JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

SOLUTION MANUAL

GRADE 12

2023

LAST PUSH PAPER 1 AND 2

JENN MAREMATLOU TRAINING INSTITUTE

A Division of Marematlou Group Holdings

PAPER 1

ALGEBRA

PAPER A

QUESTION 1

(a) (1) $x^2 - 5x = -6$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2 \quad \text{or} \quad x = 3$$

(2) $(3x+1)(x-4) < 0$

$$-\frac{1}{3} < x < 4$$

(3) $\log_2(x+6) = 1$

$$x + 6 = 2$$

$$x = -4$$

(4) $2x + \sqrt{x+1} = 1$

$$\sqrt{x+1} = 1 - 2x$$

$$x + 1 = 1 - 4x + 4x^2$$

$$4x^2 - 5x = 0$$

$$x(4x-5) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{5}{4}$$

Check $x = 0$:

$$\text{LHS} = 2 \times 0 + \sqrt{0+1}$$

$$= 1 = \text{RHS}$$

Check $x = \frac{5}{4}$

$$\text{LHS} = 2\left(\frac{5}{4}\right) = \sqrt{\frac{5}{4} + 1}$$

$$= 4 \neq \text{RHS}$$

(5) $12^{5+3x} = 1$

$5 + 3x = 0$

$x = \dfrac{-5}{3}$

(b) $2x - y = 8$ .... ①

$x^2 - xy + y^2 = 19$ ...... ②

① : $y = 2x - 8$

② : $x^2 - x(2x-8) + (2x-8)^2 = 19$

$x^2 - 2x^2 + 8x + 4x^2 - 32x + 64 = 19$

$3x^2 - 24x + 45 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ or $x = 5$

$y = 2\times 3 - 8$ or $y = 2\times 5 - 8$

$= -2$ $\quad$ $= 2$

(c) $f(x) = x^{10} - 2x^5 + c$

$f(-1) = (-1)^{10} - 2(-1)^5 + c = 0$

$1 + 2 + c = 0$

$c = -3$

(d) $y = x^2$

$\dfrac{dy}{dx} = 2x$

At $(-1;\ 1)$: $m = 2\,(-1)$

$= -2$

PAPER B

QUESTION 1

(a) (1) $\dfrac{4x}{2} - \dfrac{2x+1}{3} = 5$

$\dfrac{12x - 4x - 2}{6} = \dfrac{30}{6}$ **OR** $12x - 2\,(2x + 1) = 30$

$8x = 32$

$x = 4$

(2) $(x-5)(x-6)\leq 56$

$x^2-11x+30\leq 56$

$x^2-11x-26\leq 0$

$(x-13)(x+2)\leq 0$

Critical Values: 13 ; -2

$-2\leq x\leq 13$

(d) $c=-1$ or $c=-\dfrac{1}{4}$ (other answers possible)

(e) $3-k<0 \therefore k>3$

PAPER C

(a) (1)

$$(x-1)^2=2(1-x)$$
$$(x-1)^2=-2(x-1)$$
$$(x-1)^2+2(x-1)=0$$
$$(x-1)(x-1+2)=0$$
$$(x-1)(x+1)=0$$
$$x=1 \qquad x=-1$$

(2)

$$5^{-x}.5^{x-2}=\frac{25^{2x}}{5}$$
$$5^{-x}.5^{x-2}=\frac{5^{4x}}{5^1}$$
$$5^{-x+x-2}=5^{4x-1}$$
$$-2=4x-1$$
$$x=-\frac{1}{4}$$

PAPER D

1.1	1.1.1	$(2x-3)^2=1$ $2x-3=\pm 1$ $x=\dfrac{3\pm 1}{2}$ $x=1$ or $x=2$ **OR** $4x^2-12x+9=1$ $4x^2-12x+8=0$ $4(x-1)(x-2)=0$ $x=1$ or $x=2$

1.1.2

$$2x^2 + 4x - 7 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)} \\ &= \frac{-4 \pm \sqrt{72}}{4} \\ &= \frac{-2 \pm 3\sqrt{2}}{2} \end{aligned}$$

1.1.3

$$\begin{aligned} x - \sqrt{2x-1} &= 2 \\ (x-2)^2 &= \left(\sqrt{2x-1}\right)^2 \\ x^2 - 4x + 4 &= 2x - 1 \\ x^2 - 6x + 5 &= 0 \\ (x-1)(x-5) &= 0 \end{aligned}$$

$$x \neq 1 \quad \text{or} \quad x = 5$$

$$\therefore \quad x = 5$$

1.2.1

$$x^2 + 5xy + 6y^2 = 0$$

$$(x+3y)(x+2y) = 0$$

$$x + 3y = 0 \qquad\qquad x + 2y = 0$$

$$x = -3y \qquad \text{OR} \qquad x = -2y$$

$$\frac{x}{y} = -3 \qquad\qquad \frac{x}{y} = -2$$

OR

$$x^2 + 5xy + 6y^2 = 0$$

$$x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$$

$$x = \frac{-5y \pm \sqrt{y^2}}{2}$$

$$x = \frac{-5y \pm y}{2}$$

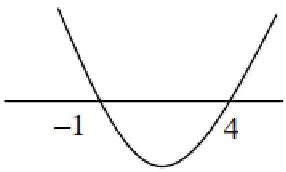
$$x = -3y \qquad x = -2y$$

$$\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$$

1.2.2	$\begin{aligned} x+y&=8 \\ -3y+y&=8 \\ -2y&=8 \\ y&=-4 \\ x&=12 \end{aligned}$ OR $\begin{aligned} x+y&=8 \\ -2y+y&=8 \\ -y&=8 \\ y&=-8 \\ x&=16 \end{aligned}$

1.3.1	$\begin{aligned} \sqrt{2p+5}&=0 \\ 2p+5&=0 \\ 2p&=-5 \\ p&=-\frac{5}{2} \end{aligned}$
1.3.2	$\begin{aligned} 2p+5&<0 \\ p&<-\frac{5}{2} \end{aligned}$

PAPER E

1.1.4	$(x+1)(4-x)>0$ $(x+1)(x-4)<0$ $+ \quad 0 \quad - \quad 0 \quad +$ $-1 \qquad\quad 4$ or  $-1<x<4$

1.2.1	$\begin{aligned} 2^x+2^{x+2}&=-5y+20 \\ 2^x(1+2^2)&=-5y+20 \\ 2^x&=\frac{-5y+20}{5} \end{aligned}$

1.2.2	If $y=-4$, $\begin{aligned} 2^x+2^{x+2}&=-5y+20 \\ 2^x+2^{x+2}&=40 \\ 2^x\left(1+2^2\right)&=40 \\ 2^x&=8 \\ 2^x&=2^3 \\ x&=3 \end{aligned}$

1.2.3	$-y+4>0$ $y<4$ Largest integer value of y is 3 $2^x=-3+4$ $2^x=1$ $x=0$

PAPER F

QUESTION 1

1.1.1 $(x-3)(x+4)=18$

$\therefore x^2+x-12=18$ ✓

$\therefore x^2+x-30=0$ ✓

$\therefore (x+6)(x-5)=0$ ✓

$\therefore x=-6$ or $x=5$ ✓

1.1.2 $x^2=6(x+2)$

$\therefore x^2=6x+12$ ✓

$\therefore x^2-6x-12=0$ ✓

$$\therefore x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-12)}}{2(1)}$$ ✓

$$\therefore x=\frac{6\pm\sqrt{84}}{2}$$

$\therefore x=7{,}58$ ✓ or $x=-1{,}58$ ✓

1.1.3 $2-\frac{1}{x}=\frac{3}{x+2}$

$\therefore 2x(x+2)-(x+2)=3x$ ✓

$\therefore 2x^2+4x-x-2=3x$ ✓✓

$\therefore 2x^2-2=0$ ✓

$\therefore x^2-1=0$

$\therefore (x+1)(x-1)=0$ ✓

$\therefore x=-1$ or $x=1$ ✓

1.2 $y^2 + 2y - \frac{8}{y^2 + 2y} = 7$

Let $y^2 + 2y = k$

$\therefore k - \frac{8}{k} = 7$ ✓

$\therefore k^2 - 7k - 8 = 0$ ✓

$\therefore (k+1)(k-8) = 0$ ✓

$\therefore k = -1$ or $k = 8$ ✓

$\therefore y^2 + 2y = -1$ or $y^2 + 2y = 8$

$\therefore y^2 + 2y + 1 = 0$ or $y^2 + 2y - 8 = 0$ ✓

$\therefore (y+1)(y+1) = 0$ ✓ or $(y+4)(y-2) = 0$ ✓

$\therefore y = -1$ or $y = -4$ or $y = 2$ ✓

1.3 $3x^2 - 6px - 9p^2 = 0$

$\therefore x^2 - 2px - 3p^2 = 0$ ✓

$\therefore x^2 - 2px = 3p^2$

$\therefore x^2 - 2px + p^2 = 3p^2 + p^2$ ✓

$\therefore (x-p)^2 = 4p^2$ ✓

$\therefore x - p = \pm\sqrt{4p^2}$ ✓

$\therefore x = p \pm \sqrt{4p^2}$

$\therefore x = p + 2p$ or $x = p - 2p$ ✓

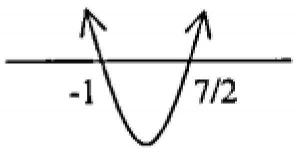
$\therefore x = 3p$ or $x = -p$ ✓

1.4 $2x^2 - 5x \geq 7$

$\therefore 2x^2 - 5x - 7 \geq 0$ ✓

$\therefore (x+1)(2x-7) \geq 0$ ✓

$\therefore x \leq -1$ ✓ or $x \geq \frac{7}{2}$ ✓



QUESTION 2

2.1 $(2y+3)(x^2+4)=0$

$\therefore y=-\frac{3}{2}$ ✓

or $x^2=-4$ ✓

$\therefore$ no real value for x ✓

2.2 $2a-b=7$

$\therefore b=2a-7$ ✓

Subs $b=2a-7$ into $a^2+ab+b^2=7$

$a^2+a(2a-7)+(2a-7)^2=7$ ✓

$\therefore a^2+2a^2-7a+4a^2-28a+49=7$ ✓✓

$\therefore 7a^2-35a+42=0$ ✓

$\therefore a^2-5a+6=0$

$\therefore (a-2)(a-3)=0$ ✓

$\therefore a=2$ or $a=3$ ✓

$\therefore b=-3$ or $b=-1$ ✓

2.3 $xy=20$ ✓

$\therefore y=\frac{20}{x}$

$(x+3)(y+1)=40$ ✓

$\therefore xy+x+3y+3=40$

Subs $y=\frac{20}{x}$ into $xy+x+3y+3=40$

$\therefore x\left(\frac{20}{x}\right)+x+3\left(\frac{20}{x}\right)+3=40$

$\therefore 20+x+\frac{60}{x}+3=40$

$\therefore 20x+x^2+60+3x-40x=0$

$\therefore x^2-17x+60=0$ ✓

$\therefore (x-5)(x-12)=0$

$\therefore x=5$ or $x=12$ ✓

$\therefore y=4$ or $y=\frac{5}{3}$ ✓

Dimensions $\therefore 5\times 4$ or $12\times\frac{5}{3}$

PATTERNS, SEQUENCES AND SERIES

PAPER A

4.1 The second, third, fourth and fifth terms are 1 ; -6 ; T_4 and -14

First differences are: -7 ; $T_4 + 6$; $-14 - T_4$

So $T_4 + 6 + 7 = -14 - 2T_4 - 6$

$T_4 = -11$

$d = -11 + 6 + 7 = 2$ or $-14 + 22 - 6 = 2$

OR

$$
\begin{array}{ccccccc}
T_2 & & T_3 & & T_4 & & T_5 \\
1 & & -6 & & & & -14 \\
 & -7 & & -7+d & & -7+2d & \\
 & & d & & d & &
\end{array}
$$

$$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$$
$$-15 = (-7 + 2d) + (-7 + d) + -7$$
$$-15 = -21 + 3d$$
$$6 = 3d$$
$$d = 2$$

OR

$$4a + 2b + c = 1$$
$$9a + 3b + c = -6$$
$$5a + b = -7$$

$$25a + 5b + c = -14$$
$$16a + 2b = -8$$
$$10a + 2b = -14$$
$$6a = 6$$
$$a = 1$$
$$d = 2a = 2$$

4.2

$T_1 \qquad\qquad 1 \qquad\qquad -6$

$\qquad -9 \qquad\qquad -7$

$\qquad\qquad 2$

$T_1 = 10$

OR

$$\begin{aligned}
a &= 1 \\
5a + b &= -7 \\
5(1) + b &= -7 \\
b &= -12 \\
a + b + c &= 1 \\
4(1) + 2(-12) + c &= 1 \\
c &= 21 \\
T_n &= n^2 - 12n + 21 \\
T_1 &= (1)^2 - 12(1) + 21 \\
&= 10
\end{aligned}$$

QUESTION/VRAAG 7

7.1	29

7.2

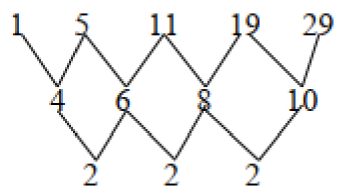
$$T_n = an^2 + bn + c$$
$$1 = a + b + c$$
$$\therefore c = 1 - a - b$$
$$5 = 4a + 2b + c$$
$$5 = 4a + 2b + 1 - a - b$$
$$4 = 3a + b \qquad (1)$$
$$11 = 9a + 3b + c$$
$$11 = 9a + 3b + 1 - a - b$$
$$\therefore 10 = 8a + 2b \qquad (2)$$

Solving (1) and (2) simultaneously.
Los (1) en (2) gelyktydig op.

$8 = 6a + 2b$ (1) x 2
$\underline{10 = 8a + 2b \quad (2)}$

$$\therefore 2 = 2a$$
$$\therefore a = 1$$
$$\therefore b = 1$$
$$\therefore c = -1$$
$$T_n = n^2 + n - 1$$

Or/Of



2a = 2	3a + b = 4	a + b + c = 1
a = 1	3 + b = 4	1 + 1 + c = 1
	b = 1	c = -1

$T_n = n^2 + n - 1$

7.3	$T_n = n^2 + n - 1$ or/of $\therefore T_{100} = 100^2 + 100 - 1$ $= 10\,099$	$T_n = 100(101) - 1$ $= 10\,099$

PAPER B

QUESTION 4

4.1	$-7 \quad 0 \quad 9 \quad 20$ $7 \quad 9 \quad 11$ $2 \quad 2$ $2a = 2$ $a = 1$ $3(1) + b = 7$ $b = 4$ $(1) + (4) + c = -7$ $c = -12$ $\therefore T_n = n^2 + 4n - 12$ **OR** $2a = 2$ $a = 1$ $T_2 = 2^2 + b(2) + c = 0$ $2b + c = -4 \quad (1)$ $T_3 = 3^2 + b(3) + c = 9$ $3b + c = 0 \quad (2)$ *OR* $3(1) + b = 7$ $b = 4$ $1 + a + c = -7$ $c = -12$ $(2) - (1) \quad b = 4$ $\therefore c = -4 - 2(4) = -12$ $T_n = n^2 + 4n - 12$
4.2	$n^2 + 4n - 12 = 128$ $n^2 + 4n - 140 = 0$ $(n + 14)(n - 10) = 0$ $n \neq -14$ or $n = 10$ invalid $\quad \therefore n = 10$

4.3	$-7 ; 0 ; 9 ; 20 ; \ldots$ first difference7 9 11 second difference2 2 $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ this difference will be between term 297 and term 298

QUESTION 5

5.1

Pattern	1	2	3
White squares	4	12	24

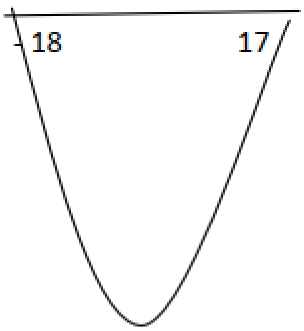
40

5.2

$$W_n = 2n^2 + 2n$$
$$W_{157} = 2(157)^2 + 2(157)$$
$$= 49612$$

5.3

$$2n^2 + 2n + 1 < 613$$
$$2n^2 + 2n - 612 < 0$$
$$n^2 + n - 306 < 0$$
$$(n-17)(n+18) < 0$$



$\therefore n = 16$

5.4	$P_n = 4n^2 + 4n + 1$ $= (2n)^2 + 2(2n) + 1$ $2n$ is even for all $n \in Z$ $\therefore$ Total squares used in the n^{th} pattern is always odd. **OR** $P_n = 4n^2 + 4n + 1$ $= 2(2n^2 + 2n) + 1$ $2(2n^2 + 2n)$ is odd for all $n \in Z$ $2(2n^2 + 2n) + 1$ is odd for all $n \in Z$ $\therefore$ Total squares used in the n^{th} pattern is always odd.

PAPER C

QUESTION 3

3.1	$T_2 - T_1 = T_3 - T_2$ $p + 5 - 2p + 3 = 2p + 7 - p - 5$ $-p + 8 = p + 2$ $p = 3$
3.2	Pattern is $3 ; 8 ; 13 ; \ldots$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{120} = \frac{120}{2}[2(3) + 119(5)]$ $= 36060$
3.3.1	$x = k + 1$ and $y = k + 2$

3.3.2	$T_x = a + (x-1)d = 3 + 5k$ $T_y = a + (k+1)d$ $= 3 + (k+1)(5)$ $= 8 + 5k$ $T_x + T_y = 11 + 10k$

QUESTION 4

4.1.1	15 ; 5
4.1.2	$S_\infty = \frac{a}{1-r}$ $= \frac{15}{1-\frac{1}{3}}$ $= \frac{45}{2} = 22{,}5$
4.2	$\sin 30°\, ;\, \cos 30°\, ;\, \frac{3}{2}$ $\frac{1}{2}\, ;\, \frac{\sqrt{3}}{2}\, ;\, \frac{3}{2}$ $a = \frac{1}{2}\, ;\, r = \sqrt{3}$ $ar^{n-1} = 40{,}5\sqrt{3}$ $\frac{1}{2}\left(\sqrt{3}\right)^{n-1} = \frac{81}{2}\sqrt{3}$ $3^{\frac{n-1}{2}} = 3^4.3^{\frac{1}{2}}$ $\frac{n-1}{2} = 4\frac{1}{2} = \frac{9}{2}$ $n - 1 = 9$ $n = 10$

OR

	$\sin 30°\, ;\, \cos 30°\, ;\, \frac{3}{2}$ $\frac{1}{2}\, ;\, \frac{\sqrt{3}}{2}\, ;\, \frac{3}{2}$ $a = \frac{1}{2}\, ;\, r = \sqrt{3}$ $ar^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{1}{2}\left(\sqrt{3}\right)^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{\left(\sqrt{3}\right)^n}{\sqrt{3}} = 81\sqrt{3}$ $243 = \left(\sqrt{3}\right)^n$ $3^5 = 3^{\frac{1}{2}n}$ $n = 10$

PAPER D

QUESTION 3	
3.1	$r = \frac{2(3x-1)^2}{2(3x-1)}$ $r = 3x - 1$ $-1 < 3x - 1 < 1$ $0 < 3x < 2$ $0 < x < \frac{2}{3}$

3.2	$T_2 = ar$ $6 = kr$ $r = \frac{6}{k} \quad \ldots \quad (1)$ $S_\infty = 25$ $S_\infty = \frac{a}{1-r}$ $25 = \frac{k}{1-r} \quad \ldots \quad (2)$ sub. (1) into (2) $25 = \frac{k}{1-\frac{6}{k}}$ $k = 25\left(1 - \frac{6}{k}\right)$ $k = 25 - \frac{150}{k}$ $0 = k^2 - 25k + 150$ $0 = (k-10)(k-15)$ $\therefore k = 10$ **or** $k = 15$

3.3	$(1\times2)+(5\times6)+(9\times10)+(13\times14)+\ldots+(81\times82)$ $(4n-3)\times(4n-2)$ $=16n^2-20n+6$ $4n-3=81$ **OR** $4n-2=82$ $4n=84$ $\quad$ $4n=84$ $n=21$ $\quad$ $n=21$ $\sum_{n=1}^{21} 16n^2-20n+6$ **OR** $\sum_{n=1}^{21}(4n-3)(4n-2)$

PAPER E

QUESTION 2

2.1.1	$\frac{1}{16}$; 13
2.1.2	$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)$ $\quad (4+7+10+13+\ldots to\ 25\ terms)$ $\frac{a(r^n-1)}{r-1}$ $\quad = \frac{n}{2}[2a+(n-1)d]$ $= \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$ $\quad = \frac{25}{2}[2(4)+24(3)]$ $= 0,9999999$ $\quad = 1\ 000$ $S_{50} = 1001,00$
	OR S_{50} = 25 terms of 1st sequence + 25 terms of 2nd sequence $S_{50} = \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)+(4+7+10+13+\ldots to\ 25\ terms)$ $S_{50} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}+\frac{25}{2}[2(4)+24(3)]$ $S_{50} = 0,999999\ldots+1000$ $S_{50} = 1001,00$

2.2.1

60 ; 78

2.2.2

$$\begin{array}{ccccccc} 8 & & 18 & & 30 & & 44 \\ & 10 & & 12 & & 14 & \\ & & 2 & & 2 & & \end{array}$$

$2a = 2$

$a = 1$

$T_n = n^2 + bn + c$

$8 = 1 + b + c$

$7 = b + c \qquad \ldots\text{(i)}$

$18 = 4 + 2b + c$

$14 = 2b + c \qquad \ldots\text{(ii)}$

(ii) – (i): $\qquad 14 = 2b + c$

$\qquad\qquad 7 = b + c$

$\qquad\qquad \therefore\ 7 = b$

$\qquad\qquad c = 0$

$T_n = n^2 + 7n$

OR

$T_1 = 8$

$T_2 - T_1 = 10$

$T_3 - T_2 = 12$

$T_n - T_{n-1} = n$th term of sequence with $a = 8$ and $d = 2$

Add both sides

$T_n = 8 + 10 + 12 + \ldots +$ to 25 terms

$T_n = \frac{n}{2}[16 + 2(n-1)]$

$T_n = n(n+7)$

2.2.3

$n(n+7) = 330$

$n^2 + 7n - 330 = 0$

$(n+22)(n-15) = 0$

$n = -22 \quad$ or $\quad n = 15$

$n = 15$

$\therefore 15^{th}$ term is 330.

2.3	$S_n = a + [a+d] + [a+2d] + \ldots + [a+(n-2)d] + [a+(n-1)d]$ $S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + [a+d] + a$ $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] + [2a+(n-1)d]$ $= n[2a+(n-1)d]$ $S_n = \frac{n}{2}[2a+(n-1)d]$

QUESTION 3

3.1	$T_n = (8x^2)\left(\frac{x}{2}\right)^{n-1}$

3.2	$ratio = \frac{x}{2}$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$
3.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$ $S_\infty = 72$

	OR $18 + \frac{27}{2} + \frac{81}{8} + \ldots$ $S_\infty = \frac{18}{1-\frac{3}{4}}$ $S_\infty = \frac{18}{\frac{1}{4}}$ $S_\infty = 72$

PAPER F

QUESTION 2

2.1	20 ; 24 ; 28 ; 32 ; … 4 4 4 $T_n = 20 + (n-1)4$ $100 = 20 + 4n - 4$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. **OR** $T_n = 4n + 16$ $100 = 4n + 16$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. **OR** $100 = 20 + 80$ $= 20 + 4(21-1)$ $\therefore n = 21$
2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{14} = \frac{14}{2}[2(20) + (14-1)4]$ $= 644$ km
2.3	No. It will not be humanly possible to just keep on increasing the distance covered indefinitely. For example: $T_{1\,000} = 4\ 016$ km in one day.

QUESTION 3

3.1	45
3.2	$T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $3a + b = 7$ $3 + b = 7$ $b = 4$ $a + b + c = 5$ $1 + 4 + c = 5$ $c = 0$ $T_n = n^2 + 4n$
	OR $T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $T_0 = 0 = c$ $T_n = n^2 + bn + 0$ $5 = (1)^2 + (1)b$ $b = 4$ $T_n = n^2 + 4n$ **OR** If $T_n = an^2 + bn + c$ $5 = T_1 = a + b + c$ $12 = T_2 = 4a + 2b + c \Rightarrow 3a + b = 7$ $21 = T_3 = 9a + 3b + c \Rightarrow 5a + b = 9$ $\Rightarrow a = 1,\ b = 4,\ c = 0$

QUESTION 4

4.1	$S = a + ar + ar^2 + \ldots + ar^{n-1}$ $rS = ar + ar^2 + \ldots + ar^{n-1} + ar^n$ $S - rS = a - ar^n$ $S(1-r) = a(1-r^n)$ $S = \dfrac{a(1-r^n)}{1-r}$
4.2.1	$15 \; ; \; 5 \; ; \; \dfrac{5}{3} \; ; \ldots$ $r = \dfrac{5}{15} = \dfrac{1}{3}$ The series converges because $-1 < r < 1$
4.2.2	$S_\infty = \dfrac{15}{1 - \dfrac{1}{3}}$ $= \dfrac{45}{2}$

4.3.1	$S_{24} = 2^{24+2} - 4$ $= 67108860$
4.3.2	$S_{24} = 2^{24+2} - 4 = 67108860$ $S_{23} = 2^{23+2} - 4 = 33554428$ $T_{24} = 33554432$ **OR** $T_{24} = S_{24} - S_{23}$ $= 2^{26} - 2^{25}$ $= 2 \times 2^{25} - 2^{25}$ $= 2^{25}$

FUNCTIONS AND GRAPHS

PAPER A

QUESTION 5

5.1

$$x = -\frac{b}{2a}$$
$$= -\frac{1}{2(-2)}$$
$$= \frac{1}{4}$$
$$\therefore y = -2\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right) + 6$$
$$y = \frac{49}{8}$$

5.2

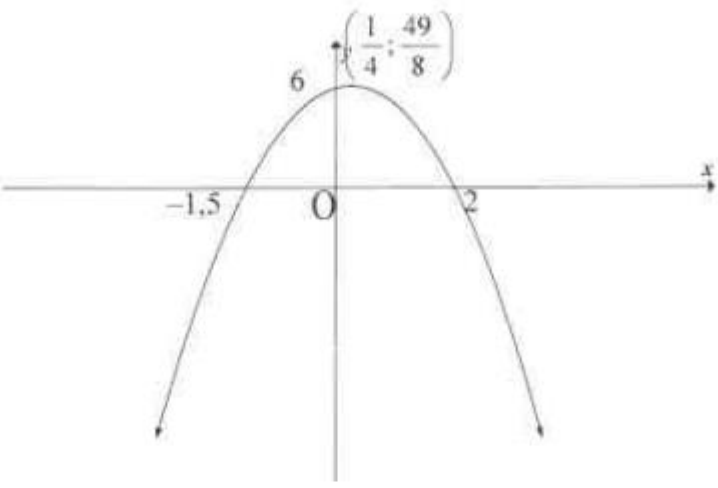
$$y = -2(0)^2 + 0 + 6$$
$$\therefore y \text{ intercept } (0;6)$$

5.3

x intercepts

$$0 = -2x^2 + x + 6$$
$$0 = 2x^2 - x - 6$$
$$0 = (2x + 3)(x - 2)$$
$$\therefore x = 2 \text{ or } x = -\frac{3}{2}$$
$$(2;0) \text{ and } \left(-\frac{3}{2};0\right)$$

5.4



5.5	$k = \frac{49}{8}$
5.6	New/*Nuwe* turning point/*drpn.t* $\left(\frac{9}{4};\frac{57}{8}\right)$ Equation/*verg.* of h $y = -2\left(x - \frac{9}{4}\right)^2 + \frac{57}{8}$

QUESTION/*VRAAG* 6

6.1	$x = -3$ and $y = -1$
6.2	$x \in R\,;\, x \neq -3$ OR $x \in (-\infty;-3) \cup (-3:\infty)$

6.3.1	At B, $x = 0$ $\therefore y = \frac{1}{0+3} - 1$ $y = -\frac{2}{3}$ $\therefore$ OB $= \frac{2}{3}$ units
6.3.2	At A, $y = 0$ $0 = \frac{1}{x+3} - 1$ $1 = \frac{1}{x+3}$ $x + 3 = 1$ $x = -2$ $\therefore$ OA $= 2$ units/ eenhede

6.4	$\frac{1}{x+3} - 1 = \frac{1}{2}x$ $2 - 2(x+3) = x(x+3)$ $x^2 + 3x - 2 + 2x + 6 = 0$ $x^2 + 5x + 4 = 0$ $(x+4)(x+1) = 0$ $x = -4$ or/*of* $x = -1$ when/*wanneer* $x = -1$; $y = -\frac{1}{2}$ when/*wanneer* $x = -4$; $y = -2$ $\therefore C\ (-1; -\frac{1}{2})$ and $D\ (-4; -2)$

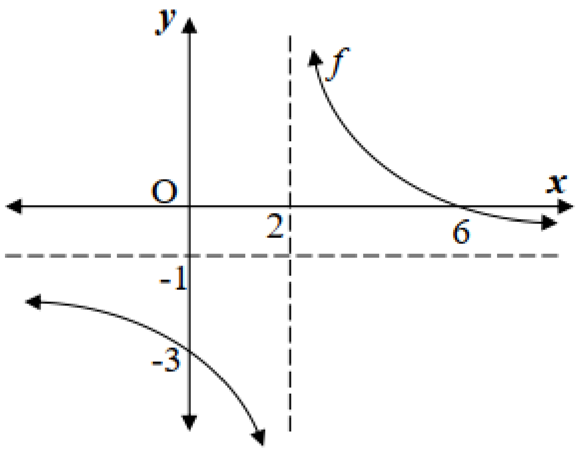
6.5	$\frac{1}{x+3} \geq \frac{x+2}{2}$ $\frac{1}{x+3} \geq \frac{x}{2} + 1$ $\frac{1}{x+3} - 1 \geq \frac{x}{2}$ $\therefore f(x) \geq g(x)$ $\therefore x \leq -4$ or $-3 < x \leq -1$

QUESTION/*VRAAG* 7

7.1	$q=2$ $f(x)=2.b^{x+1}+2$ $20=2.b^{1+1}+2$ $18=2.b^2$ $9=b^2$ $b=3$ $f(x)=2.3^{x+1}+2$
7.2	$y=2.3^{-1+1}+2$ $y=2.1+2$ $y=4$
7.3	$m=\dfrac{y_2-y_1}{x_2-x_1}$ $=\dfrac{20-4}{1-(-1)}$ $=8$
7.4	$h(x)=-2.3^{x+1}+2$
7.5	$y<2$

PAPER B

QUESTION 5

5.1	$x = 2 \ and \ y = -1$
5.2	$y -$ intercept$:(0\,;-3)$ $x -$ intercept: $\frac{-4}{2-x} - 1 = 0$ $\frac{-4}{2-x} = \frac{1}{1}$ $-4 = 2 - x$ $x = 6$ $(6\,;\ 0)$
5.3	

QUESTION 6

6.1	$\mathrm{A}(0; 6)$
6.2	$x = -\frac{b}{2a} = 2{,}5$ S(5 ; 6)
6.3	$-x^2 + 5x + 6 = 0$ $x^2 - 5x - 6 = 0$ $(x+1)(x-6) = 0$ $x = -1 \ or \ x = 6$ $\mathrm{B}(-1; 0), \mathrm{C}(6; 0)$
6.4	$(-x^2 + 5x + 6) - (x + 1) = 5$ $-x^2 + 5x + 6 - x - 1 = 5$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0 \ or \ 4$ $\mathrm{OR} = 4\mathrm{units}$

6.5.1	$x = \frac{-1+6}{2} = \frac{5}{2}$ $y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) + 6 = \frac{49}{4} = 12{,}25$ $\left(\frac{5}{2}; 12{,}25\right)$

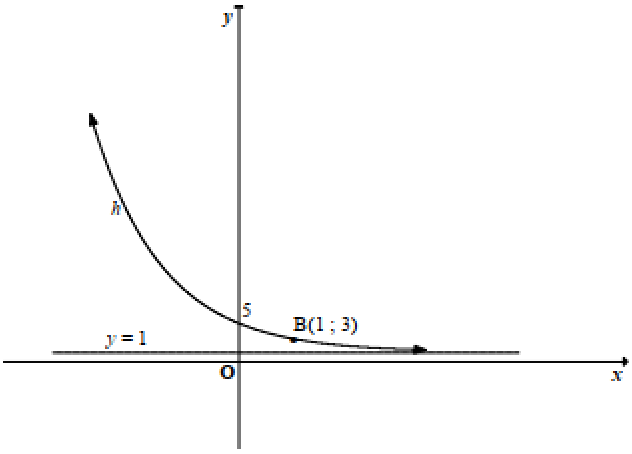
6.5.2	$PQ = -x^2 + 4x + 5$ $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$ Max. $PQ = -(2)^2 + 4(2) + 5 = 9$units

QUESTION 7

7.1	$y = 5^x$
7.2	$y > 0$ or $y \in (0\,;\,\infty)$
7.3	$log_5 x = -4$ $x = 5^{-4} = \frac{1}{625}$ $0 < x \leq \frac{1}{625}$

PAPER C

5.2.1	For y-intercept/*Vir y-afsnit* substitution $x = 0$: $y = 4.2^0 + 1$ $= 5$ $H(0\,;5)$
5.2.2	For x-intercept/*Vir y-afsnit* $y = 0$ i.e./*d.i.* $4.2^{-x} + 1 = 0$ $4.2^{-x} = -1$ $2^{-x} = -\frac{1}{4}$, which is impossible, since $2^{-x} > 0$ for $x \in R$, *wat onmoontlik is omdat* $2^{-x} > 0$ *vir* $x \in R$ Therefore/*Dus*: no solution/*geen oplossing*, which means there will be no x-intercept/*wat beteken daar sal geen x-afsnit wees nie*.

