $\sin 14°.\ \cos 14°$ (3)

5.2 Prove the following identity:

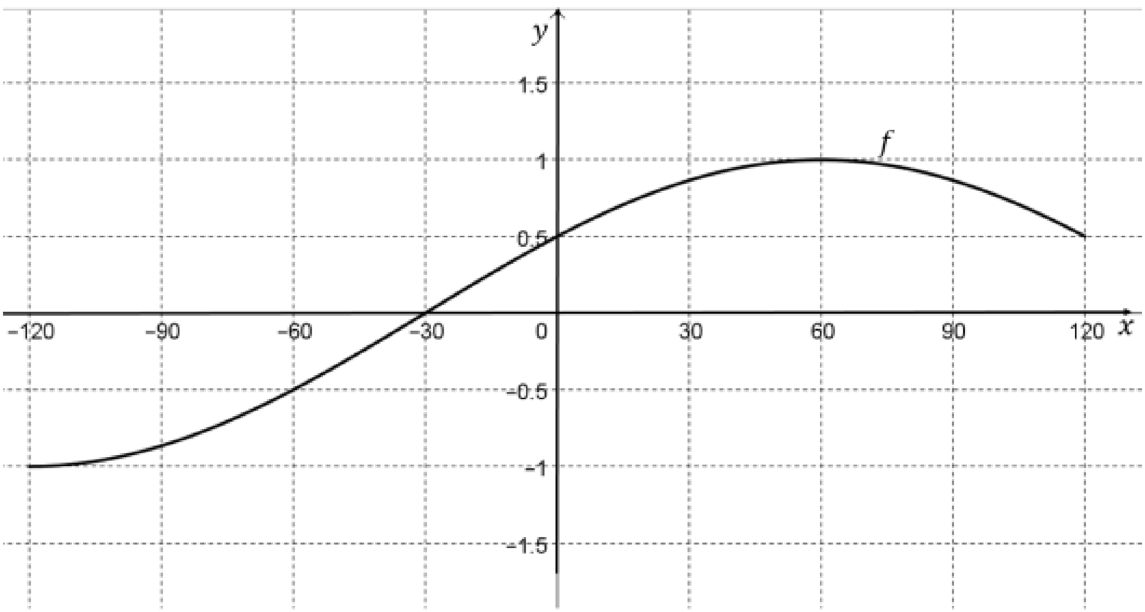
$$\frac{1-\cos 2\theta}{\sin 2\theta \times \tan\theta} = 1$$ (4)

5.3 For which value(s) of A will the following expression be real?

$$\sqrt{\sin(180° + \mathrm{A})\,.\cos(90° + \mathrm{A}) - \tan 45°}$$ (6)

QUESTION 6

In the diagram is the graph of $f(x) = \sin(x + a)$ for the interval [-120°;120°]



6.1 Determine the numerical value of a. (1)

6.2 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(3x)$ for the interval $x \in [-120°; 120°]$. Clearly show ALL intercepts with the axes, the turning point(s) and endpoint(s) of the graph. (4)

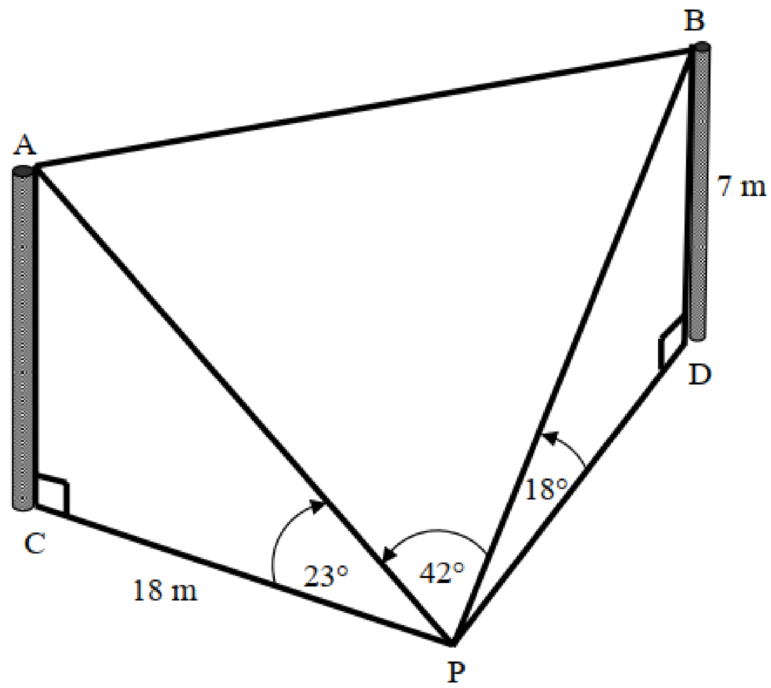
6.3 Determine the general solution for the following: $f(x) = g(x)$ (5)

6.4 Determine the values of x in the interval $x \in [0°; 120°]$, for which $f(x) > g(x)$. (2)

6.5 Describe the transformation from graph g to the graph of $k(x) = \cos(60° - 3x)$. (2)

QUESTION 7

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

7.1 The distance from Thandi to the top of pole BD (2)

7.2 The distance from Thandi to the top of pole AC (2)

7.3 The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$ (4)

QUESTION 8

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2}\tan x$ for $x \in [-90° ; 180°]$.

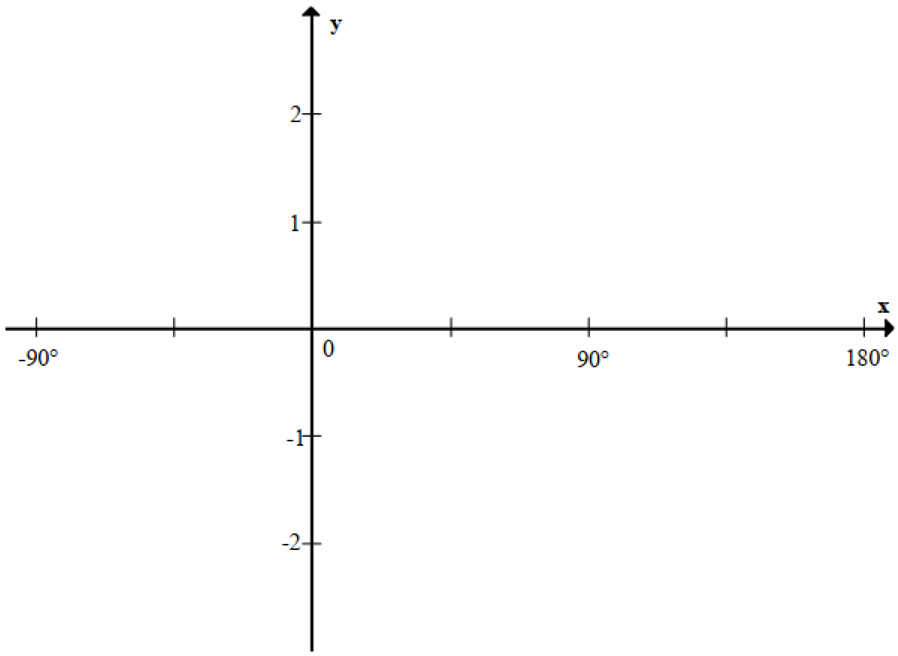
8.1 Sketch the graphs of f and g on the same system of axes on DIAGRAM SHEET 1. (6)

8.2 Calculate the x-coordinates of the points of intersection of f and g. (10)

8.3 Determine the values of x for which $g(x) > f(x)$. (3)

DIAGRAM SHEET 1

QUESTION 8.1



PAPER F

QUESTION 6

6.1 If $\sin 23° = p$, write down the following in terms of p. Do NOT use a calculator.

6.1.1 $\cos 113°$ (2)

6.1.2 $\cos 23°$ (2)

6.1.3 $\sin 46°$ (2)

6.2 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°\,;270°]$ and $\beta \in [90°\,;270°]$. Determine, without using a calculator, the values of the following:

6.2.1 $\cos\alpha$ (3)

6.2.2 $\cos(\alpha+\beta)$ (5)

6.3 Solve for $x \in [0°\,;360°]$ if $\frac{1}{2}\cos x = 0{,}435$. (3)

QUESTION 9

9.1 If $4\tan\theta = 3$ and $180° < \theta < 360°$, determine with the aid of a diagram:

9.1.1 $\sin\theta + \cos\theta$ (4)

9.1.2 $\tan 2\theta$ (5)

9.2 9.2.1 Show that: $\dfrac{\cos(360° - x)\tan^2 x}{\sin(x - 180°)\cos(90° + x)} = \dfrac{1}{\cos x}$ (5)

9.2.2 Hence, calculate without the use of a calculator, the value of:

$\dfrac{\cos 330° \tan^2 30°}{\sin(-150°)\cos 120°}$ (Leave your answer in surd form.) (2)

QUESTION 11

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

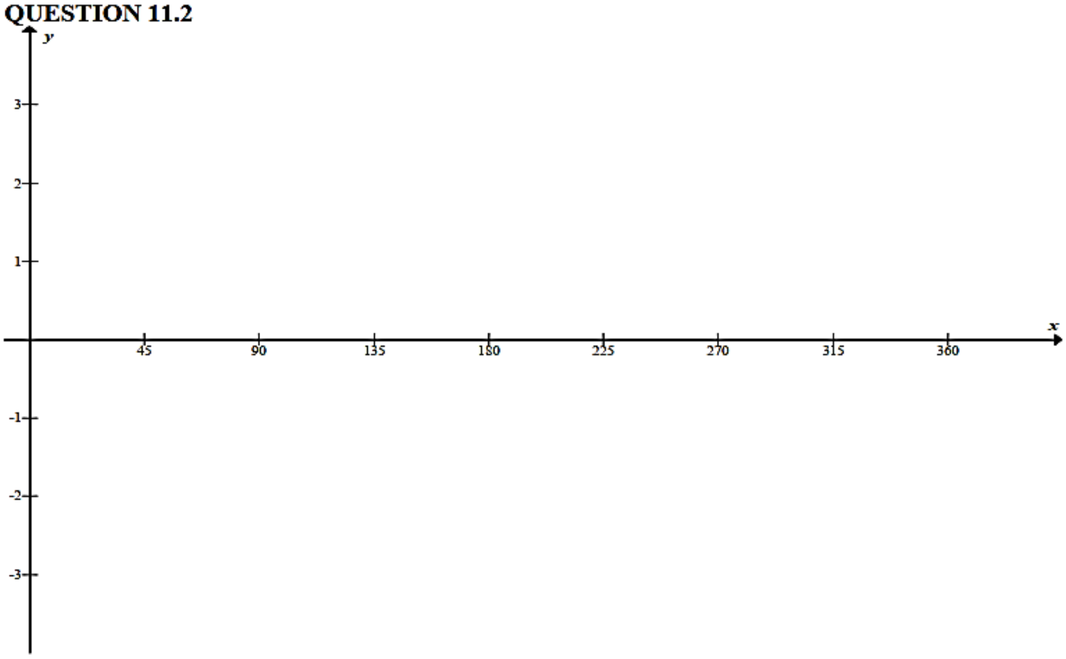
11.1 Calculate the points of intersection of the graphs f and g for $x \in [180°\,;360°]$. (7)

11.2 Draw sketch graphs of f and g for $x \in [180°\,;360°]$ on the same system of axes provided on DIAGRAM SHEET 3. (4)

11.3 For which values of x will $f(x) \le g(x)$ for $x \in [180°\,;360°]$? (3)

DIAGRAM SHEET 3

QUESTION 11.2



PAPER G

QUESTION 10

10.1 If $\sin 28° = a$ and $\cos 32° = b$, determine the following in terms of a and/or b:

10.1.1 $\cos 28°$ (2)

10.1.2 $\cos 64°$ (3)

10.1.3 $\sin 4°$ (4)

10.2 Prove without the use of a calculator, that if $\sin 28° = a$ and $\cos 32° = b$, then $b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2}$. (4)

QUESTION 11

11.1 If $\sin 61° = \sqrt{p}$, determine the following in terms of p:

11.1.1 $\sin 241°$ (2)

11.1.2 $\cos 61°$ (2)

11.1.3 $\cos 122°$ (3)

11.1.4 $\cos 73° \cos 15° + \sin 73° \sin 15°$ (3)

11.2 11.2.1 Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$$ (6)

11.2.2 Determine a value of x in the interval [0° ; 180°] for which the identity is not valid. (2)

11.3 11.3.1 Given: $\sin x = \cos 2x - 1$. Show that $2\sin^2 x + \sin x = 0$. (1)

11.3.2 Determine the general solution of the equation: $\sin x = \cos 2x - 1$. (6)

11.4 Determine the value of:

$$\tan 1° \times \tan 2° \times \tan 3° \times \tan 4° \times ... \times \tan 87° \times \tan 88° \times \tan 89° .$$ (4)

EUCLIDEAN GEOMETRY

PAPER A

A1

QUESTION 7

7.1 Complete the statements below by filling in the missing word(s) so that the statements are CORRECT:

7.1.1 The angle subtended by a chord at the centre of a circle is … . (1)

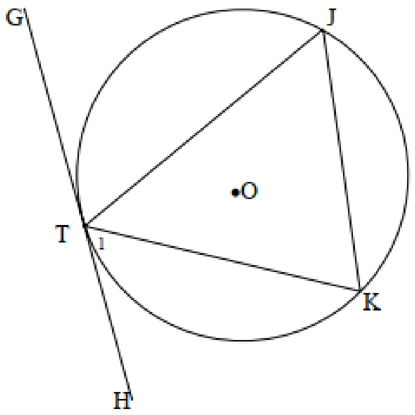
7.1.2 The angle between the tangent and a chord is … . (1)

7.1.3 The opposite angles of a cyclic quadrilateral are … . (1)

A2

QUESTION 8

8.1 In the diagram below O is the centre of the circle. GH is a tangent to the circle at T. J and K are points on the circumference of the circle. TJ, TK and JK are joined.



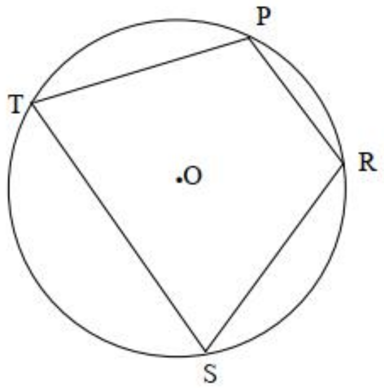
Prove the theorem that states $\hat{T}_1 = T\hat{J}K$. (5)

A3

QUESTION 9

9.1 In the figure below O is the centre of the circle and PRST is a cyclic quadrilateral.

Prove the theorem that states $P\hat{R}S + P\hat{T}S = 180°$.



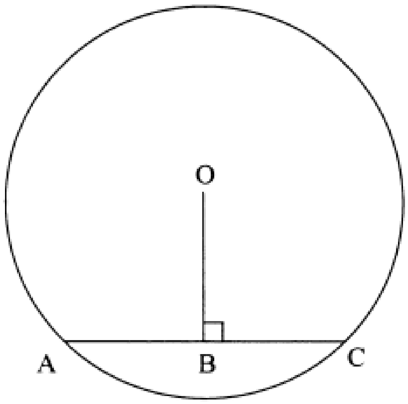
(5)

A4

QUESTION 11

In the diagram below, O is the centre of the circle and OB is perpendicular to the chord AC.