5.2.3	
5.2.4	$g(x)=4(2^{-x}+2)$ $=4.2^{-x}+8$ The graph of *h* is translated 7 units upwards to form *g*/ *Die grafiek van h word 7 eenhede na bo getransleer om g te vorm.*

QUESTION 6

6.1	A(0; 2)
6.2	$-x^2+x+2=\frac{1}{2}x^2-x$ $0=\frac{3}{2}x^2-2x-2$ $0=3x^2-4x-4$ $(3x+2)(x-2)=0$ $x=-\frac{2}{3}$ *or* $x=2$ $y=\frac{8}{9}$ *or* $y=0$ $C\left(-\frac{2}{3};\frac{8}{9}\right)$ & $D(2;0)$
6.3	$x\leq-\frac{2}{3}$ or $x\geq 2$

6.4	Length of PQ $= -x^2 + x + 2 - \left(\frac{1}{2}x^2 - x\right)$ $L = -\frac{3}{2}x^2 + 2x + 2$ $x = -\frac{b}{2a} = -\frac{2}{2\left(\frac{-3}{2}\right)} = \frac{2}{3}$ OR $L' = -3x + 2 = 0 \ \therefore x = \frac{2}{3}$ Maximum value of PQ $= -\frac{3}{2}\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) + 2$ $= \frac{8}{3} = 2\frac{2}{3} = 2{,}67$ units
6.5	$f(x) = -x^2 + x + 2$ $f^{/}(x) = -2x + 1 = 3$ $x = -1$

6.6	Axis of symmetry: $x = \frac{1}{2}$ Maximum value: $y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 2 = 2\frac{1}{4}$ $2 < k < 2\frac{1}{4}$

QUESTION/*VRAAG* 7

7.1	$CD = 2x + 3 - \left(-2x^2 + 14x + k\right)$ $= 2x + 3 + 2x^2 - 14x - k$ $= 2x^2 - 12x + 3 - k$
7.2	Minimum value occurs at/*Minimum waarde vind plaas by* $x = \frac{-b}{2a}$ $= \frac{12}{2(2)}$ $= 3$ Minimum value/*Minimum waarde* $5 = 2(3)^2 - 12(3) + 3 - k$ $5 = 18 - 36 + 3 - k$ $k = -20$

PAPER D

QUESTION 6	
6.1	$f(x)=-2x^2-5x+3$ $x=\frac{-(-5)}{2(-2)}$ $x=-\frac{5}{4}$ $f(\frac{-5}{4})=-2(\frac{-5}{4})^2-5(\frac{-5}{4})+3$ $f(\frac{-5}{4})=\frac{49}{8}$ $\therefore TP(\frac{-5}{4};\frac{49}{8})$
6.2	$y\in(-\infty;\frac{49}{8}]$ **OR** $y\leq\frac{49}{8}$
6.3	$\tan 135^0=-1$ $m=-1$ $f'(x)=-1$ $-4x-5=-1$ $\therefore x=-1$ $f(-1)=-2(-1)^2-5(-1)+3$ $y=6$ $\therefore P(-1\ ;\ 6)$
6.4	$k<-\frac{49}{8}$ **or** $k>-\frac{49}{8}$

QUESTION 7	
7.1	$f(x) = a^x$ $\frac{1}{4} = a^2$ $\sqrt{\frac{1}{4}} = a$ $\frac{1}{2} = a$
7.2	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$
7.3	$y = \left(\frac{1}{2}\right)^x$ $-y = \left(\frac{1}{2}\right)^x$ $h(x) = -\left(\frac{1}{2}\right)^x$ **OR** $h(x) = -a^x$ **OR** $h(x) = -f(x)$
7.4	$x \leq 0$ **OR** $x > 0$ **OR** $x < 0$ **OR** $x \geq 0$

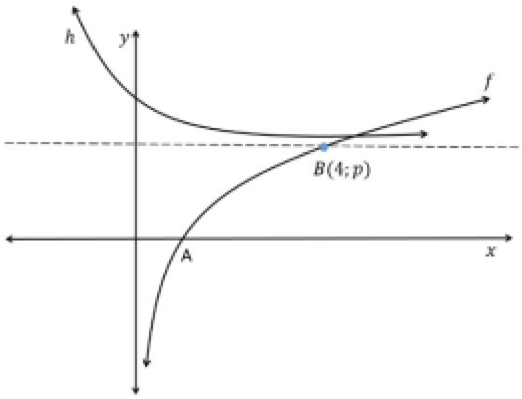
QUESTION 8		
8.1	8.1.1	$x < 2$
	8.1.2	$0 < x \leq 1$
8.2	8.2.1	$g(x) = \log_2 x$ $y = \log_2 x$ $\therefore x = 2^y$ $\therefore y = 2^x$ $\therefore g^{-1}(x) = 2^x$
	8.2.2	$\log_2(3 - x) = x$ $2^x = 3 - x$ therefore point of intersection of g^{-1} and f
	8.2.3	$x = 1$

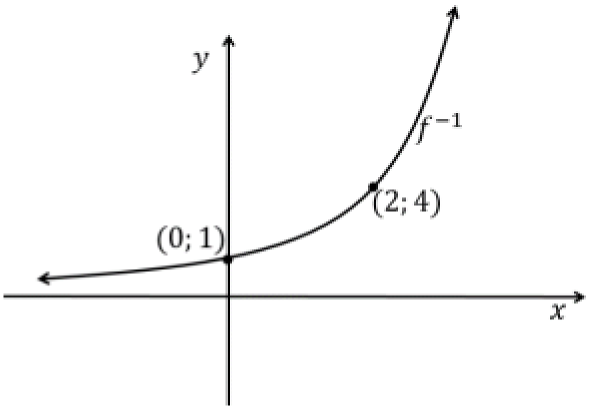
PAPER E

5.1	$g(x) = -3x + 20$ $g(3) = -3(3) + 20 = 11$ $\therefore k = 11$
5.2	$y \geq -11$ OR $x \in [-11; \infty)$
5.3	$y = a(x-3)^2 + 11$ $subst. (6; 2)$ $2 = a(6-3)^2 + 11$ $-9 = 9a$ $-1 = a$ $y = -1(x-3)^2 + 11$ $y = -(x^2 - 6x + 9) + 11$ $y = -x^2 + 6x + 2$ $\therefore a = -1\ ; b = 6\ and\ /en\ \ c = 2$

5.4	$3 < x < 6$ OR $x \in (3; 6)$
5.5	• Real *Reëel* • Rational *Rasionaal* • Equal *Gelyk*
5.6	$x > 3$ OR $x \in (3; \infty)$

QUESTION 6



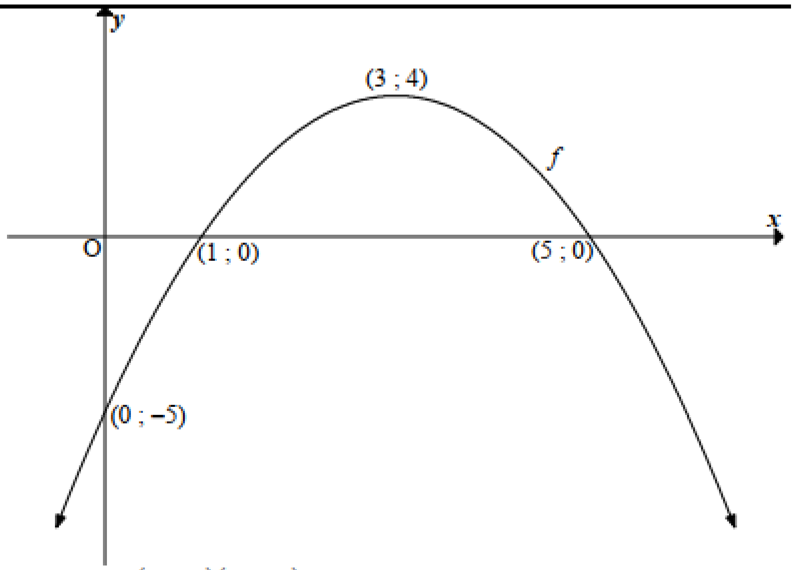
6.1	$A(1;0)$
6.2	$x \in R\,;\, x > 0$ or $x \in (0;\infty)$
6.3	$x = \log_2 y$ $y = 2^x$
6.4	
6.5	$p = \log_2 4 = 2$ $\therefore y = 2$
6.6	Reflection in the y –axis and translate 2 units down.

PAPER F

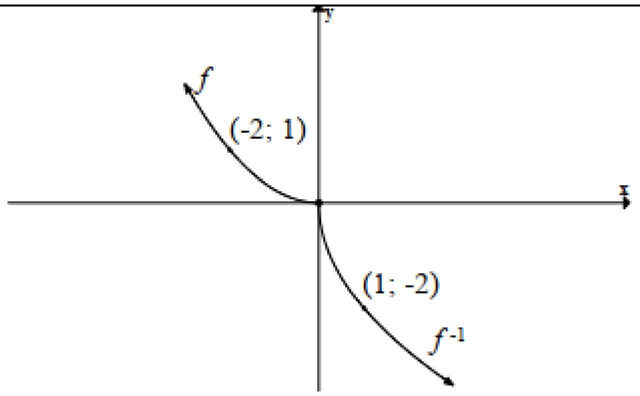
QUESTION 5

5.1.1	C(0 ; –3)
5.1.2	$f(x)=x^2-2x-3$ $(x-3)(x+1)=0$ $x=-1$ or $x=3$ $AB=3-(-1)$ $AB=4$ units
5.1.3	$x=\frac{2}{2(1)}$ or $2x-2=0$ or $x=\frac{-1+3}{2}$ $=1$ $y=(1)^2-2(1)-3$ $=-4$ $D(1;-4)$
5.1.4	C(0 ; –3) D(1 ; – 4) Average gradient / *Gemiddelde gradiënt* $=\frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ $=-1$
5.1.5	OC = OB = 3 $O\hat{C}B=45°$ isosceles right angled triangle *Gelykbenige reghoekige driehc* **OR / OF** $\tan\beta=m_g$ $\tan\beta=1$ $\beta=45°$ $O\hat{B}C=45°$ $O\hat{C}B=45°$
5.1.6	$-4<k<-3$ **OR** $(-4;-3)$

5.1.7	$f'(x)\ .f''(x) > 0$ $(2x-2).2 > 0$ $2x-2 > 0$ $x > 1$

5.2	 $f(x) = a(x-1)(x-5)$ $4 = a(3-1)(3-5)$ $4 = -4a$ $a = -1$ $f(x) = -x^2 + 6x - 5$

QUESTION/*VRAAG* 6

6.1	$f: \quad y = \frac{1}{4}x^2$ $f^{-1}: \quad x = \frac{1}{4}y^2$ $y^2 = 4x$ $y = \pm\sqrt{4x}$ $f^{-1}(x) = -\sqrt{4x}$ **OR**/OF $f^{-1}(x) = -2\sqrt{x}$
6.2	
6.3	Yes. No value of x in the domain of f^{-1} maps onto more than one y-value. *Ja. Geen waarde van x in die definisieversameling van f^{-1} assosieer met meer as een y-waarde nie.* **OR/*OF*** Yes. One to one function./*Ja. Een-tot-een-funksie.* **OR/*OF*** Yes. Vertical line test holds./*Ja. Die vertikale lyntoets werk.*

FINANCE, GROWTH AND DECAY

PAPER A

7.1	$A = P(1+i)^n$ $23000 = 1570(1.12)^n$ $(1.12)^n = 14,64968153..$ $n\log(1,12) = \log 14,64968153..$ $n = 23,69$ years or $n = 24$ years or $n = 23$ years 8 months or $n = 23,7$ years

7.2.1	$A = P(1+i)^n$ $= 800000(1.08)^5$ $= R1175462,46$ $\therefore R1175462,46 - R200\,000$ $= R975462,46$ Some calculators give R 975 462,50
7.2.2	$F = \dfrac{x[(1+i)^n - 1]}{i}$ $975462,46 = x\dfrac{[1,01]^{60} - 1}{0,01}$ $\dfrac{975462,46 \times 0.01}{[1,01]^{60} - 1} = x$ $x = R\,11944,00$

7.2.3	$Service = [5000(1{,}01)^{48} + 5000(1{,}01)^{36} + 5000(1{,}01)^{24} + 5000(1{,}01)^{12} + 5000]$ $= 32197{,}77$ $975462{,}46 = x\dfrac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R\,12338{,}24$ **OR** $Service = \dfrac{5000[1{,}01^{60} - 1]}{1{,}01^{12} - 1}$ $= 32197{,}77$ $975462{,}46 = x\dfrac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R\,12338{,}24$

PAPER B

QUESTION 7

7.1	$A = P(1-i)^n$ $\dfrac{P}{2} = P(1-0{,}07)^n$ $\dfrac{1}{2} = 0{,}93^n$ $\log\dfrac{1}{2} = n\log 0{,}93$ $n = \dfrac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years	**OR**	$A = P(1-i)^n$ $\dfrac{P}{2} = P(1-0{,}07)^n$ $\dfrac{1}{2} = 0{,}93^n$ $\log_{0{,}93}\dfrac{1}{2} = n$ $n = 9{,}55$ years

7.2	**Radesh:** $A = P(1+in)$ $= 6\ 000(1+0{,}085\times 5)$ $= 8\ 550$ OR $A = 6\ 000 + 8{,}5\% \text{ of } 6000\times$ $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\ 550$ $\text{Bonus} = 0{,}05\times 6\ 000$ $= 300$ $\text{Received} = 8\ 550 + 300$ $= \text{R}8\ 850$ **Thandi:** $A = P(1+i)^n$ $= 6\ 000\left(1+\frac{0{,}08}{4}\right)^{20}$ $= \text{R}8\ 915{,}68$ Thandi's investment is bigger.

7.3	F_v = initial deposit with interest + annuity $= 1\ 000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ $= 1\ 250{,}58 + 14\ 032{,}33$ $= \text{R}15\ 282{,}91$

PAPER C

QUESTION 8

8.1	$\text{Depreciation value} = 7\ 200(1-0{,}25)^3$ $= \text{R}3\ 037{,}50$

8.2.1	$300\ 000=\frac{5\ 000\left[1-(1{,}015)^{-n}\right]}{0{,}015}$ $4\ 500=5\ 000-5\ 000(1{,}015)^{-n}$ $5\ 000(1{,}015)^{-n}=500$ $(1{,}015)^{-n}=0{,}1$ or $(1{,}015)^{n}=10$ $-n=\frac{\log 0{,}1}{\log 1{,}015}$ $n=154{,}65$ Number of payments = 155
8.2.2	Balance outstanding $=300\ 000\left(1+\frac{0.18}{12}\right)^{154}-\frac{5\ 000\left[\left(1+\frac{0.18}{12}\right)^{154}-1\right]}{\frac{0.18}{12}}$ $=\text{R}3\ 230{,}50$

8.2.3	Amount paid in last month $=3\ 230{,}50\left(1+\frac{0.18}{12}\right)$ $=\text{R}3\ 278{,}96$
8.2.4	Total repaid = (154 × 5 000) + 3 278,96 = R773 278,96

PAPER D

QUESTION 8

8.1	$A=P(1-i)^n$ $250000=P(1-13{,}5\%)^5$ $P=\frac{250000}{(1-13{,}5\%)^5}$ $=R516249$

8.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $950000 = \frac{x\left[1-\left(1+\frac{14,25\%}{12}\right)^{-240}\right]}{\frac{14,25\%}{12}}$ $x = R11986,33$

8.2.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{11986,33\left[1-\left(1+\frac{14,25\%}{12}\right)^{-140}\right]}{\frac{14,25\%}{12}}$ $= R816048,67$ **OR** $A = P(1+i)^n$ $A = 950\ 000\left(1+\frac{14,25\%}{12}\right)^{100}$ $=R3093215,766$ $F = \frac{x[(1+i)^n-1]}{i}$ $F = \frac{11986,33\left[\left(1+\frac{14,25\%}{12}\right)^{100}-1\right]}{\frac{14,25\%}{12}}$ $=R2277167,107$ Balance on Loan $= R3093215,766 - R2277167,107$ $= R816048,67$

8.2.3	$A = P(1+i)^n$ $= 816\ 048,67\left(1+\frac{14,25\%}{12}\right)^4$ $= R855\ 506,92$ $855\ 506,92 = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{x\left[1-\left(1+\frac{14,25\%}{12}\right)^{-136}\right]}{\frac{14,25\%}{12}}$ $x = R12711,51$

PAPER E

QUESTION 8

8.1	$A = P(1+i)^n$ $3P = P\left(1+\frac{i}{12}\right)^{72}$ $i = 12(\sqrt[72]{3}-1)$ $i = 0{,}1845$ Annual interest rate is 18,45 % p.a.
8.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $192\ 000 = \frac{x\left[1-\left(1+\frac{0.12}{12}\right)^{-60}\right]}{\frac{0.12}{12}}$ $x = R4270{,}93$
8.2.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{4270{,}93\left[1-\left(1+\frac{0.12}{12}\right)^{-15}\right]}{\frac{0.12}{12}}$ $= R59216{,}72421$

	OR $A = P(1+i)^n$ $A = 192\ 000\left(1+\frac{0.12}{12}\right)^{45}$ =R300 443,6635 $F = \frac{x[(1+i)^n-1]}{i}$ $F = \frac{4270{,}934\left[\left(1+\frac{0.12}{12}\right)^{45}-1\right]}{\frac{0.12}{12}}$ =R241 226,9424 Balance on Loan = R300 443,6635 −R241 226,9424 = R59216,7211

PAPER F

QUESTION 4	
4.1	$4500 = 3000\left(1+\frac{0,08}{12}\right)^n$ $\frac{3}{2} = \left(1+\frac{0,08}{12}\right)^n$ $\log_{\left(1+\frac{0,08}{12}\right)}\frac{3}{2} = n$ $n = 61,02$ months / (accept 62) $n = 5,09$ years (accept 5,17) **NOTE: (5,08 is NOT accepted.)**
4.2.1	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $40\,000 = \frac{x[1-(1+\frac{0,24}{12})^{-240}]}{\frac{0,24}{12}}$ $x = $ R806,96
4.2.2	$P_o = \frac{x[1-(1+i)^{-n}]}{i}$ $P_o = \frac{806,96\left[1-\left(1+\frac{0,24}{12}\right)^{-180}\right]}{\frac{0,24}{12}}$ $= R39205,67$ **OR** $P_o = P(1+i)^n - \frac{x[(1+i)^n-1]}{i}$ $P_o = 40000(1+\frac{0,24}{12})^{60} - \frac{806,96\left[\left(1+\frac{0,24}{12}\right)^{60}-1\right]}{\frac{0,24}{12}}$ $= R39\,206,20$

4.2.3	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $39205{,}67 = \frac{x[1-(1+\frac{0{,}18}{12})^{-180}]}{\frac{0{,}18}{12}}$ $x = \text{R}631{,}38$ **NOTE: If the candidate uses R39 206,20 then the answer is R631,39.**

QUESTION 7

7.1	$1+i_{eff} = \left(1+\frac{0{,}11}{2}\right)^2$ $i_{eff} = \left(1+\frac{0{,}11}{2}\right)^2 - 1$ $i_{eff} = 11{,}30\%$ $\therefore$ Mary has secured the better rate.
7.2.1	$Fv = \frac{10\ 000\left[\left(1+\frac{0{,}0772}{12}\right)^{114}-1\right]}{\frac{0{,}0772}{12}}$ $= R1\ 674\ 501{,}44$

7.2.2	$R1\ 674\ 501{,}44 = \dfrac{30000\left[1-\left(1+\dfrac{0{,}1}{12}\right)^{-n}\right]}{\dfrac{0{,}1}{12}}$ $0{,}46513\ldots = \left[1-\left(1+\dfrac{0{,}1}{12}\right)^{-n}\right]$ $0{,}53486\ldots = \left(1+\dfrac{0{,}1}{12}\right)^{-n}$ $\log_{\left(1+\frac{0{,}1}{12}\right)} 0{,}53486\ldots = -n$ $n = 75{,}4$ She will be able to receive the money in 75 full months.
7.2.3	$Pv = \dfrac{30\ 000\left[1-\left(1+\dfrac{0{,}1}{12}\right)^{-55}\right]}{\dfrac{0{,}1}{12}}$ $Pv =$ R 1 319 260,60 $\therefore$ No

DIFFERENTIAL CALCULUS

PAPER A

9.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^2-\frac{1}{2}(x+h)-\left(x^2-\frac{1}{2}x\right)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-\frac{1}{2}x-\frac{1}{2}h-x^2+\frac{1}{2}x}{h}$ $=\lim_{h\to 0}\frac{h\left(2x+h-\frac{1}{2}\right)}{h}$ $=2x-\frac{1}{2}$
9.2.1	$\frac{d}{dx}[3x^4+\sqrt[5]{x}+a^2]$ $\frac{d}{dx}\left[3x^4+x^{\frac{1}{5}}+a^2\right]$ $=12x^3+\frac{1}{5}x^{-\frac{4}{5}}$
9.2.2	$xy=x+x^2-1$ $y=1+x-x^{-1}$ $\frac{dy}{dx}=1+x^{-2}$

QUESTION 10

10.1.1	$x^3 + 5x^2 - 8x - 12 = 0$ $(x+1)$ is a factor $f(-1) = 0$ $(x+1)(x^2 - 4x - 12) = 0$ $(x+6)(x+1)(x-2) = 0$ $x = -6 \;\; or \;\; x = -1 \;\; or \;\; x = 2$
10.1.2	$f(x) = x^3 + 5x^2 - 8x - 12$ $f'(x) = 3x^2 + 10x - 8 = 0$ $(3x-2)(x+4) = 0$ $x = \frac{2}{3} \; or \; x = -4$ $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) - 12 = -\frac{400}{27}$ $= -14{,}81$ $B\left(\frac{2}{3}; -14{,}81\right)$

10.1.3	$f''(x) = 6x + 10 = 0$ $x = -\frac{5}{3}$ **OR** $x = \frac{\frac{2}{3} + (-4)}{2} = -\frac{5}{3}$ **OR** $x = -\frac{b}{3a} = -\frac{5}{3}$

10.2.1	$f'(0) = -8$ $y = -8x - 12$
10.2.2	$f'(x).g'(x) > 0$ Since $g'(x) < 0$ for all $x \in R$ $(3x^2 + 10x - 8) < 0$ $(3x - 2)(x + 4) < 0$ $-4 < x < \frac{2}{3}$ **OR** $-4 < x < \frac{2}{3}$

PAPER B

9.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h}$ $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - bx - bh - x^2 + bx}{h}$ $f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - bh}{h}$ $f'(x) = \lim_{h \to 0} \frac{h(2x + h - b)}{h}$ $f'(x) = 2x - b$

9.2.1	$\frac{d}{dx}\left[\frac{x^4}{4} - 3\sqrt[3]{x} + 7\right]$ $\frac{d}{dx}\left[\frac{x^4}{4} - 3x^{\frac{1}{3}} + 7\right]$ $= x^3 - x^{-\frac{2}{3}}$
9.2.2	$y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ $y = x^{\frac{2}{3}} - 4x + 4x^{\frac{4}{3}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - 4 + \frac{16}{3}x^{\frac{1}{3}}$

QUESTION 10

10.1		
	10.1.1	$x = -5$ or $x = 1$
	10.1.2	$x = -4$ or $x = 0$

10.2	$y = a(x+5)(x-1)$ $-15 = a(0+5)(0-1)$ $-15 = -5a$ $3 = a$ $y = 3(x+5)(x-1) = 3x^2 + 12x - 15$ **OR** $y = a(x+p)^2 + q$ $y = a(x+2)^2 + q$ $(0\,;\,-15)$: $-15 = a(0+2)^2 + q$ $-15 = 4a + q$ .... (1) $(1\,;\,0)$: $0 = a(1+2)^2 + q$ $0 = 9a + q$ .... (2) (2) – (1) : $15 = 5a \quad \therefore \quad a = 3$ $0 = 27 + q \quad \therefore \quad q = -27$ $y = 3(x+2)^2 - 27$ $y = 3(x^2 + 4x + 4) - 27$ $y = 3x^2 + 12x - 15$

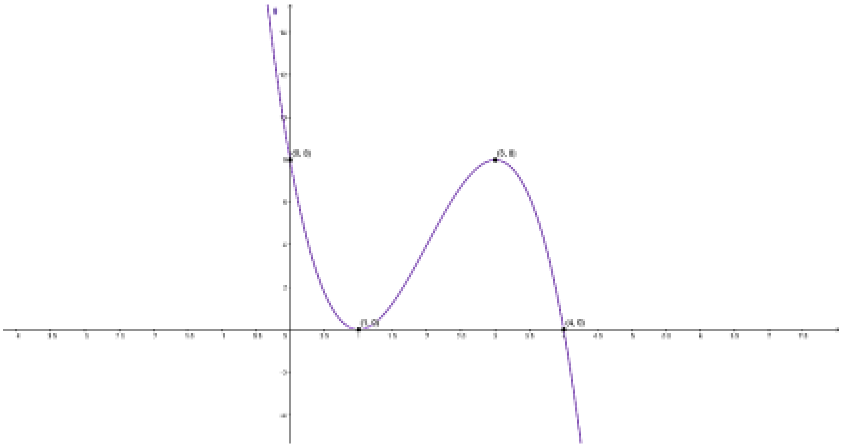
10.3	$f(x) = ax^3 + bx^2 + cx + d$ $f'(x) = 3ax^2 + 2bx + c$ $f'(x) = 3x^2 + 12x - 15$ Equating coefficients of equal polynomials $3a = 3$ *and* $2b = 12$ *and* $c = -15$ $a = 1$ *and* $b = 6$ *and* $c = -15$ $f(x) = x^3 + 6x^2 - 15x + d$

	$f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$

10.4	$x = -5$: $y = (-5)^3 + 6(-5)^2 - 15(-5) - 72 = 28$ $(-5\,;28)$ maximum point $x = 1$: $y = (1)^3 + 12(1)^2 - 15(1) - 72 = -74$ $(1\,;-74)$ minimum point
10.5	$3x^2 + 12x - 15 = t$ $3x^2 + 12x - 15 - t = 0$ $\Delta = b^2 - 4ac = 0$ $\Delta = (12)^2 - 4(3)(-15 - t) = 0$ $144 + 180 + 12t = 0$ $12t = -324$ $t = -27$

PAPER C

9.1	Volume of Sphere $=\frac{4}{3}\pi(8)^3$ or $=\frac{2048\pi}{3}$ or $=2144{,}66$
9.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$
9.3	$V_{cone} = \frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(64 - x^2\right)(8 + x)$ $=\frac{\pi}{3}\left(512 + 64x - 8x^2 - x^3\right)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3}$ $x \neq -8$ $\frac{V_{cone}}{V_{sphere}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $=\frac{8}{27} = 0{,}3$

QUESTION 10	
10.1	
10.2	$f(x) = a(x-1)^2(x-4)$ $8 \;= a(3-1)^2(3-4)$ $8 \;= a(-4)$ $a \;= -2$ $f(x) = -2(x-1)^2(x-4)$ $f(x) = -2(x^2-2x+1)(x-4)$ $f(x) = -2x^3+12x^2-18x+8$
10.3	$f(x) = -2x^3+12x^2-18x+8$ $f'(x) = -6x^2+24x-18$ $f''(x) = -12x+24$ $f''(x) < 0$ $-12x+24 < 0$ $-x < -2$ $x > 2$

QUESTION 12

12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \ldots \text{(i)}$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \ldots \text{(ii)}$ (ii) + (i): $\quad 6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$

12.2	Average Gradient $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= \frac{13{,}5}{3}$ $= \frac{9}{2}$

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\ ;\ 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ **OR** $x = \frac{-1+2}{2}$ $x = \frac{1}{2}$
12.5	$p > 3{,}5$ or $p < -10$

PAPER D

QUESTION 8

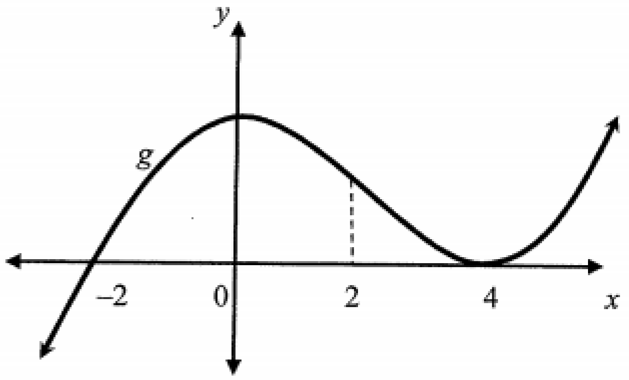
8.1	$f(x) = 2x^2 - 5x + 3$ $f(x+h) = 2(x+h)^2 - 5(x+h) + 3$ $= 2(x^2 + 2xh + h^2) - 5x - 5h + 3$ $= 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $- (2x^2 - 5x + 3)$ $= 4xh + 2h^2 - 5h$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h}$ $= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$ $= \lim_{h \to 0} (4x + 2h - 5)$ $= 4x - 5$

8.2	$y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3+1}{x^3}$ $= \frac{2x^2}{3x^{\frac{1}{2}}} - 2 - \frac{1}{x^3}$ $= \frac{2}{3}x^{\frac{3}{2}} - 2 - x^{-3}$ $\frac{dy}{dx} = x^{\frac{1}{2}} + 3x^{-4}$

QUESTION 9

9.1.1	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $0 = (x-3)(-2x^2 - x + 1)$ $x - 3 = 0$ or $-2x^2 - x + 1 = 0$ $x = 3$ $\quad 2x^2 + x - 1 = 0$ $(3; 0)$ $\quad (2x-1)(x+1) = 0$ $2x = 1$ or $x = -1$ $x = \frac{1}{2}$ $\quad (-1; 0)$ $\left(\frac{1}{2}; 0\right)$

9.1.2	$f'(x) = -6x^2 + 10x + 4$ $0 = -6x^2 + 10x + 4$ $3x^2 - 5x - 2 = 0$ $(3x+1)(x-2) = 0$ or $x = \frac{-10 \pm \sqrt{10^2 - 4(-6)(4)}}{2(-6)}$ $3x = -1$ or $x = 2$ $x = -\frac{1}{3}$
9.1.3	$f''(x) = -12x + 10$ $0 = -12x + 10$ $12x = 10$ $x = \frac{10}{12} = \frac{5}{6}$ $\therefore x < \frac{5}{6}$

9.2.1	
9.2.2	$x < -2$ or $x > 2;\ x \neq 4$

QUESTION 10

10.1	$N(t) = t^3 - 12t^2 + 36t + 8$ $N(0) = 8$ $\therefore$ 8 people
10.2	$N'(t) = 3t^2 - 24t + 36$ increasing $N'(t) \geq 0$ $3t^2 - 24t + 36 \geq 0$ $t^2 - 8t + 12 \geq 0$ $(t-6)(t-2) \geq 0$ $t \geq 6$ or $t \leq 2$ $\therefore$ for first 2 hours after opening or 6 hours after opening until closing time
10.3	Minimum turning point at $t = 6$ hours after opening

QUESTION 11

11.1	$Area = x^2 + 3x^2 + 4xh$ $Area = 4x^2 + 4xh$ $V = x^2h = 1000$ $h = \dfrac{1000}{x^2}$ $A = 4x^2 + 4x\left(\dfrac{1000}{x^2}\right)$ $A = 4x^2 + \dfrac{4000}{x}$

11.2	$A = 4x^2 + 4000x^{-1}$ $A' = 8x - 4000x^{-2} = 0$ $8x = \frac{4000}{x^2}$ $x^3 = 500$ $x = \sqrt[3]{500} = 7{,}94\ cm$ $h = \frac{1000}{(7{,}94)^2} = 15{,}86\ cm$

PAPER E

QUESTION/*VRAAG* 8

8.1	$f(x+h) = 3 - 2(x+h)^2$ $= 3 - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2$ $= -4xh - 2h^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \to 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \to 0} (-4x - 2h)$ $= -4x$

8.2	$y=\dfrac{12x^2+2x+1}{6x}$ $=2x+\dfrac{1}{3}+\dfrac{1}{6x}$ $=2x+\dfrac{1}{3}+\dfrac{1}{6}x^{-1}$ $\dfrac{dy}{dx}=2-\dfrac{1}{6}x^{-2}$ $=2-\dfrac{1}{6x^2}$