Prove, using Euclidean geometry methods, the theorem that states AB = BC.

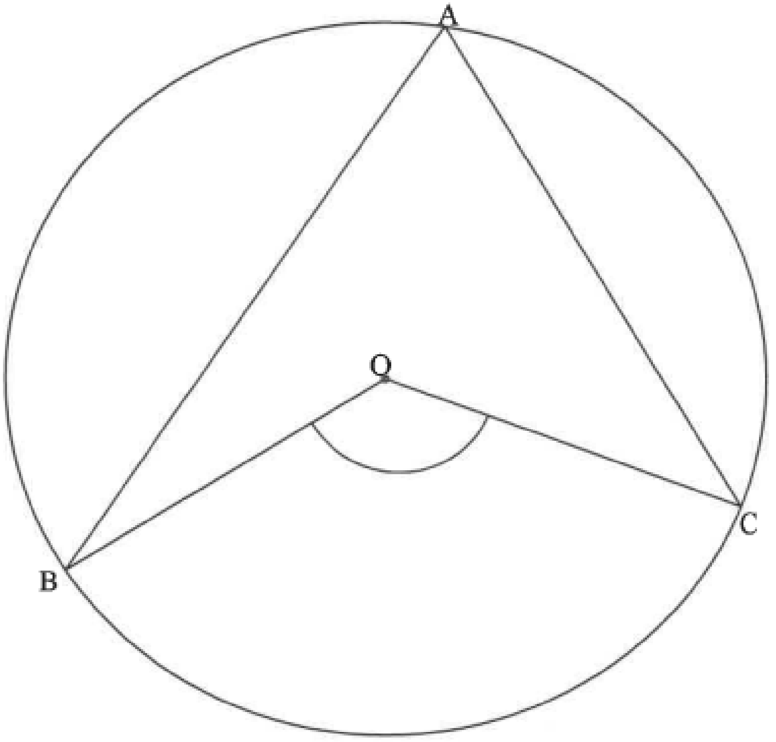


[5]

A5

QUESTION 10

10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.

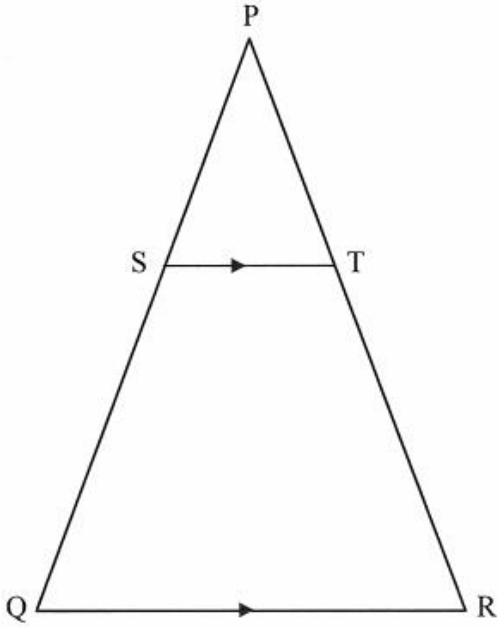


Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

A6

QUESTION 10

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that $ST \parallel QR$.

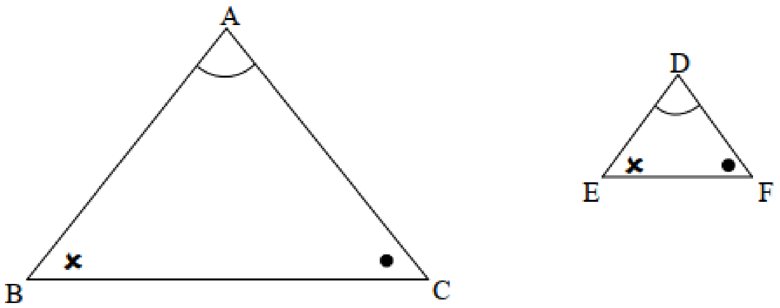


Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

A7

QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.

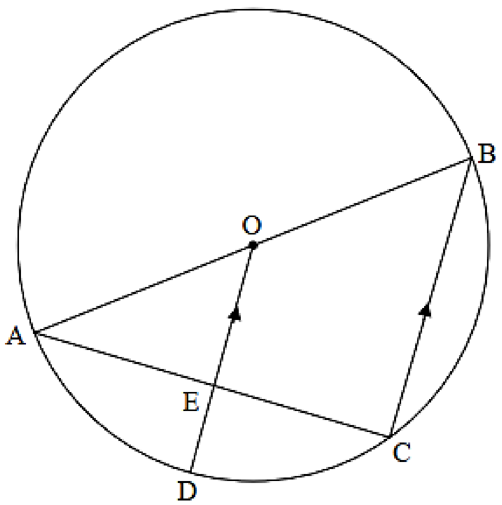


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

PAPER B

QUESTION 9

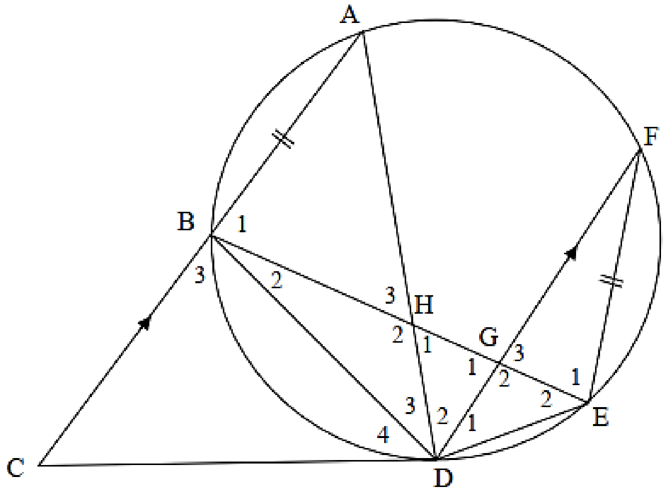
AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.



If the radius is 10 cm and AC = 16 cm, calculate the length of ED. **[5]**

QUESTION 10

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. AC || FD and FE = AB. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



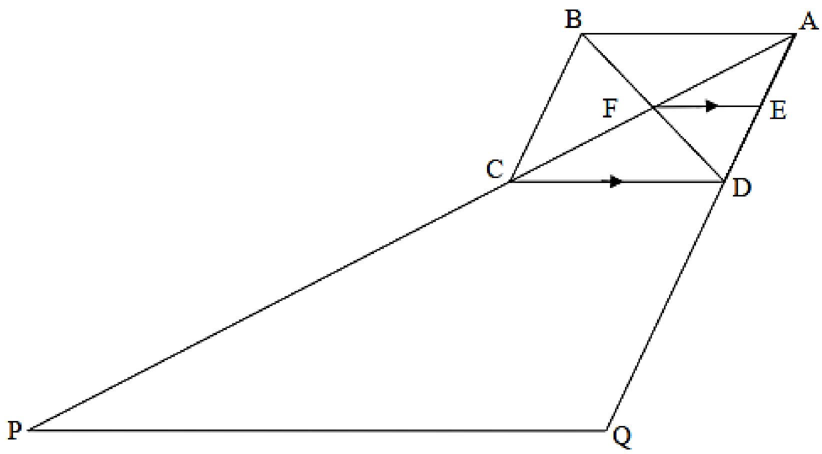
10.1 Determine THREE other angles that are each equal to x. (6)

10.2 Prove that $\Delta BHD \parallel\!| \Delta FED$. (5)

10.3 Hence, or otherwise, prove that AB.BD = FD.BH. (2)

QUESTION 11

ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that PC = 2AC and AD is produced to Q such that DQ = 2AD.



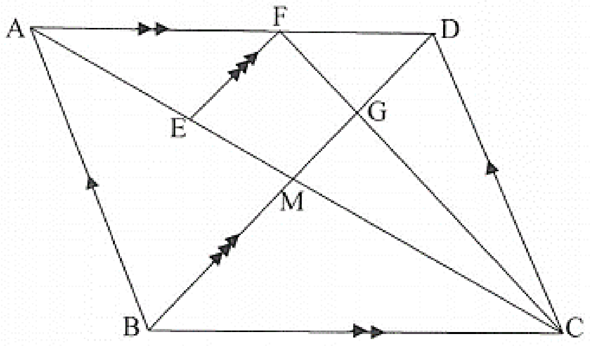
11.1 Show that E is the midpoint of AD. (2)

11.2 Prove PQ || FE. (3)

11.3 If PQ is 60 cm, calculate the length of FE. (5)

PAPER C

9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF : FD = 4 : 3. E is a point on AM such that EF || BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\dfrac{EM}{AM}$ (3)

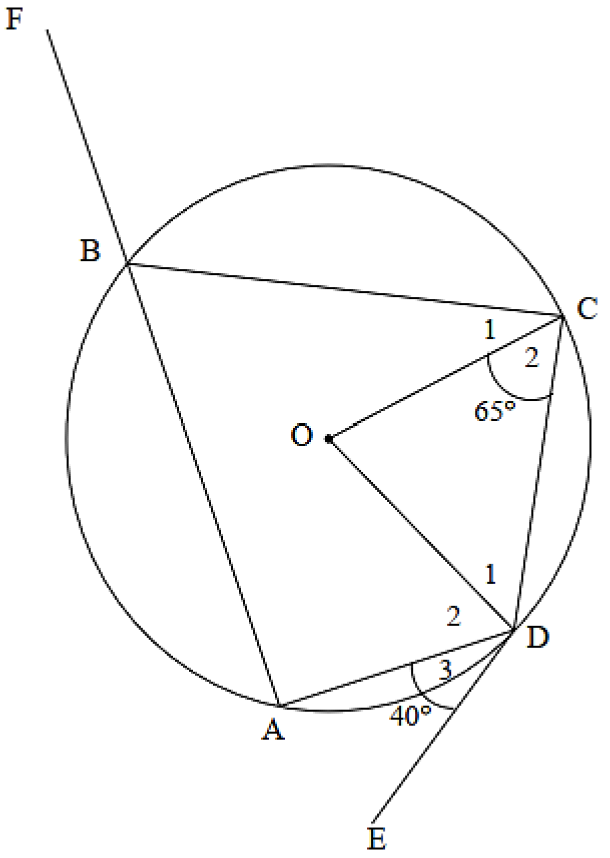
9.2.2 $\dfrac{CM}{ME}$ (3)

9.2.3 $\dfrac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4)

PAPER D

QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O. ED is a tangent to the circle at D. Chord AB is produced to F. Radii OC and OD are drawn. $A\hat{D}E = 40°$ and $\hat{C}_2 = 65°$,

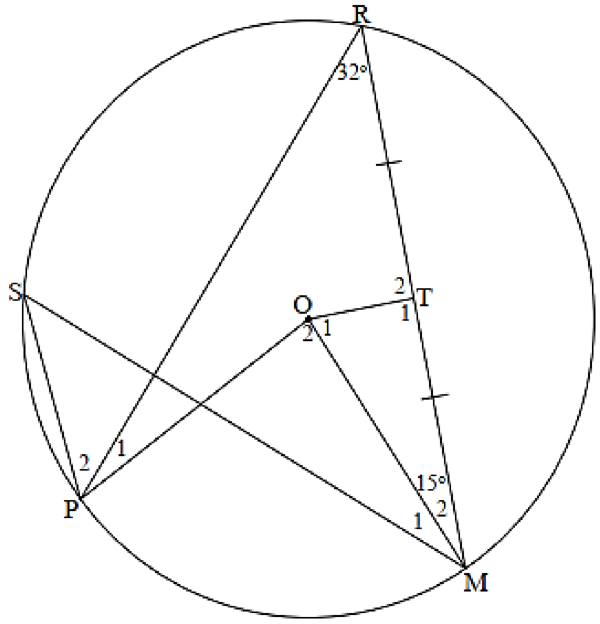


Determine, giving reasons, the size of each of the following angles:

8.1.1 $\hat{D}_2$ (3)

8.1.2 $F\hat{B}C$ (4)

8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM. $P\hat{R}M = 32°$. SP, SM and radii OP and OM are drawn. $O\hat{M}T = 15°$.



Calculate, with reasons, the size of the angles:

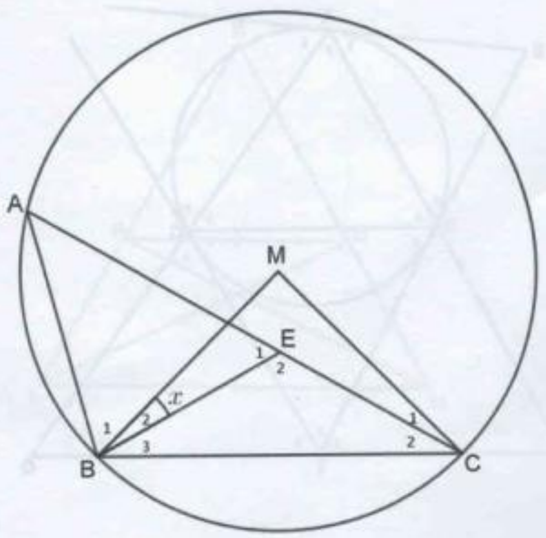
8.2.1 $\hat{S}$ (2)

8.2.2 $\hat{O}_2$ (2)

8.2.3 $\hat{O}_1$ (3)

QUESTION 9

In the diagram, M is the centre of the circle through A, B and C. E is on AC. AC bisects $M\hat{C}B$ and EB bisects $M\hat{B}C$. $\hat{B}_2 = x$



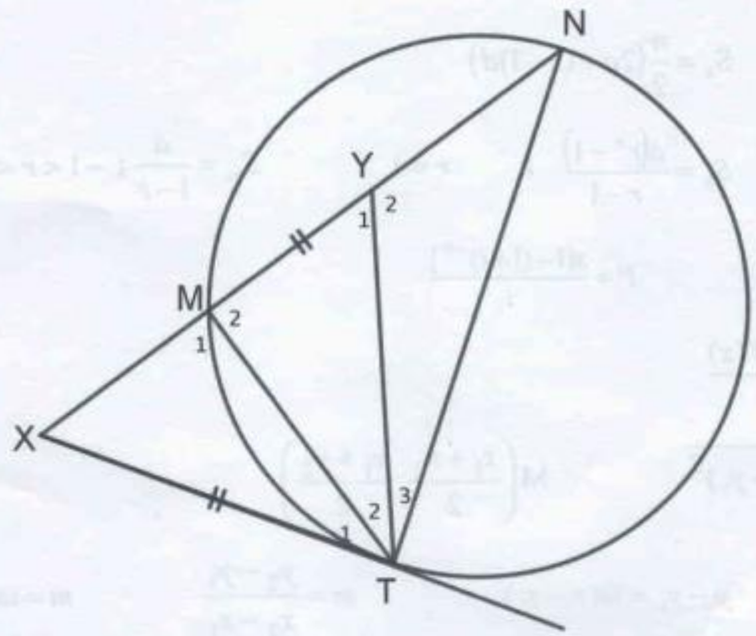
9.1 Determine the size of $\hat{E}_2$ in terms of x. (4)

9.2 Show $B\hat{A}C = 90^\circ - 2x$ (3)

9.3 Prove that AE is a diameter of circle ABE. (5)

QUESTION 10

10.1 In the diagram XMN is a straight line and XT is a tangent to the circle. Y is a point on XN so that XY = XT.



Prove that:

10.1.1 YT bisect $M\hat{T}N$. (5)

10.1.2 $\dfrac{XM}{XT} = \dfrac{XT}{XN}$ (6)

10.2 Given that MY = 20 mm, YN = 50 mm and XT = k mm:

10.2.1 Express XM in terms of k. (3)

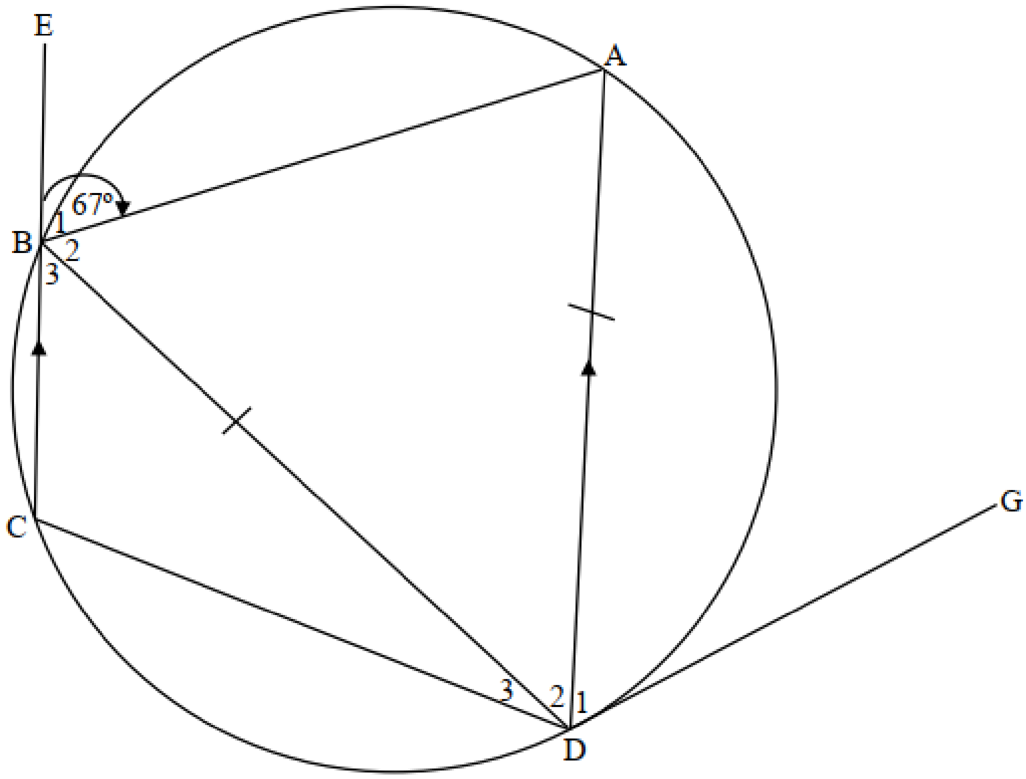
10.2.2 Calculate the length of k. (4)

PAPER E

QUESTION 8

In the diagram below, points A, B, C and D lie on the circumference of a circle with AD || EC. CB is produced to E. GD is a tangent to the circle at D and DB = AD.

$E\hat{B}A = 67°$.



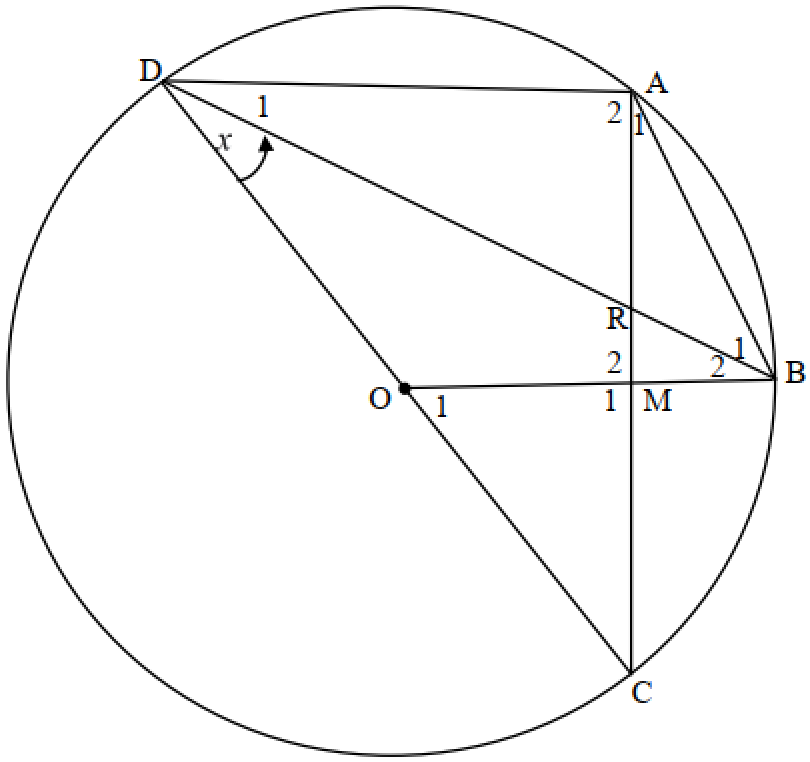
8.1 Calculate, with reasons, the size of the following angles:

8.1.1	$A\hat{D}C$	(2)
8.1.2	$\hat{C}$	(1)
8.1.3	$\hat{A}$	(1)
8.1.4	$\hat{D}_2$	(3)
8.1.5	$B\hat{D}G$	(2)

8.2 Prove that AB = CD. (2)

QUESTION 9

9.1 In the diagram below, A, B, C and D are points on a circle with centre O. OB intersects AC at M, the midpoint of chord AC.
Let $B\hat{D}C = x$.



9.1.1 Determine, with reasons, in terms of x:

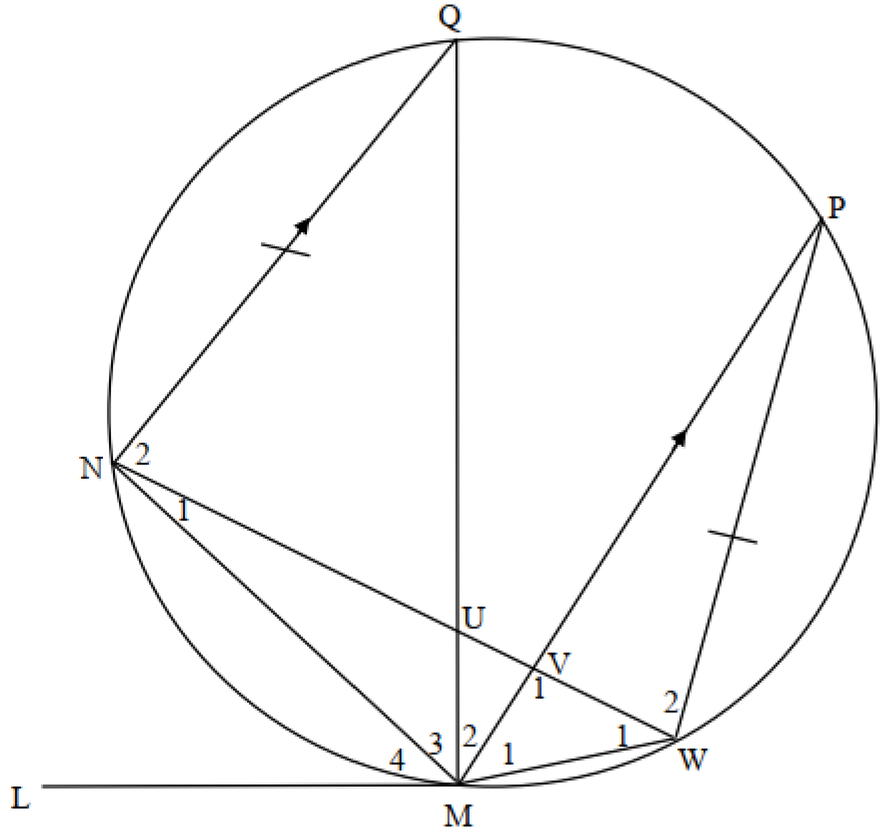
(a) $\hat{O}_1$ (1)

(b) $A\hat{B}O$ (4)

9.1.2 Prove that AB is a tangent to the circle that passes through points A, D and R. (6)

9.1.3 Prove that $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$. (4)

9.2 In the diagram below, LM is a tangent to circle QNMWP at M. NW cuts QM and PM at U and V respectively.
NQ = WP and NQ || MP.

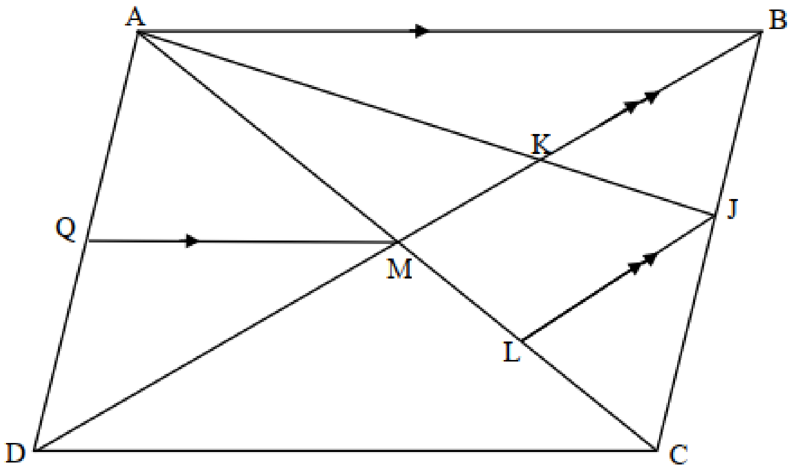


9.2.1 State, with reasons, THREE angles equal to $\hat{M}_2$. (3)

9.2.2 Prove that $\Delta WMV \parallel\!\!| \Delta QMN$. (3)

9.2.3 Prove that $\frac{MV}{WV} = \frac{MN}{PW}$. (3)

10.2 ABCD is a parallelogram with diagonals that intersect at M. J is a point on BC. BJ : JC is 2 : 3. AJ meets BD at K. BD || JL and JL meets AC at L. Q is a point on AD such that AB || QM.



10.2.1 Determine, with reasons, the following ratios:

(a) $\frac{ML}{LC}$ (2)

(b) $\frac{AK}{KJ}$ (3)

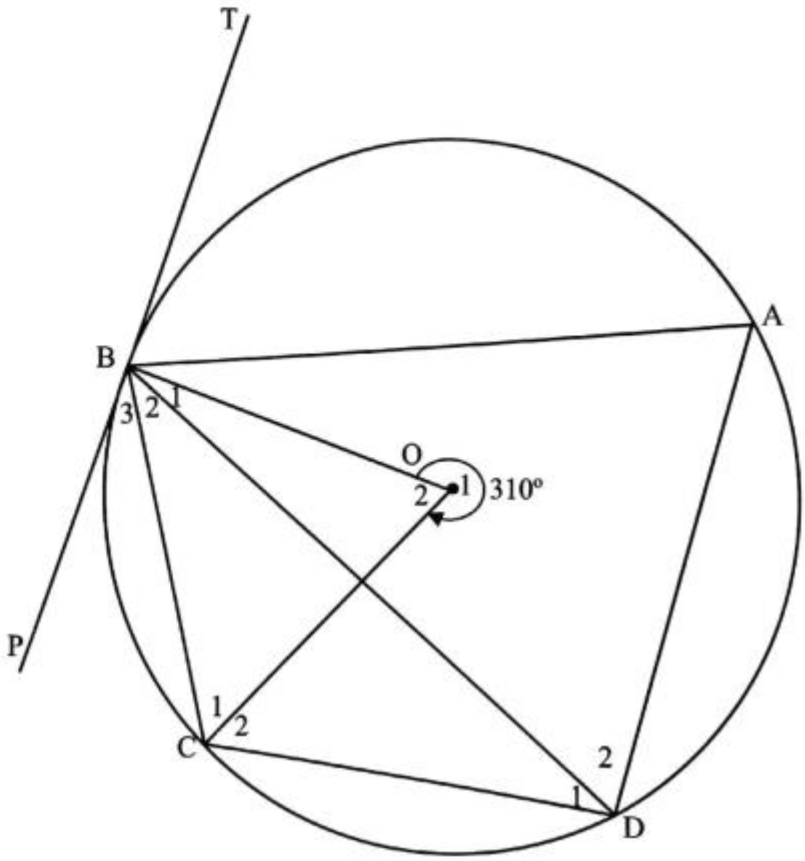
10.2.2 If $AB = \sqrt{10}$ units and $BC = \frac{2}{3}AB$.

Calculate the length of AQ. (4)

PAPER F

QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O.
PBT is a tangent to the circle at B.
Reflex $B\hat{O}C = \hat{O}_1 = 310°$ as shown in the diagram below.