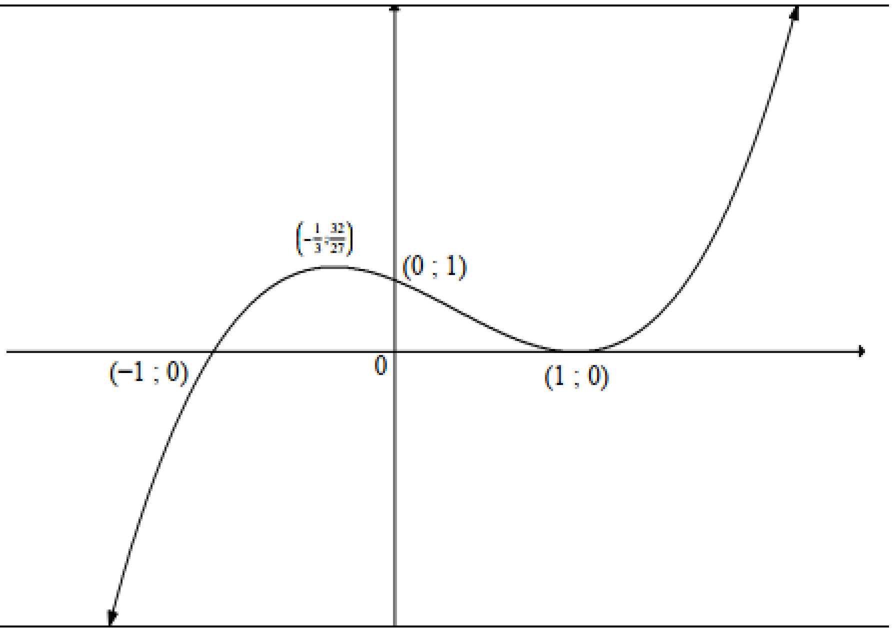
8.3	$y=x^3+bx^2+cx-4$ $y'=3x^2+2bx+c$ $y''=6x+2b$ At point of inflection: $y''=6x+2b=0$ Substitute $x=2$: $6(2)+2b=0$ $2b=-12$ $b=-6$ $y=x^3-6x^2+cx-4$ Substitute $(2;\,4)$: $4=2^3-6(2)^2+c(2)-4$ $2c=24$ $c=12$ $y=x^3-6x^2+12x-4$

QUESTION/*VRAAG* 9

9.1	$(0\,;\,1)$
9.2	$f(x)=x^3-x^2-x+1$ $f(x)=x^2(x-1)-(x-1)$ $f(x)=(x-1)(x^2-1)$ $f(x)=(x-1)(x-1)(x+1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: $(-1;\,0);\,(1;\,0)$

OR

	$f(x)=x^3-x^2-x+1$ $f(x)=(x-1)(x^2-1)$ $f(x)=(x-1)(x-1)(x+1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: $(-1;\,0);\,(1;\,0)$
9.3	$f(x)=x^3-x^2-x+1$ $f'(x)=3x^2-2x-1$ $f'(x)=0$ $(3x+1)(x-1)=0$ $x=-\frac{1}{3}$ or $x=1$ $y=\frac{32}{27}$ $\quad y=0$ $\left(-\frac{1}{3};\frac{32}{27}\right)$ $\quad (1;0)$

9.4	
9.5	$f'(x) < 0$ $-\frac{1}{3} < x < 1$

QUESTION 10

10.1	$s(t) = 2t^2 - 18t + 45$ $s'(t) = 4t - 18$ $s'(0) = 4(0) - 18$ $= -18\ m/s$
10.2	$s''(t) = 4\,\text{m/s}^2$

10.2	$s''(t) = 4\,\text{m/s}^2$
10.3	$4t - 18 = 0$ $4t = 18$ $t = \dfrac{9}{2}$ seconds or $4{,}5$ seconds **OR** $s(t) = 2\left(t - \dfrac{9}{2}\right)^2 + \dfrac{9}{2}$ $t = \dfrac{9}{2}$ seconds or $4{,}5$ seconds **OR** $s(t) = 2t^2 - 18t + 45$ $t = -\dfrac{-18}{2(2)}$ $t = \dfrac{9}{2}$ seconds or $4{,}5$ seconds

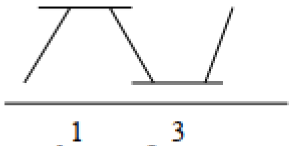
PAPER F

8.1	$f(x+h)=\dfrac{4}{x+h}$ $f(x+h)-f(x)=\dfrac{4}{x+h}-\dfrac{4}{x}$ $=\dfrac{4x\text{-}4(x+h)}{x(x+h)}$ $=\dfrac{4x-4x-4h}{x(x+h)}$ $=\dfrac{-4h}{x(x+h)}$ $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{-4h}{x(x+h)}}{h}$ $=\dfrac{-4h}{xh(x+h)}$ $\dfrac{-4}{x(x+h)}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-4}{x(x+h)}$ $=\dfrac{-4}{x^2}$

8.2.1	$y=5x^2+5x+2$ $\dfrac{dy}{dx}=10x+5$
8.2.2	$D_x\left[\sqrt[3]{x^2}-\dfrac{1}{2}x\right]$ $=D_x\left[x^{\frac{2}{3}}-\dfrac{1}{2}x\right]$ $=\dfrac{2}{3}x^{\frac{-1}{3}}-\dfrac{1}{2}$

8.3	$p(x)=x^3+2x$ $p'(x)=3x^2+2$ $3x^2\geq 0$ or / *of* $x^2\geq 0$ for all/*vir alle* $x\in\mathbf{R}$ $\therefore 3x^2+2\geq 2>0$ for all/*vir alle* $x\in\mathbf{R}$ i.e. $p'(x)>0$ for all/*vir alle* $x\in\mathbf{R}$ i.e. all tangents to p have gradient greater than (or equal to) 2. Thus there is no tangent to p that has negative gradient.

QUESTION/*VRAAG* 9

9.1	$x=1$ or $x=3$
9.2	$1<x<3$
9.3	For a point x close to 3/*Vir 'n punt naby aan 3*: If $x<3$, $f'(x)<0 \Rightarrow f$ decreasing/*dalend* If $x>3$, $f'(x)>0 \Rightarrow f$ increasing/*stygend* Therefore: f has a local minimum at/*f het lokale minimum by* $x=3$  **OR/*OF*** At $x=3$, the gradient function changes from negative to positive therefore the function will have a local minimum point at $x=3$/ *By* $x=3$ *verander die gradiëntfunksie van negatief na positief dus sal die funksie 'n lokale minimum punt hê by* $x=3$.
	OR/*OF* $f'(3)=0$ and $f''(3)>0$ therefore the function will have a local minimum point at $x=3$ / $f''(3)>0$ *dus sal die funksie 'n lokale minimum punt hê by* $x=3$.
9.4	$f''(x)=0$ at the turning point of/*by die draaipunt van* $f'(x)$ Using symmetry/*Deur simmetrie* $x=\dfrac{1+3}{2}$ $=2$
9.5	Concave up if/*Konkaaf op as* $f''(x)>0$ $x>2$

QUESTION/*VRAAG* 10

	Given: $M(t)=t^3-9t^2+3000 \quad ; \quad 0\le t\le 30$
10.1	$M(0)=0^3-9(0)^2+3000$ $=3000g$ or $3kg$
10.2	$t^3-9t^2+3000=3000$ $t^3-9t^2=0$ $t^2(t-9)=0$ $t=0$ or $t=9$ Baby's mass will return to the birth mass on the 9th day/ *Baba se massa keer terug na massa by geboorte op die 9de dag.*
10.3	$M'(t)=0$ $3t^2-18t=0$ $3t(t-6)=0$ $t=0$ or $t=6$ Baby's mass will be a minimum on the 6th day/ *Baba se massa sal 'n minimum wees op die 6de dag.*
10.4	$M'(t)=3t^2-18t$ $M''(t)=6t-18$ $0=6t-18$ $t=3$
	OR / *OF* Using symmetry/*Deur simmitrie*: $t=\frac{0+6}{2}$ $=3$

PROBABILITY AND COUNTING

BASIC PROBABILITY QUESTIONS AND/OR VENN DIAGRAMS

PAPER A

QUESTION 4

4.1	P(A or B) = 0,3 + 0,5 = 0,8
4.2	Since A and B are independent P(A or B) = P(A) + P(B) – P(A and B) = 0,3 + 0,5 – 0,15 = 0,65

PAPER B

QUESTION 4

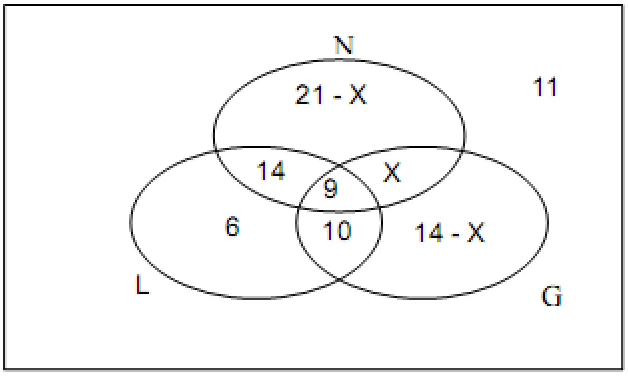
4.1	Since A and C are mutually exclusive, there is no intersection of A and C $\therefore$ P(A and C) = 0.
4.2	Since B and C are independent, P(B and C) = P(B).P(C). P(B and C) = (0,4)(0,2) = 0,08
4.3	Since A and B are independent, P(A and B) = P(A).P(B). P(A and B) = (0,3)(0,4) = 0,12 P(A or B) = P(A) + P(B) – P(A and B) = 0,3 + 0,4 – 0,12 = 0,58

PAPER D

QUESTION 4

4.1.1 11 students

4.1.2 Let N represent students reading the *National Geographic* magazine, G represent students reading the *Getaway* magazine and L represent students reading the *Leadership* magazine.



4.1.3 $21 - x + x + 14 - x + 9 + 14 + 10 + 6 + 11 = 80$

$$85 - x = 80$$
$$x = 5$$

4.1.4 P(student reads at least two magazines) $= \dfrac{5+14+10+9}{80} = 0{,}475$

4.2.1

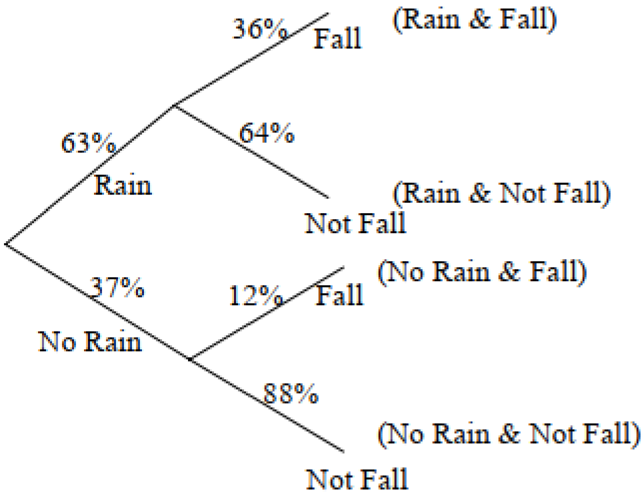
P(smoke detected by device A or device B)
= P(smoke detected by A) + P(smoke detected by B) – P(smoke detected by both)
$= 0{,}95 + 0{,}98 - 0{,}94$
$= 0{,}99$

4.2.2 P(smoke not detected) $= 1 - 0{,}99 = 0{,}01$

PAPER E

QUESTION 3

3.1

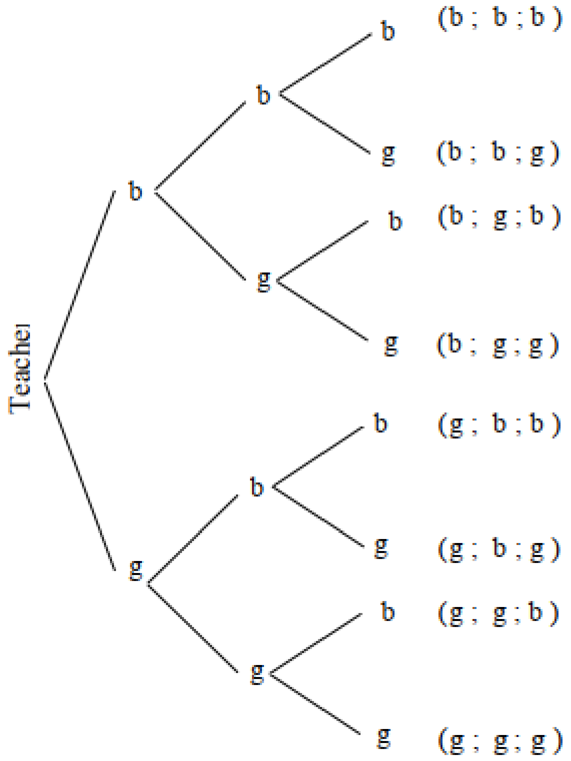


3.2	P(Not Fall)	$= \left(\frac{37}{100} \times \frac{88}{100}\right) + \left(\frac{63}{100} \times \frac{64}{100}\right)$ $= \frac{407}{1250} + \frac{252}{625}$ $= \frac{911}{1250}$ $= 0{,}7288$
3.3	P(Dry & Fall)	$= \frac{37}{100} \times \frac{12}{100}$ $= \frac{111}{2500}$ $= 0{,}0444$

4.2.1 $P(\text{boy chosen first}) = \frac{20}{35} = \frac{4}{7} = 0{,}57$.

4.2.2

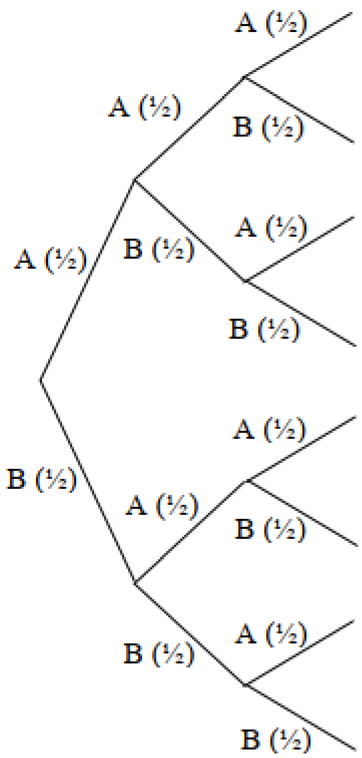
Outcomes



4.2.3 $P(b\ ;\ g\ ;\ b\ ) = \frac{20}{35} \times \frac{15}{34} \times \frac{19}{33} = \frac{190}{1309} = 0{,}15$

4.2.4 $P(g\ ;\ g\ ;\ g\ ) = \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = \frac{13}{187} = 0{,}07$

4.2.5 $P(\text{at least one boy})$
$$\begin{aligned} &= 1 - P(\text{three girls chosen}) \\ &= 1 - 0{,}07 \\ &= 0{,}93 \end{aligned}$$

5.1	Let A represent Alfred winning a point and B represent Barry winning a point.  Outcomes (A ; A ; A) (A ; A ; B) (A ; B ; A) (A ; B ; B) (B ; A ; A) (B ; A ; B) (B ; B ; A) (B ; B ; B)
5.2	P(Barry wins three points) $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$
5.3	P(Alfred wins two points and Barry wins one point) = P(A; A; B) + P(A; B; A) + P(B; A; A) $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{3}{8}$
5.4	P(Alfred wins 3 of the four points) = P(AAAB) + P(AABA) + P(ABAA) + P(BAAA) $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= 4\left(\frac{1}{2}\right)^4$ $= \frac{1}{4}$

CONTINGENCY TABLES

PAPER A

5.1.1	$P(\text{male}) = \frac{120}{236}$ $= \frac{30}{59}$ $= 0{,}51\ (0{,}508474...)$
5.1.2	$P(\text{female and plays sport})$ $= \frac{67}{236}$ $= 0{,}28\ (0{,}2838983051...)$
5.2	No. From the table, $P(\text{male and do not play sport}) = \frac{51}{236}$, which is greater than zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.

5.3	$P(\text{male}) = \frac{120}{236}$ $P(NS) = \frac{100}{236}$ $P(\text{male}) \times P(NS) = \frac{120}{236} \times \frac{100}{236}$ $= \frac{750}{3\,481}$ $= 0{,}22 \quad (0{,}215455...)$ $P(\text{male and NS}) = \frac{51}{236}$ $= 0{,}22 \quad (0{,}2161016949...)$ So, $P(\text{male}) \times P(NS) = P(\text{male and NS})$ Therefore the events 'male' and 'do not play sport' are independent (correct to TWO decimal places). **OR** The events are not independent as there is a discrepancy from the third decimal place.

PAPER B

5.1	$a = 450$ $b = 319$ $c = 298$ $d = 748$
5.2	P(Female who has not broken a limb) $= \frac{298}{1530}$ $= \frac{149}{765}$
5.3	P(Female & broken a limb) $= \frac{450}{1530}$ $= \frac{5}{17}$ $= 0{,}2941176471...$ $= 0{,}29$ P(Female) × P(Broken a limb) $= \frac{748}{1530} \times \frac{913}{1530}$ $= 0{,}29$ The events of being female and having broken a limb are independent. If a candidate answers not independent due to the fact that the answers are not accurate to more than 2 decimal places, award full marks.

PROBABILITY AND COUNTING PRINCIPLE

PAPER A

10.2.1	$5!$ $= 120$
10.2.2	5^5 $= 3125$
10.3	$n(\mathrm{E}) = 5! \times 2! \times 2!$ $n(\mathrm{S}) = 7!$ $P(\mathrm{E}) = \dfrac{5! \times 2! \times 2!}{7!}$ $= \dfrac{2}{21}$

PAPER B

QUESTION 7

7.1	Number of ways $= 8 \times 8$ $= 64$
7.2	Number of ways for a 4-digit number $= 8 \times 7 \times 6 \times 5$ $= 1\ 680$ **OR** Number of ways for a 4-digit number $= \dfrac{8!}{(8-4)!}$ $= \dfrac{8!}{4!}$ $= 1680$
7.3	Numbers between 4 000 and 5 000 $= 1 \times 8 \times 8 \times 8$ $= 512$

PAPER C

QUESTION/*VRAAG* 12

12.1.1	$26\times25\times24\times23\times22$ $=7\ 893\ 600$

12.1.2	$24\times23\times22$ $=12\ 144$

PAPER D

5.1.1	Number of PIN codes $= 10 \times 10 \times 10 \times 10 \times 10$ $= 10^5$ $= 100\ 000$
5.1.2	Number of PIN codes $= 10 \times 9 \times 8 \times 7 \times 6$ $= 30\ 240$ **OR** Number of PIN codes $=\dfrac{10!}{5!}$ $= 30\ 240$
5.2	Number of PINs that DO NOT contain 9s $=9\times9\times9\times9\times9$ $=59\ 049$ P(at least one 9) $=1-\text{P(no 9s)}$ $=1-\dfrac{59049}{100000}$ $=0{,}41$

PAPER E

5.2.1 Any learner seated in any position in: $6! = 6\times5\times4\times3\times2\times1$
$= 720$ different ways.

5.2.2 $2 \times 5! = 240$

PAPER F

QUESTION 6

6.1	Number of different ways the shirts and trousers can be arranged $=(7+4)!$ $=11!$ $=39\ 916\ 800$
6.2	Number of ways so that the shirts are together and trousers are together $=7!.4!.2$ $=241\ 920$
6.3	P(Shirt at beginning and trouser at the end) $=\frac{9!\times4\times7}{11!}$ $=\frac{14}{55}$

PAPER G

QUESTION 5

5.1	Number of arrangements $=7!$ $=5040$
5.2	Number of arrangements $=5!$ $=120$
5.3	Number of arrangements $=3!\times5!$ $=720$

PAPER H

QUESTION/*VRAAG* 11

11.1	$8\times7\times6\times5\times4$ or $\frac{8!}{3!}$ $=6720$

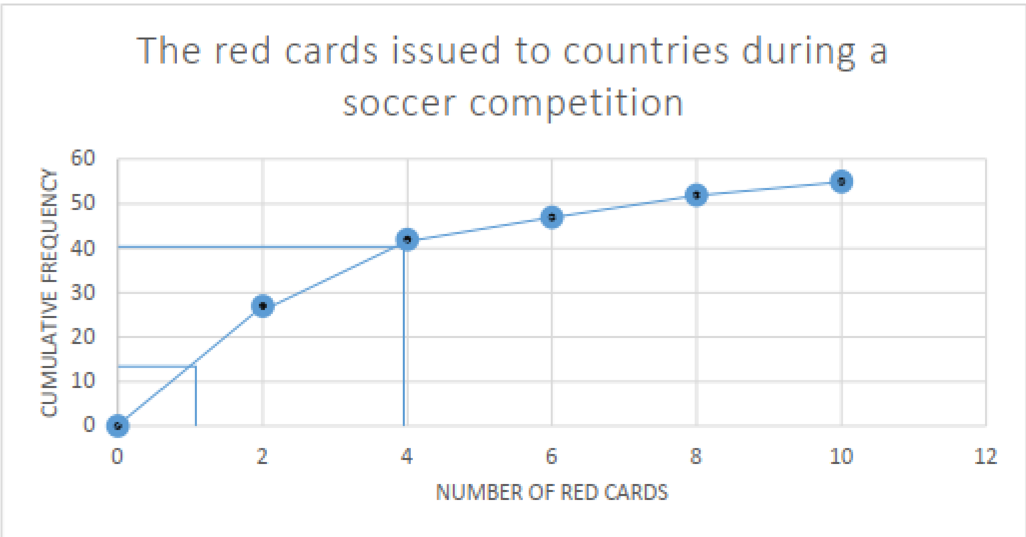
PAPER 2

STATISTICS

PAPER A

QUESTION 1

NUMBER OF RED CARDS	NUMBER OF COUNTRIES (f)	MIDPOINT OF INTERVAL (x)	$f.x$
$0 < x \leq 2$	27	1	27
$2 < x \leq 4$	15	3	45
$4 < x \leq 6$	5	5	25
$6 < x \leq 8$	5	7	35
$8 < x \leq 10$	3	9	27
TOTAL	55		159

1.1	Estimated mean $= \frac{159}{55} = 2,89 \approx 3$ red cards Answer only full marks
1.2	

1.3	$Q_3 = 4$ and $Q_1 = 1$ $\therefore IQR = 4 - 1 = 3$ red cards

QUESTION 2

2.1	$A = 5{,}97$; $B = 2{,}18$ $Y = 5{,}97 + 2{,}18\,\boldsymbol{x}$ Answer only full marks
2.2	Estimated monthly income $y = 5{,}97 + 2{,}18\,(9)$ $= 25{,}59$ $\therefore$ Monthly income = R25598,89 If 9000 is used only 1 mark
2.3	$r = 0{,}94$
2.4	Very strong positive relationship between the monthly rent and the monthly income.

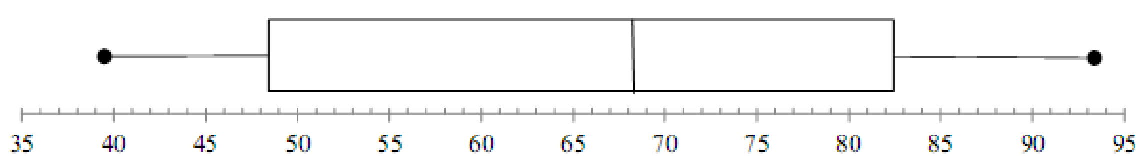
PAPER B

QUESTION 1

39	42	48	54	62	68	78	78	82	91	93

1.1	$\bar{x} = 66{,}82$ or 66,82 thousands
1.2	$\sigma = 18{,}3$
1.3	$(\bar{x} - \sigma;\ \bar{x} + \sigma) = (48{,}52\ ;\ 85{,}12)$ $\therefore$ 6 countries

1.4



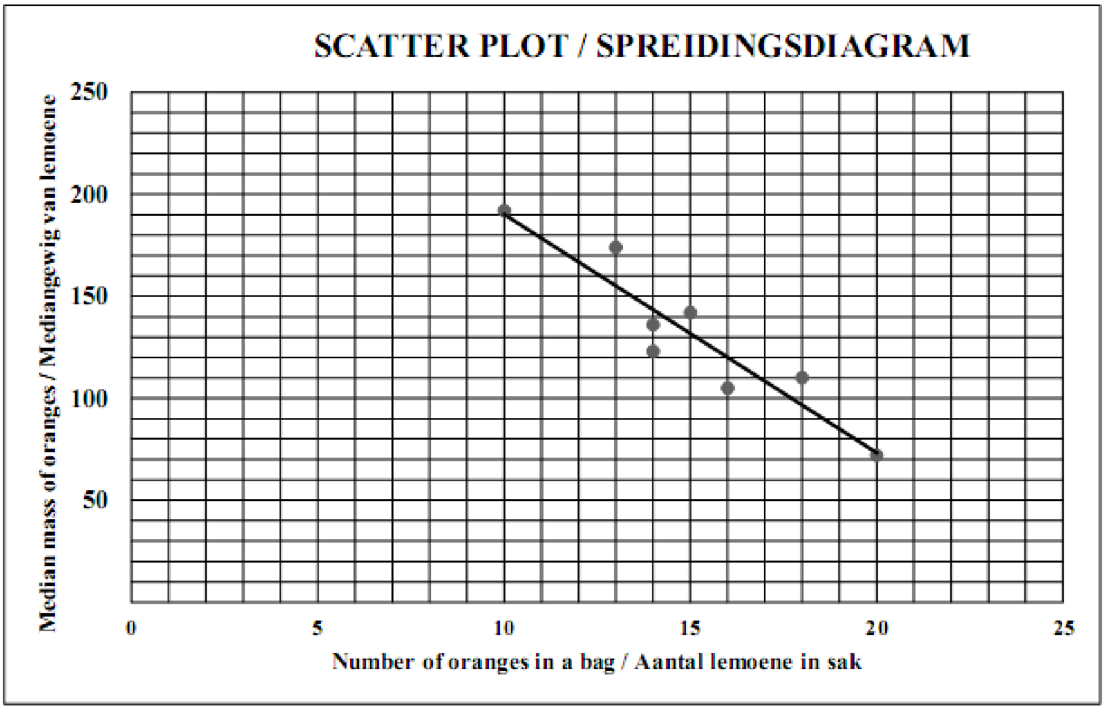
1.5	The data is skewed to the left/negatively skewed
1.6.1	The sum of the data provided for the years must increase.
1.6.2	IQR will remain the same or change.

QUESTION 2

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1	$a = 5{,}65$ *and* $b = 3{,}65$ $y = 5{,}65 + 3{,}65x$
2.2	$y = 5{,}65 + 3{,}65x$ $= 5{,}65 + 3{,}65(1{,}2)$ $= 10{,}03\%$
2.3	strong positive correlation of the data $r = 0{,}7$

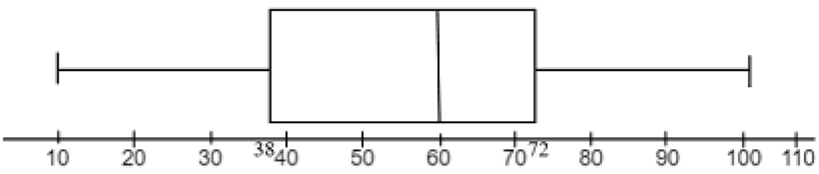
PAPER C



1.1	$a = 307{,}20$ $b = -11{,}70$ $\hat{y} = 307{,}20 - 11{,}7x$
1.2	$r = -0{,}93$

1.3	See scatter plot above/*sien spreidingsdiagram hierbo* (10 ; 190,2) (20 ; 73,2)
1.4	Negative strong association / *Negatiewe sterk assosiasie*
1.5	$\hat{y}=307,20-11,7(12)$ $=166,8$

QUESTION / *VRAAG* 2

2.1.1	100
2.1.2	Median / *Mediaan* = ±62
2.1.3	
2.1.4	Skewed to the left / *Skeef na links*
2.2	$b=20$ $\dfrac{d-a}{2}=8$ $2a=d$ $sub\ \dfrac{2a-a}{2}=8$ $a=16$ $d=32$ $5+16+19+20+c+32+35=7\times 22$ $\therefore c=27$

PAPER D

QUESTION 1

1.1	$\bar{x} = \dfrac{1\,581}{31}$ $= 51$ **OR / *OF*** $\bar{x} = 51$ (calculator method / *sakrekenaar metode*)
1.2	$\therefore$ skewed to the left
1.3	Physical Sciences performed better. Q_1 is 40% in Physical Sciences and 28% in Mathematics which indicates the lower 25% of the class performed much better in Physical Sciences than in Mathematics. *Fisiese Wetenskappe presteer beter.* *Q_1 is 40% in Fisiese Wetenskappe en 28% in Wiskunde wat aandui dat die onderste 25% van die klas heelwat beter presteer in Fisiese Wetenskappe as in Wiskunde.*
1.4	Accept any mark between 40 – 50. *Aanvaar enige punt tussen 40 – 50 .*
1.5	The greatest difference is $87\% - 71\% = 16\%$

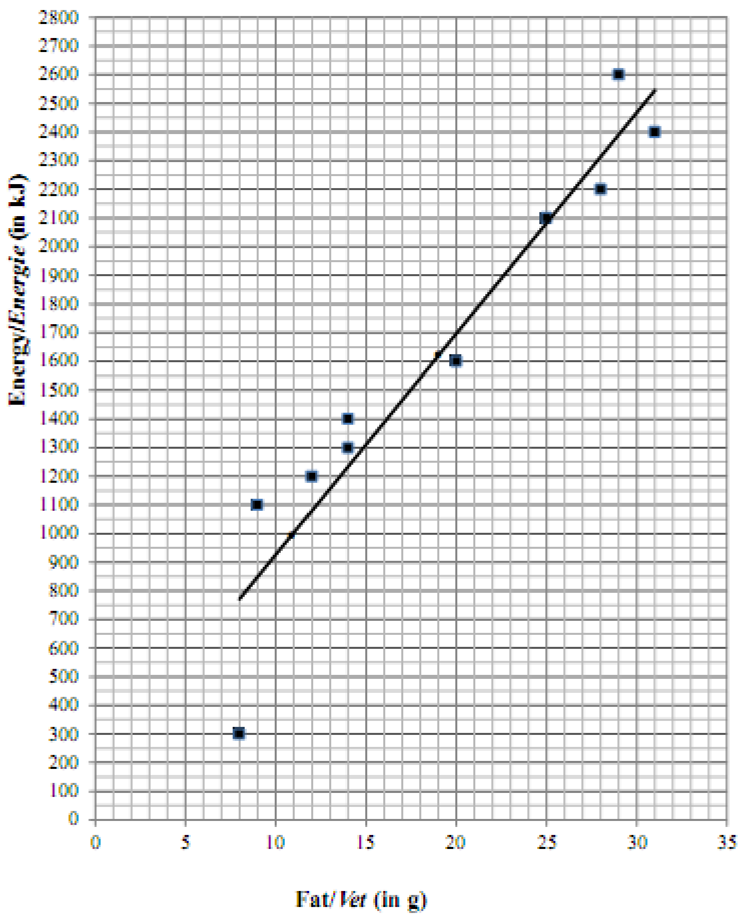
QUESTION / *VRAAG* 2

2.1	$a = 12{,}41$ $b = 0{,}49$ $\hat{y} = 12{,}41 + 0{,}49x$
2.2	$\hat{y} = 12{,}41 + 0{,}49x$ $= 12{,}41 + 0{,}49(150)$ $= 85{,}91 \approx 86\%$ **OR/OF** $\hat{y} = 85{,}17$
2.3	$\hat{y} = 12{,}41 + 0{,}49x$ The y-intercept is 12,41 which means that a learner who did not begin the exam achieved 12,41%. This is clearly impossible.