Calculate, giving reasons, the size of:

8.1 $\hat{D}_1$ (3)

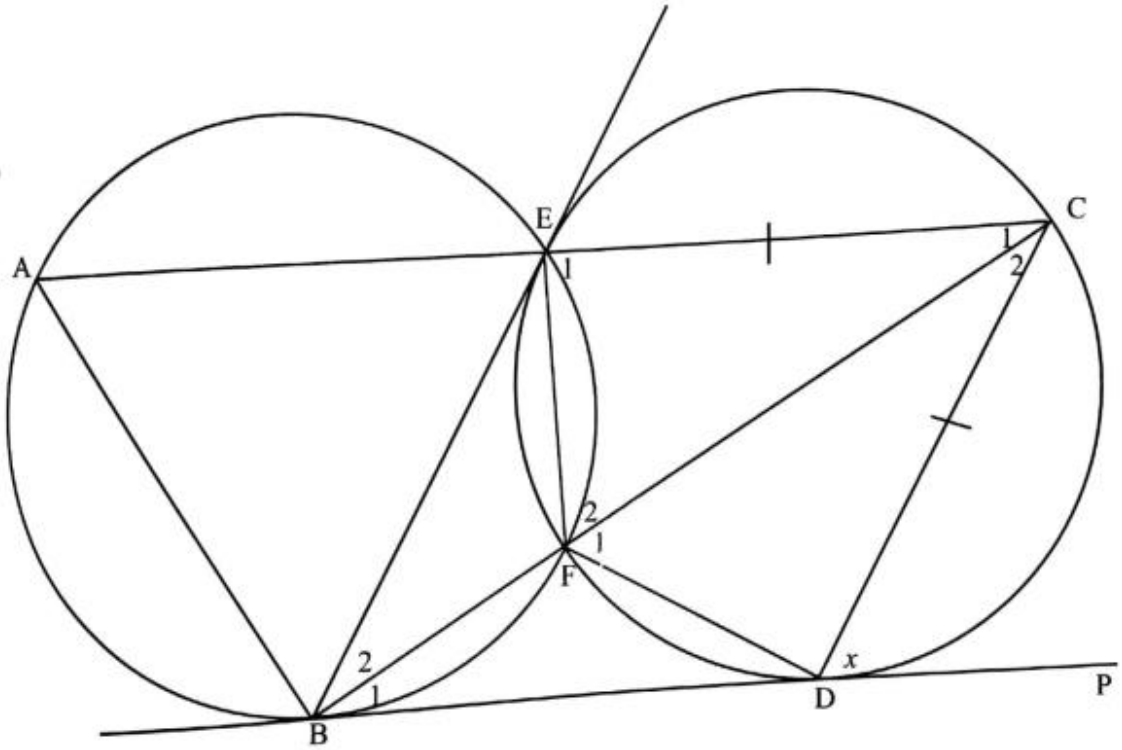
8.2 $\hat{B}_3$ (2)

8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60°$. (4)

QUESTION 9

9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are … (1)

9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. EC = CD.
Let $C\hat{D}P = x$



Prove, giving reasons, that:

9.2.1 $\hat{F}_1 = \hat{F}_2$. (3)

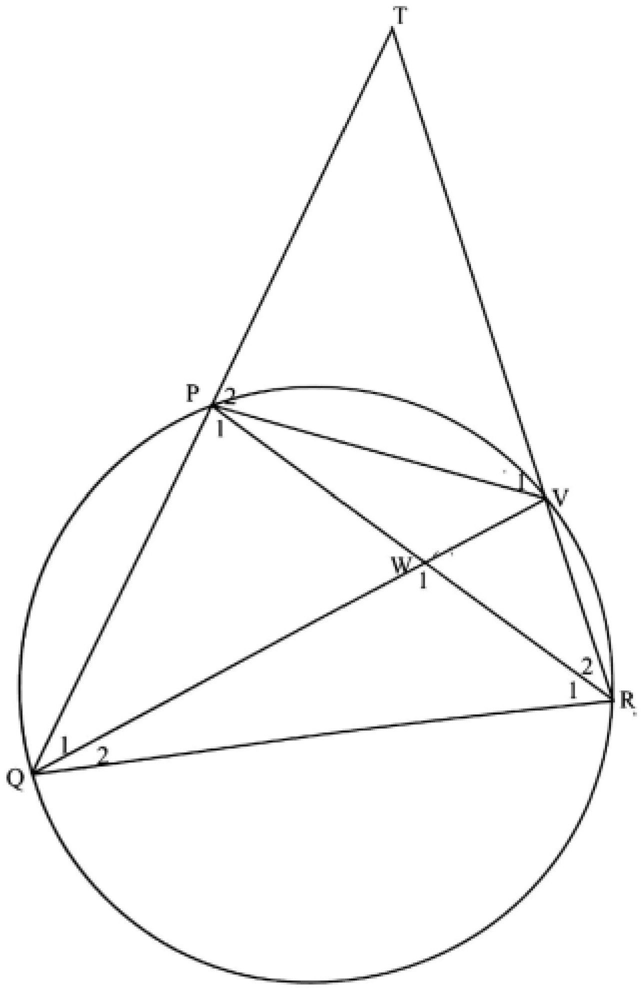
9.2.2 ABDC is a cyclic quadrilateral. (3)

9.2.3 BE || CD. (2)

9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)

Question 10

10.2 In the diagram below, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T. PR and QV intersect at W.



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = T\hat{R}Q$ (3)

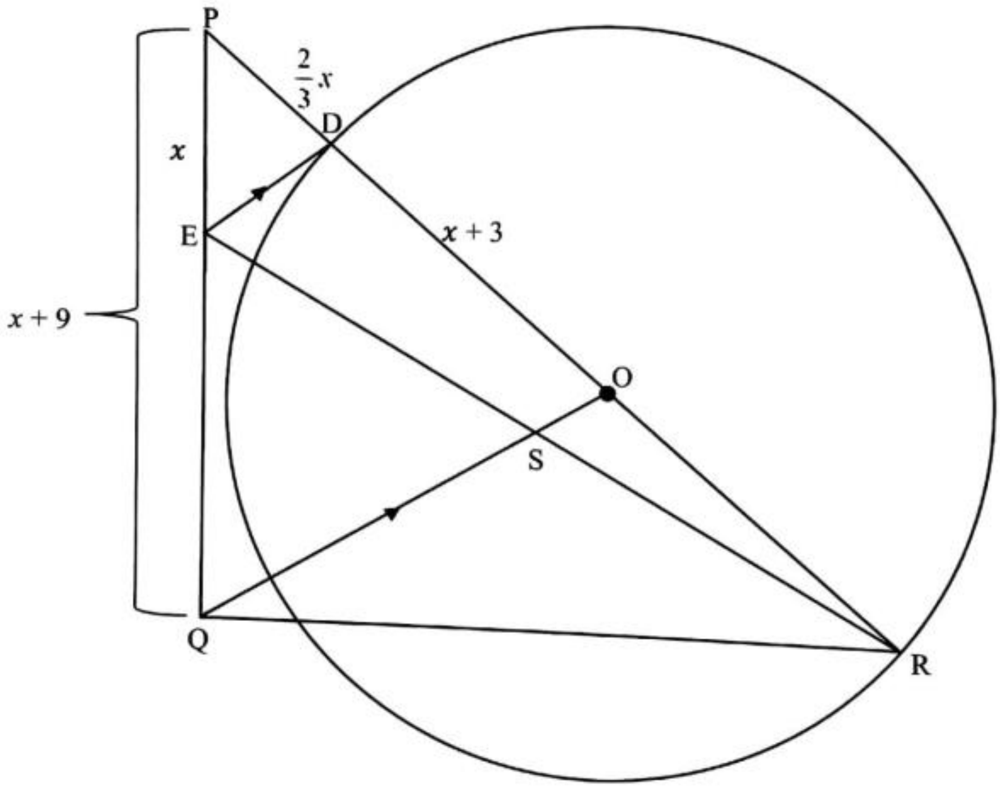
10.2.2 $\Delta TQR \, ||| \, \Delta QRW$ (3)

10.2.3 $\dfrac{PT}{QW} = \dfrac{PV}{WR}$ (6)

QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ.
RD is produced to P and PQ is joined.

PE = x units, PQ = $x + 9$ units, PD = $\frac{2}{3}x$ units and DO = $x + 3$ units.



11.1 Calculate the length of RO. (4)

11.2 If OS = 1,4 units and S is the midpoint of ER, determine the length of DE. (2)

11.3 If the area of $\Delta PED = 2,7$ units^2, find the area of ΔPER. (4)

ANNEXURE A – NOVEMBER 2021 PAPER 1

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 2x - 24 = 0$ (3)

1.1.2 $2x^2 - 3x - 3 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 + 5x \leq -4$ (4)

1.1.4 $\sqrt{x+28} = 2 - x$ (4)

1.2 Solve simultaneously for x and y in:

$2y = 3 + x$ and $2xy + 7 = x^2 + 4y^2$ (6)

1.3 The roots of an equation are $x = \frac{-n \pm \sqrt{n^2 - 4mp}}{2m}$ where m, n and p are positive real numbers. The numbers m, n and p, in that order, form a geometric sequence. Prove that x is a non-real number. (4)

[24]

QUESTION 2

Given the geometric series: $x + 90 + 81 + \ldots$

2.1 Calculate the value of x. (2)

2.2 Show that the sum of the first n terms is $S_n = 1\,000\left(1 - (0{,}9)^n\right)$. (2)

2.3 Hence, or otherwise, calculate the sum to infinity. (2)

[6]

QUESTION 3

Consider the quadratic number pattern: $-145\ ;-122\ ;-101\ ;\ldots$

3.1 Write down the value of T_4. (1)

3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

3.3 Between which TWO terms of the quadratic number pattern will there be a difference of -121? (4)

3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)

[11]

QUESTION 4

Consider the linear pattern: $5\ ;7\ ;9\ ;\ldots$

4.1 Determine T_{51}. (3)

4.2 Calculate the sum of the first 51 terms. (2)

4.3 Write down the expansion of $\sum_{n=1}^{5000}(2n+3)$. Show only the first 3 terms and the last term of the expansion. (1)

4.4 Hence, or otherwise, calculate $\sum_{n=1}^{5000}(2n+3)+\sum_{n=1}^{4999}(-2n-1)$.
ALL working details must be shown. (4)

[10]

QUESTION 5

Given: $f(x)=\dfrac{-1}{x-3}+2$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Write down the domain of f. (1)

5.3 Determine the coordinates of the x-intercept of f. (2)

5.4 Write down the coordinates of the y-intercept of f. (2)

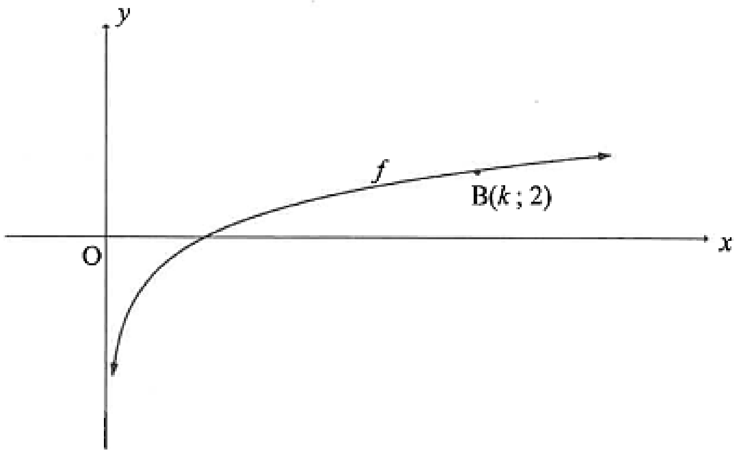
5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)

[10]

QUESTION 6

The graph of $f(x) = \log_4 x$ is drawn below.

B$(k\,;2)$ is a point on f.



6.1 Calculate the value of k. (2)

6.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

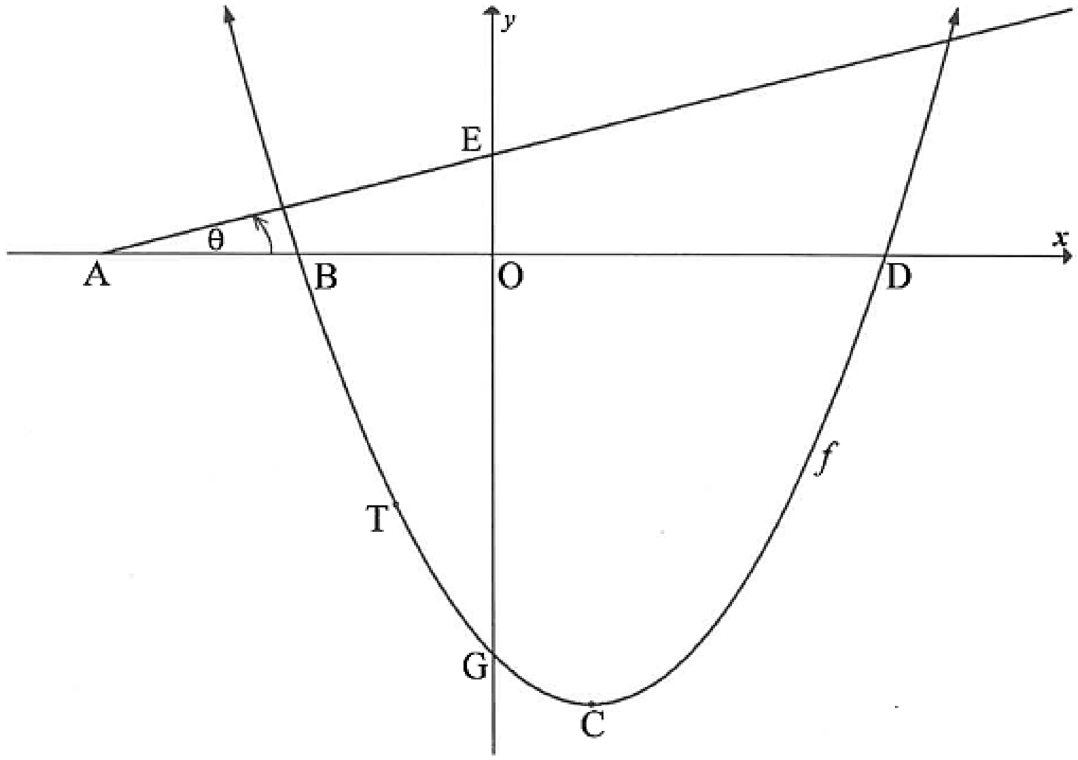
6.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

6.4 For which values of x will $x.f^{-1}(x) < 0$? (2)

[8]

QUESTION 7

The graph of $f(x) = (x+4)(x-6)$ is drawn below.
The parabola cuts the x-axis at B and D and the y-axis at G.
C is the turning point of f.
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.
T is a point on f between B and G.



7.1 Write down the coordinates of B and D. (2)

7.2 Calculate the coordinates of C. (2)

7.3 Write down the range of f. (1)

7.4 Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

7.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

7.4.2 Calculate the coordinates of T. (5)

7.5 A straight line, g, parallel to AE, cuts f at $K(-3\,;-9)$ and R.
Calculate the x-coordinate of R. (6)

[17]

QUESTION 8

8.1 A farmer bought a tractor for R980 000. The value of the tractor depreciates annually at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of the tractor after 7 years. (3)

8.2 How many years will it take for an amount of R75 000 to accrue to R116 253,50 in an account earning interest of 6,8% p.a., compounded quarterly? (4)

8.3 Thabo wanted to save R450 000 as a deposit to buy a house on 30 June 2018.

8.3.1 He deposited a fixed amount of money at the end of every month into an account earning interest of 8,35% p.a., compounded monthly. His first deposit was made on 31 July 2013 and his 60th deposit on 30 June 2018. Calculate the amount he deposited monthly. (3)

8.3.2 Thabo bought a house costing R1 500 000 and used his savings as the deposit. He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years. He made his first monthly instalment of R11 058,85 towards the loan on 31 July 2018.

(a) What will the balance outstanding on the loan be on 30 June 2039, 21 years after the loan was granted? (3)

(a) Calculate the interest Thabo will have paid over the first 21 years of the loan. (3)

[16]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

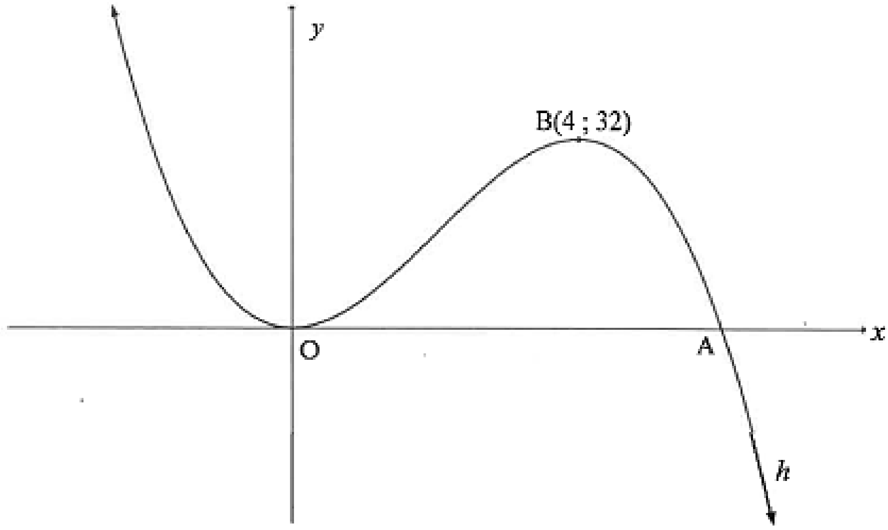
9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.