2.4	10,28
2.5	$63{,}9-\sigma = p$ $63{,}9+\sigma = 103{,}59$ $127{,}92 = p + 103{,}59$ $p = 24{,}33$ **OR / *OF*** $\sigma = 103{,}59 - 63{,}96$ $= 39{,}63$ $p = 63{,}96 - 39{,}63$ $= 24{,}33$

PAPER E

Question 1

1.1 1.2.2	**Scatter plot/*Spreidiagram*** 

1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\ 542{,}94 \approx 1\ 500$ kJ
1.3	(8 ; 300)
1.4	$r = 0{,}9520... \approx 0{,}95$
1.5	very strong positive relationship/

Question 2

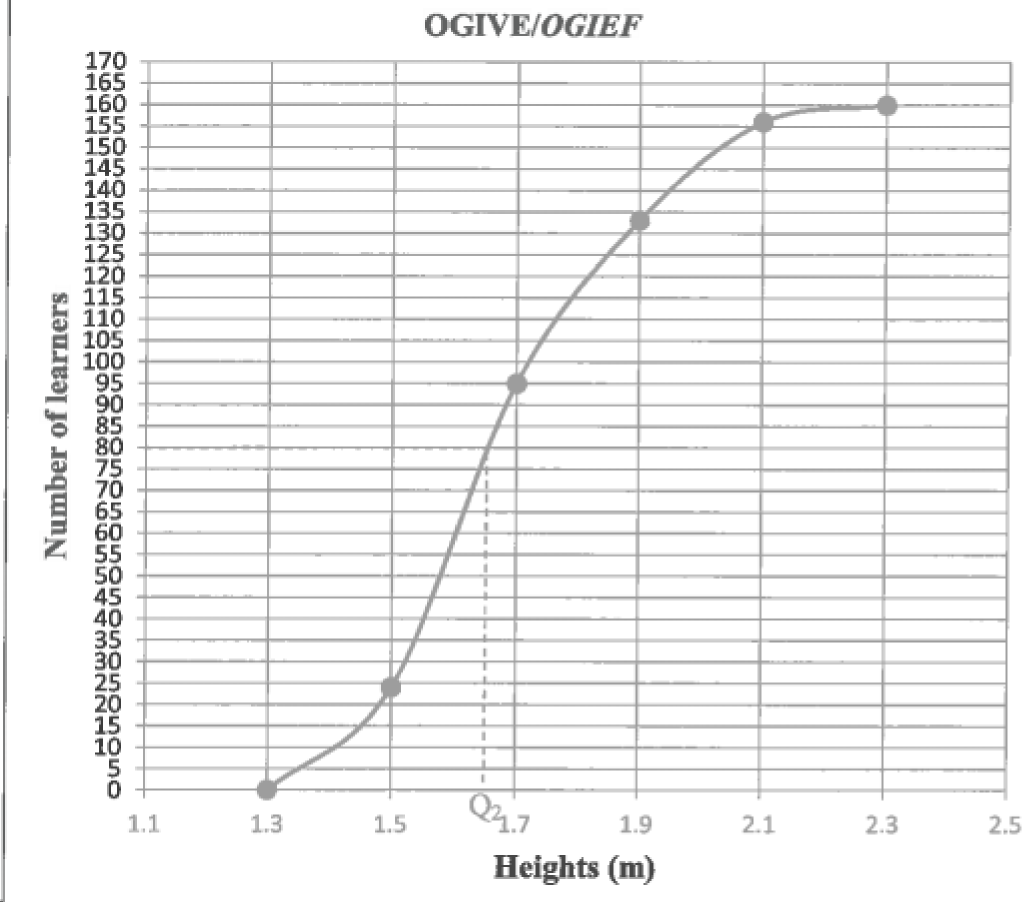
2.1	mean/*gemiddelde* $= \dfrac{2(0)+3(3)+4(2)+...12(1)}{30} = \dfrac{202}{30}$ $= 6{,}73$
2.2	median/*mediaan* $= \dfrac{T_{15}+T_{16}}{2} = \dfrac{7+7}{2} = 7$
2.3	SD/*SA* $= 2{,}264... \approx 2{,}26$
2.4	$(6{,}73 - 2{,}26 ; 6{,}73 + 2{,}26)$ $= (4{,}47 ; 8{,}99)$ $\therefore 4 + 4 + 8 + 3 = 19$ times/*keer*

PAPER F

QUESTION/*VRAAG* 2

2.1	Positively skewed **OR** skewed to the right/*positief skeef* ***OF*** *skeef na regs*
2.2	Range/*Omvang* = 2,21 – 1,39 = 0,82 m
2.3	(see table below)

Intervals *Klasse*	**Cumulative frequency** ***Kumulatiewe frekwensie***
$1{,}3 \le x < 1{,}5$	24
$1{,}5 \le x < 1{,}7$	95
$1{,}7 \le x < 1{,}9$	133
$1{,}9 \le x < 2{,}1$	156
$2{,}1 \le x < 2{,}3$	160

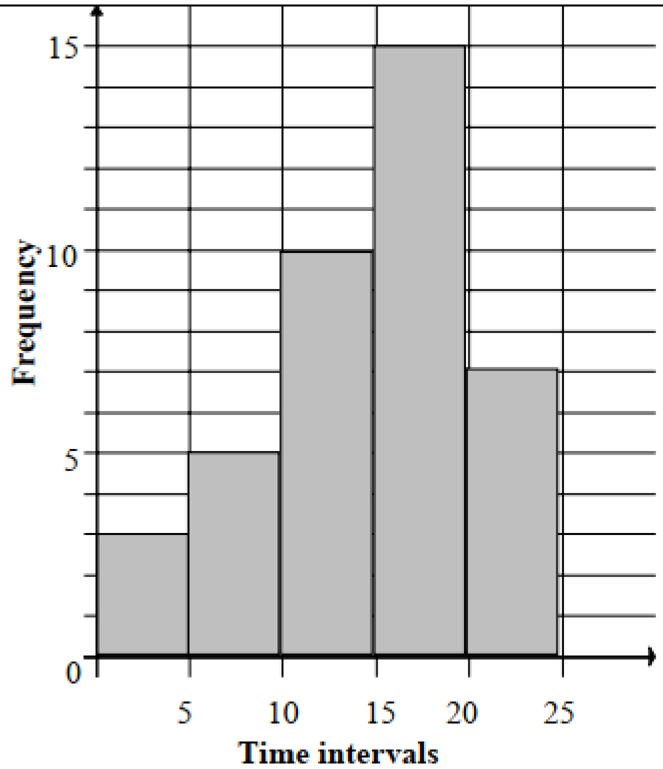
2.4	
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)
2.6.1	The mean would change by 0,1 *m* *Die gemiddelde sal met 0,1 m verander*
2.6.2	No influence/change as there is no difference in variation of data./*Geen invloed*

PAPER G

QUESTION 3

3.1	40
3.2	see table below

Time, t, in minutes	Frequency
$0 \leq t < 5$	3
$5 \leq t < 10$	5
$10 \leq t < 15$	10
$15 \leq t < 20$	15
$20 \leq t < 25$	7

3.3	Frequency 0 5 10 15 5 10 15 20 25 Time intervals

ANALYTICAL GEOMETRY

PAPER A

QUESTION 3

3.1.1	$m_{LM} = \frac{0-1}{4-1} = -\frac{1}{3}$ $m_{MN} = \frac{2-0}{8-4} = \frac{1}{2}$
3.1.2	$KM = \sqrt{(4-4)^2 + (10-0)^2}$ $= \sqrt{100}$ $= 10 \text{ units}$ Answer only full marks
3.1.3	$m_{MN} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$
3.1.4	$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ $\left(\frac{1+8}{2} ; \frac{1+2}{2}\right)$ $\left(\frac{9}{2} ; \frac{3}{2}\right)$
3.2	$m_{KL} = \frac{10-1}{4-1} = 3$ $m_{KL} \times m_{LM} = 3 \times \left(-\frac{1}{3}\right)$ $= -1$ $\therefore KL \perp LM$
3.3	$m_{KN} = \frac{10-2}{4-8}$ $= -2$ $\therefore KN \perp NM$ $\therefore K\hat{L}M + K\hat{N}M = 180°$ $\therefore KLMN$ is cyclic quadrilateral (converse, opp $\angle^s$ of a cyclic quad are supplementary)

QUESTION 4

4.1	$M\left(\frac{-5+3}{2};\frac{4+2}{2}\right)=M(-1;3)$
4.2	$r^2=BM^2=(-5+1)^2+(4-3)^2=17$ $\therefore (x+1)^2+(y-3)^2=17$
4.3	$m_{AB}=\frac{2-3}{3+1}=-\frac{1}{4}$ $m_{AN}=\frac{2+2}{3-2}=4$ $m_{AB}\times m_{AN}=-1$ $\therefore B\hat{A}T=90°$ $\therefore TA$ is a tangent (conv. tangent and diameter)
4.4.1	$m_{TA}=m_{AN}=4$ $y=4x+c$ Subst. (3; 2): $2=4(3)+c$ $-10=c$ $\therefore y=4x-10$

4.4.2	Let C(x; y) $\therefore (x+1)^2+(y-3)^2=17$ At C; $x=0$ $\therefore (0+1)^2+(y-3)^2=17$ $(y-3)^2=16$ $y-3=\pm 4$ $y=7$ or $y=-1$ $\therefore C(0;\ -1)$ $m_{BC}=\frac{-1-4}{0+5}=-1$ Now $y=-x-1$
4.5	Lines AT and BT intersect at C $\therefore 4x-10=-x-1$ $5x=9$ $x=\frac{9}{5}=a$ $b=-\frac{9}{5}-1=-2\frac{4}{5}$

PAPER B

3.1

3.1.1	$m_{KL} = \frac{6-4}{3-(-1)}$ $= \frac{2}{4}$ or $\frac{1}{2}$
3.1.2	midpoint of R : $\left(\frac{x_N + x_L}{2} ; \frac{Y_N + Y_L}{2}\right)$: $\left(\frac{3+4}{2} ; \frac{6+1}{2}\right)$ $R\left(\frac{7}{2} ; \frac{7}{2}\right)$
3.1.3	co-ordinates of M: $\frac{x-1}{2} = \frac{7}{2}$ $x - 1 = 7 \quad \therefore x = 8$ $\frac{y+4}{2} = \frac{7}{2}$ $y = 7 - 4 \quad \therefore y = 3$ $\therefore M(8; 3)$

3.2	The equation of NM $m_{NM} = \frac{1}{2}$ [NM ‖ KL] $y - y_1 = m(x - x_1)$ $y - 1 = \frac{1}{2}(x - 4)$ $y = \frac{1}{2}x - 2 + 1 \therefore y = \frac{1}{2}x - 1$

3.3

3.3.1	$0 = \frac{1}{2}x - 1$ $x = 2$ $\therefore$ P $(2; 0)$
3.3.2	$\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$
3.3.3	$m_{KP} = \frac{0-4}{2-(-1)} = -\frac{4}{3}$ $\tan^{-1}\left(\frac{-4}{3}\right) = 180° - 53{,}13° = 126{,}87°$ $K\hat{P}N = 126{,}87° - 26{,}57°$ $= 100{,}3°$

Question 4

4.1.	$x^2 + 2x + y^2 - 4y = 27$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 27 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 32$ $\therefore$ T $(-1; 2)$ **OR** $x = \frac{2}{-2} = -1$ $y = \frac{-4}{-2} = 2$
4.2	$m_{TB} \times m_{BD} = -1$ [tangent $\perp$ radius] $m_{TB} = 1$ $y = mx + c$ $2 = 1.(-1) + c$ $\therefore c = 3$ $\therefore$ Equation of TB is $y = x + 3$
4.3	equating TB and BD: $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $y = 1 + 3$ *or* $y = -1 + 5$ $\therefore B(1 ; 4)$

4.4	$(x+1)^2+(y-2)^2=r^2$ $(1+1)^2+(4-2)^2=r^2$ $8=r^2$ $(x+1)^2+(y-2)^2=8$
4.5	Draw a horizontal line TE with E on BD: Point D(5 ; 0) and Point E: $2=-x+5$ $\therefore$ E(3 ; 2) $\text{Area of trap TEDO} = \frac{1}{2}(\text{TE}+\text{OD})\times\perp h$ $=\frac{1}{2}(4+5)\times 2$ $=9$ units $\text{Area of } \Delta\text{TBE} = \frac{1}{2}(\text{TB}\times\text{BE})$ $=\frac{1}{2}\left(\sqrt{8}\right)\left(\sqrt{8}\right)$ $=4$ units Area of OTBD = 9 + 4 = 13 units^2

PAPER C

3.1	$\text{E}\left(\frac{12}{2};\frac{6}{2}\right)$ E(6; 3)
3.2	$m_{BA}=\frac{6-0}{7-5}$ $=3$ $y=mx+c$ $y=3x+c$ $6=3(7)+c$ **OR / *OF*** $c=-15$ $y=3x-15$ $y-y_1=m(x-x_1)$ $y-6=3(x-7)$ $y=3x-21+6$ $y=3x-15$

3.3	$rx-3y+5=0$ $-3y=-rx-5$ $y=\frac{r}{3}x+\frac{5}{3}$ $3=\frac{r}{3}$ $r=9$

3.4	Area Δ AOP $=10$ $\frac{1}{2}\times \text{AO}\times\perp h=10$ $\frac{1}{2}\times 5\times\perp h=10$ $\perp h=4$ but / *maar* $y<0$ $\therefore y=-4$ $\text{AP}=\text{BP}$ $\text{AP}^2=\text{BP}^2$ $(x-5)^2+(-4-0)^2=(x-7)^2+(-4-6)^2$ $x^2-10x+25+16=x^2-14x+49+100$ $4x=108$ $x=27$ P $(27\,;-4)$

Question 4

4.1	$\alpha=135°$ $\tan(135°)=m$ $m=-1$ $y=mx+c$ $y=-1x+c$ $4=-1(-2)+c$ $c=2$ $y=-x+2$	**OR / *OF***	ext $\angle$ of Δ / *buite* $\angle$ *van* Δ $y-y_1=m(x-x_1)$ $y-4=-1(x+2)$ $y=-1x-2+4$ $y=-x+2$

4.2	$P(-4;0)$ $a=-4$ $m_{BA}.m_{AS}=-1$ $m_{BA}=1$ $m_{BA}=\dfrac{4-b}{-2+4}$ $1=\dfrac{4-b}{2}$ $2=4-b$ $b=2$
4.3	$(x-a)^2+(y-b)^2=r^2$ $(x+4)^2+(y-2)^2=r^2$ $(-2+4)^2+(4-2)^2=r^2$ $4+4=r^2$ $(x+4)^2+(y-2)^2=8$
4.4	$x^2-2x+y^2-2y=0$ $(x^2-2x+1)+(y^2-2y+1)=1+1$ $(x-1)^2+(y-1)=2$
4.5	D (1 ; 1)
4.6	$DE=\sqrt{2}$ $DA=\sqrt{(-2-1)^2+(4-1)^2}$ $=\sqrt{9+9}$ $=\sqrt{18}$ **OR/*OF*** $=3\sqrt{2}$ $D\hat{E}A=90°$ radius $\perp$ tangent $AD^2=DE^2+AE^2$ pythagoras $(\sqrt{18})^2=(\sqrt{2})^2+AE^2$ $18-2=AE^2$ $AE=4$

PAPER D

3.1.1	$1=\frac{3+x}{2}$ $2=3+x$ $x=-1$ $-2=\frac{4+y}{2}$ $-4=4+y$ $y=-8$ $B(-1;-8)$
3.1.2	$m_{CD}=\frac{0-4}{6-3}$ $=-\frac{4}{3}$
3.1.3	$y-2=\frac{-4}{3}(x-11)$ $y=\frac{-4}{3}x+\frac{50}{3}$ **OR / *OF*** $y=\frac{-4}{3}x+c$ $2=\frac{-4}{3}(11)+c$ $c=\frac{50}{3}$ $y=\frac{-4}{3}x+\frac{50}{3}$

3.1.4	$CD = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$ $= \sqrt{(0-4)^2+(6-3)^2}$ $= \sqrt{25}$ $CD = 5$ **OR / *OF*** $R(5;10)$ midpoint / *middelpun* $RQ = \sqrt{(2-10)^2+(11-5)^2}$ $RQ = 10$ D is the midpoint of PR / *D is die middelpunt van PR* C is the midpoint of PQ (line from midpoint of 1 side || to 2nd side)/ *C is die middelpunt van PQ (lyn van middelpunt van 1 sy || aan 2de sy)* $RQ = 2CD = 10$ (midpoint theorem/ *middelpuntstelling*) $PK = RQ$ $\sqrt{(y+2)^2+(4-1)^2} = 10$ $(y+2)^2+(4-1)^2 = 10^2$ $(y+2)^2 = 91$ $y+2 = \pm\sqrt{91}$ $y = \pm\sqrt{91}-2$ $y = -11{,}54$ **or / *of*** $y \neq 7{,}54$ **or / *of*** $\sqrt{(y+2)^2+(4-1)^2} = 10$ $y^2+4y+4+9 = 100$ $y^2+4y-87 = 0$ $y = \dfrac{-4\pm\sqrt{4^2-4(1)(-87)}}{2(1)}$ $y = \dfrac{-4\pm\sqrt{364}}{2}$ $y = -11{,}54$ **or / *of*** $y \neq 7{,}54$

3.2.1	$m_{PQ} = \tan\theta$ $\tan\theta = 1$ $\theta = 45°$ $\hat{P}_1 = 35°$ vertical opp $\angle$s / *regoorst* $\angle e$ QR || to the x-axis / *aan die x-as* $\hat{T}_1 = 35° + 45°$ ext $\angle$ of Δ / *buite* $\angle$ v Δ $\hat{T}_1 = 80°$ $\alpha = \hat{T}_1 = 80°$ corr $\angle$s ST||QR / *ooreenkomstige* $\angle e$ *ST||QR*

3.2.2	x at/*by* U: $\dfrac{U_2+(-8)}{2}=1$ $\therefore U_x = 10$ units / *eenhede* QU $=18$ units / *eenhede* x at/*by* W $= x$ at / *by* U $= 10$ y at/*by* W: $y = 10+\dfrac{2}{3}$ $=\dfrac{32}{3}$ $\text{WU} = \dfrac{32}{3}+5=\dfrac{47}{3}$ $\therefore \text{Area } \Delta \text{QWU} = \dfrac{1}{2}(18)\left(\dfrac{47}{3}\right)$ $=141$ square units / *eenhede kwadraat*

Question 4

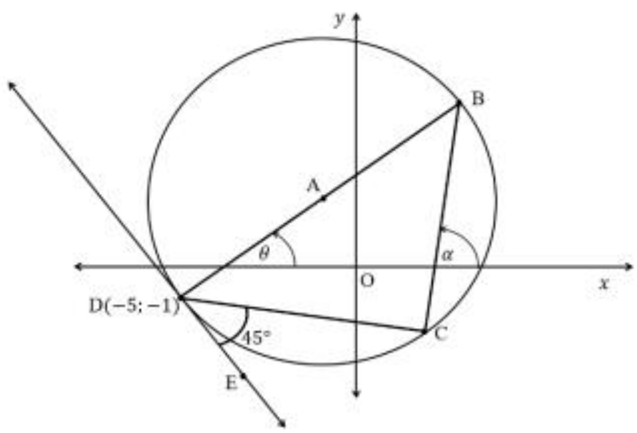
4.1	$\text{T}\hat{\text{U}}\text{S} = 180° - 101{,}31° = 78{,}69°$ adj supp ∠s / *aangrensende suppl* ∠e $m_{\text{TU}} = \tan 78{,}69° = 5$ $c = 6$ $y = 5x+6$
4.2	$x-$int / *afsnit* $y=0$ $\dfrac{-1}{5}x+\dfrac{4}{5}=0$ $-x+4=0$ $x=4$ $\therefore \text{S}(4;0)$ $\text{M} = \left(\dfrac{-6+4}{2};\dfrac{2+0}{2}\right)$ $\therefore \text{M}(-1;1)$

4.3	$(x+1)^2+(y-1)^2=r^2$ $(-6+1)^2+(2-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$ **OR / *OF*** $(x+1)^2+(y-1)^2=r^2$ $(4+1)^2+(0-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$
4.4	$m_{MP}=-\frac{1}{5}$ $\quad$ $m_{MP}\times m_{KL}=-1$ $m_{KL}=5$ $\quad$ radius $\perp$ tan / *radius* $\perp$ *raaklyn* $m_{TU}=5$ $\quad$ proven / *reeds bewys* $\therefore m_{TU}=m_{KL}=5$ KL || TU
4.5	$\text{VM}=\sqrt{\left(-1+\frac{1}{2}\right)^2+(1-7)^2}$ $=6{,}02$ $\text{radius}=\sqrt{26}=5{,}1$ $6{,}02>5{,}1$ $\therefore \text{V}\left(-\frac{1}{2};7\right)$ does not lie within the circle / *lê nie binne die sirkel nie.*

PAPER E

3.1	AC=BD $\sqrt{50} = \sqrt{(3-(-2))^2 + (p-(-4))^2}$ $50 = 25 + p^2 + 8p + 16$ $p^2 + 8p - 9 = 0$ $(p+9)(p-1) = 0$ $p \neq -9\,;p = 1$
3.2	$\text{M}\left(\frac{3-2}{2};\frac{1-4}{2}\right)$ $\text{M}\left(\frac{1}{2};-\frac{3}{2}\right)$
3.3	$m_{DC} = \frac{1-(-2)}{3-4}$ $m_{DC} = \frac{3}{-1}$ $m_{DC} = -3$
3.4	$y = -3x + c$ $-4 = -3(-2) + c$ $-10 = c$ $y = -3x - 10$

QUESTION 4

		
4.1.1	$A(-1;2)$	

4.1.2	$\frac{x+(-5)}{2}=-1$ $x-5=-2$ $x=3$ $\frac{y+(-1)}{2}=2$ $y-1=4$ $y=5$ $B(3;5)$
4.1.3	$m_{AD}=\frac{2-(-1)}{-1-(-5)}$ $=\frac{3}{4}$
4.1.4	$tan\theta=\frac{3}{4}$ $\theta=36{,}87°$

4.1.5	$m_{radius}=\frac{3}{4}$ $m_{tangent}=\frac{-4}{3}$ $radius \perp tangent$ $y=-\frac{4}{3}x+c$ $-1=-\frac{4}{3}(-5)+c$ $-1=\frac{20}{3}+c$ $c=-\frac{23}{3}$ $\therefore y=-\frac{4}{3}x-\frac{23}{3}$

4.2	$\hat{B} = 45°$ [tan;chord theorem / raaklyn;koord stelling] $\alpha = 45° + 36{,}87°$ $[ext \angle of \Delta / buite \angle van \Delta]$ $\alpha = 81{,}87°$ $m_{BC} = \tan 81{,}87°$ $m_{BC} = 7$
4.3.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M $(3; -1)$
4.3.2	M$(3;-1)$ and A $(-1;2)$ $\text{MA} = \sqrt{(3-(-1))^2 + (-1-2)^2}$ $MA = \sqrt{16+9}$ $MA = \sqrt{25}$ $MA = 5$ $r_M + r_A = \sqrt{18} + 5$ or $= 9{,}24$ MA$< r_M + r_A$ $\therefore$ *circles intersect/sirkels sny*

PAPER F

2.2.1	$x^2 + 2x + \left(\frac{1}{2}(2)\right)^2 + y^2 - 4y + \left(\frac{1}{2}(-4)\right)^2 = 5 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 10$ But $(x;y) \rightarrow (x-2; y+4)$ $\therefore (-1;2) \rightarrow (-1-2; 2+4)$ $= (-3;6)$ $\therefore (x+3)^2 + (y-6)^2 = 10$ **OR** $(x+2)^2 + 2(x+2) + (y-4)^2 - 4(y-4) - 5 = 0$ $x^2 + y^2 + 6x - 12y + 35 = 0$ $(x+3)^2 + (y-6)^2 = 10$

2.2.2	Distance from origin to centre $= \sqrt{(-3-0)^2 + (6-0)^2} = \sqrt{45}$ Since $\sqrt{45} > \sqrt{10}$, the origin lies outside the circle.

Question 3

3.1	$\text{AB}: y + 3x - 2 = 0$ $\therefore y = -3x + 2$ $\therefore k = 2$
3.2	$\text{AC} = \sqrt{(1-6)^2 + (-1-4)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$
3.3	$m_{AB} = \dfrac{-1-2}{1-0} = -3$ $m_{BC} = \dfrac{4-2}{6-0} = \dfrac{1}{3}$ $m_{AB} \times m_{BC} = -3 \times \dfrac{1}{3} = -1$ $\therefore \text{A}\hat{\text{B}}\text{C} = 90°$.

3.4	$\tan\beta = m_{AC} = 1$ $\beta = 45°$ $\tan\phi = m_{BC} = \dfrac{1}{3}$ $\phi = 18{,}43°$ $\theta = 45° - 18{,}43° \quad [\text{ext } \angle \text{of } \Delta]$ $= 26{,}57°$

3.5	$B\hat{D}C = 90^\circ$ altitude $\therefore$ BC is the diameter (line subtends $90^\circ \angle$) midpoint of BC : $\left(\frac{0+6}{2};\frac{2+4}{2}\right)$ $= (3;3)$ $(x-3)^2 + (y-3)^2 = r^2$ subst $B(0;2)$ $(0-3)^2 + (2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2 + (y-3)^2 = 10$

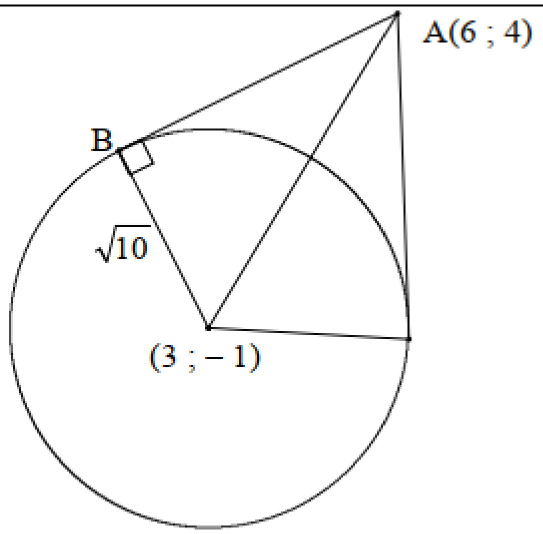
3.6	$M_{AC} = \left(\frac{1+6}{2}\ ;\ \frac{2+1}{2}\right)$ $= \left(\frac{7}{2}\ ;\ \frac{3}{2}\right)$ $\frac{7}{2} = \frac{x+0}{2}$ $x = 7$ D(7 ; 1)	 $\frac{3}{2} = \frac{2+y}{2}$ $y = 1$

QUESTION 5

5.1.1	$x^2+y^2-8x+6y$ $=(2)^2+(-9)^2-8(2)+6(-9)$ $=4+81-16-54$ $=15$ Hence, the point lies on the circumference of the circle **OR** $x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ $(x-4)^2+(y+3)^2$ $=(2-4)^2+(-9+3)^2$ $=2^2+6^2$ $=40$ $\therefore$ The point lies on the circumference of the circle.

5.1.2	$x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ Circle centre $(4\,;-3)$ $m_{rad}=\dfrac{-3-(-9)}{4-2}$ $m_{rad}=3$ $m_{\tan}=-\dfrac{1}{3}$ $y+9=-\dfrac{1}{3}(x-2)$ $y=-\dfrac{1}{3}x-\dfrac{25}{3}$

5.2



Radius $= \sqrt{10}$

Distance from A to centre of circle is

$= \sqrt{(6-3)^2 + (4+1)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

$AB^2 = 34 - 10$

$AB^2 = 24$

$AB = \sqrt{24}$

$AB = 2\sqrt{6}$

$AB = 4{,}90$

OR

$r^2 = 10$

$r = \sqrt{10}$

Radius $\perp$ tangent

By Pythagoras

$AB^2 = (6-3)^2 + (4+1)^2 - 10$

$= 24$

$AB = 4{,}90$

QUESTION 6

6.1	$x^2+y^2+8x+4y-38=0$ $x^2+8x+16+y^2+4y+4=16+4+38$ $(x+4)^2+(y+2)^2=58$ Centre is $(-4\ ;-2)$ and the radius is $\sqrt{58}$
6.2	Centre of second circle is $(4\ ;6)$ Distance between centres is $\sqrt{(4+4)^2+(6+2)}=\sqrt{128}=11{,}31$
6.3	Sum of radii $=\sqrt{58}+\sqrt{26}=12{,}71$ Distance between centres is $11{,}31$. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.
6.4	Equation of second circle: $(x-4)^2+(y-6)^2=26$ $x^2-8x+16+y^2-12y+36=26$ $x^2-8x+y^2-12y+26=0$ Let $(x\ ;y)$ be either of the two points on intersection. Then $x^2+y^2+8x+4y-38=0$ and $x^2+y^2-8x-12y+26=0$ Subtract $16y+16x-64=0$ $y=-x+4$ Both points of intersection lie on this line. $\therefore\ y=-x+4$ is the equation of the common chord. **OR**

Check that the line $y = -x + 4$ cuts the two circles at the same points:

$(x-4)^2 + (-x-2)^2 = 26$

$x^2 - 8x + 16 + x^2 + 4x + 4 = 26$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

$x^2 + y^2 + 8x + 4y - 38 = 0$

$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$

$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$x = 3$ or $x = -1$

TRIGONOMETRY

PAPER A

QUESTION 5

5.1	$\cos 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$ $= \cos 79^\circ \cos 49^\circ + \sin 79^\circ \sin 49^\circ$ $= \cos(79^\circ - 49^\circ)$ $= \cos 30^\circ$ $= \frac{\sqrt{3}}{2}$ Answer only no marks, used calculator
5.2	$\sin(x+y) = 3\sin(x-y)$ $\sin x \cos y + \cos x \sin y = 3(\sin x \cos y - \cos x \sin y)$ $\sin x \cos y + \cos x \sin y = 3\sin x \cos y - 3\cos x \sin y$ $-2\sin x \cos y = -4\cos x \sin y$ $\div -2\cos x \cos y$: $\frac{\sin x}{\cos x} = 2\left(\frac{\sin y}{\cos y}\right)$ $\therefore \tan x = 2\tan y$

5.3.1	$\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$ LHS: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x}$ $= \frac{\cos x}{2\sin x \cos x} - \frac{1-2sin^2 x}{2\sin x}$ $= \frac{1}{2\sin x} - \frac{(1-2sin^2 x)}{2\sin x}$ $= \frac{1-1+2sin^2 x}{2\sin x}$ $= \frac{2sin^2 x}{2\sin x}$ $= \sin x$ =RHS

5.3.2	$1 + 2\cos 2x = \dfrac{\cos 2x}{2\sin x} - \dfrac{\cos x}{\sin 2x}$ $1 + 2\cos 2x = -\sin x$ $1 + 2(1 - 2sin^2x) = -\sin x$ $1 + 2 - 4sin^2x = -\sin x$ $4sin^2x - \sin x - 3 = 0$ $(\sin x - 1)(4\sin x + 3) = 0$ $\sin x = 1$ OR $\sin x = -\frac{3}{4}$ $x = 90°$ $\text{ref}\angle = 48{,}59°$ $x = 228.59$ OR $x = 311{,}41°$

QUESTION 6

6.1	$a = 1$ $b = 2$ $c = 2$ $d = 1$
6.2	360°
6.3.1	$x \in [-90°;\ 90°]$ *or* $x \in [270°;\ 360°]$
6.3.2	$x \in (-45°;\ 0°)$ *or* $x \in (45°;\ 90°)$ *or* $x \in (315°; 360°)$

QUESTION 7

7.1	n ΔPQR: $\hat{Q}_1 = x$ $(PR = QR)$ $\hat{R} = 180° - 2x$ $(sum\ of\ \angle\ \Delta PQR)$ $Area\ of\ \Delta PQR = \frac{1}{2}pq\sin\hat{R}$ $= \frac{1}{2}m.m\sin(180° - 2x)$ $= \frac{1}{2}m^2\sin 2x$

7.2	$\therefore \dfrac{PQ}{\sin(180^\circ - 2x)} = \dfrac{m}{\sin x}$ $\therefore PQ = \dfrac{m\,.\sin(180^\circ - 2x)}{\sin x}$ $\therefore PQ = \dfrac{m\,.\sin 2x}{\sin x}$ $\therefore PQ = \dfrac{m\,.2\sin x\,.\cos x}{\sin x}$ $\therefore PQ = 2m\cos x$
7.3	In ΔSPQ: $\tan y = \dfrac{SP}{PQ}$ $\therefore SP = PQ\tan y$ $\therefore SP = 2m\cos x\tan y$

PAPER B

QUESTION 5

5.1.1	$\tan\theta = \dfrac{-\sqrt{3}}{1} = -\sqrt{3}$
5.1.2	$r^2 = (1)^2 + \left(-\sqrt{3}\right)^2$ Pyth $r^2 = 4$ $r = 2$ $\sin(-\theta) = -\sin\theta$ $= -\left(\dfrac{-\sqrt{3}}{2}\right)$ *or* $\dfrac{\sqrt{3}}{2}$
5.1.3	$\sin(\theta - 60^\circ)$ $= \sin\theta\cos 60^\circ - \cos\theta\sin 60^\circ$ $= \left(-\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) - \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{-\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4} = \dfrac{-2\sqrt{3}}{4} = \dfrac{-\sqrt{3}}{2}$

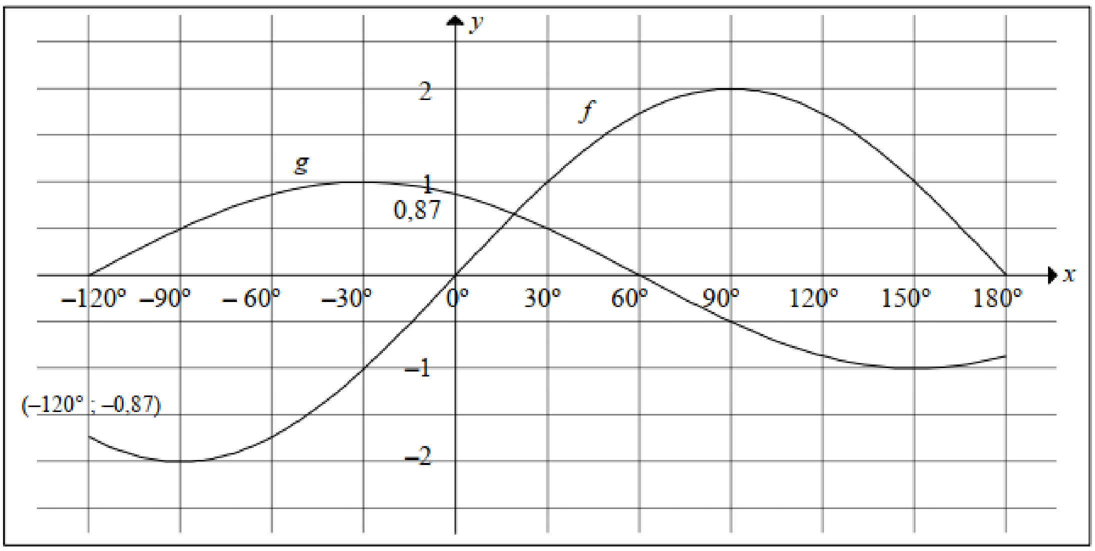
5.2	$\dfrac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300° \sin(\theta - 360°)}$ $= \dfrac{-\tan\theta \, . \cos\theta}{\cos 60° \, . \sin\theta}$ $= \dfrac{-\dfrac{\sin\theta}{\cos\theta} . \cos\theta}{\dfrac{1}{2} . \sin\theta}$ $= -2$

5.3.1	$LHS = \dfrac{\cos 2x - 1}{\sin 2x}$ $= \dfrac{1 - 2\sin^2 x - 1}{2\sin x \cos x}$ $= -\dfrac{\sin x}{\cos x}$ $= -\tan x$ $= RHS$
5.3.2	$x = 90° \ ; \ 180° \ ; \ 270°$

5.3.3	$-\tan 2x = \dfrac{1}{4}$ $\tan 2x = -\dfrac{1}{4}$ $2x = 165{,}96° + k.180°$ $x = 82{,}98° + k.90° \ ; k \in Z$

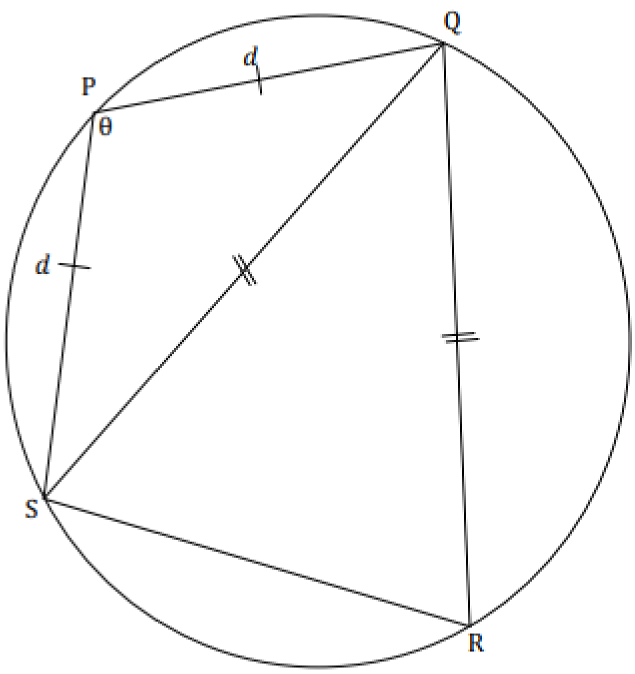
QUESTION 6

6.1



6.2	Period $= 360^\circ$
6.3.1	$-30^\circ < x < 150^\circ$
6.3.2	$x \in (0^\circ\ ;60^\circ)$
6.4	$y = \cos\left(x + 30^o + 60^o\right)$ $y = \cos\,(x + 90^o)$ $y = -\sin x$

QUESTION 7



7.1	$QS^2 = d^2 + d^2 - 2d.d.\cos\theta$ $QS^2 = 2d^2 - 2d^2.\cos\theta$ $QS^2 = 2d^2(1 - \cos\theta)$ $QS\ = d\sqrt{2(1 - \cos\theta)}$

7.2	$\hat{R} = 180° - \theta$ opp. $\angle^s$ cyclic quad suppl $= Q\hat{S}R$ equal sides, equal angles $S\hat{Q}R = 2\theta - 180°$ sum $\angle^s \Delta$ $\Delta QRS = \frac{1}{2}.QS.QR\sin S\hat{Q}R$ $= \frac{1}{2}.d\sqrt{2(1-\cos\theta)}.d\sqrt{2(1-\cos\theta)}\sin(2\theta - 180°)$ $= \frac{1}{2}d^2.2(1-\cos\theta)(-\sin 2\theta)$ $= -d^2(1-\cos\theta).\sin 2\theta$

PAPER C

QUESTION / *VRAAG* 5

5.1	$1-4\sin^2 15°$ $=1-4\sin^2(45°-30°)$ $=1-4[\sin 45°\cos 30° - \cos 45°\sin 30°]^2$ $=1-4\left[\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}\right]^2$ $=1-4\left[\frac{\sqrt{6}-\sqrt{2}}{4}\right]^2$ $=1-4\left[\frac{6-4\sqrt{3}+2}{16}\right]$ $=1-4\left[\frac{8-4\sqrt{3}}{16}\right]$ $=1-\left[\frac{8-4\sqrt{3}}{4}\right]$ $=\sqrt{3}-1$

5.2

$$\frac{\sqrt{3}\sin x.\sin^2 72°+\sin^2 198°.\sqrt{3}\cos(x-90°)}{\tan 120°.\sin x}$$

$$=\frac{\sqrt{3}\sin x.\sin^2(90°-18°)+\sin^2(180°+18°).\sqrt{3}\sin x}{\tan(180°-60°).\sin x}$$

$$=\frac{\sqrt{3}\sin x.\cos^2 18°+\sin^2 18°.\sqrt{3}\sin x}{-\tan 60°.\sin x}$$

$$=\frac{\sqrt{3}\sin x\left(\cos^2 18°+\sin^2 18°\right)}{-\sqrt{3}.\sin x}$$

$$=-1$$

5.3

$$6\sin x.\cos x+3\cos x-4\sin^2 x-2\sin x=0$$

$$3\cos x(2\sin x+1)-2\sin x(2\sin x+1)=0$$

$$(2\sin x+1)(3\cos x-2\sin x)=0$$

$\sin x=-\dfrac{1}{2}$ **OR/*OF*** $3\cos x=2\sin x$

$$\tan x=\frac{3}{2}$$

$\mathrm{RA}=30°$	$\mathrm{RA}=56{,}31°$
$x=210°+k.360°$	$x=56{,}31°+k.180°$
$x=330°+k.360°$	$x=236{,}31°+k.180°;k\in Z$

5.4

$$(1-\tan A)\left(\frac{\cos A}{\cos 2A}\right)=\frac{1}{\cos A+\sin A}$$

$$\text{LHS}/LK=(1-\tan A)\left(\frac{\cos A}{\cos 2A}\right)$$

$$=\left(1-\frac{\sin A}{\cos A}\right)\left(\frac{\cos A}{\cos^2 A-\sin^2 A}\right)$$

$$=\left(\frac{\cos A-\sin A}{\cos A}\right)\left(\frac{\cos A}{(\cos A-\sin A)(\cos A+\sin A)}\right)$$

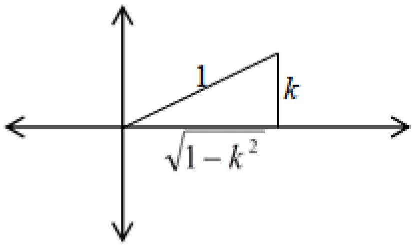
$$=\frac{1}{\cos A+\sin A}$$

$$\text{LHS}\,/\,LK=\text{RHS}\,/\,RK$$

5.5.1

$\cos 2\theta$

$= \sqrt{1 - k^2}$



OR/*OF*

$$\cos^2 2\theta = 1 - \sin^2 2\theta$$
$$= 1 - k^2$$
$$\cos 2\theta = \sqrt{1 - k^2}$$

5.5.2

$$\frac{\sin 2\theta}{\tan\theta}$$

$$= \frac{2\sin\theta.\cos\theta}{\frac{\sin\theta}{\cos\theta}}$$

$$= 2\sin\theta.\cos\theta.\frac{\cos\theta}{\sin\theta}$$

$$= 2\cos^2\theta$$

But/*maar* $\cos 2\theta = \sqrt{1-k^2}$

$$2\cos^2\theta - 1 = \sqrt{1-k^2}$$
$$2\cos^2\theta = \sqrt{1-k^2} + 1$$
$$\frac{\sin 2\theta}{\tan\theta} = \sqrt{1-k^2} + 1$$

QUESTION / *VRAAG* 6

6.1	$a=-1$ $d=2$
6.2	$D\left(-150°;\frac{1}{2}\right)$
6.3.1	$-90°<x<90°$ **OR / *OF*** $x\in(-90°;90°)$
6.3.2	$-135°<x<-45°$ **OR / *OF*** $x\in(-135°;-45°)$

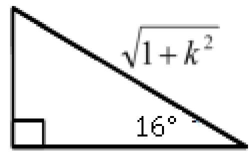
Question 7

7.1	In Δ KLM $\frac{KM}{LM}=\tan x$ $\frac{KM}{r}=\tan x$ $KM=r\tan x$ In Δ KMN $\frac{MN}{KM}=\tan x$ $\frac{2r}{KM}=\tan x$ **OR/*OF*** $KM=\frac{2r}{\tan x}$ $\frac{2r}{r\tan x}=\tan x$ $r\tan x=\frac{2r}{\tan x}$ $2=\tan^2 x$ $\sqrt{2}=\tan x$ $x=54{,}74°$

7.2	$LN^2 = LM^2 + MN^2 - 2LM.MN\cos M$ $LN^2 = (5)^2 + (10)^2 - 2(5).(10)\cos 110°$ $LN^2 = 159,20$ $LN = \sqrt{159,20}$ $LN = 12,62\,m$

PAPER D

QUESTION / *VRAAG* 5

5.1.1	$x^2 = \left(\sqrt{1+k^2}\right)^2 - (1)^2$ (Pythagoras) $x^2 = k^2$ $x = k$ $\tan 16° = \dfrac{1}{k}$ 
5.1.2	$\cos 32°$ $= \cos 2(16°)$ $= 2\cos^2 16° - 1$ $= 2\left(\dfrac{k}{\sqrt{1+k^2}}\right)^2 - 1$

5.2	$\dfrac{\cos(90°+x)\,\sin(x-180°) - \cos^2(180°-x)}{\cos(-2x)}$ $= \dfrac{(-\sin x)(-\sin x) - \cos^2 x}{\cos 2x}$ $= \dfrac{\sin^2 x - \cos^2 x}{\cos^2 x - \sin^2 x}$ **OR / *OF*** $\dfrac{-\cos 2x}{\cos 2x}$ $= \dfrac{-(\cos^2 x - \sin^2 x)}{\cos^2 x - \sin^2 x}$ $= -1$ $= -1$

5.3	$\cos 75^\circ.\cos 45^\circ - \cos 15^\circ.\cos 45^\circ$ $= \cos 75^\circ.\cos 45^\circ - \sin 75^\circ.\sin 45^\circ$ $= \cos(75^\circ + 45^\circ)$ $= \cos 120^\circ$ $= -\cos 60^\circ$ $= -\frac{1}{2}$

5.4.1	$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right)$ $= \frac{\sin\theta}{\cos\theta}\left(2\sin\theta\cos\theta + \frac{3\cos^2\theta}{\sin\theta}\right)$ $= 2\sin^2\theta + 3\cos\theta$ $= 2(1-\cos^2\theta) + 3\cos\theta$ $= -2\cos^2\theta + 3\cos\theta + 2$
5.4.2	$-2\cos^2\theta + 3\cos\theta + 2 = 0$ $2\cos^2\theta - 3\cos\theta - 2 = 0$ $(2\cos\theta + 1)(\cos\theta - 2) = 0$ $\cos\theta = -\frac{1}{2}$ **or / *of*** $\cos\theta = 2$ no solution / *geen oplossing* ref / *verwy* $\angle = 60^\circ$ $\theta = \pm 120^\circ + k360^\circ;\ k \in Z$ **OR / *OF*** $\theta = 120^\circ + k360^\circ; k \in Z$ $\theta = 240^\circ + k360^\circ; k \in Z$

5.5	$\cos(a+b) = -\frac{\sqrt{2}}{2}$ ref $\angle$ / *verw* $\angle = 45^\circ$ $a + b = 180^\circ - 45^\circ$ $a + b = 135^\circ$ .... (1) $\cos(a-2b) = \frac{1}{2}$ ref $\angle$ / *verw* $\angle = 60^\circ$ $a - 2b = 60^\circ$ .... (2) $3b = 75^\circ$ (1)−(2) $b = 25^\circ$ $a = 110^\circ$

QUESTION 6

6.1	$x = -45°$ $x = 135°$

6.2	$h(x) = \tan(45° - x)$ $h(x) = -\tan(x - 45°) = -f(x)$ h is the reflection of f about the x-axis **OR** h is the reflection of f about the line $y = 0$

6.3	$y = 3\sin 2x$

QUESTION 7

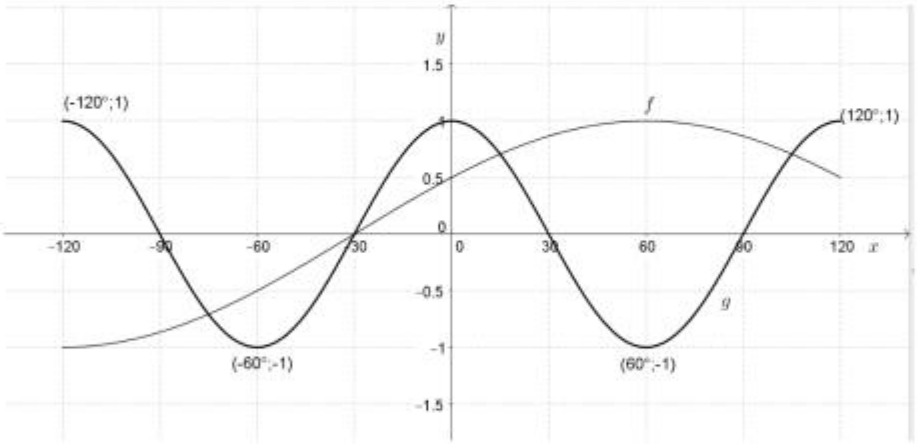
7.1	$\text{M}\hat{\text{A}}\text{G} = 2\theta$
7.2	$\dfrac{MG}{sin2\theta} = \dfrac{k}{sin90°}$ $MG = \dfrac{ksin2\theta}{1}$ ……… (1)

7.3	$\frac{MC}{\sin 150°} = \frac{MG}{\sin\theta}$ $\frac{MC}{½} = \frac{MG}{\sin\theta}$ $MC = \frac{½MG}{\sin\theta} \ldots\ldots\ldots (1)$ Subst. (2) in (1) $MC = \frac{k\sin 2\theta}{2\sin\theta}$ $MC = \frac{k2\sin\theta.\cos\theta}{2\sin\theta}$ $MC = k\cos\theta$
7.4	ΔMGC$= \frac{1}{2}MG.CG\ \sin 150°$ ΔMGC$= \frac{1}{2}k\sin 2\theta.8\ \frac{1}{2}$ ΔMGC$= 2k\sin 2\theta$ **OR** ΔMGC$= \frac{1}{2}MC.CG\ \sin\theta$ ΔMGC$= \frac{1}{2}k\cos\theta.8\ \sin\theta$ ΔMGC$= 4k\cos\theta\sin\theta$ ΔMGC$= 2k(2\cos\theta\sin\theta)$ ΔMGC$= 2k\sin 2\theta$

PAPER E

QUESTION 6

6.1	$f(x) = \sin(x + a)$

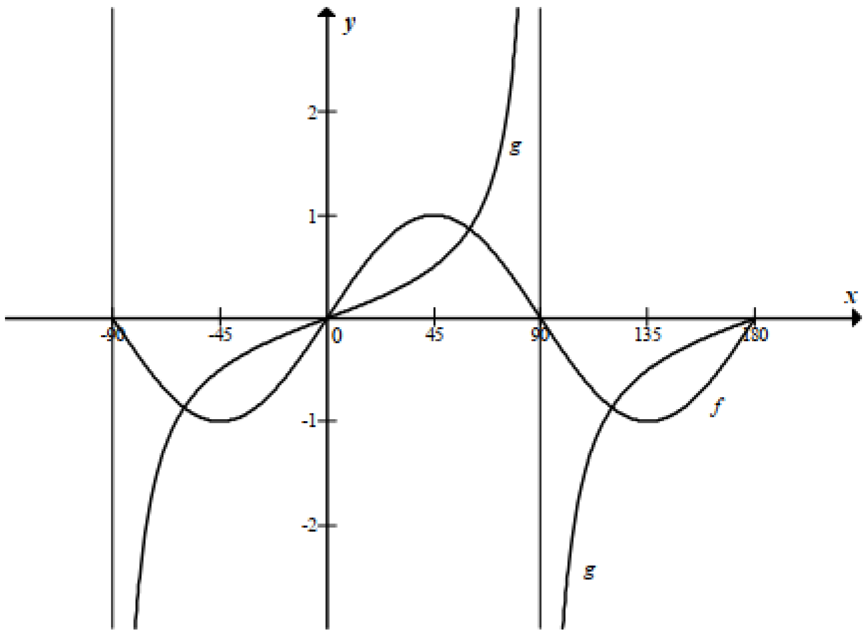


6.1	$a = 30°$
6.2	Sketch
6.3	$f(x) = g(x)$ $\sin(x + 30°) = \cos(3x)$ $\sin(x + 30°) = \sin(90° - 3x)$ $x + 30° = 90° - 3x \ + 360°n \quad n \in Z$ $4x = 60° + 360°n$ $x = 15° + 90°n$ or $x + 30° = 180° - (90° - 3x) + 360°n$ $x + 30° = 90° + 3x + 360°n$ $-2x = 60° + 360°n$ $x = -30° - 180°n$
6.4	$15° < x < 105°$ or $x \in (15°; 105°)$
6. 5	$g(x) = \cos(3x)$ $k(x) = \cos(60° - 3x)$ $k(x) = \cos(3x - 60°)$ $k(x) = cos3(x - 20°)$ $\therefore translated\ 20°\ to\ the\ right.$

QUESTION 7

7.1	$\frac{7}{PB} = \sin 18°$ $PB = \frac{7}{\sin 18°}$ $PB = 22{,}65$ m (22,65247584...)
7.2	$\frac{18}{PA} = \cos 23°$ $PA = \frac{18}{\cos 23°}$ $PA = 19{,}55$ m (19,55448679..)
7.3	$AB^2 = (22{,}65)^2 + (19{,}55)^2 - 2(22{,}65)(19{,}55).\cos 42°$ $= 237{,}0847954...$ $AB = 15{,}40$ m (15,3975581...)

QUESTION 8

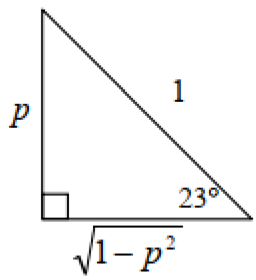
8.1	

8.2	$\sin 2x = \frac{1}{2}\tan x$ $2\sin x.\cos x = \frac{\sin x}{2\cos x}$ $4\sin x.\cos^2 x - \sin x = 0$ $\sin x(4\cos^2 x - 1) = 0$ $\sin x = 0$ or $\cos^2 x = \frac{1}{4}$ $x = 0°$ or $180°$ $\cos x = \pm\frac{1}{2}$ $x = 60°\ ;\ -60°$ or $120°$

8.3	$\{x \mid -60° < x < 0°\} \cup \{x \mid 60° < x < 90°\} \cup \{x \mid 120° < x < 180°\}$ **OR** $x \in (-60°\ ;\ 0°) \cup (60°\ ;\ 90°) \cup (120°\ ;\ 180°)$ **OR** $-60° < x < 0°$ or $60° < x < 90°$ or $120° < x < 180°$

PAPER F

QUESTION 6

6.1.1	$\cos 113°$ $= \cos(90° + 23°)$ $= -\sin 23°$ $= -p$	
6.1.2	$\cos 23°$ $= \sqrt{1-p^2}$ **OR** $\cos^2 23° + \sin^2 23° = 1$ $\cos^2 23° = 1 - p^2$ $\cos 23° = \sqrt{1-p^2}$	

6.1.3

$\sin 46°$

$= 2\sin 23°.\cos 23°$

$= 2p\sqrt{1-p^2}$

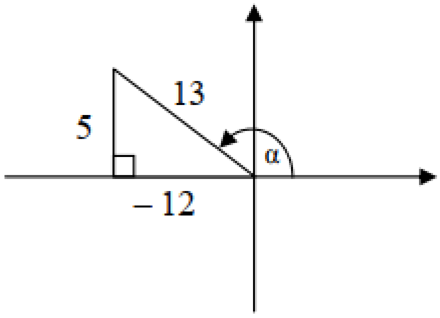
6.2.1

$\sin\alpha = \frac{5}{13}$

$y_\alpha = 5 \quad r_\alpha = 13$

$x_\alpha = -12$

$\cos\alpha = -\frac{12}{13}$

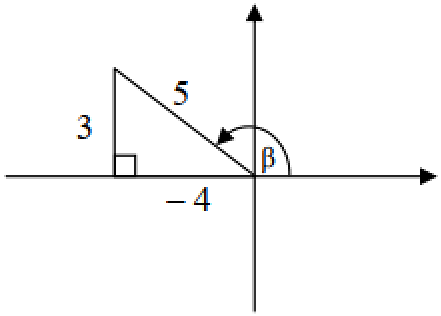


6.2.2

$\tan\beta = -\frac{3}{4}$

$y_\beta = 3 \quad x_\beta = -4$

$r = 5$



$\cos(\alpha+\beta)$

$= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{48-15}{65}$

$= \frac{33}{65}$

6.3

$\frac{1}{2}\cos x = 0{,}435$

$\cos x = 0{,}87$

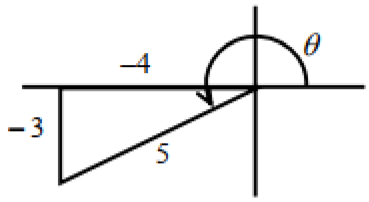
$x = 29{,}54°$ or $x = 330{,}46°$

QUESTION 9

9.1.1

$\sin\theta = -\frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$

$\sin\theta + \cos\theta = -\frac{7}{5}$



9.1.2

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$$

$$= \frac{24}{7}$$

9.2.1

$$\frac{\cos(360° - x).\tan^2 x}{\sin(x - 180°).\cos(90° + x)}$$

$$= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$$

$$= (\cos x)\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\sin^2 x}\right)$$

$$= \frac{1}{\cos x}$$

9.2.2

$x = 30°$

$$\frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

QUESTION 11

11.1

$1 + \sin x = \cos 2x$

$1 + \sin x = 1 - 2\sin^2 x$

$\sin x + 2\sin^2 x = 0$

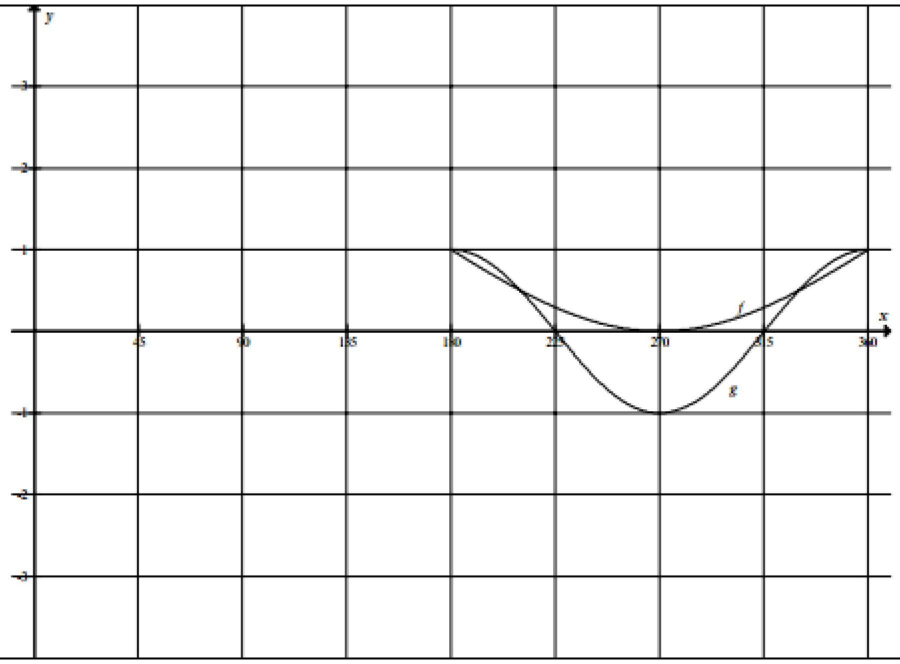
$\sin x(1 + 2\sin x) = 0$

$\sin x = 0$ or $\sin x = -\frac{1}{2}$

$x = k.180$ or $x = -30° + k.360$ $k \in Z$

$x = 210° + k.360$

$x \in \{180°; 210; 330°; 360°\}$

11.2	
11.3	$180° \leq x \leq 210°$ or $330° \leq x \leq 360°$

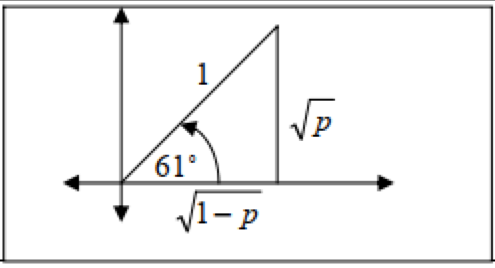
PAPER G

QUESTION 10

10.1.1	$\cos 28° = \sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$
10.1.2	$\cos 64°$ $= \cos 2(32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$
10.1.3	$\sin 4°$ $= \sin(32° - 28°)$ $= \sin 32° \cos 28° - \cos 32° \sin 28°$ $= \sqrt{1-b^2}.\sqrt{1-a^2} - ab$

10.2	$b\sqrt{1-a^2}-a\sqrt{1-b^2}$ $=\cos 32°.\sqrt{1-\sin^2 28°}-\sin 28°\sqrt{1-\cos^2 32°}$ $=\cos 32°.\cos 28°-\sin 28°.\sin 32°$ $=\cos(32°+28°)$ $=\cos 60°$ $=\frac{1}{2}$

QUESTION 11

11.1.1	$\sin 61° = \sqrt{p}$ $\sin 241° = \sin(180° + 61°)$ $= -\sin 61°$ $= -\sqrt{p}$	
11.1.2	$\cos 61° = \sqrt{1-\sin^2 61°}$ $=\sqrt{1-p}$	
11.1.3	$\cos 122° = \cos 2(61°)$ $=2\cos^2 61°-1$ $=2\left(\sqrt{1-p}\right)^2-1$ $=2(1-p)-1$ $=2-2p-1$ $=1-2p$	
11.1.4	$\cos 73°\cos 15° + \sin 73°.\sin 15°$ $=\cos(73°-15°)$ $=\cos 58° = (\cos 180° - 122°)$ $=-(\cos 122°)$ $=-(1-2p)$ $=2p-1$	

11.2.1	$\text{LHS} = \dfrac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{\cos^2 x+2\cos x\sin x+\sin^2 x-(\cos^2 x-2\sin x\cos x+\sin^2 x)}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{4\cos x\sin x}{\cos^2 x-\sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan x$ $= \text{RHS}$
11.2.2	$\cos x=\sin x$ or $\cos x=-\sin x$ $x=45°$ $\quad x=135°$
11.3.1	$\sin x=\cos 2x-1$ $\sin x=1-2\sin^2 x-1$ $\sin x=-2\sin^2 x$ $2\sin^2 x+\sin x=0$

11.3.2	$\sin x=\cos 2x-1$ $2\sin^2 x+\sin x=0$ $\sin x(2\sin x+1)=0$ $\sin x=0$ or $\sin x=-\dfrac{1}{2}$ $\therefore x=0°+180°k;\ k\in Z$ or $x=\{210° \text{ or } 330°\}+360°k;\ k\in Z$ **OR** $x=n.180°$ $x=n.360°-30°$ $x=(2n+1).180°+30°, n\in Z$

11.4	$\tan 1°\times\tan 2°\times\tan 3°\times\tan 4°\times\ldots\ldots\times\tan 87°\times\tan 88°\times\tan 89°$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\sin 88°}{\cos 88°}\right)\left(\dfrac{\sin 89°}{\cos 89°}\right)$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\sin(90°-2°)}{\cos(90°-2°)}\right)\left(\dfrac{\sin(90°-1°)}{\cos(90°-1°)}\right)$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\cos 2°}{\sin 2°}\right)\left(\dfrac{\cos 1°}{\sin 1°}\right)$ $=\tan 45°$ $=1$

EUCLID'S GEOMETRY

PAPER A

A1

QUESTION 7

7.1.1 equal to twice the angle subtended by the same chord at the circle.

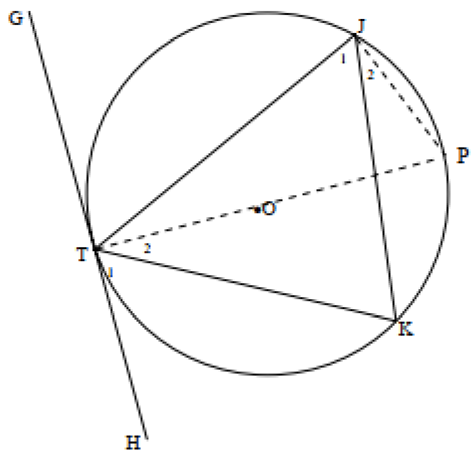
7.1.2 equal to the angle subtended by the same chord in the alternate segment.

7.1.3 supplementary.

A2

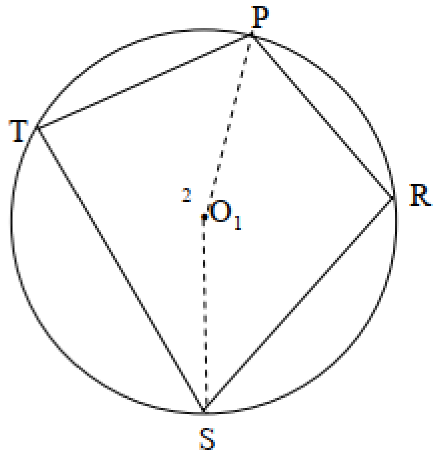
QUESTION 8

8.1	Draw diameter TP. Join P to J. $\hat{T}_1 + \hat{T}_2 = 90°$ (tan ⊥ diameter) $\hat{J}_1 + \hat{J}_2 = 90°$ (∠ in semi-circle) $\hat{J}_2 = \hat{T}_2$ (∠ in same seg) $T\hat{J}K = \hat{T}_1$



A3

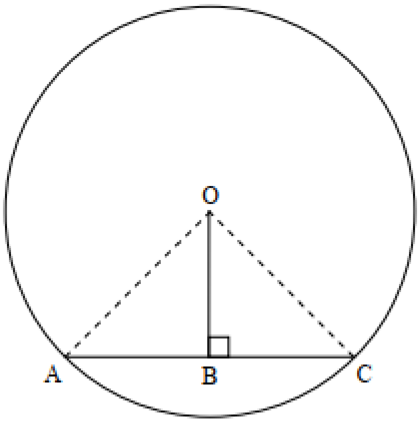
QUESTION 9



9.1	Join PO and OS Let $\hat{O}_1 = 2x$ $\hat{T} = x$ (∠ at circ centre = 2 ∠ at circumference) $\hat{O}_2 = 360° - 2x$ (∠s round a point) $\hat{R} = 180° - x$ (∠ at circ centre = 2 ∠ at circumference) $\hat{T} + \hat{R} = x + 180° - x$ $= 180°$

A4

QUESTION/*VRAAG* 11



Construct radii OA and OC.