10.1 Show that $a = -1$ and $b = 6$. (5)

10.2 Calculate the coordinates of A. (3)

10.3 Write down the values of x for which h is:

10.3.1 Increasing (2)

10.3.2 Concave down (2)

10.4 For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots? (3)

[15]

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2 + \frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

QUESTION 12

12.1 A and B are independent events. It is further given that:

P(A and B) = 0,3 and P(only B) = 0,2

12.1.1 Are A and B mutually exclusive? Motivate your answer. (1)

12.1.2 Determine:

(a) P(only A) (4)

(b) P(not A or not B) (2)

12.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

12.2.1 Write down the probability that a novel will be the first book placed on the shelf. (1)

12.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position. (2)

12.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position. (4)

[14]

TOTAL: 150

ANNEXURE B – NOVEMBER 2021 PAPER 2

QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

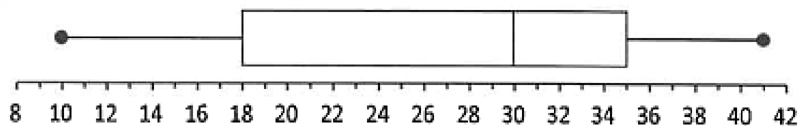
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

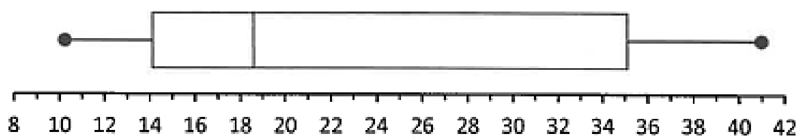
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)

[10]

QUESTION 2

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)

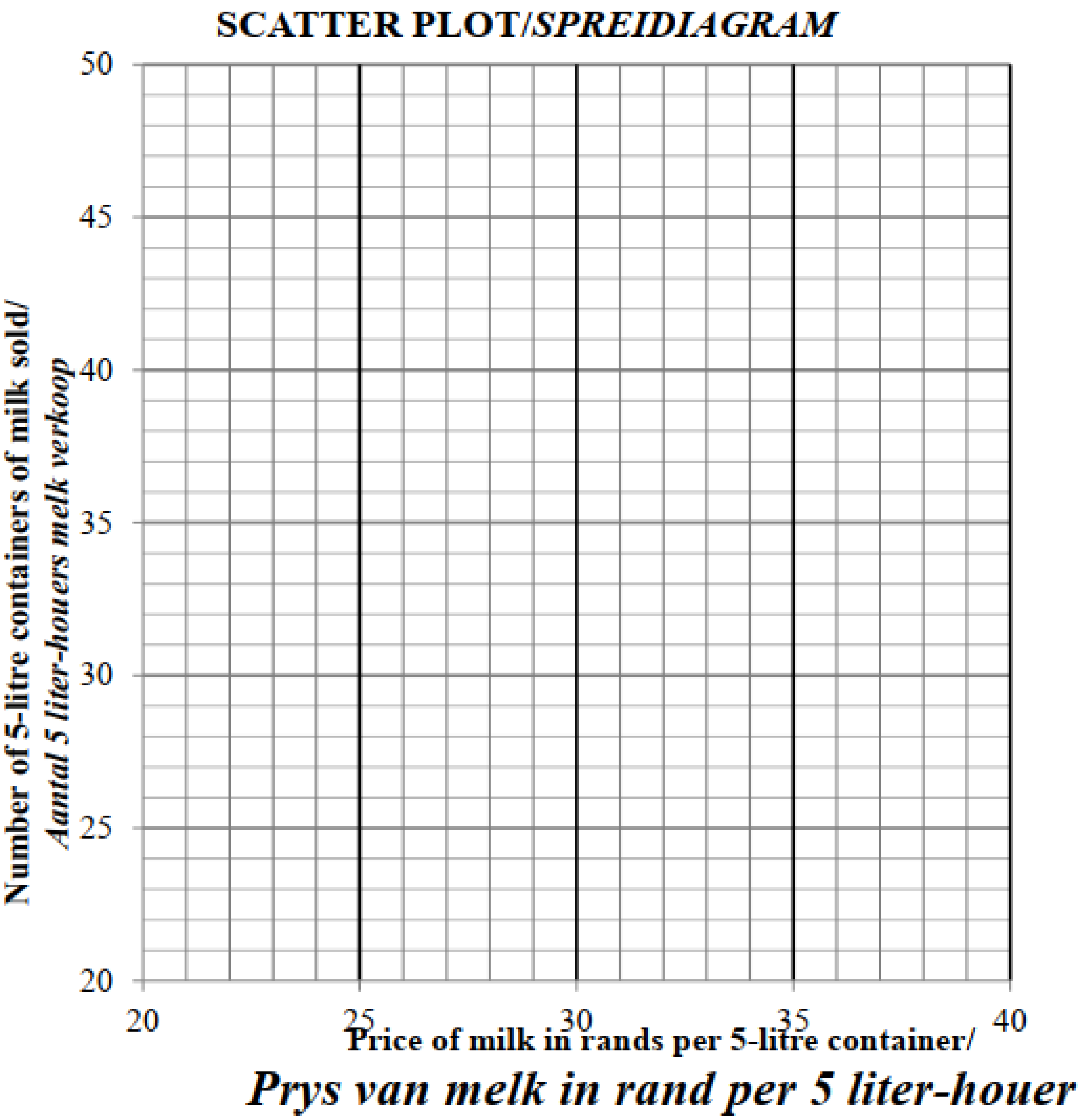
2.2 Determine the equation of the least squares regression line for the data. (3)

2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)

2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)

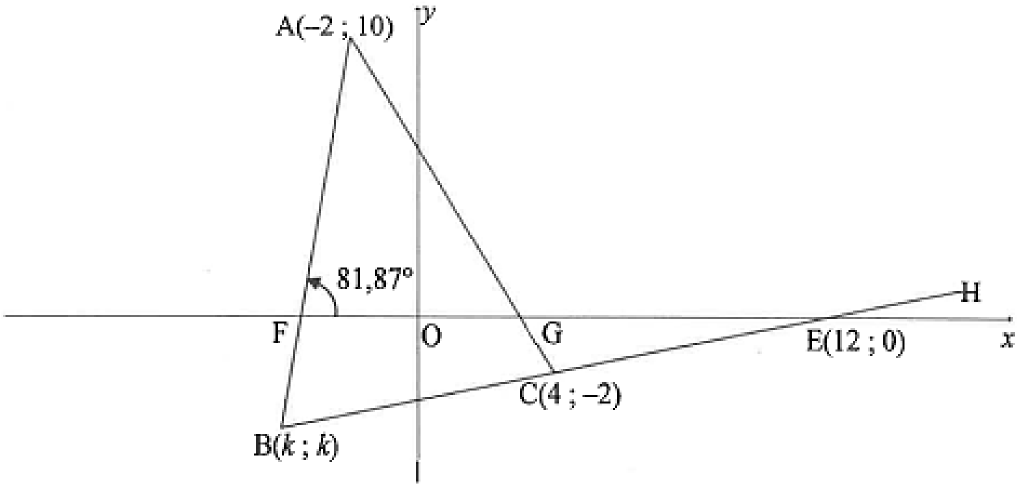
[10]

GRID FOR QUESTION 2.1



QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

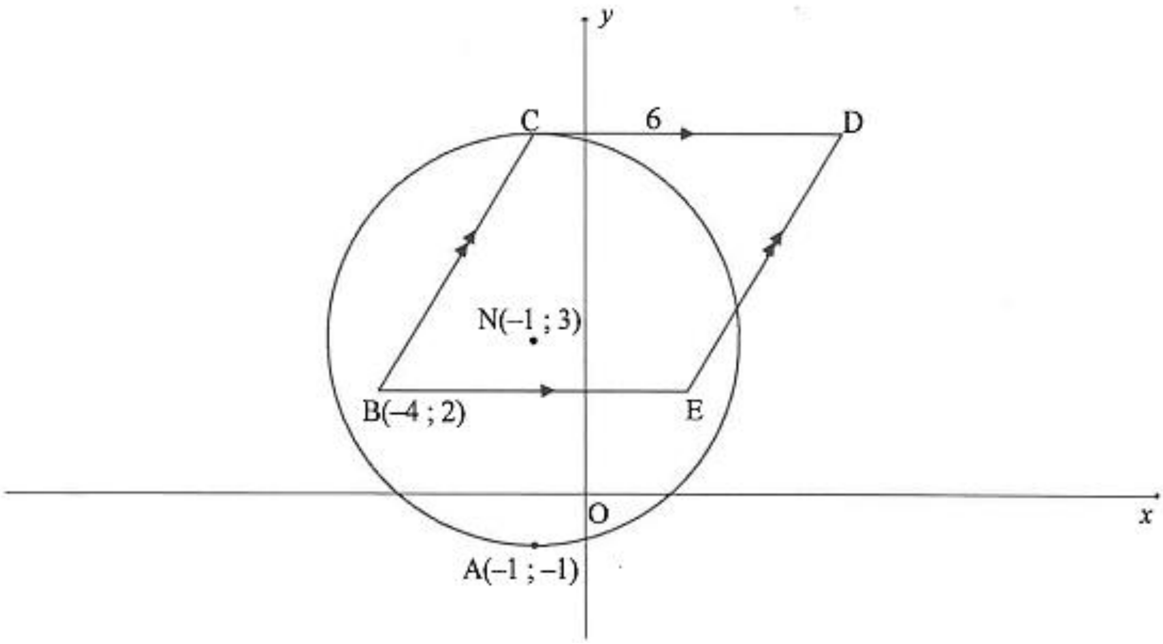
(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

(b) Tangent to the circle at point B(k ; k) (3)

[24]

QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1	Write down the length of the radius of the circle.		(1)
4.2	Calculate the:		
	4.2.1	Coordinates of C	(2)
	4.2.2	Coordinates of D	(2)
	4.2.3	Area of ΔBCD	(3)
4.3	The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.		
	Calculate the:		
	4.3.1	Length of NM	(3)
	4.3.2	Midpoint of AF	(4)
			[15]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$$ (6)

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)
[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

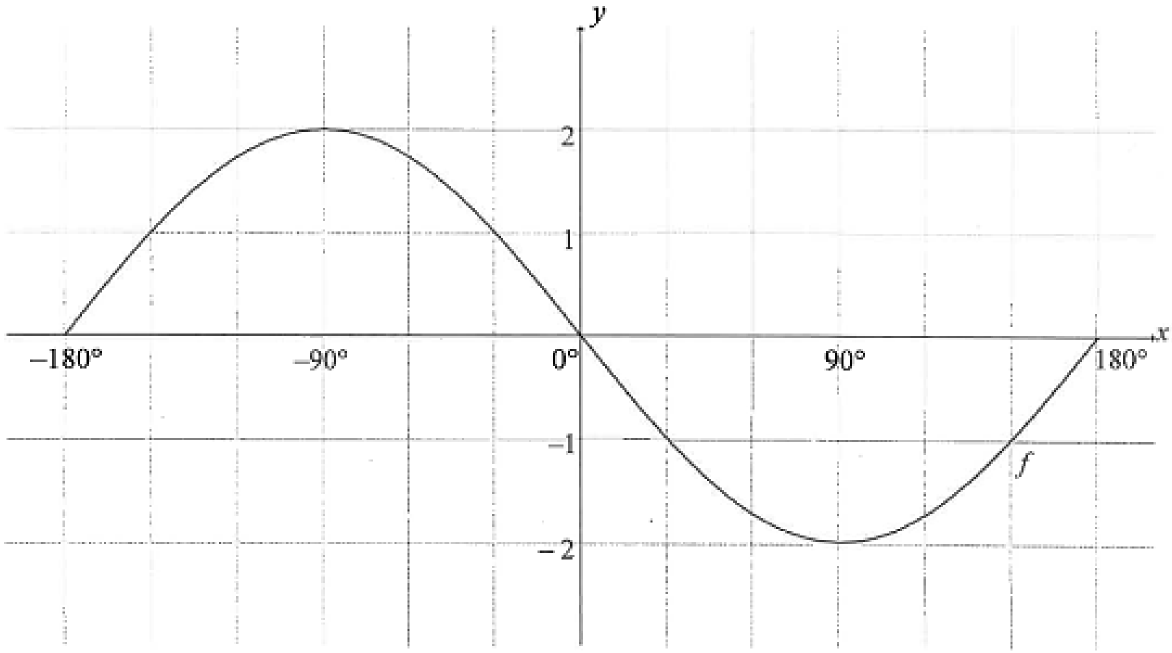
6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

6.1.2 Simplify completely: $2\cos 6x \cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)
[15]

QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180°\,;\,180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180°\,;\,180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

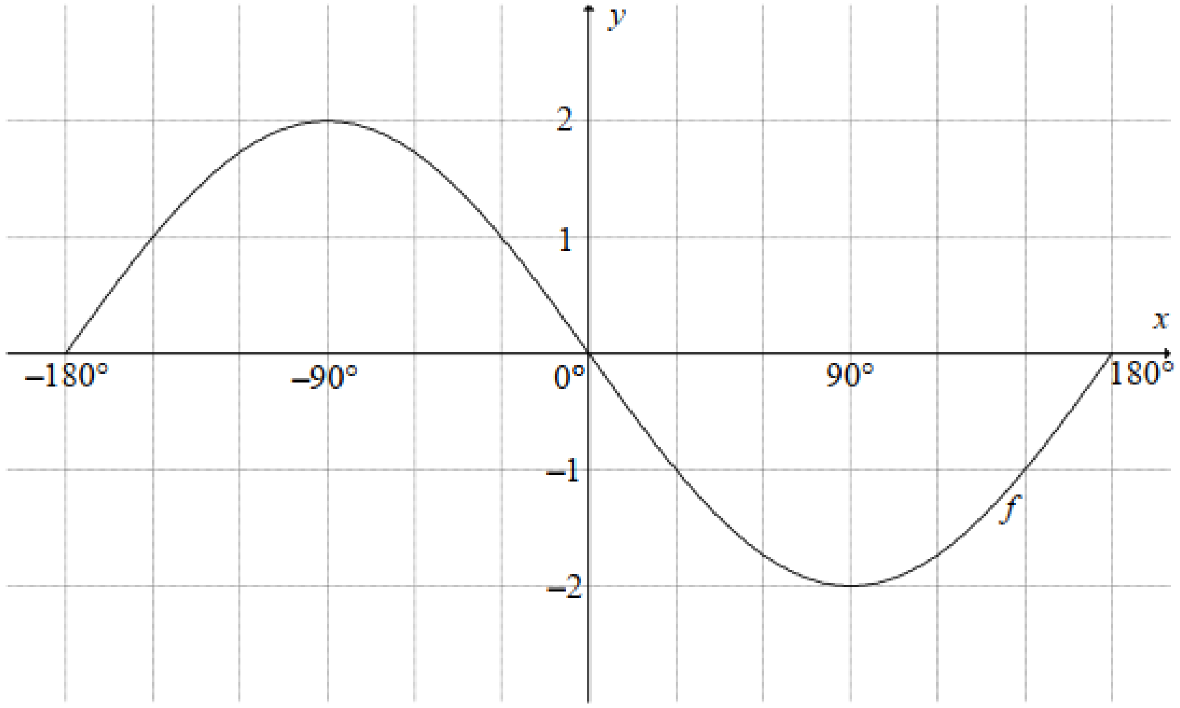
7.2 Write down the period of $f(3x)$. (2)