In ΔOAB and ΔOCB

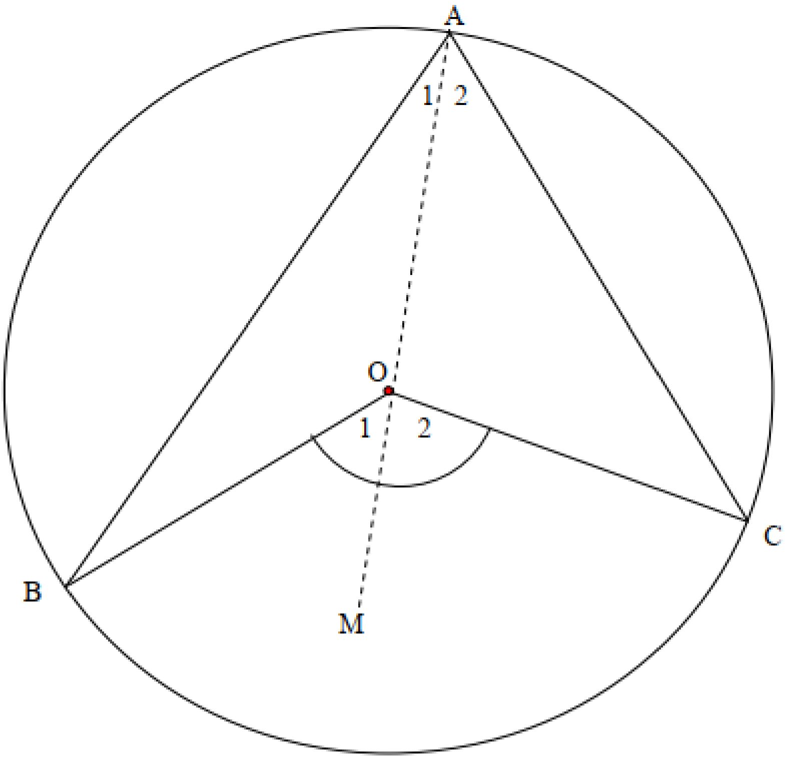
i. OB is common
ii. OA = OC (radii)
iii. $O\hat{B}A = O\hat{B}C = 90°$ (given)

ΔOAB ≡ ΔOCB (90°HS)

AB = BC (≡ Δs)

A5

QUESTION/*VRAAG* 10

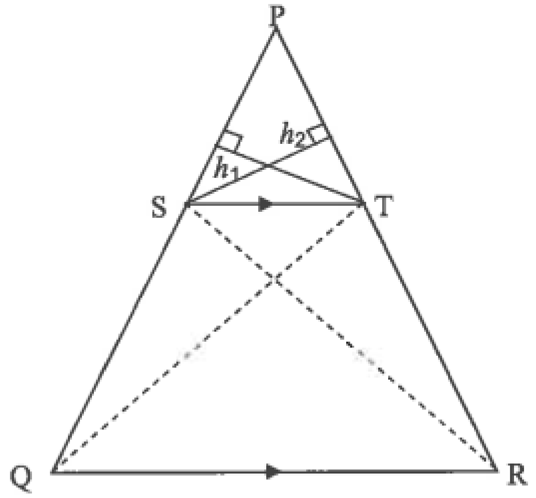


10.1	Construction: AO is drawn and produced to M $\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext $\angle$ of Δ/*buite* $\angle$ *van* Δ] But $\hat{A}_1 = \hat{B}$ [$\angle$s opp = radii/$\angle$*e teenoor* =*radii*] $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly/*Netso*: $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$ $= 2(\hat{A}_1 + \hat{A}_2)$ $B\hat{O}C = 2B\hat{A}C$

A6

QUESTION/*VRAAG* 10

10.1

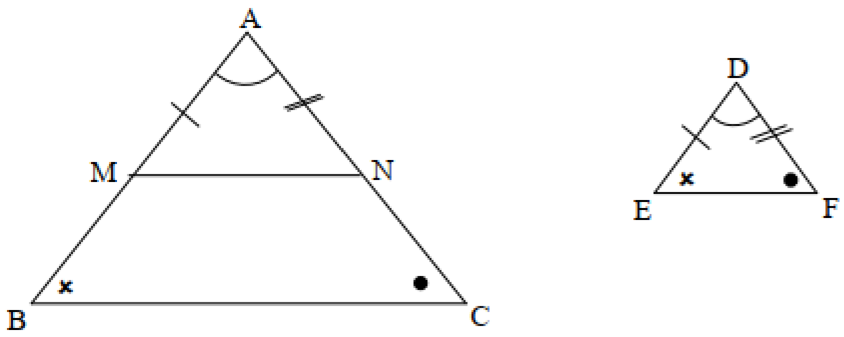


10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ *Verbind SR en TQ en trek h_1 van S $\perp$ PT en h_2 van T $\perp$ PS*]
	Proof :
	$\dfrac{\text{area } \Delta PST}{\text{area } \Delta QST} = \dfrac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \dfrac{PS}{SQ}$ equal altitudes
	$\dfrac{\text{area } \Delta PST}{\text{area } \Delta STR} = \dfrac{\frac{1}{2} PT \times h_1}{\frac{1}{2} TR \times h_1} = \dfrac{PT}{TR}$ equal altitudes
	area ΔPST = area ΔPST [common]
	But area ΔQST = area ΔSTR [same base, height; ST $\parallel$ QR]
	$\therefore \dfrac{\text{area } \Delta PST}{\text{area } \Delta QST} = \dfrac{\text{area } \Delta PST}{\text{area } \Delta STR}$
	$\therefore \dfrac{PS}{SQ} = \dfrac{PT}{TR}$

A7

QUESTION/*VRAAG* 10

10.1



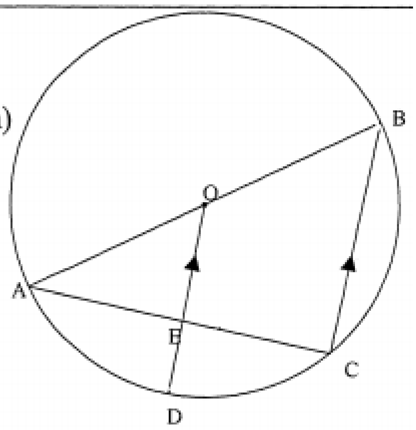
10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \Delta AMN \equiv \Delta DEF$ (SAS) $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ∥ BC [corresp ∠'s are equal/*ooreenkomstige ∠e* =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ∥ one side of Δ **OR** prop theorem; MN ∥BC] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]

PAPER B

QUESTION 9

9.

$\hat{C} = 90°$ (∠s in semi circle)
$O\hat{E}A = 90°$ (corres ∠s; OD ∥ BC)
AE = 8 cm (line from circ cent ⊥ ch bis ch)
OE = 6 cm (Pythagoras)
ED = 10 – 6
= 4 cm



OR

$\hat{C} = 90°$ (∠s in semi circle)
$O\hat{E}A = 90°$ (corres ∠s; OD ∥ BC)
OE ∥ BC (given)
OA = OB (radii)
AE = EC = 8cm (midpoint theorem)
OE = 6 cm (Pythagoras)
ED = 10 – 6
= 4 cm

OR

$\hat{C} = 90°$ (∠s in semi circle)
$BC^2 = (20)^2 - (16)^2$
$BC^2 = 144$
$BC = 12$
$OE = \frac{1}{2}BC$ (midpoint theorem)
OE = 6 cm
OD = 10cm
ED = 10 – 6
= 4 cm

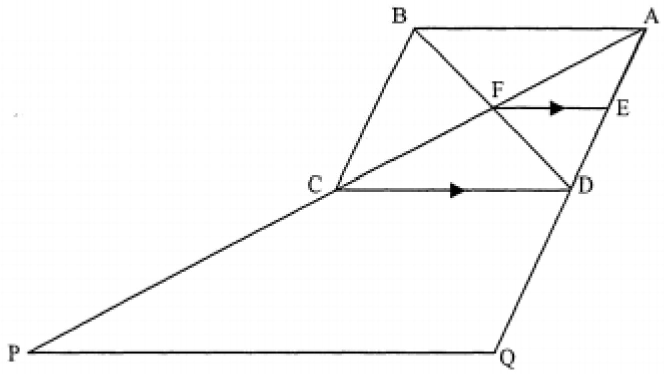
OR

$\hat{C} = 90°$ (∠s in semi circle)
$BC^2 = (20)^2 - (16)^2$
$BC^2 = 144$
$BC = 12$
$OE = \frac{1}{2}BC$ (midpoint theorem)
OE = 6 cm
ED = 4cm

Question 10

10.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) **OR** ($\angle$s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$s; CA $\parallel$ DF)
10.2	In ΔBHD and Δ FED 1. $\hat{B}_2 = \hat{F}$ ($\angle$s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$s) ΔBHD $\vert\vert\vert$ Δ FED ($\angle\angle\angle$)
10.3	$\frac{FE}{BH} = \frac{FD}{BD}$ ($\vert\vert\vert$ Δs) But FE = AB (given) $\frac{AB}{BH} = \frac{FD}{BD}$ AB.BD = FD.BH

QUESTION 11



11.1	AF = FC FE ∥ CD AE = ED	(diags of parallelogram bisect) (Prop Th; FE ∥ CD) **OR** (Midpoint Theorem)
11.2	$\frac{AC}{CP}=\frac{1}{2}$ $\frac{AD}{DQ}=\frac{1}{2}$ $\frac{AC}{CP}=\frac{AD}{DQ}$ CD ∥ PQ CD ∥ FE ∴ PQ ∥ FE	(given) (given) (converse proportionality theorem) (given)

	OR $\frac{AC}{AP}=\frac{1}{3}$ $\frac{AD}{AQ}=\frac{1}{3}$ $\frac{AC}{AP}=\frac{AD}{AQ}$ CD ∥ PQ (converse proportionality theorem) CD ∥ FE (given) ∴ PQ ∥ FE **OR** $\frac{AF}{AP}=\frac{1}{6}$ $\frac{AE}{AQ}=\frac{1}{6}$ $\frac{AF}{AP}=\frac{AE}{AQ}$ ∴ PQ ∥ FE (converse proportionality theorem)

11.3	In ΔAEF and ΔAPQ 1. $\hat{A}$ is common 2. $A\hat{E}F = A\hat{Q}P$ (corres ∠s; FE ∥ PQ) 3. $A\hat{F}E = A\hat{P}Q$ (corres ∠s; FE ∥ PQ) ∴ ΔAEF ||| ΔAQP (∠∠∠) $\frac{FE}{PQ}=\frac{AF}{AP}$ (||| Δs) $\frac{FE}{60}=\frac{1}{6}$ FE = 10 cm

	OR In ΔADC and ΔAPQ 1. $\hat{A}$ is common 2. $A\hat{D}C = A\hat{Q}P$ (corres $\angle$s; CD ∥ PQ) 3. $A\hat{C}D = A\hat{P}Q$ (corres $\angle$s; CD ∥ PQ) $\therefore \Delta ADC \mid\mid\mid \Delta AQP$ ($\angle\angle\angle$) $\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3}$ (||| Δs) $CD = \frac{1}{3}PQ$ CD = 20 cm But AF = FC AE = ED (Midpoint Theorem) $FE = \frac{1}{2}CD$ FE = 10 cm

PAPER C

9.2.1	$\frac{EM}{AM} = \frac{FD}{AD}$ $\frac{EM}{AM} = \frac{3}{7}$	(Line parallel one side of Δ **OR** prop th; EF | | BD) (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredigst; EF ||BD)*
9.2.2	CM = AM $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$	(diags of parm bisect/*hoekl parm halv*) (from 9.2.1/*vanaf 9.2.1*)
9.2.3	h of $\Delta FDC = h$ of ΔBDC $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{\frac{1}{2}FD.h}{\frac{1}{2}BC.h}$ $= \frac{FD}{AD}$ $= \frac{3}{7}$	(AD | | BC) (opp sides of parm =) *(tos sye v parm =)*

PAPER D

8.1.1	$\hat{D}_2 + \hat{D}_3 = 90^{O}$ (tan $\perp$ radius) $\therefore\ \hat{D}_2 = 90^\circ - 40^\circ = 50^\circ$
8.1.2	$\hat{D}_1 = \hat{C}_2 = 65^\circ$ (angles opposite equal sides) $\hat{D}_1 + \hat{D}_2 = F\hat{B}C$ (ext angle of a cyclic quad) $65^\circ + 50^\circ = F\hat{B}C$ $\therefore F\hat{B}C = 115^\circ$

8.2.1	$\hat{S} = 32^{O}$ ($\angle$s in the same segment)
8.2.2	$\hat{O}_2 = 64^\circ$ ($\angle$ at centre $= 2 \times \angle$ at circumference)
8.2.3	$\hat{T}_1 = 90^\circ$ (line from centre to midpoint of chord) $\hat{O}_1 + \hat{T}_1 + \hat{M}_2 = 180^{O}$ (sum of angles of Δ) $\hat{O}_1 + 90^\circ + 15^\circ = 180^\circ$ $\hat{O}_1 = 75^\circ$

QUESTION 9

9.1	In ΔMBC: $\hat{B}_2 = \hat{B}_3 = x$ BE bisects $M\hat{B}C$ $\therefore M\hat{B}C = 2x$ $M\hat{B}C = M\hat{C}B = 2x$ angles opposite equal sides In ΔBEC: $\hat{E}_2 = 180^\circ - (x + x)$ Sum of angles of a Δ $= 180^\circ - 2x$

9.2	In ΔMBC: $B\hat{M}C = 180^\circ - (2x+2x)$ Sum of angles of a Δ
	$= 180^\circ - 4x$
	But $B\hat{A}C = \frac{1}{2}B\hat{M}C$ $\angle$ at centre twice angle
	$= \frac{1}{2}(180^\circ - 4x)$
	$= 90 - 2x$
9.3	In ΔABE:
	$\hat{E}_1 + \hat{E}_2 = 180^\circ$ Straight line
	$\hat{E}_1 = 180^\circ - E_2$
	$= 180^\circ - (180^\circ - 2x)$
	$= 2x$
	In ΔABE:
	$A\hat{B}E + B\hat{A}C + \hat{E} = 180^\circ$ Sum of $\angle$s of Δ
	$A\hat{B}E = 180^\circ - (B\hat{A}C + \hat{E}_1)$
	$= 180^\circ - (90^\circ - 2x + 2x)$
	$= 90^\circ$
	$\therefore$ AE is a diameter of circle ABE (Subtends) $\angle$ 90°

QUESTION 10

10.1.1	Let $\hat{Y}_1 = a$ and $\hat{N} = b$	
	$\therefore \hat{T}_3 = a - b$	(ext. ∠ of∆= sum opp. ∠s)
	$\hat{T}_1 = \hat{N} = b$	(tan XT; chord MT)
	$X\hat{T}Y = a$	(angles opposite equal sides)
	$\hat{T}_2 = X\hat{T}Y - \hat{T}_1$	
	$= a - b$	
	$\therefore \hat{T}_3 = \hat{T}_2$	
	$\therefore$ YT bisects $M\hat{T}N$	
10.1.2	In ΔXMT and ΔXTN:	
	$\hat{X}$ is common	
	$\hat{T}_1 = \hat{N}$	tan XT; chord MT
	$\hat{M}_1 = X\hat{T}N$	remaining ∠
	$\therefore \Delta XMT \parallel\!\mid \Delta XTN$	∠∠∠
	$\therefore \frac{XM}{XT} = \frac{XT}{XN} = \frac{MT}{TN}$	similar ∆'s
	$\therefore \frac{XM}{XT} = \frac{XT}{XN}$	

10.2.1	$XM = XY - 20$	XY = XT
	$= k - 20$	
10.2.2	$\frac{XM}{XT} = \frac{XT}{XN}$	
	$\therefore \frac{k-20}{k} = \frac{k}{k+50}$	
	$\therefore (k-20)(k+50) = k^2$	
	$\therefore k^2 + 30k - 1000 = k^2$	
	$\therefore 30k - 1000 = 0$	
	$\therefore 30k = 1000$	
	$\therefore k = 33{,}3$ mm	

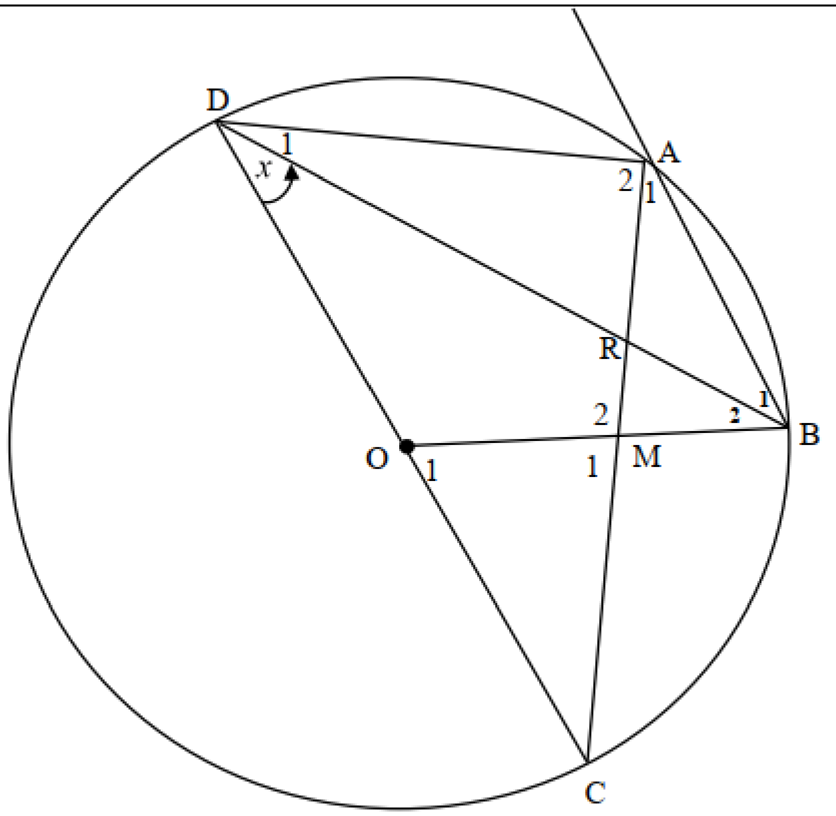
PAPER E

8.1.1	$A\hat{D}C = 67°$ **OR / *OF*** $\hat{B}_2 + \hat{B}_3 = 113°$ $A\hat{D}C = 67°$	ext. ∠ of cyclic quad / *buite* ∠ *van kvh* ∠s straight line / ∠e *op reguit lyn* opp ∠s of cyclic quad / *oorst* ∠e *van kvh*
8.1.2	$\hat{C} = 180° - 67°$ $= 113°$	co-int ∠s BC ∥ AD / *ko-binne* ∠e *BC ∥ AD*
8.1.3	$\hat{A} = 67°$	opp ∠s of cyclic quad / alt ∠s BC∥AD / alt ∠s EC ∥ AD *oorst* ∠e *van kvh* / *verwisselende* ∠e *BC∥AD* / *verwis* ∠e *EC ∥ AD*
8.1.4	$\hat{B}_2 = 67°$ $\hat{D}_2 = 180° - 67° - 67°$ $= 46°$	∠s opposite = sides / ∠e *teenoor = sye* sum of ∠s in Δ / *som vd* ∠e *v* Δ
8.1.5	$B\hat{D}G = 113°$ **OR / *OF*** $\hat{D}_1 = 67°$ $B\hat{D}G = 113°$	tan chord theorem / *raaklyn koordstelling* tan chord theorem / *raaklyn koordstelling*

8.2	$\hat{B}_3 = \hat{D}_2 = 46°$ $AB = CD$	alt ∠s BC∥AD / *verwisselende* ∠e BC∥AD ∠s subtend = chords / ∠e *onderspan = koorde*

QUESTION / *VRAAG* 9

9.1

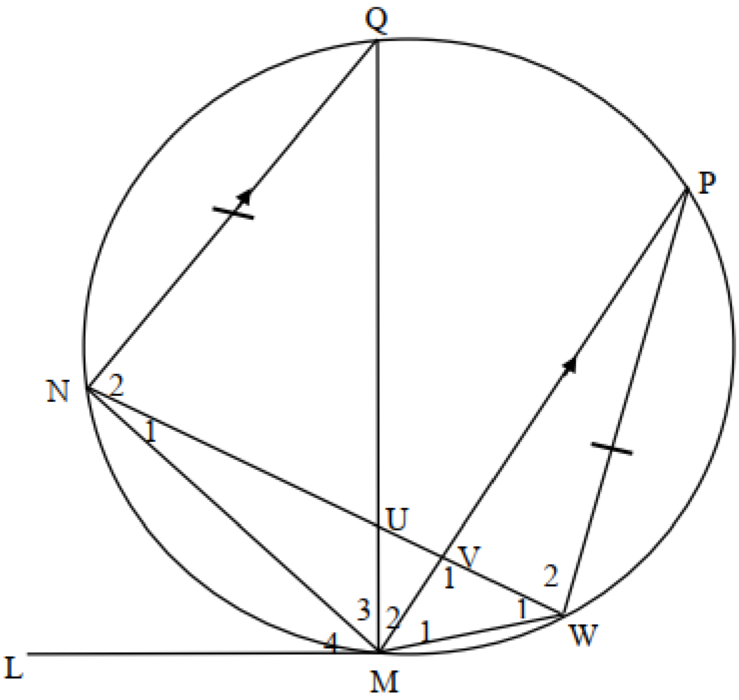


9.1.1(a)	$\hat{O}_1 = 2x$	$\angle$ centre = 2 × $\angle$ circumference / *middelpunts* $\angle$ *= 2 × omtreks* $\angle$

9.1.1(b)	$\hat{A}_1 = C\hat{D}B = x$	$\angle^s$ in the same segment / *$\angle^e$ in dies. segment*
	$\hat{M}_2 = 90°$	line from centre to midpoint of chord / *lyn van middelpunt van sirkel na middelpunt van koord*
	$\therefore A\hat{B}O = 90° - x$	sum of $\angle^s$ in Δ / ext $\angle$ of a Δ / *som vd $\angle^e$ v Δ / buite $\angle$ v Δ*
	OR/*OF*	
	$\hat{O}_1 = 2x$	proved/*reeds bewys*
	$\hat{M}_1 = 90°$	line from centre to midpoint of chord / *lyn van middelpunt van sirkel na middelpunt van koord*
	$\hat{C} = 90° - 2x$	sum of $\angle^s$ in Δ / *som vd $\angle^e$ v Δ*
	$\hat{B}_1 = 90° - 2x$	$\angle^s$ in the same segment / *$\angle^e$ in dies. segment*
	$A\hat{B}O = 90° - x$	

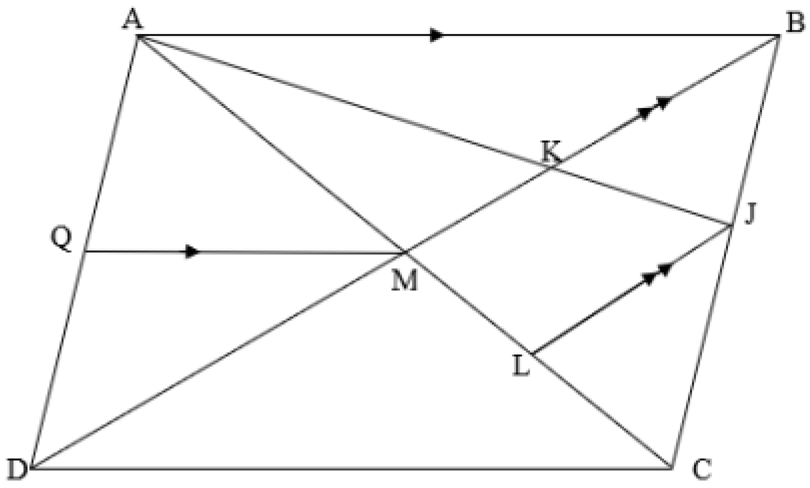
9.1.2	AD‖OB $\hat{O}_1 = A\hat{D}C = 2x$ $\therefore \hat{D}_1 = x$ $\hat{A}_1 = x$ $\therefore \hat{D}_1 = \hat{A}_1$ AB is a tangent / *is 'n raaklyn* **OR / *OF*** $\hat{A}_2 = 90°$ $\therefore$ AD‖OB $C\hat{D}A = \hat{O}_1 = 2x$ $\therefore \hat{D}_1 = x$ $\hat{D}_1 = \hat{A}_1$ AB is a tangent / *is 'n raaklyn*	midpoint theorem / *middelpunt stelling* corresponding ∠s AD ‖ OB / *ooreenkom ∠e AD ‖ OB* proved / *reeds bewys* converse tan chord theorem / *omgekeerde raaklyn koordstelling* ∠ in a semi-circle / *∠ in halwe sirkel* corr ∠s are equal / *ooreenk ∠e gelyk* corr ∠s DA ‖ OB / *ooreenk ∠e* DA ‖ OB converse tan chord theorem / *omgekeerde raaklyn koord*

9.1.3	$DC^2 = AD^2 + AC^2$ but / *maar* $AC = 2AM$ and / *en* $DC = 2DO$ $(2DO)^2 = AD^2 + (2AM)^2$ $4DO^2 = AD^2 + 4AM^2$ but / *maar* In ΔABM $AM^2 = AB^2 - MB^2$ $\therefore 4DO^2 = AD^2 + 4(AB^2 - MB^2)$ $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$	Pythagoras Pythagoras

9.2	

9.2.1	$\hat{M}_2 = \hat{Q}$	alternate $\angle^s$ NQ || MP / *verwissel* $\angle^e$ *NQ ||MP*
	$\hat{M}_4 = \hat{Q} = \hat{M}_2$	tan chord theorem / *raaklyn koordstelling*
	$\hat{W}_1 = \hat{Q} = \hat{M}_2$	$\angle^s$ in the same segment / tan chord theorem / $\angle^e$ *dieselfde sirkel segment / raaklyn koord stelling*
	OR/*OF*	
	$\hat{W}_1 = \hat{M}_4 = \hat{M}_2$	tan chord theorem / *raaklyn koordstelling*
9.2.2	In ΔWMV and/*en* ΔQMN	
	$\hat{W}_1 = \hat{Q}$	proved / *reeds bewys*
	$\hat{M}_1 = \hat{M}_3$	equal chords subtend equal $\angle^s$ / *gelyke koorde onderspan gelyke* $\angle^e$
	$\hat{V}_1 = \hat{N}_1 + \hat{N}_2$	sum of $\angle^s$ in Δ / *som vd* $\angle^e$ *v* Δ
	$\therefore \Delta WMV \parallel\!\mid \Delta QMN$	$\angle\angle\angle$

9.2.3	$\frac{MV}{MN}=\frac{WV}{QN}$	$\Delta WMV \parallel\mid \Delta QMN / \parallel\mid \Delta^s$
	$\frac{MV}{WV}=\frac{MN}{QN}$	
	$MV \times QN = MN \times WV$	given / *gegee*
	but / *maar* $QN = PW$	
	$MV \times PW = MN \times WV$	
	$\frac{MV}{WV}=\frac{MN}{PW}$	

10.2	

10.2.1 (a)	$\frac{ML}{LC}=\frac{BJ}{JC}=\frac{2}{3}$	line ∥ one side ΔBCM **OR** prop theorem MB∥JL / *lyn* ∥ *aan een sy van* ΔBCM ***OF*** *eweredigheidsstelling MB*∥*JL*

10.2.1 (b)	$\frac{MC}{ML}=\frac{BC}{BJ}=\frac{5}{2}$	line ∥ one side ΔBMC **OR** prop theorem MB∥JL / *lyn* ∥ *aan een sy van* ΔBMC ***OF*** *eweredigheidsstelling MB* ∥ *JL*
	$AM = MC$	diagonals of a parm bisect / *hoeklyne van parm halveer*
	$\frac{AM}{ML}=\frac{5}{2}$	
	$\frac{AK}{KJ}=\frac{AM}{ML}=\frac{5}{2}$	line ∥ one side ΔAJL **OR** prop theorem MK∥JL / *lyn* ∥ *aan een sy van* ΔAJL ***OF*** *eweredigheidsstelling MK* ∥ *JL*

10.2.2	$AB \parallel CD$	opposite sides of parm / *oorst sye van parm*
	$AB \parallel QM$	
	In ΔADC	
	$\therefore QM \parallel CD$	
	$AM = MC$	proved / *reeds bewys*
	$\therefore AQ = QD$	line passing through the midpoint of 1 side || to second side / *lyn sny die middelpunt van 1 sy || aan tweede sy*
	but $AD = BC$	opposite sides of parm / *oorst sye van parm*
	$AQ = \frac{1}{2}AD$	
	$= \frac{1}{2}\left(\frac{2\sqrt{10}}{3}\right)$	
	$\therefore AQ = QD = \frac{2}{3}\sqrt{10} \div 2$	
	$= \frac{\sqrt{10}}{3}$ units	

PAPER F

8.1	$\hat{O}_2 = 50^\circ$	$\angle$s around a point / *$\angle$e om 'n punt*
	$\hat{D}_1 = 25^\circ$	$\angle$ centre $= 2\times \angle$ at circumference *midpts $\angle$ = 2 × omtreks $\angle$*
8.2	$\hat{B}_3 = 25^\circ$	tan chord theorem / *raaklyn koordstelling*
8.3	$B\hat{C}D = 120^\circ$	opp $\angle$s of a cyclic quad /*teenoorst $\angle$e v kvh*
	$\hat{B}_2 = 35^\circ$	sum of $\angle$s of a triangle / *som $\angle$e v Δ*
	$O\hat{B}C = O\hat{C}B = 65^\circ$	$\angle$s opp. equal radii / *$\angle$e teenoor gelyke radiuse*
	$\therefore \hat{B}_1 = 65^\circ - 35^\circ$	
	$\hat{B}_1 = 30^\circ$	
	OR / *OF*	**OR / *OF***
	$B\hat{C}D = 120^\circ$	opp $\angle$s of a cyclic quad/*teenoorst $\angle$e vkvh*
	$\hat{B}_2 = 35^\circ$	sum of $\angle$s of a triangle / *som $\angle$e v Δ*
	$\hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90^\circ$	radius $\perp$ tangent / *radius $\perp$ raaklyn*
	$\hat{B}_1 = 30^\circ$	

QUESTION / *VRAAG* 9

9.1	Equal / *gelyk.*

9.2.1	$\hat{D}_1 = \hat{F}_1 = x$ $\hat{F}_1 = \hat{F}_2 = x$	tan chord theorem / *raaklyn koordstelling* = chords subtend = $\angle s$ = *koorde onderspan* = $\angle e$
9.2.2	$\hat{F}_2 = \hat{A} = x$ $\hat{D}_1 = \hat{A} = x$ ABDC is a cyclic quad / *ABDC is 'n kvh*	ext. $\angle$ of cyclic quad / *buite $\angle$ v kvh* ext $\angle$ = opp int $\angle$ **OR** converse of ext. $\angle$ of cyclic quad / *buite $\angle$ = oorst binne $\angle$* ***OF*** *omgekeerde buite $\angle$ v kvh*
9.2.3	$\hat{B}_1 + \hat{B}_2 = \hat{A}$ $\hat{A} = \hat{D}_1$ $\hat{B}_1 + \hat{B}_2 = \hat{D}_1$ BE || CD	tan chord theorem / *raaklyn koordstelling* proved / *reeds bewys* correspond $\angle s$ = / *ooreenkomst $\angle e$ =*

9.2.4	$\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180°$	opp $\angle s$ of a cyclic quad / *teenoorst $\angle e$ v kvh*
	$\hat{C}_1 = \hat{C}_2$	diag rhombus bisect $\angle$ / *diag ruit halveer $\angle$*
	$\hat{F}_1 = \hat{F}_2$	proved / *reeds bewys*
	$2\hat{C}_1 + 2\hat{F}_2 = 180°$	
	$\hat{C}_1 + \hat{F}_2 = 90°$	
	$\hat{E}_1 = 90°$	sum of $\angle s$ of Δ / *som van $\angle e$ v Δ*
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde $\angle$ in half sirkel*

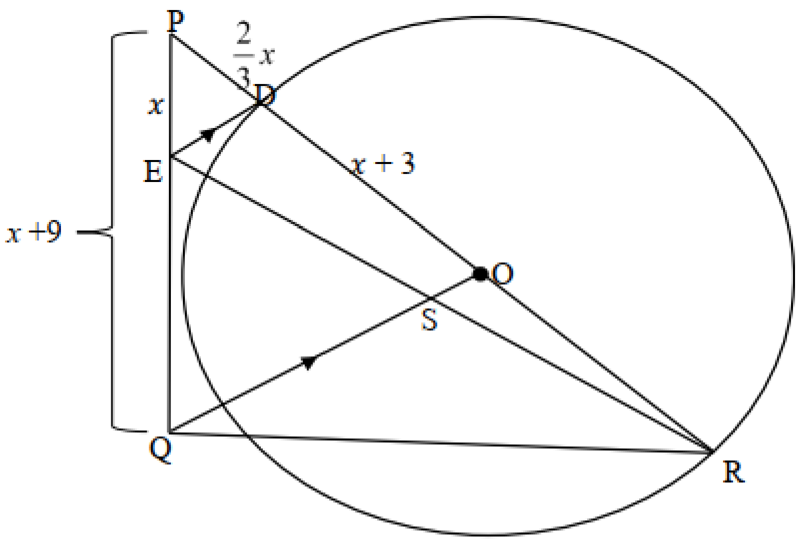
	OR / *OF*	
	Let $\hat{F}_1 = \hat{F}_2 = x$	proved / *reeds bewys*
	$\hat{C} = 180° - 2x$	opp $\angle s$ of a cyclic quad / *teenoorst* $\angle e$ *v kvh*
	$\hat{C}_1 = \hat{C}_2 = 90° - x$	diag rhombus bisect $\angle$ / *diag ruit halveer* $\angle$
	In ΔFDC **or / *of*** ΔEFC	
	$\hat{D} = 90°$ **or / *of*** $\hat{E} = 90°$	sum of $\angle s$ of Δ / *som van* $\angle e$ *v* Δ
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde* $\angle$ *in half sirkel*

10.2

10.2.1	$\hat{R}_1 = 60°$	equilateral Δ / *gelyksydige* Δ
	$\hat{W}_1 = \hat{P}_1 + \hat{Q}_1$	ext. $\angle$ of a Δ / *buite* $\angle$ *v* Δ
	$= 60° + \hat{Q}_1$	
	$= \hat{R}_1 + \hat{Q}_1$	
	$\hat{Q}_1 = \hat{R}_2$	$\angle$s in the same segment/ $\angle$*e in dieselfde segment*
	$\therefore \hat{W}_1 = T\hat{R}Q$	
10.2.2	In ΔTQR and / *en* ΔQRV	
	1. $\hat{W}_1 = T\hat{R}Q$	proved / *reeds bewys*
	2. $\hat{R}_1 = T\hat{Q}R$	equilateral Δ / *gelyksydige* Δ
	3. $\hat{Q}_2 = \hat{T}$	sum $\angle s$ of Δ / *som van* $\angle e$ *v* Δ
	$\therefore \Delta WRQ \,\|\|\, \Delta RQT$	$\angle\angle\angle$

10.2.3	In ΔTPV and / *en* ΔWQR	
	1. $P\hat{Q}R = \hat{R}_1$	both 60° / *albei* 60°
	$P\hat{Q}R = \hat{V}_1$	ext. $\angle$ of a cyclic quad. / *buite* $\angle$ *v kvh*
	$\hat{V}_1 = \hat{R}_1$	
	2. $\hat{P}_2 = T\hat{R}Q$	ext. $\angle$ of a cyclic quad. / *buite* $\angle$ *v kvh*
	but / *maar* $\hat{W}_1 = T\hat{R}Q$	proved / *reedsbewys*
	$\hat{P}_2 = \hat{W}_1$	
	3. $\hat{T} = \hat{Q}_2$	sum of $\angle$s of Δ / *som v* $\angle$*e v* Δ
	$\Delta VPT \parallel\!\mid \Delta RWQ$	$\angle\angle\angle$
	$\dfrac{VP}{RW} = \dfrac{PT}{WQ} = \dfrac{VT}{RQ}$	corresponding sides in prop/ *ooreenkomstige sye in verhouding*
	$\therefore \dfrac{PT}{WQ} = \dfrac{PV}{WR}$	

QUESTION / *VRAAG* 11



11.1	$\frac{PE}{EQ}=\frac{PD}{DO}$	line || one side ΔPOQ **OR** prop theorem ED||OQ / *lyn || een sy* ΔPOQ ***OF*** *eweredigheid stelling* ED||OQ
	$\frac{x}{9}=\frac{\frac{2}{3}x}{x+3}$	
	$x^2+3x=6x$	
	$x^2-3x=0$	
	$x(x-3)=0$	
	$x=0$ or/*of* $x=3$	
	N.A / *n.v.t*	
	$DO=6$	
	$DO=OR$	radii / *radiusse*
	$OR=6$ units / *eenhede*	
11.2	S is the midpoint of RE / *S is die middelpunt van RE*	
	DE =2OS	
	DE = 2,8 units / *eenhede*	midpoint theorem / *middelpunt stelling*
11.3	$\frac{\text{Area } \Delta PED}{\text{Area } \Delta PER}=\frac{PD}{PR}$	same height (DE) / *dieselfde hoogte (DE)*
	$=\frac{2}{14}$	
	$=\frac{1}{7}$	
	Area ΔPER $= 7 \times$ Area ΔPED	
	$= 18{,}9$ units^2 / *eenhede*^{2}	

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 16 pages.
Hierdie nasienriglyne bestaan uit 16 bladsye.