7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180°\,;\,180°]$ for which $f(x) - g(x) = 1$. (1)

7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)

[8]

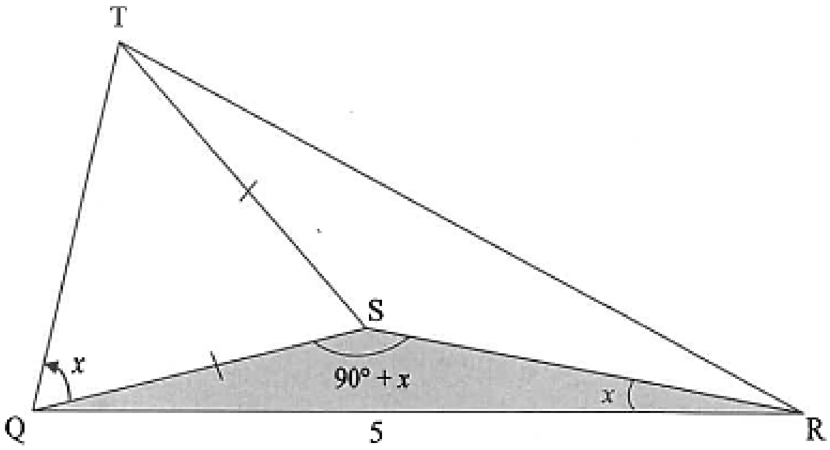
GRID FOR QUESTION 7.1



QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.

$Q\hat{S}R = 90° + x$, $Q\hat{R}S = x$, QR = 5 units and TS = SQ.



8.1 Prove that $QS = 5 \tan x$ (3)

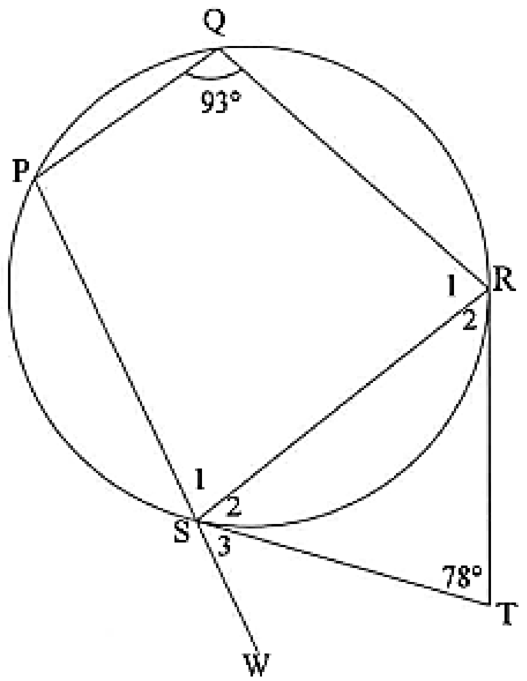
8.2 Prove that the length of $QT = 10 \sin x$ (5)

8.3 Calculate the area of ΔTQR if $T\hat{Q}R = 70°$ and $x = 25°$. (2)

[10]

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T}=78°$ and $\hat{Q}=93°$.



9.1 Give a reason why $ST = TR$. (1)

9.2 Calculate, giving reasons, the size of:

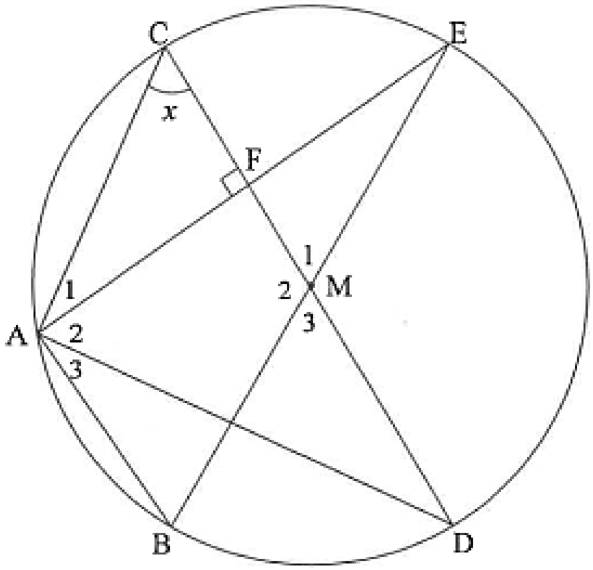
9.2.1 $\hat{S}_2$ (2)

9.2.2 $\hat{S}_3$ (2)

[5]

QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. $AE \perp CD$. Let $\hat{C} = x$.



10.1 Give a reason why $AF = FE$. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

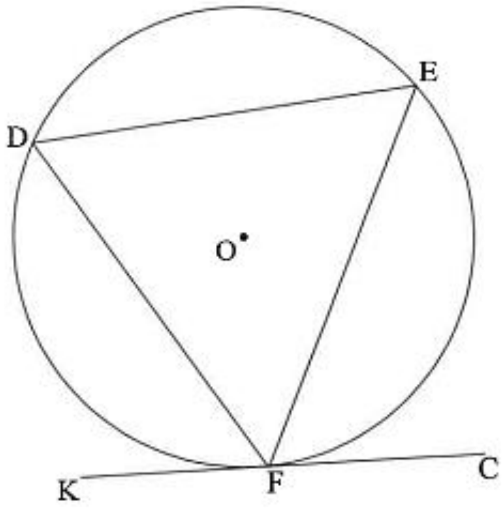
10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

10.4 Given that $CF = 6$ units and $AB = 24$ units, calculate, giving reasons, the length of AE. (5)

[13]

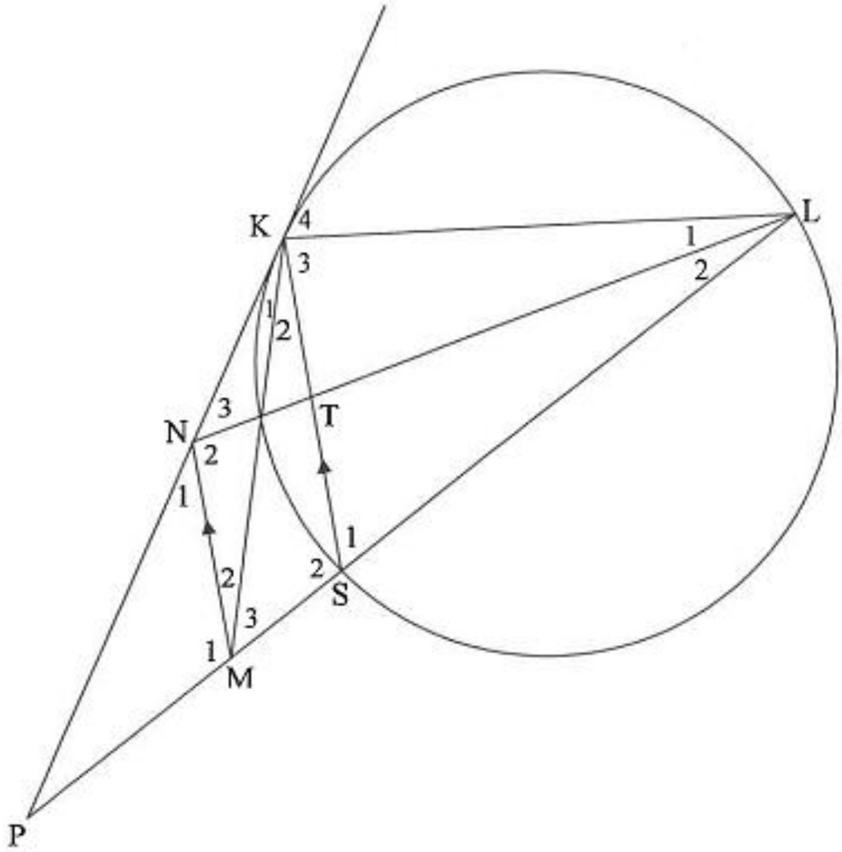
QUESTION 11

11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN ∥ SK. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\Delta LKN \mathbin{|||} \Delta KSM$. (5)

11.2.3 If $LK = 12$ units and $3KN = 4SM$, determine the length of KS. (4)

11.2.4 If it is further given that $NL = 16$ units, $LS = 13$ units and $KN = 8$ units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150

ANNEXURE C – NOVEMBER 2022 PAPER 1

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+2)=0$ (2)

1.1.2 $2x^2-6x+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-90>x$ (4)

1.1.4 $x-7\sqrt{x}=-12$ (4)

1.2 Solve for x and y simultaneously:

$2x-y=2$
$xy=4$ (5)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all positive integer values of n. (3)

1.4 Determine the values of x and y if: $\frac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[25]

QUESTION 2

2.1 The first term of a geometric series is 14 and the 6^{th} term is 448.

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.1.3 If the first term of another series is 448 and the 6^{th} term is 14, calculate the sum to infinity of the new series. (3)

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k. (5)

[14]

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

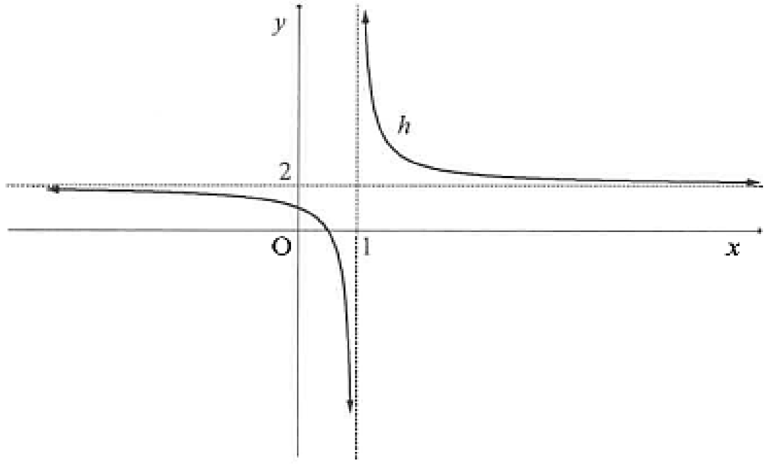
3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

[10]

QUESTION 4

4.1 Sketched below is the graph of $h(x) = \dfrac{1}{x+p} + q$. The asymptotes of h intersect at (1 ; 2).



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

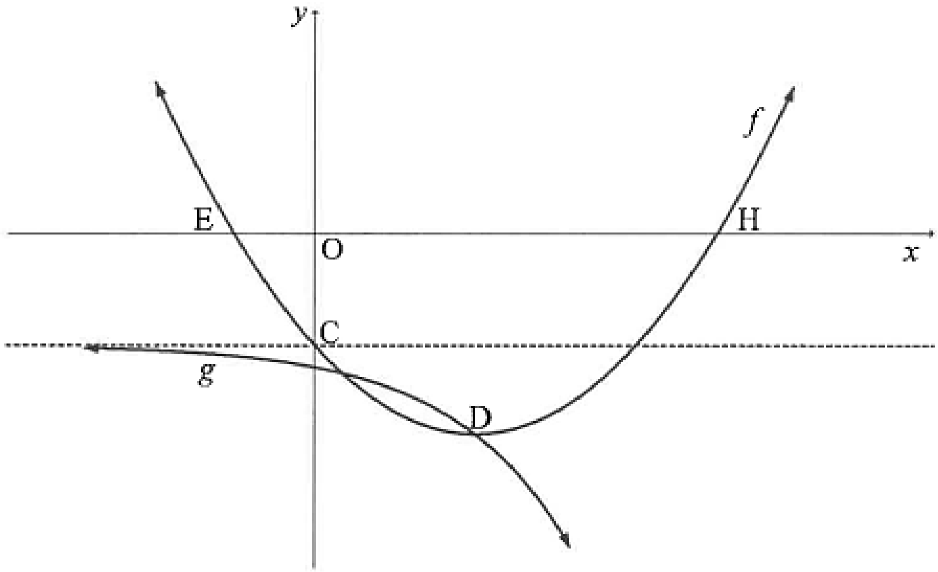
4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x) = h(x+3)$. (2)

4.1.4 The equation of an axis of symmetry of h is $y = x + t$. Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2 \leq \dfrac{1}{x-1}$. (3)

4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.