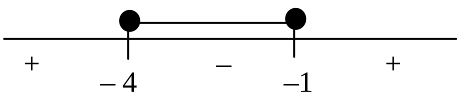
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking guidelines.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/VRAAG 1

1.1.1	$x^2-2x-24=0$ $(x-6)(x+4)=0$ $x=6$ or $x=-4$	✓factors ✓$x = 6$ ✓$x = -4$ (3)
1.1.2	$2x^2-3x-3=0$ $x=\frac{3\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$ $x=\frac{3\pm\sqrt{33}}{4}$ $x=2{,}19$ or $x=-0{,}69$	✓substitution into the correct formula ✓$x = 2{,}19$ ✓$x = -0{,}69$ (3)
1.1.3	$x^2+5x\le-4$ $x^2+5x+4\le0$ $(x+4)(x+1)\le0$ Critical values: $x=-4$ or $x=-1$  $-4\le x\le-1$ **OR/*OF*** $x\in[-4\,;-1]$	✓ standard form ✓ critical values ✓✓answer (4)
1.1.4	$\sqrt{x+28}=2-x$ $\left(\sqrt{x+28}\right)^2=(2-x)^2$ $x+28=4-4x+x^2$ $x^2-5x-24=0$ $(x-8)(x+3)=0$ $x\ne8$ or $x=-3$	✓ squaring both sides ✓ standard form ✓ factors ✓ answers with selection (4)

1.2	$2y = 3 + x$ $x = 2y - 3$ … (1) $2xy + 7 = x^2 + 4y^2$ … (2) $2y(2y-3) + 7 = (2y-3)^2 + 4y^2$ $4y^2 - 6y + 7 = 4y^2 - 12y + 9 + 4y^2$ $4y^2 - 6y + 2 = 0$ $2y^2 - 3y + 1 = 0$ $(2y-1)(y-1) = 0$ $y = \frac{1}{2}$ or $y = 1$ $x = -2$ or $x = -1$	✓equation 1 ✓substitution ✓simplification ✓standard form ✓y – values ✓x – values (6)
	OR/*OF*	**OR/*OF***
	$2y = 3 + x$ $y = \frac{3}{2} + \frac{x}{2}$ …(1) $2xy + 7 = x^2 + 4y^2$ … (2) $2x\left(\frac{3}{2} + \frac{x}{2}\right) + 7 = x^2 + 4\left(\frac{3}{2} + \frac{x}{2}\right)^2$ $3x + x^2 + 7 = x^2 + 9 + 6x + x^2$ $x^2 + 3x + 2 = 0$ $(x+2)(x+1) = 0$ $x = -2$ or $x = -1$ $y = \frac{1}{2}$ or $y = 1$	✓equation 1 ✓substitution ✓simplification ✓standard form ✓x – values ✓y – values (6)
1.3	$\frac{n}{m} = \frac{p}{n}$ $n^2 = mp$ $\Delta = b^2 - 4ac$ $\Delta = n^2 - 4mp$, but $n^2 = mp$ $\Delta = n^2 - 4n^2$ **OR/*OF*** $\Delta = mp - 4mp$ $\Delta = -3n^2$ $\quad$ $\Delta = -3mp$ $n^2 > 0$ $\quad$ $mp > 0$ $\therefore -3n^2 < 0$ $\quad$ $\therefore -3mp < 0$ $\therefore \Delta < 0 \Rightarrow x$ is a non-real number	✓ $\frac{n}{m} = \frac{p}{n}$ ✓ $n^2 = mp$ ✓ $\Delta = -3n^2$ or $-3mp$ ✓ $\Delta < 0$ (4)
		[24]

QUESTION/VRAAG 2

2.1	$\frac{90}{x}=\frac{81}{90}$ $81x=8\,100$ $x=100$ **OR/OF** $x=90\times\frac{10}{9}$ $x=100$	✓ $\frac{90}{x}=\frac{81}{90}$ ✓ answer (2) **OR/OF** ✓ $\frac{10}{9}$ ✓ answer (2)
2.2	$S_n=\frac{a(1-r^n)}{1-r}$ $S_n=\frac{100(1-(0,9)^n)}{1-0,9}$ $S_n=\frac{100(1-(0,9)^n)}{0,1}$ $\therefore S_n=1\,000(1-(0,9)^n)$	✓$r=0,9$ ✓substitution into correct formula (2)
2.3	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{100}{1-\frac{9}{10}}$ $S_\infty=1000$ **OR/OF** $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0,9)^n\right)\right]$ $S_\infty=1000$	✓ substitution ✓ answer (2) **OR/OF** ✓ $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0,9)^n\right)\right]$ ✓ answer (2)
		[6]

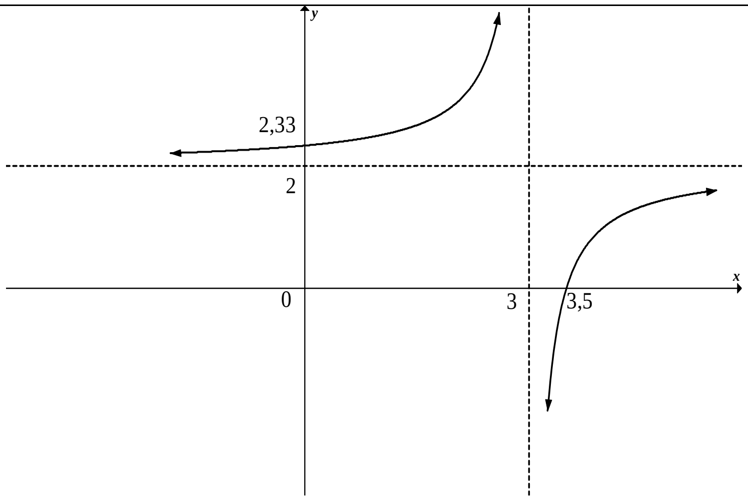
QUESTION 3

3.1	–82	✓ answer (1)
3.2	–145 ; –122 ; –101 ; . . . 23 21 –2 –2 $2a=-2 \quad \therefore\ a=-1$ $3a+b=23 \quad \therefore\ 3(-1)+b=23 \quad \therefore b=26$ $a+b+c=-145\ \therefore -1+26+c=-145\ \therefore\ c=-170$ $\therefore T_n=-n^2+26n-170$ **OR/OF** $2a=-2 \quad \therefore\ a=-1$ $c=-145+(-2)-23=-170$ $\therefore T_n=-n^2+bn-170$ $-145=-1+b-170$ $b=26$ $\therefore T_n=-n^2+26n-170$	✓ $2a=-2$ ✓ $3(-1)+b=23$ ✓ $-1+26+c=-145$ (3) **OR/OF** ✓ $2a=-2$ ✓ $c=-145+(-2)-23$ ✓ $-145=-1+b-170$ (3)
3.3	$T_n=bn+c$ $T_n=-2n+25$ or $T_n=a+(n-1)d$ $=23+(n-1)(-2)$ $=25-2n$ $-2n+25=-121$ $-2n=-146$ $n=73$ Between T_{73} and T_{74} **OR/*OF*** $T_{n+1}-T_n=-(n+1)^2+26(n+1)-170-(-n^2+26n-170)$ $-121=-2n+25$ $n=73$ Between T_{73} and T_{74}	✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4) **OR/*OF*** ✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4)
3.4	$n=\dfrac{-b}{2a}=\dfrac{-26}{2(1)}=13$ $T_{13}=-1$ $\therefore$ add 2 **OR/*OF*** $T'_n=-2n+26=0$ $n=13$ $T_{13}=-(13)^2+26(13)-170=-1$ $\therefore$ add 2	✓13 ✓ $T_{13}=-1$ ✓ add 2 (3) **OR/*OF*** ✓13 ✓ $T_{13}=-1$ ✓ add 2 (3)
		[11]

QUESTION/*VRAAG* 4

4.1	$a = 5$ and/*en* $d = 2$ $T_{51} = 5 + (51-1)(2)$ $= 105$	✓ *a* and *d* ✓ substitution into correct formula ✓ answer (3)
4.2	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{51} = \frac{51}{2}\left[2(5) + (51-1)2\right]$ or/*of* $S_{51} = \frac{51}{2}\left[5 + 105\right]$ $= 2\ 805$ $= 2\ 805$	✓ substitution into correct formula ✓ answer (2)
4.3	$\sum_{n=1}^{5\ 000}(2n+3) = 5 + 7 + 9 + ... + 10\ 003$	✓ expansion (1)
4.4	$T_1 = -3$ $T_{4\ 999} = -2(4\ 999) - 1 = -9\ 999$ $\therefore \sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$ = (5 + 7 + 9 + … + 9 999 + 10 001 + 10 003) + (–3 – 5 – 7 – 9 – … – 9 999) = 10 001 + 10 003 –3 = 20 001 **OR/OF** $S_{4\ 999} = \frac{4\ 999}{2}\left[2(-3) + (4\ 999 - 1)(-2)\right] = -24\ 999\ 999$ $S_{5\ 000} = \frac{5000}{2}\left((2)(5) + (5\ 000 - 1)(2)\right) = 25\ 020\ 000$ $\sum_{n=1}^{5\ 000}(2n+3) + \sum_{n=1}^{4\ 999}(-2n-1) = 25\ 020\ 000 - 24\ 999\ 999$ $= 20\ 001$	✓ $T_1 = -3$ ✓ $T_{4999} = -9\ 999$ ✓both expansions ✓ answer (A) (4) **OR/OF** ✓ $T_1 = -3$ ✓ $S_{4\ 999} = -24\ 999\ 999$ ✓ $S_{5\ 000} = 25\ 020\ 000$ ✓ answer (A) (4)
		[10]

QUESTION/VRAAG 5

5.1	$x = 3$ $y = 2$	✓ $x = 3$ ✓ $y = 2$ (2)
5.2	$x \in R,\ x \neq 3$ **OR/*OF*** $x \in (-\infty\ ;3) \cup (3\ ;\infty)$ **OR/OF** $x < 3$ or $x > 3$	✓answer (1) **OR/OF** ✓answer (1) **OR/OF** ✓answer (1)
5.3	$0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ x-int: $\left(\frac{7}{2}\ ;\ 0\right)$	✓ $y = 0$ ✓ answer (2)
5.4	y-int: $\left(0\ ;\ \frac{7}{3}\right)$	✓ $x = 0$ ✓ $\frac{7}{3}$ (2)
5.5		✓ asymptotes ✓ intercepts with the axes ✓ shape (3)
		[10]

QUESTION/VRAAG 6

6.1	$f(x)=\log_4 x$ $2=\log_4 k$ $4^2=k$ $\therefore k=16$	✓ substitution of $(k\ ;\ 2)$ ✓ answer (2)
6.2	$-1=\log_4 x \quad \therefore\ x=\frac{1}{4}$ $\frac{1}{4}\le x\le 16$ or/of $x\in\left[\frac{1}{4}\ ;\ 16\right]$	✓ $x=\frac{1}{4}$ ✓ answer (2)
6.3	$f(x)=\log_4 x$ $y=\log_4 x$ $x=\log_4 y$ $y=4^x$	✓swopping x and y ✓answer (2)
6.4	$x<0$ **OR/*OF*** $x\in(-\infty\ ;\ 0)$	✓✓answer (2) **OR/*OF*** ✓✓answer (2)
		[8]

QUESTION 7

7.1	B(–4 ; 0) D(6 ; 0)	✓ B(–4 ; 0) ✓ D(6 ; 0) (2)
7.2	$f(x)=x^2-2x-24$ $x_{tp}=\frac{-b}{2a}$ **OR/*OF*** $2x-2=0$ **OR/OF** $x=\frac{-4+6}{2}$ $x=\frac{-(-2)}{2(1)}$ $\therefore\ x_{tp}=1$ $y_{tp}=f(1)$ $=1^2-2(1)-24$ $=-25$ C(1 ; –25)	✓ $x_{tp}=1$ ✓ $y_{tp}=-25$ (2)
7.3	$y\geq -25$ **OR/*OF*** $y\in[-25\ ;\ \infty)$	✓ answer (1) **OR/OF** ✓ answer (1)
7.4.1	$m_{AE}=\tan 14,04°=0,25=\frac{1}{4}$	✓ answer (1)
7.4.2	$m_{\text{tang}}=-4$ $f^{/}(x)=2x-2$ $2x-2=-4$ $x_T=-1$ $y_T=-21$	✓ $m_{\text{tang}}=-4$ ✓ $f^{/}(x)=2x-2$ ✓ equating ✓ $x_T=-1$ ✓ $y_T=-21$ (5)
7.5	$m_{line}=\frac{1}{4}$ $y+9=\frac{1}{4}(x+3)$ **OR/OF** $-9=\frac{1}{4}(-3)+c$ $y+9=\frac{1}{4}x+\frac{3}{4}$ $c=-\frac{33}{4}=-8,25$ $y=\frac{1}{4}x-\frac{33}{4}$ **OR/OF** $y=0,25x-8,25$ $x^2-2x-24=\frac{1}{4}x-\frac{33}{4}$ $4x^2-8x-96=x-33$ $4x^2-9x-63=0$ $(4x-21)(x+3)=0$ $\therefore x=\frac{21}{4}=5,25$ or $x\neq -3$	✓ $m_{line}=\frac{1}{4}$ ✓ substitution *m* and K(– 3 ; – 9) ✓ $y=\frac{1}{4}x-\frac{33}{4}$ ✓ equating ✓ standard form ✓ answer with selection (6)
		[17]

QUESTION/VRAAG 8

8.1	$A = P(1-i)^n$ $A = 980\ 000(1-0,092)^7$ $A = R498\ 685,82$	✓ correct formula ✓ substitution ✓ answer (A) (3)
8.2	$A = P(1+i)^n$ $116\ 253,50 = 75\,000\left(1+\frac{0,068}{4}\right)^{4n}$ $1,550\ 046\ 667 = (1,017)^{4n}$ $\log(1,550\ 046\ 667) = 4n\log(1,017)$ $4n = \frac{\log(1,550\ 046\ 667)}{\log(1,017)}$ or $4n = \log_{1,017}(1,550\ 046\ 667)$ $4n = 25,99\ \ldots$ $n = 6,50$ years	✓ $\frac{0,068}{4}$ ✓ substitution in correct formula ✓ correct use of logs ✓ answer (4)
8.3.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $450\ 000 = \frac{x\left[\left(1+\frac{0,0835}{12}\right)^{60} - 1\right]}{\frac{0,0835}{12}}$ $x = R6\ 068,69$	✓ $i = \frac{0,0835}{12}$ ✓substitution into correct formula ✓answer (3)
8.3.2(a)	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{11\ 058,85\left[1-\left(1+\frac{0,12}{12}\right)^{-4\times12}\right]}{\frac{0,12}{12}}$ $P = R419\ 948,32$ **OR/*OF*** Balance = A – F $= P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$ $=1\ 050\ 000\left(1+\frac{0,12}{12}\right)^{12\times21} - \frac{11\ 058,85\left[\left(1+\frac{0,12}{12}\right)^{12\times21} - 1\right]}{\frac{0,12}{12}}$ =R12 887 702,20 – R12 467 749,81 =R419 952,39	✓ $n = 48$ in P-formula ✓substitution into correct formula ✓answer (A) (3) **OR/*OF*** ✓n = 252 in both formulae ✓ subst into correct formulae ✓answer (A) (3)

8.3.2(b)	Total paid $= 11\ 058,85 \times 21 \times 12 = 2\ 786\ 830,20$ Loan Paid $= 1\ 050\ 000 - 419\ 948,32 = 630\ 051,68$ Interest paid = 2 786 830,20 – 630 051,68 = R2 156 778,52 **OR/OF** Total paid $= 11\ 058,85 \times 21 \times 12 = 2\ 786\ 830,20$ Loan Paid $= 1\ 050\ 000 - 419\ 952,39 = 630\ 047,61$ Interest paid = 2 786 830,20 – 630 047,61 = R2 156 782,59 **OR/OF** Interest paid = 11 058,85 × 21 × 12 – (1 050 000 – 419 948,32) = 2 786 830,20 – 630 051,68 = R2 156 778,52	✓ $11\ 058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\ 058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\ 058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3)
		[16]

QUESTION/VRAAG 9

9.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$ **OR/OF** $f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5) **OR/OF** ✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\frac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓3 (3)

9.2.2	$D_x\left[\dfrac{-\sqrt[3]{x}}{2}+\left(\dfrac{1}{3x}\right)^2\right]$ $D_x\left[\dfrac{-x^{\frac{1}{3}}}{2}+\dfrac{x^{-2}}{9}\right]$ $D_x\left[-\dfrac{1}{2}x^{\frac{1}{3}}+\dfrac{1}{9}x^{-2}\right]$ $=-\dfrac{1}{6}x^{-\frac{2}{3}}-\dfrac{2x^{-3}}{9}$ $=-\dfrac{1}{6x^{\frac{2}{3}}}-\dfrac{2}{9x^3}$	✓ $\dfrac{-x^{\frac{1}{3}}}{2}$ ✓ $\dfrac{x^{-2}}{9}$ ✓ $-\dfrac{1}{6}x^{-\frac{2}{3}}$ ✓ $-\dfrac{2x^{-3}}{9}$ (4)
		[12]

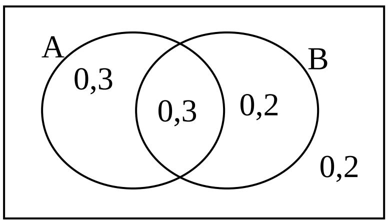
QUESTION/VRAAG 10

10.1	$h(x)=ax^3+bx^2$ $h^/(x)=3ax^2+2bx$ $h^/(4)=0$ $48a+8b=0$ $6a+b=0$ ...(1) $h(4)=32$ $64a+16b=32$ $4a+b=2$ ...(2) (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h^/(x)$ ✓ $h^/(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A$(6\,;0)$	✓ $-x^3+6x^2=0$ ✓ factors ✓ A$(6\,;0)$ (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0\,;\,4)$ or $x\in[0\,;\,4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2\,;\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7\,;32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

QUESTION/VRAAG 11

11	Time = $\frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x)=1{,}6\times\left(\frac{20}{x}\right)+20\left(1{,}2+\frac{x}{4000}\right)$ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ $C^{/}(x)=-\frac{32}{x^2}+\frac{1}{200}=0$ $x^2=6\,400$ $x=80\,km/h$	✓ $\frac{20}{x}$ ✓ $1{,}6\times\left(\frac{20}{x}\right)$ ✓ $20\left(1{,}2+\frac{x}{4000}\right)$ ✓ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ ✓ $C^{/}(x)=-\frac{32}{x^2}+\frac{1}{200}$ ✓ $C^{/}(x)=0$ ✓ answer (A) (7)
		[7]

QUESTION/VRAAG 12

12.1.1	No, because P(A and B) $\neq 0$	✓ answer and reason (1)
12.1.2(a)	$P(A \text{ and } B)=0{,}3$ $P(\text{only } B)=0{,}2$ $P(A \text{ and } B)=P(A)\times P(B)$ $0{,}3=P(A)\times 0{,}5$ $P(A)=0{,}6$ $P(\text{only } A)=0{,}3$	✓ $P(A \text{ and } B)=P(A)\times P(B)$ ✓ 0,5 ✓ $P(A)=0{,}6$ ✓answer (4)
12.1.2(b)	 $P(\text{not A or not B})=0,2+0,2+0,3=0,7$ **OR/OF** $P(\text{not A or not B})=1-P(\text{A and B})=1-0,3=0,7$ **OR/OF** P(A$^{/}$ or B$^{/}$) = P(A$^{/}$) + P(B$^{/}$) – P(A$^{/}$ and B$^{/}$) = 0,4 + 0,5 – 0,2 = 0,7	✓method ✓answer (2) **OR/OF** ✓method ✓answer (2) **OR/OF** ✓method ✓answer (2)

<table>
<tr><td>12.2.1</td><td>P(novel) = $\frac{3}{12}=\frac{1}{4}$</td><td>✓answer
(1)</td></tr>
<tr><td>12.2.2</td><td>12! = 479 001 600</td><td>✓✓ answer
(2)</td></tr>
<tr><td>12.2.3</td><td>
<table>
<tr><td>5</td><td>3!</td><td>8!</td><td>4</td></tr>
<tr><td>(Poetry)</td><td>(Novels all together)</td><td>(Arrangements of rest of the books including the novels)</td><td>(Drama)</td></tr>
</table>
P(start with poetry, end with drama AND all novels together)
$= \frac{5\times 3!\times 8!\times 4}{12!}$
$= \frac{1}{99}$</td><td>✓ 5×4
✓ $3! = 6$
✓8!

✓ $\frac{5\times 3!\times 8!\times 4}{12!}=\frac{1}{99}$ (A)
(4)</td></tr>
<tr><td></td><td></td><td>**[14]**</td></tr>
</table>

TOTAL/TOTAAL: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

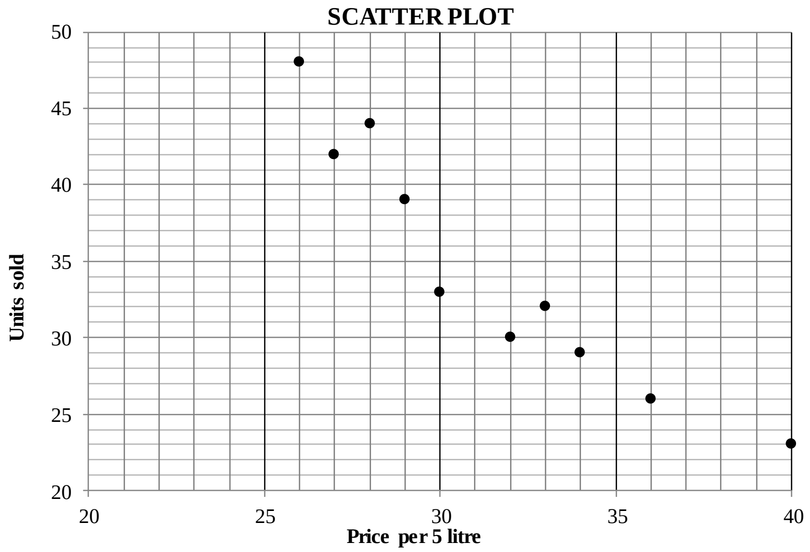
QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

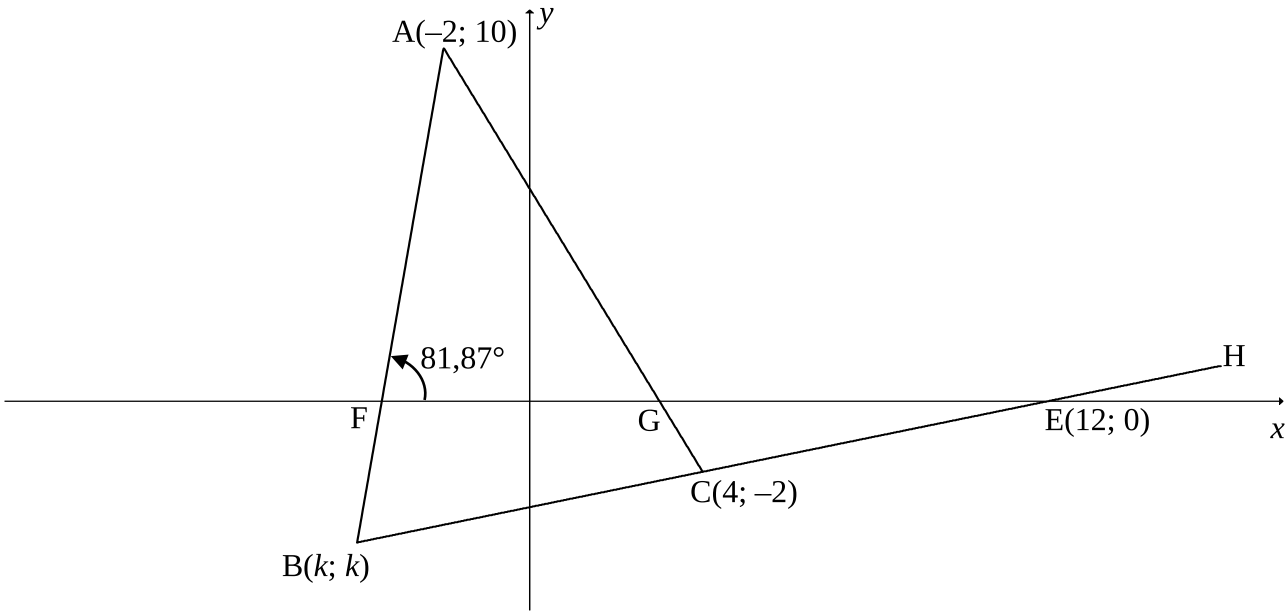
1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$ Answer only: Full marks *Slegs antw: Volpunte*	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore 5$ days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** 22–20 = 2 $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* Any one of the following reasons/*Enige een van die vlg redes*: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (x) ***Prys van melk in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

2.1		1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly (3)
2.2	$a = 90{,}478... \approx 90{,}48$ $b = -1{,}773... \approx -1{,}77$ $\hat{y} = 90{,}48 - 1{,}77x$ Answer only: Full marks *Slegs antw: Volpunte*	✓ a ✓ b ✓ equation (3)
2.3	$y = 23{,}069... \approx 23{,}07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90{,}48 - 1{,}77(38)$ $y = 23{,}22$ units/*eenhede*	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	$r = -0{,}94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold $\therefore$ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan* $\therefore$ *daar is 'n goeie kans dat die voorspelling akkuraat is*	✓ value of r **OR/OF** strong relationship/ *sterk verwantskap* ✓ accurate/*akkuraat* (2)
		[10]

QUESTION/*VRAAG* 3

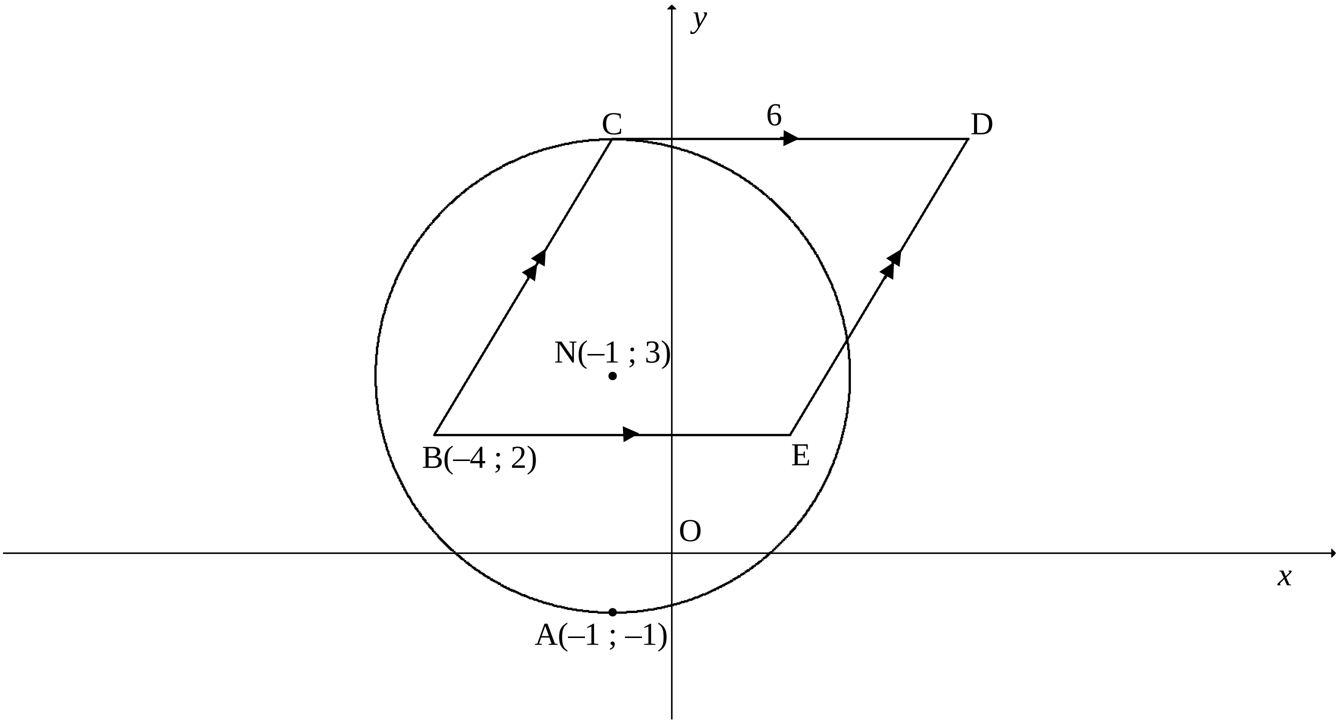


3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

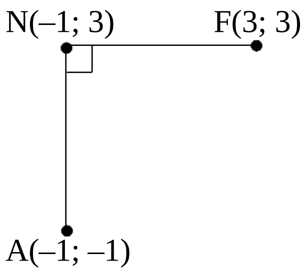
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* $m_{\text{BE}}=\frac{1}{4}$ **OR/*OF*** $m_{\text{BE}}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $-4k=12-k$ $4k=k-12$ $k=-4$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* $m_{\text{AB}}=\tan 81{,}87°$ $m_{\text{AB}}=7$ $m_{\text{AB}}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore$ B(–4; –4)	✓ equating EB & AB ✓ answer (2)

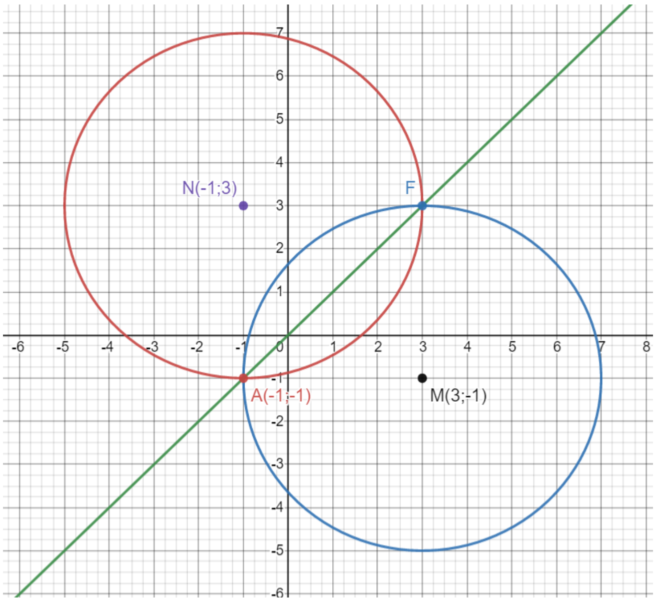
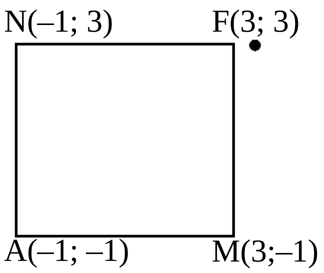
3.3.2	In ΔAFG: $m_{AC} = \dfrac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34{,}70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17};\ b = AC = 6\sqrt{5};\ c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \dfrac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\dfrac{12+(-2)}{2};\dfrac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$ T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \dfrac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $-4 = -4(-4) + c$ $y-(-4) = -4(x-(-4))$ $c = -20$ $y = -4x - 20$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