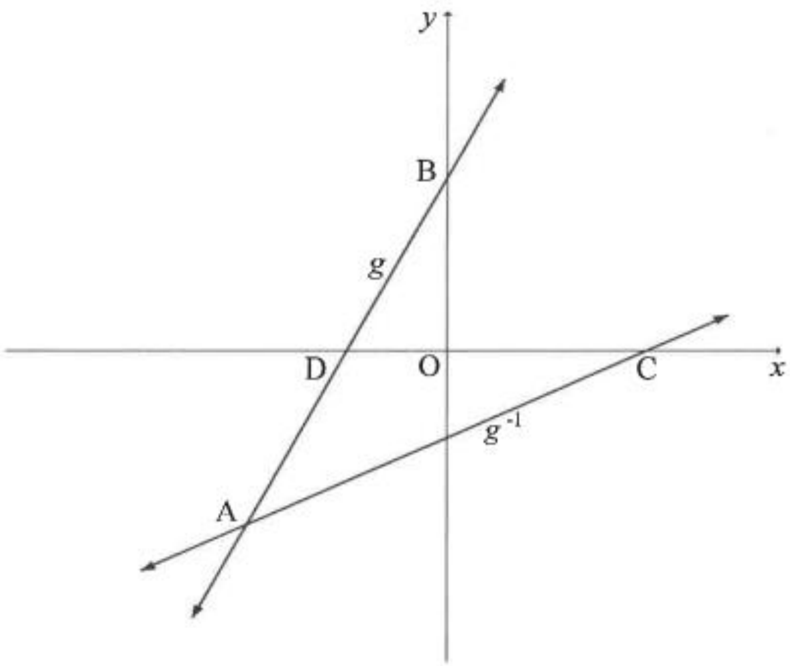


4.2.1	Write down the y-coordinate of C.	(1)
4.2.2	Determine the coordinates of D.	(2)
4.2.3	Determine the values of a and q.	(3)
4.2.4	Write down the range of g.	(1)
4.2.5	Determine the values of k for which the value of $f(x) - k$ will always be positive.	(2)
		[20]

QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

5.3 Determine the coordinates of A. (3)

5.4 Calculate the length of AB. (2)

5.5 Calculate the area of ΔABC. (5)

[13]

QUESTION 6

6.1 R12 000 was invested in a fund that paid interest at $m\%$ p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)

[14]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

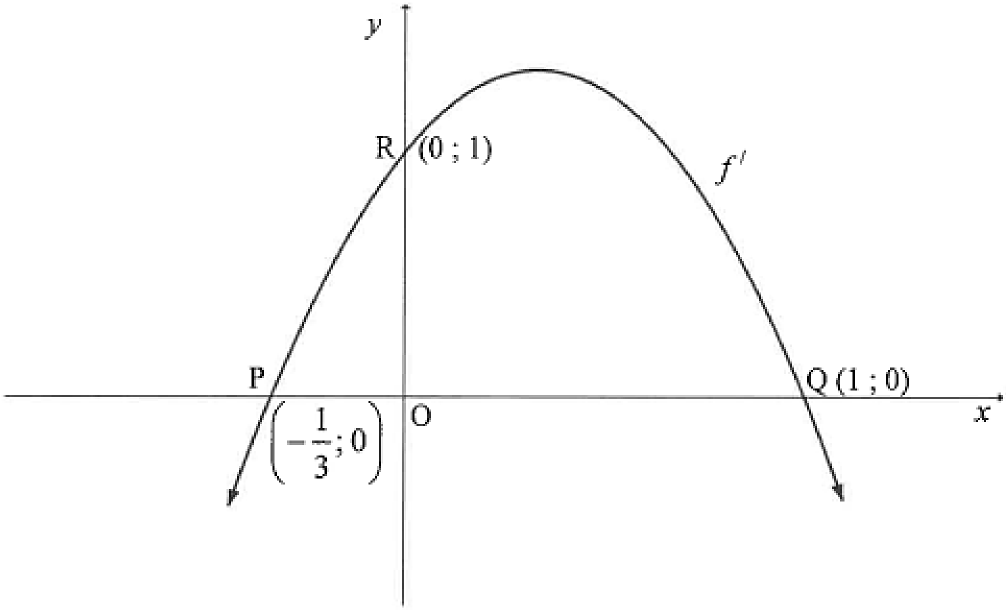
7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1\,;-7)$. For which values of x will g be concave up? (4)

[12]

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}\,;0\right)$, $Q(1\,;0)$ and $R(0\,;1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at $D(a\,;b)$.

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

[17]

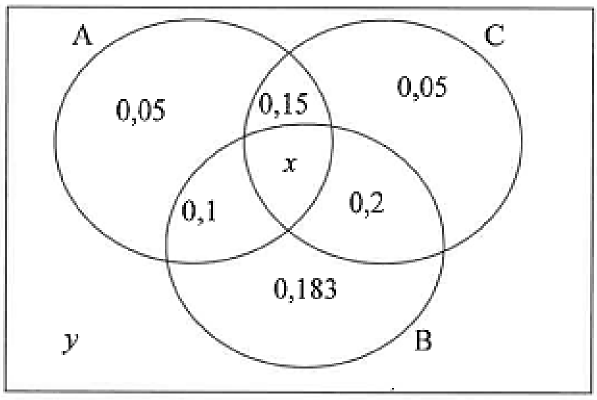
QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10\,;2)$ and a point on f. **[8]**

QUESTION 10

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)

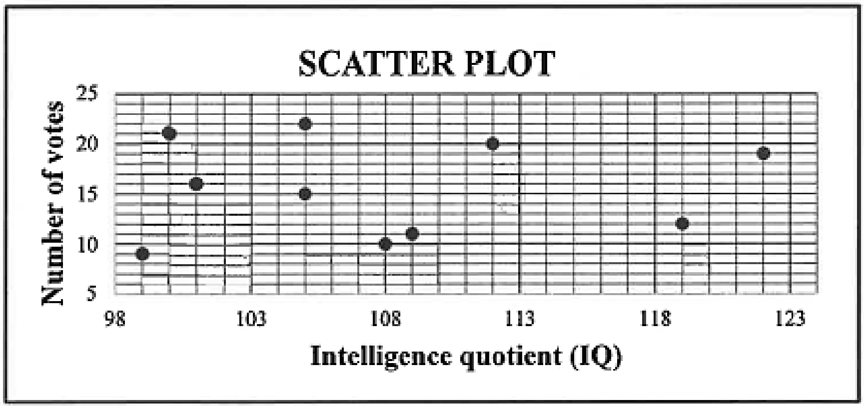
[17]

TOTAL: 150

ANNEXURE D – NOVEMBER 2022 PAPER 2

QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

1.1.1 Mean number of votes that these 10 learners received (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received (1)

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)

1.5 Using the scatter plot and table above, provide a reason why:

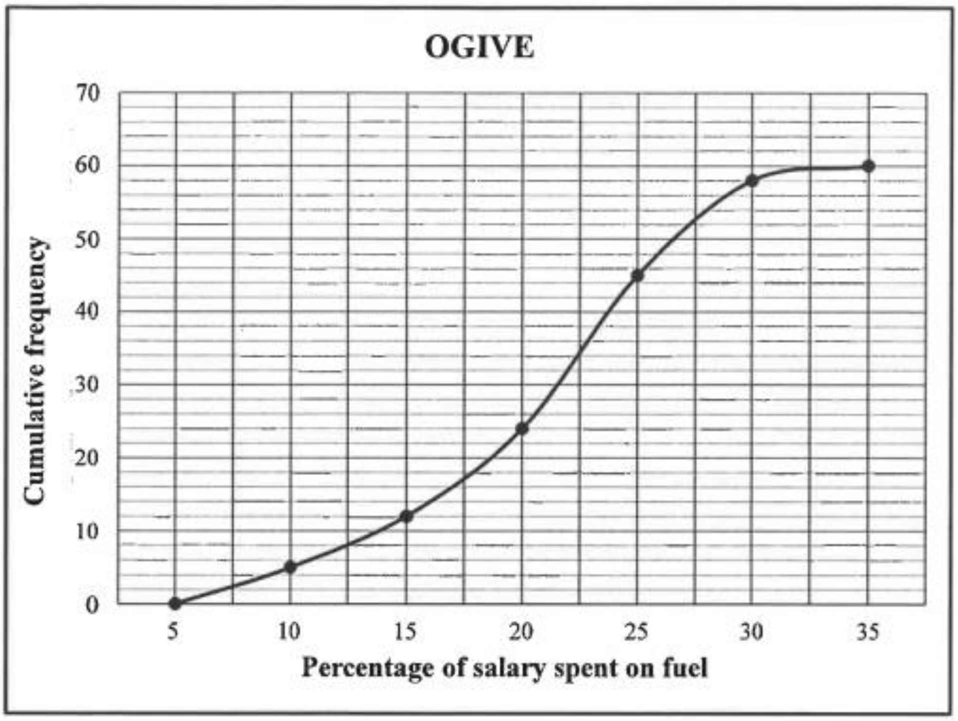
1.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1)

1.5.2 The prediction in QUESTION 1.4 is reliable (1)

[12]

QUESTION 2

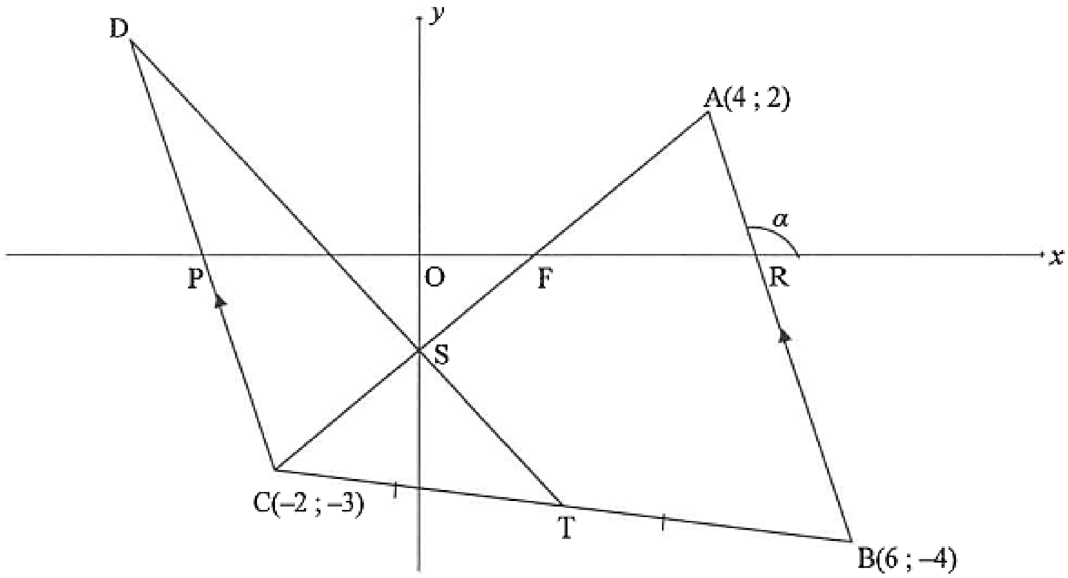
A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1	How many people are employed at this company?	(1)
2.2	Write down the modal class of the data.	(1)
2.3	How many employees spent more than 22,5% of their monthly salary on fuel?	(2)
2.4	An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel.	(2)
2.5	The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change?	(2)
		[8]

QUESTION 3

In the diagram, A(4 ; 2), B(6 ; –4) and C(–2 ; –3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x-6y=8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD ∥ BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

3.2 Determine the equation of CD in the form $y=mx+c$. (3)

3.3 Calculate the:

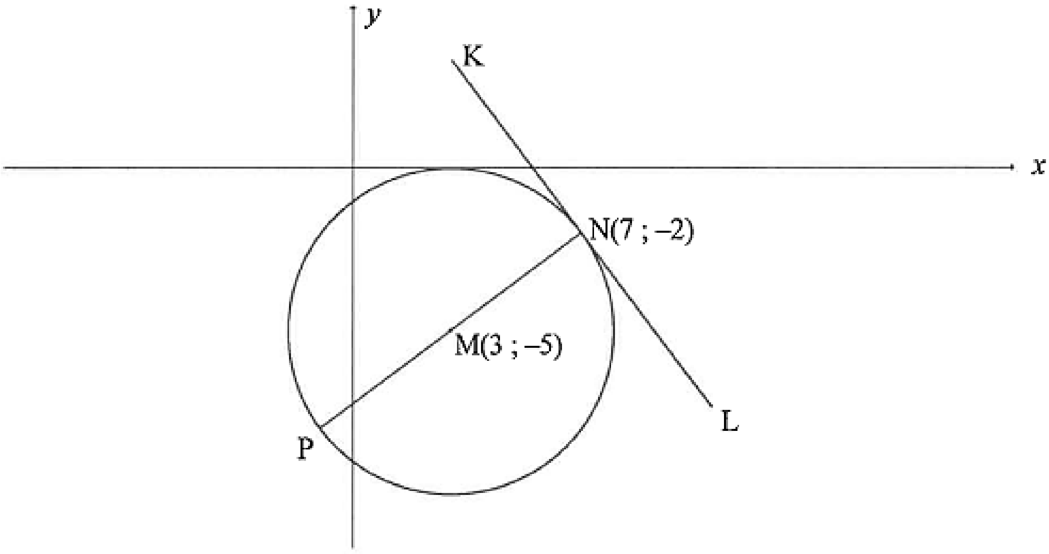
3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)

[20]

QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)

[20]

QUESTION 5

5.1 Given that $\sqrt{13}\sin x + 3 = 0$, where $x \in (90^\circ ; 270^\circ)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360^\circ + x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180^\circ + x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90^\circ + \theta)}{\sin(\theta - 180^\circ) + 3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ (6)

5.4 Given the identity: $\cos(x + y).\cos(x - y) = 1 - \sin^2 x - \sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1 - \sin^2 45^\circ - \sin^2 15^\circ$, **without using a calculator.** (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x - 8\sin x.\cos x$

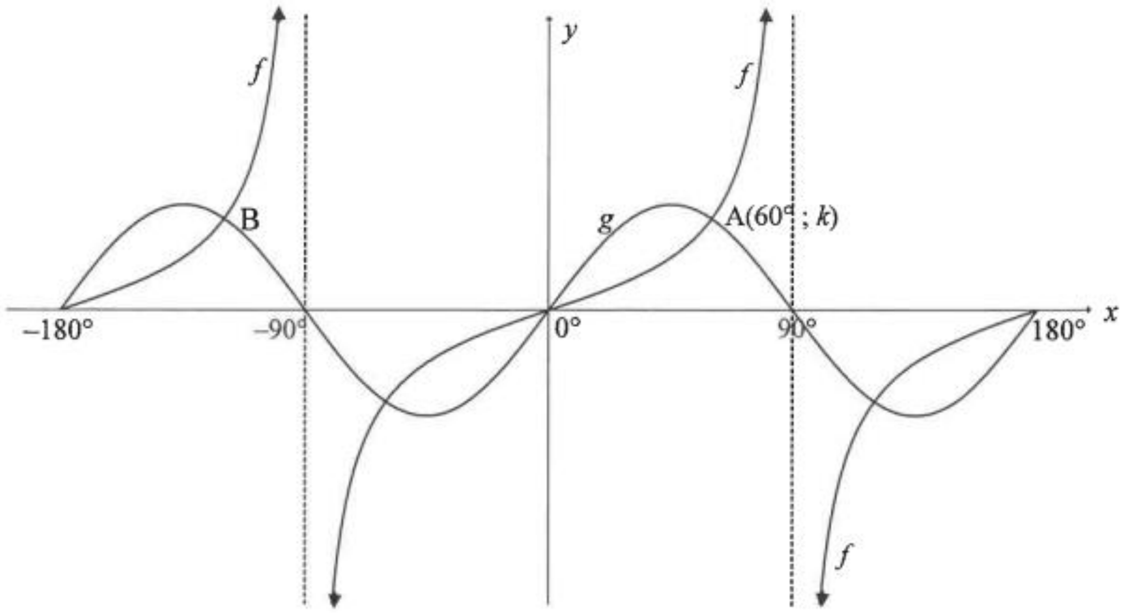
5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 For which value of x in the interval $x \in [0^\circ ; 90^\circ]$ will $16\sin x.\cos^3 x - 8\sin x.\cos x$ have its minimum value? (1)

[30]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180° ; 180°]$ A$(60° ; k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

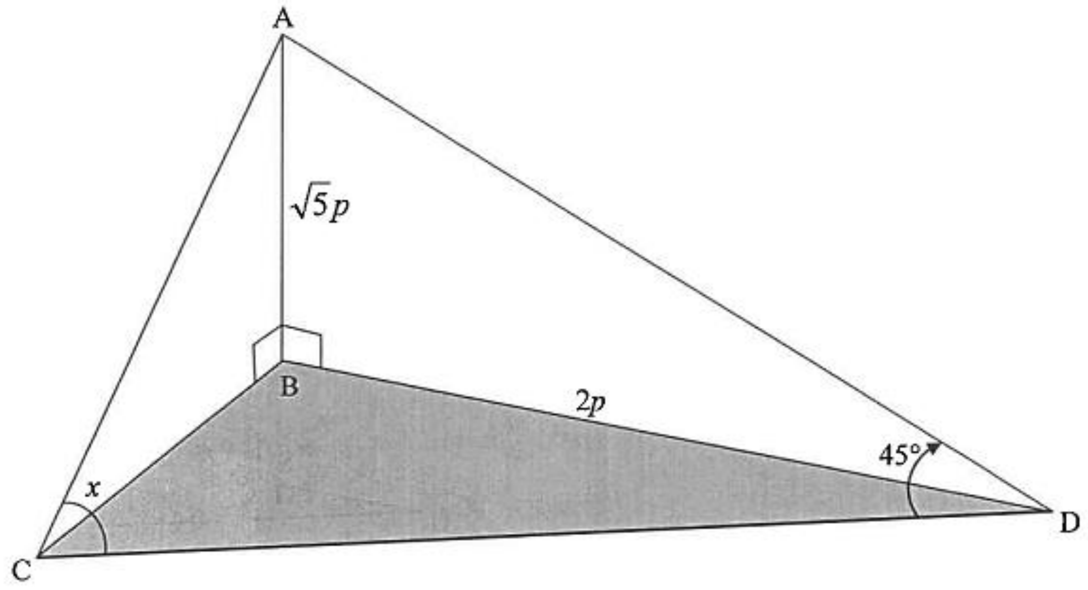
6.4 For which values of x will $g(x+5°) - f(x+5°) \leq 0$ in the interval $x \in [-90° ; 0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x = p$ will have exactly two real roots in the interval $x \in [-180° ; 180°]$. (3)

[10]

QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
$BD = 2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



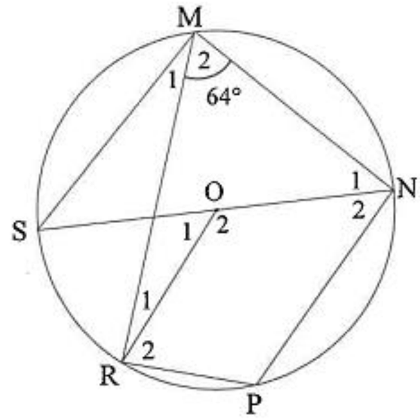
7.1 Determine the length of AD in terms of p. (2)

7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



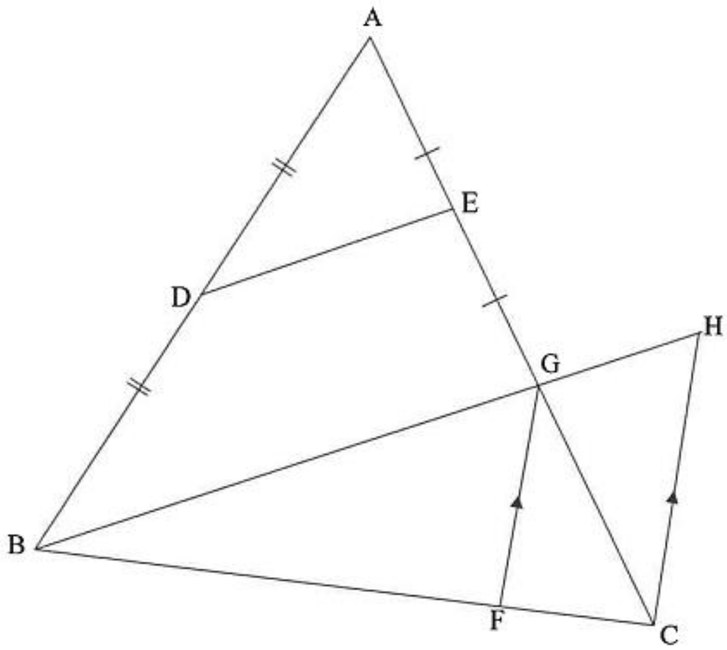
Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG || CH.



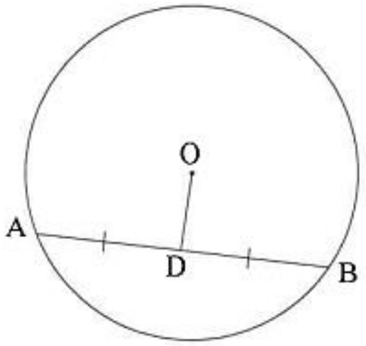
8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF}=\frac{1}{4}$, $DE = 3x - 1$ and $GH = x + 1$, calculate, giving reasons, the value of x. (6)

[13]

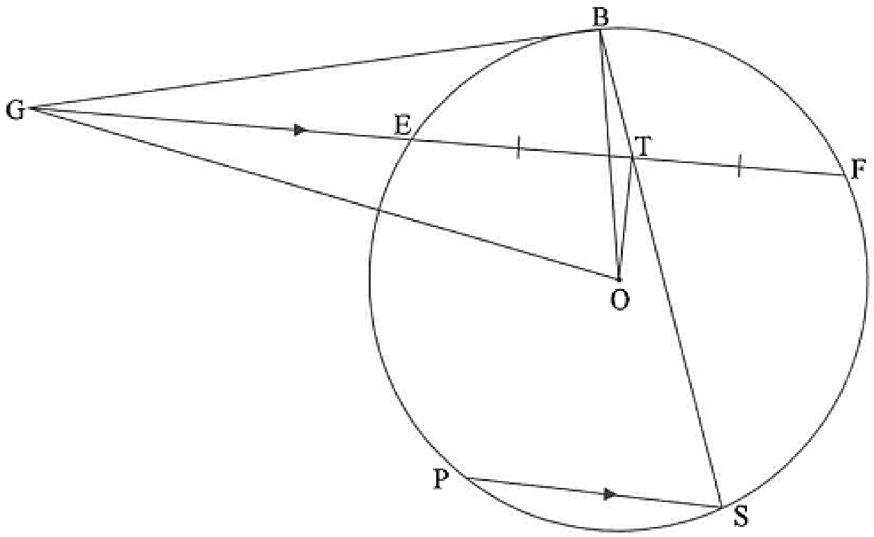
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. $OD \perp AB$. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.



Prove, giving reasons, that:

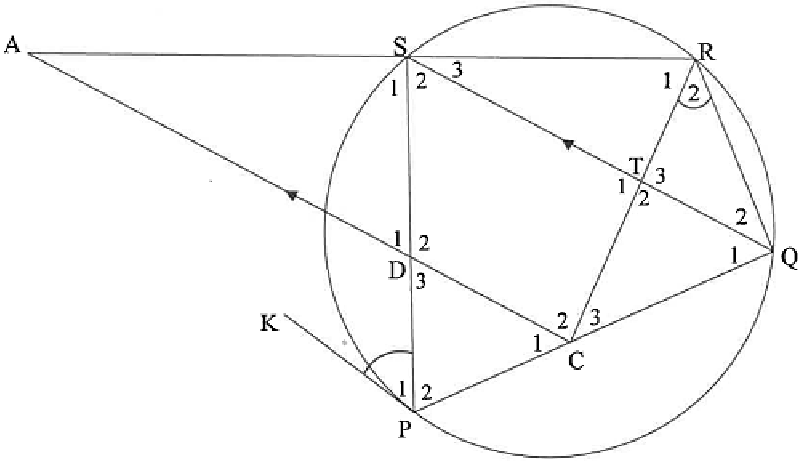
9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

TOTAL: 150

ANNEXURE E – MAY/JUNE 2022 PAPER 1

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2+2x-15=0$ (3)

1.1.2 $5x^2-x-9=0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.3 $x^2 \leq 3x$ (4)

1.2 Given: $a+\frac{64}{a}=16$

1.2.1 Solve for a. (3)

1.2.2 Hence, solve for x: $2^x+2^{6-x}=16$ (3)

1.3 **Without using a calculator**, calculate the value of $\sqrt{\frac{2^{1002}+2^{1006}}{17(2)^{998}}}$ (4)

1.4 Solve for x and y simultaneously:

$2x-y=2$ and $\frac{1}{x}-3y=1$ (6)

[26]

QUESTION 2

2.1 The first term of an arithmetic sequence is -1 and the 7^{th} term is 35.

Determine:

2.1.1 The common difference of the sequence (2)

2.1.2 The number of terms in the sequence if the last term of the sequence is 473 (3)

2.1.3 The sum of the first 40 terms in this sequence (2)

2.2 75 ; 53 ; 35 ; 21 ; ... is a quadratic number pattern.

2.2.1 Write down the FIFTH term of the number pattern. (1)

2.2.2 Determine the n^{th} term of the number pattern. (4)

2.2.3 Determine the maximum value of the following number pattern:

$-15\,;-\frac{53}{5}\,;-7\,;-\frac{21}{5}\,;\ldots$ (4)

[16]

QUESTION 3

3.1 Consider the following geometric sequence: 1 024 ; 256 ; 64 ; …

Calculate:

3.1.1 The 10^{th} term of the sequence (2)

3.1.2 $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ (4)

3.2 The first two terms of a geometric sequence are:

$-t^2-6t-9$ and $\dfrac{t^3+9t^2+27t+27}{2}$

Determine the values of t for which the sequence will converge. (5)
[11]

QUESTION 4

The graph of $g(x)=a\left(\frac{1}{3}\right)^x+7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x)=\left(\frac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y=\ldots$ (2)
[9]

QUESTION 5

Consider: $g(x)=\dfrac{a}{x+p}+q$

The following information of g is given:

- Domain: $x\in R;\ x\neq -2$
- x-intercept at K(1 ; 0)
- y-intercept at $N\left(0\ ;\ -\frac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x)=\dfrac{-3}{x+2}+1$ (6)

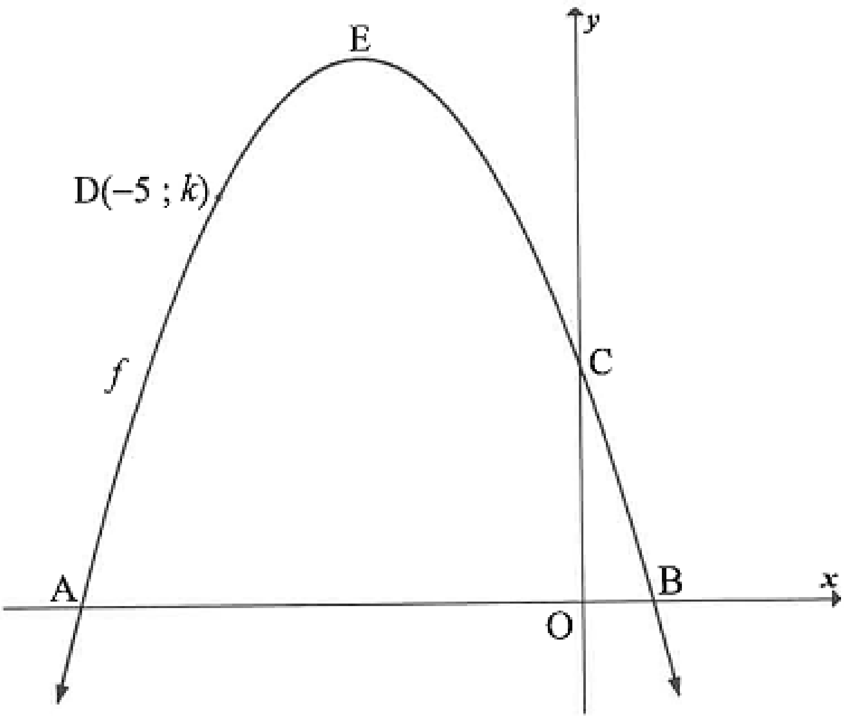
5.2 Write down the range of g. (1)

5.3 Determine the equation of h, the axis of symmetry of g, in the form $y=mx+c$, where $m>0$. (3)

5.4 Write down the coordinates of K', the image of K reflected over h. (2)
[12]

QUESTION 6

The sketch below shows the graph of $f(x)=-x^2-6x+7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5\,;k)$ is a point on f.



6.1 Calculate the coordinates of E, the turning point of f. (3)

6.2 Write down the value of k. (1)

6.3 Determine the equation of the straight line passing through C and D. (4)

6.4 A tangent, parallel to CD, touches f at P. Determine the coordinates of P. (4)

6.5 For which values of x will $f(x)-12>0$? (2)
[14]

QUESTION 7

7.1 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)

7.2 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

7.2.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)

7.2.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years? (2)

7.2.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund. (5)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

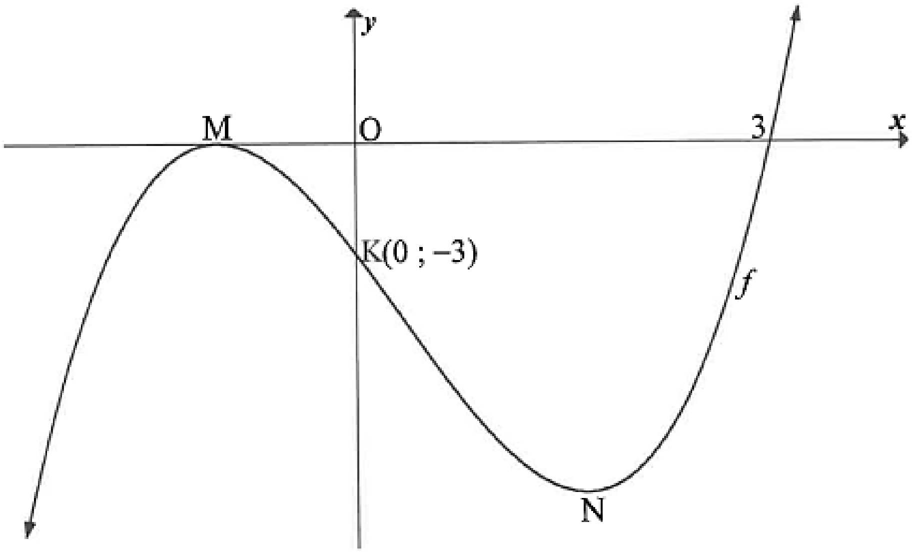
8.2.2 $D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at $(3\ ;0)$ and M, where M lies on the negative x-axis.

$K(0\ ;-3)$ is the y-intercept of f.
M and N are the turning points of f.



9.1 Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$. (5)

9.2 Calculate the coordinates of N. (5)

9.3 For which values of x will:

9.3.1 $f(x) < 0$ (2)

9.3.2 f be increasing (2)

9.3.3 f be concave up (3)

9.4 Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$. (6)

[23]

QUESTION 10

10.1 Flags from four African countries and three European countries were displayed in a row during the 2021 Olympics.

Determine:

10.1.1 The total number of possible ways in which all 7 flags from these countries could be displayed (2)

10.1.2 The probability that the flags from the African countries were displayed next to each other (3)

10.2 A and B are two independent events.

$P(A) = 0{,}4$ and $P(A \text{ or } B) = 0{,}88$

Calculate $P(B)$. (3)

10.3 There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$. Calculate the probability that the first passenger will choose a cheese sandwich. (5)

[13]

TOTAL: 150

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