QUESTION/*VRAAG* 4

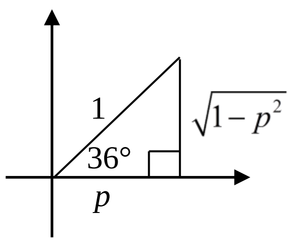


4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\ ;\ 7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\ ;\ 7)$	✓x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...°$ $B\hat{C}D = 180° - \alpha$ $B\hat{C}D = 180° - 59{,}036...°$ $B\hat{C}D = 120{,}96°$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96°$ $= 15$ units^2	✓ $B\hat{C}D = 120{,}96°$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$; $y - 3 = -x - 1$ $\therefore y = -x + 2$; $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint $(1 ; 1)$ **OR/*OF***  N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2} ; \frac{-1+3}{2}\right) = (1 ; 1)$	✓ equation of MN ✓ equating AF & MN ✓ x value ✓ y value (4) ✓✓ coordinates of F ✓ x value ✓ y value (4)

	OR/*OF*   NAMF is a square (NA=NF=AM=MF and NA $\perp$ AM) Midpoint NM = (1 ; 1) = Midpoint of AF	✓ NAMF = square ✓x ✓ y of midpt NM ✓ midpt AF (4)
		[15]

QUESTION/*VRAAG* 5

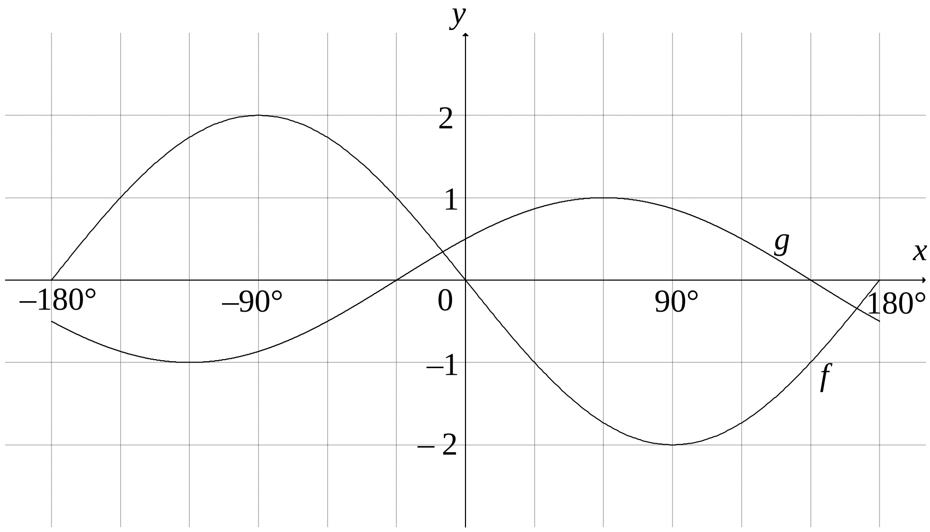
5.1	$\dfrac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$ $= \dfrac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $= \dfrac{-\sin x}{-\dfrac{\sin x}{\cos x}}$ $= \cos x$	✓ sin40° ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS} = \dfrac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)}$ $\qquad \text{RHS} = 2\cos x - 1$ $\text{LHS} = \dfrac{-2(1-\cos^2 x) + \cos x + 1}{1-(-\cos x)}$ $\text{LHS} = \dfrac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \dfrac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \dfrac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ $\therefore \text{LHS} = \text{RHS}$	✓ identity i. t. o. $\cos x$ ✓ $\cos(540° - x) = -\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36° = \sqrt{1 - p^2}$ $\tan 36° = \dfrac{\sqrt{1-p^2}}{p}$  **OR/*OF*** $\cos^2 36° = 1 - \sin^2 36°$ $\cos 36° = \sqrt{1-(1-p^2)}$ $= p$ $\tan 36° = \dfrac{\sin 36°}{\cos 36°}$ $= \dfrac{\sqrt{1-p^2}}{p}$	✓ method ✓ value of p ✓ answer (3) ✓ method ✓ cos 36° = p ✓ answer (3)

5.3.2	$\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(2\cos^2 36° - 1)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(1 - 2\sin^2 36°)$ $= -1 + 2\left(\sqrt{1-p^2}\right)^2$ $= -1 + 2(1-p^2)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(\cos^2 36° - \sin^2 36°)$ $= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$ $= -\left(p^2 - (1-p^2)\right)$ $= -2p^2 + 1$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108°$ $= \cos(2\times 54°)$ $= 2\cos^2 54° - 1$ $= 2(1-p^2) - 1$ $= 1 - 2p^2$	✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4)
	OR/*OF* $\cos 108° = \cos(72° + 36°)$ $= \cos 72°\cos 36° - \sin 72°\sin 36°$ $= (2\cos^2 36° - 1)\cos 36° - (2\sin 36°\cos 36°)\sin 36°$ $= 2\cos^3 36° - \cos 36° - 2\cos 36°\sin^2 36°$ $= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	✓ expansion ✓ both double angle identities ✓ value of sin 36° ✓ answer i. t. o. p (4)
		[17]

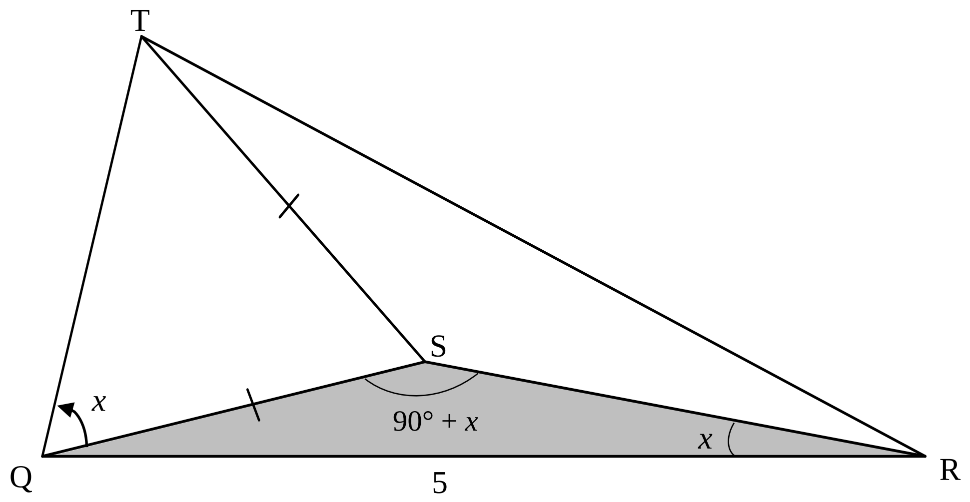
QUESTION/*VRAAG* 6

6.1.1	$\cos(\alpha+\beta)$ $=\cos(\alpha-(-\beta))$ $=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$ $=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	✓ $\cos(\alpha-(-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$ $=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$ $=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$ $=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$ $=\cos 2x+2\sin^2 x$ $=1-2\sin^2 x+2\sin^2 x$ $=1$	✓ $\cos 10x = \cos(6x + 4x)$ ✓ expansion of $\cos(6x + 4x)$ ✓ $\cos 2x$ ✓ $1-2\sin^2 x$ ✓ answer (5)
6.2	$\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=2(2\sin x\cos x)$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $\sin x(4\cos^2 x-1)=0$ $\sin x=0$ or $\cos^2 x=\frac{1}{4}$ $\cos x=-\frac{1}{2}$ $x = 180° + k.360°;\ k\in Z$ or $x = 120° + k.360°;\ k\in Z$ $x = 240° + k.360°;\ k\in Z$ **OR/*OF*** $\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=4\sin x\cos x$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $4\sin x(1-\sin^2 x)-\sin x=0$ $3\sin x-4\sin^3 x=0$ $\sin x(3-4\sin^2 x)=0$ $\sin x=0$ or $\sin^2 x=\frac{3}{4}$ $\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$ $x=180^\circ+k.360^\circ, k\in Z$ or $x=120^\circ+k.360^\circ, k\in Z$ or $x=240^\circ+k.360^\circ, k\in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x = 180°$ ✓ $x = 120°$& $240°$ **OR/OF** $x = \pm120°$ ✓ $k.360°;\ k\in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x = 180°$ ✓ $x = 120°$& $240°$ **OR/OF** $x = \pm120°$ ✓ $k.360°;\ k\in Z$ (7)
		[15]

QUESTION/*VRAAG* 7

7.1		✓ both turning points ✓ both x intercepts (–30° & 150°) ✓ shape (3)
7.2	Period = 120°	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van g*: $y \in [-1;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2};\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2};\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $\frac{1}{2} \leq y \leq \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

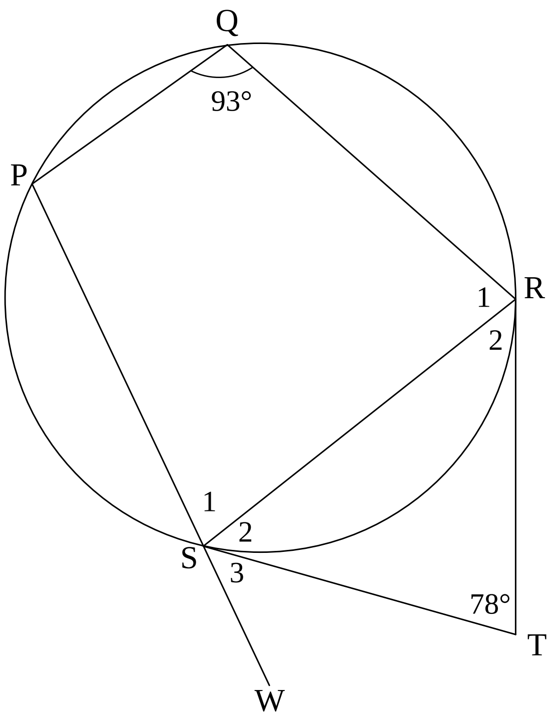
QUESTION/*VRAAG* 8



8.1	In ΔSQR: $\frac{QS}{\sin x}=\frac{QR}{\sin(90°+x)}$ $\frac{QS}{\sin x}=\frac{5}{\cos x}$ $QS=\frac{5\sin x}{\cos x}$ $QS=5\tan x$	✓ correct use of sine rule ✓ $\sin(90°+x)=\cos x$ ✓ $QS=\frac{5\sin x}{\cos x}$ (3)
8.2	$\frac{QT}{\sin(180°-2x)}=\frac{TS}{\sin x}$ $\frac{QT}{\sin 2x}=\frac{5\tan x}{\sin x}$ $QT=\frac{5\tan x\sin 2x}{\sin x}$ $QT=\frac{5\left(\frac{\sin x}{\cos x}\right)(2\sin x\cos x)}{\sin x}$ $QT=\frac{5\sin x(2\sin x)}{\sin x}$ $QT=10\sin x$	✓ correct use of sine rule ✓ TS = QS = 5tan$x$ ✓ $QT=\frac{5\tan x\sin 2x}{\sin x}$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\sin 2x=2\sin x\cos x$ (5)

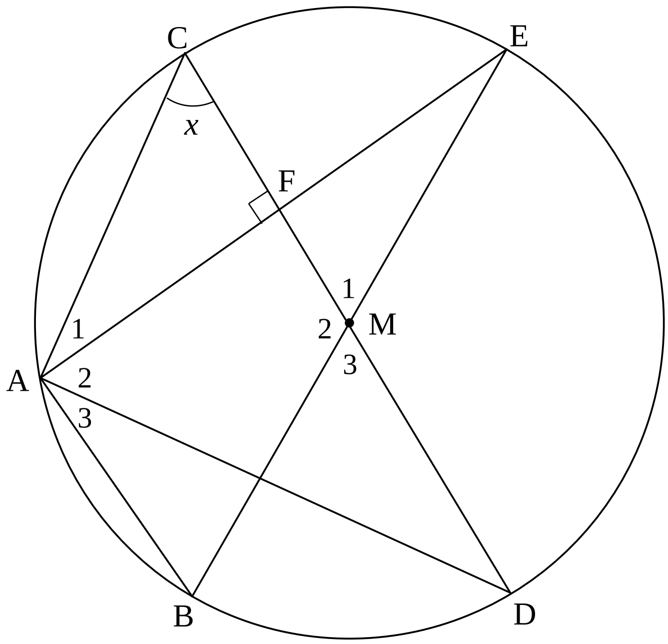
	OR/*OF* $QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$ $QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$ $QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$ $QT^2 = 50\tan^2 x(1+\cos 2x)$ $QT^2 = 50\tan^2 x(1+2\cos^2 x - 1)$ $QT^2 = 50\tan^2 x(2\cos^2 x)$ $QT^2 = 100\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)$ $QT^2 = 100\sin^2 x$ $QT = 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ $\cos 2x = 2\cos^2 x - 1$ & reduction ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $QT^2 = 100\sin^2 x$ (5)
	OR/*OF* $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ[TQ - 10\tan x.\cos x]$ $TQ = 10\tan x.\cos x \quad (TQ \neq 0)$ $= 10\dfrac{\sin x}{\cos x}.\cos x$ $= 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ quadratic equation ito TQ ✓ TQ = 10tan x . cos x ✓ $\tan x = \dfrac{\sin x}{\cos x}$ (5)
8.3	$\text{Area of } \Delta TQR = \dfrac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \dfrac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93 \text{ unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
		[10]

QUESTION/VRAAG 9



9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of $\angle$s in Δ/*som van* $\angle$*e in* Δ]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp $\angle$s of cyclic quad/*teenoorst* $\angle$*e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [$\angle$s on a str line/$\angle$*e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

QUESTION/*VRAAG* 10

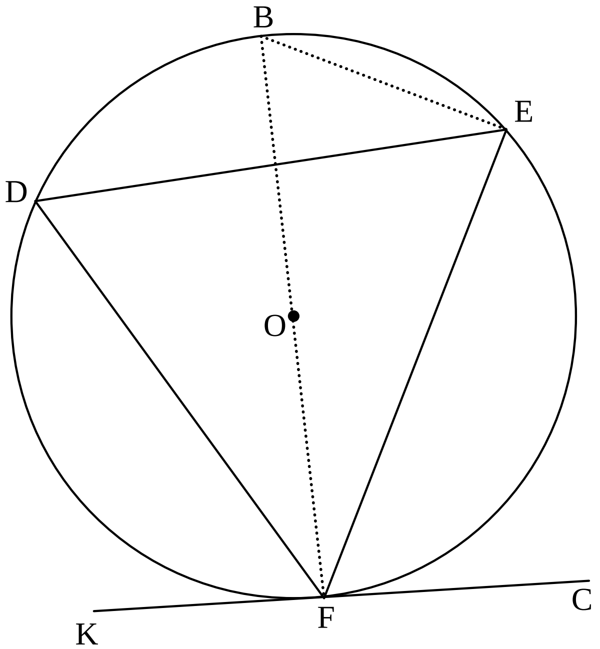


10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt* ⊥ *op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts*∠= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90° - x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ S ✓ R ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x = \hat{C}$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90^\circ$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] ∴ $AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] ∴ $FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

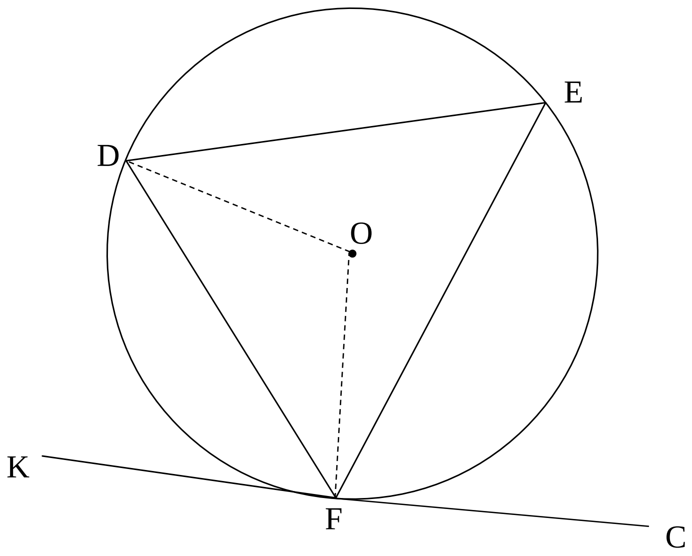
QUESTION/*VRAAG* 11

11.1



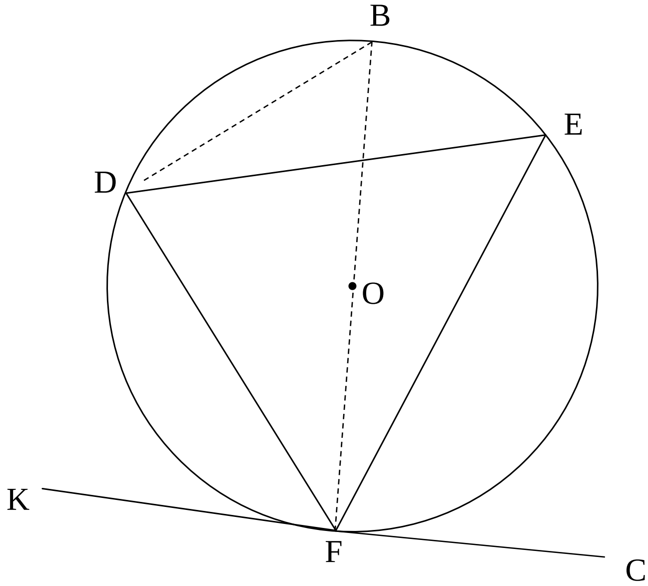
	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/*OF*



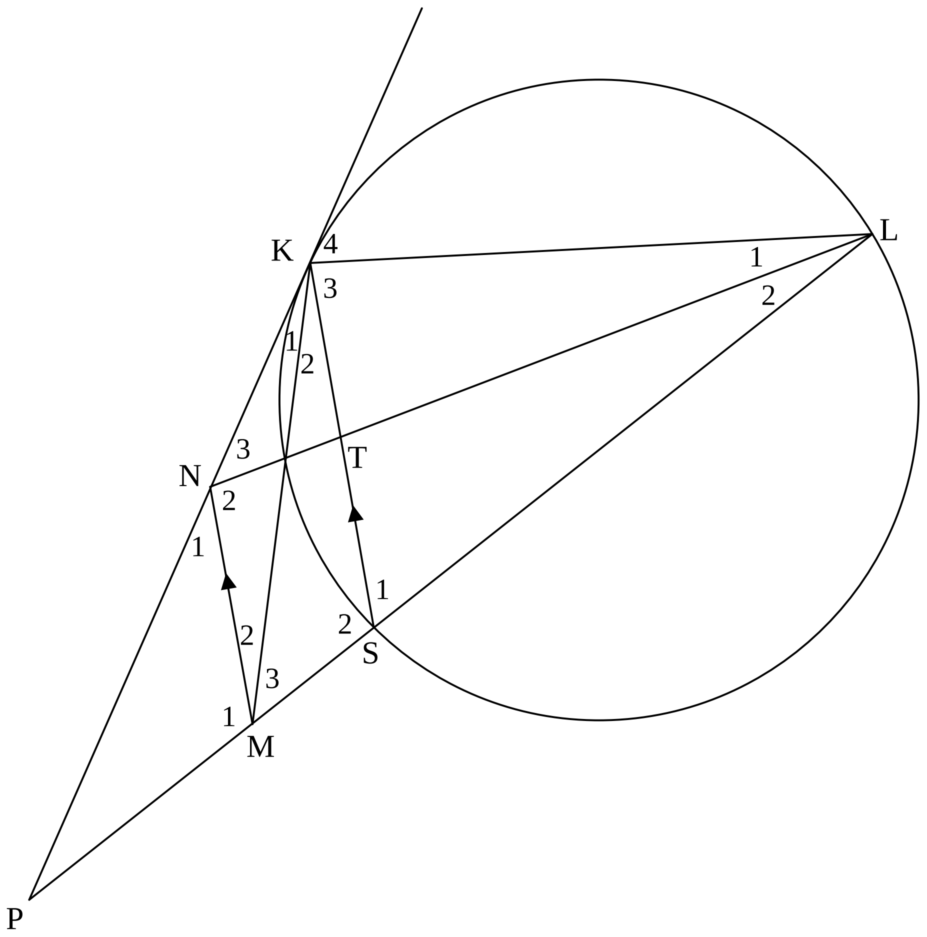
	Construction: Draw radii DO and OF *Konstruksie: Trek radii DO en OF* $\hat{OFK} = 90°$ or $\hat{DFK} = 90° - \hat{OFD}$ radius⊥ tangent/*raaklyn*] $\hat{ODF} = \hat{OFD}$ [∠s opp = sides/∠*e teenoor = sye*] $\therefore \hat{DOF} = 180° - 2\hat{OFD}$ [∠s of Δ/∠*e van Δ*] $\hat{DEF} = 90° - \hat{OFD}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore \hat{DFK} = \hat{DEF}$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

OR/OF



	Construction: Draw diameter BF and join BD. *Konstruksie: Trek middellyn BF en verbind BD.* $B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*] $F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*] $\hat{B} = 90° - B\hat{F}D$ $\therefore D\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [∠s same segment/∠*e dieselfde segment*] $\therefore D\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)

11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp $\angle$s; / *ooreenk $\angle$s;* MN || KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ $\therefore$ KLMN is a cyclic quad [ext $\angle$ of quad = opp int $\angle$ / *buite $\angle$ van vh = teenoorst binne $\angle$*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp $\angle$s; / *ooreenk $\angle$s;* MN || KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ $\therefore$ KLMN is a cyclic quad [ext $\angle$ of quad = opp int $\angle$ / *buite $\angle$ van vh = teenoorst binne $\angle$*] **OR/OF**	✓ R (1) ✓ R (1)

	$N\hat{K}L = 180^\circ - \hat{K}_4$ [adj. suppl.] $\therefore N\hat{K}L = 180^\circ - N\hat{M}L$ [proved] $\therefore$ KLMN is a cyclic quad [opp.∠s supplementary]	✓ R (1)
11.2.2	In $\Delta LKN \parallel\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{L}_1 = \hat{M}_2$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $= \hat{K}_2$ [alt ∠s; / *verw∠e;* MN‖KS] $N\hat{K}L = M\hat{S}K$ [∠s of Δ / *∠e van Δ*] $\Delta LKN \parallel\mid \Delta KSM$	✓ S ✓ R ✓ S ✓ S/R ✓ S (5)
	OR/OF In $\Delta LKN \parallel\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $N\hat{K}L = \hat{M}_1$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= \hat{S}_2$ [corresp ∠s/*ooreenk ∠e*; KS ‖ NM] $\Delta LKN \parallel\mid \Delta KSM$ [∠,∠,∠]	✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
	OR/OF In $\Delta LKN \parallel\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [∠s on straight line/*∠e op reguitlyn*] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta LKN \parallel\mid \Delta KSM$ [∠,∠,∠]	✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\Delta LKN \parallel\mid \Delta KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	4SM = 3KN $SM = \frac{3(8)}{4}$ SM = 6 $\frac{LT}{NL} = \frac{LS}{ML}$ [line ‖ one side of Δ / *lyn ‖ een sy v* Δ] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2022

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 21 pages.
Hierdie nasienriglyne bestaan uit 21 bladsye.

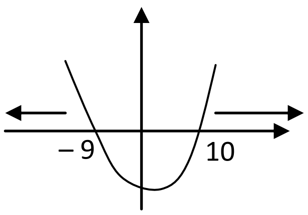
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking guidelines.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.*
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION1/*VRAAG* 1

1.1.1	$(3x-6)(x+2)=0$ $x=2$ or $x=-2$	✓ $x=2$ ✓ $x=-2$ (2)
1.1.2	$2x^2-6x+1=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\dfrac{6\pm\sqrt{(-6)^2-4(2)(1)}}{2(2)}$ $x=2{,}82$ or $x=0{,}18$	✓ correct substitution into correct formula ✓ 2,82 ✓ 0,18 (3)
1.1.3	$x^2-90>x$ $x^2-x-90>0$ $(x+9)(x-10)>0$ CV: $x=-9$ or $x=10$  $x<-9$ or $x>10$ **OR/OF** $(-\infty\,;-9)$ or $(10\,;\infty)$	✓ standard form ✓ critical values ✓✓ $x<-9$ or $x>10$ (4)

1.1.4	$x-7\sqrt{x}=-12$ $x+12=7\sqrt{x}$ $(x+12)^2=(7\sqrt{x})^2$ $x^2+24x+144=49x$ $x^2-25x+144=0$ $(x-16)(x-9)=0$ $x=16$ or $x=9$ **OR/*OF*** $x-7\sqrt{x}+12=0$ $(\sqrt{x}-3)(\sqrt{x}-4)=0$ or let $\sqrt{x}=k$ $\sqrt{x}=3$ or $\sqrt{x}=4$ $x=9$ or $x=16$	✓ isolating the root ✓ squaring both sides ✓ standard form ✓ both answers (4) **OR/*OF*** ✓ standard form ✓ factors ✓ answers ✓ both answers for *x* (4)
1.2	$2x-y=2$ $y=2x-2$ ………(1) $xy=4$ ………(2) (1) in (2): $x(2x-2)=4$ $2x^2-2x-4=0$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ $y=2$ $y=-4$	✓ eq 1 ✓ substitution ✓ standard form ✓ *x*-values ✓ *y*-values (5)

	OR/*OF* $2x - y = 2$ $x = \frac{1}{2}y + 1$ ………(1) $xy = 4$ ………(2) (1) in (2): $y\left(\frac{1}{2}y + 1\right) = 4$ $\frac{1}{2}y^2 + y - 4 = 0$ $y^2 + 2y - 8 = 0$ $(y+4)(y-2) = 0$ $y = -4$ or $y = 2$ $x = -1$ $x = 2$	**OR/*OF*** ✓ eq 1 ✓ substitution ✓ standard form ✓ y-values ✓ x-values (5)
	OR/OF $2x - y = 2$ ………(1) $y = \frac{4}{x}$ ………(2) (2) in (1): $2x - \frac{4}{x} = 2$ $2x^2 - 2x - 4 = 0$ $x^2 - x - 2 = 0$ $(x-2)(x+1) = 0$ $x = 2$ or $x = -1$ $y = 2$ $y = -4$	**OR/OF** ✓ eq 2 ✓ substitution ✓ standard form ✓ x-values ✓ y-values (5)

	OR/OF $2x - y = 2$ ………(1) $x = \frac{4}{y}$ ………(2) (2)in (1): $2\left(\frac{4}{y}\right) - y = 2$ $8 - y^2 - 2y = 0$ $y^2 + 2y - 8 = 0$ $(y+4)(y-2) = 0$ $y = -4$ or $y = 2$ $x = -1$ $x = 2$	**OR/OF** ✓ eq 2 ✓ substitution ✓ standard form ✓ y-values ✓ x-values (5)
1.3	$2.5^n - 5^{n+1} + 5^{n+2} = 2.5^n - 5^n.5^1 + 5^n.5^2$ $= 5^n(2 - 5 + 25)$ $= 5^n(22)$ $2\left(5^n(11)\right)$ **OR/OF** Any integer multiplied by an even number will be even	✓ exp law ✓common factor ✓answer/explanation (3)
1.4	$\frac{3^{y+1}}{32} = \sqrt{96^x}$ $\frac{3^{y+1}}{2^5} = (96)^{\frac{x}{2}}$ $3^{y+1}.2^{-5} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$ $-5 = \frac{5x}{2}$ $\therefore x = -2$ $y + 1 = \frac{x}{2}$ $y + 1 = \frac{-2}{2}$ $\therefore y = -2$	✓ $\frac{3^{y+1}}{2^5} = (96)^{\frac{x}{2}}$ ✓ $3^{y+1}.2^{-5} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$ ✓ $x = -2$ ✓ $y = -2$ (4)

	OR/OF $\frac{3^{y+1}}{32}=\sqrt{96^x}$ $\left(\frac{3^{y+1}}{2^5}\right)^2=\left(\sqrt{(96)^x}\right)^2$ $\frac{3^{2y+2}}{2^{10}}=2^{5x}.3^x$ $3^{2y+2}.2^{-10}=2^{5x}.3^x$ $-10=5x$ $\therefore x=-2$ $2y+2=-2$ $\therefore y=-2$	**OR/OF** ✓ $\left(\frac{3^{y+1}}{2^5}\right)^2=\left(\sqrt{(96)^x}\right)^2$ ✓ $3^{2y+2}.2^{-10}=2^{5x}.3^x$ ✓ $x=-2$ ✓ $y=-2$ (4)
		[25]

QUESTION 2/*VRAAG* 2

2.1.1	$a = 14$ $T_6 = 14r^5 = 448$ $r^5 = 32$ $\therefore r = 2$ Answer only: full marks	✓ $T_6 = 14r^5 = 448$ ✓ $r = 2$ (2)
2.1.2	$T_n = 14(2)^{n-1}$ $S_n = \frac{14(2^6 - 1)}{2-1}$ $S_6 = 882$ 114 674 – 882 = 113 792 $113\,792 = 896\left(2^n - 1\right)$ $128 = 2^n$ $n = 7$ **OR/OF** $S_n = \frac{a(r^n - 1)}{r-1}$ $114\,674 = \frac{14(2^n - 1)}{2-1}$ $8\,191 = 2^n - 1$ $2^n = 8\,192$ $n = \log_2 8\,192$ $n = 13$ $\therefore$ 7 more terms must be added to the first 6 terms.	✓ substitution into correct formula ✓ $S_6 = 882$ ✓ $128 = 2^n$ ✓ 7 (4) **OR/OF** ✓ substitution into correct formula ✓ $2^n = 8\,192$ ✓ $n = 13$ ✓ 7 (4)
2.1.3	$r = \frac{1}{2}$ OR $448r^5 = 14$ $\therefore r = \frac{1}{2}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{448}{1-\frac{1}{2}}$ $S_\infty = 896$	✓ $r = \frac{1}{2}$ ✓ substitution ✓ answer (3)

2.2	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$ $T_1=\frac{1}{6}$ $\quad$ $T_2=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}$ $d=\frac{3}{6}-\frac{1}{6}=\frac{1}{3}$ $\frac{121}{6}=\frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$ $\frac{121}{3}=n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$ $\frac{121}{3}=\frac{1}{3}n^2$ $121=n^2$ $n=11$ $\therefore k=10$	✓ $T_1=\frac{1}{6}$ ✓ d ✓ substitution ✓ value of n ✓ value of k (5)
	OR/*OF*	**OR/*OF***
	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$ $a=\frac{1}{6}$ $l=\frac{1}{3}k+\frac{1}{6}$ $n=k+1$ $S_n=\frac{n}{2}[a+l]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$ $\frac{121}{6}=\frac{(k+1)^2}{6}$ $k+1=\pm\sqrt{121}$ $k+1=11$ $k=10$	✓ $a=\frac{1}{6}$ ✓ l ✓ $n=k+1$ ✓ $\frac{121}{6}=\frac{(k+1)^2}{6}$ ✓ value of k (5)
		[14]

QUESTION 3/*VRAAG* 3

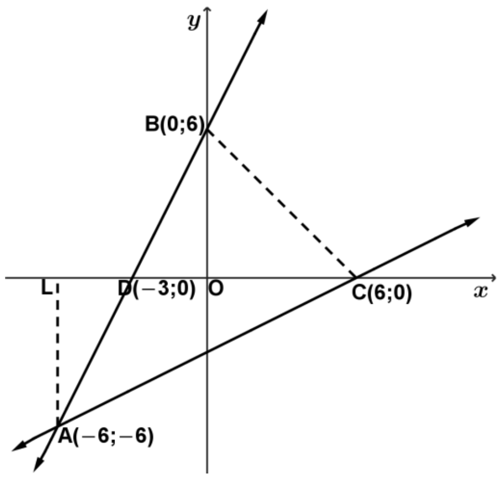
3.1	$3a+b=7$ $3+b=7$ $b=4$ **OR/OF** $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$	✓ $3a+b=7$ ✓ $3+b=7$ (2) **OR/OF** ✓ $T_2-T_1=7$ ✓ substitution (2)
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$	✓first difference ✓ 2 ✓ $2p+5$ (3) **OR/*OF*** ✓first difference ✓ 2 ✓ $2p+5$ (3)
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77} **OR/OF** $T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ $\therefore$ Between T_{76} and T_{77}	✓$157=2p+5$ ✓ $p=76$ ✓ T_{76} and T_{77} (3) **OR/OF** ✓$T_{n+1}-T_n=157$ ✓ $n=76$ ✓ T_{76} and T_{77} (3)
		[10]

QUESTION 4/*VRAAG* 4

4.1.1	$p = -1$ and $q = 2$	✓ $p = -1$ ✓ $q = 2$ (2)
4.1.2	$\frac{1}{x-1} + 2 = 0$ $-2x + 2 = 1$ $x = \frac{1}{2}$ $\left(\frac{1}{2}\ ;\ 0\right)$	✓ $= 0$ ✓ answer (2)
4.1.3	$x = \frac{1}{2} - 3$ $= \frac{-5}{2}$ Answer only: full marks	✓ -3 ✓ $x = \frac{-5}{2}$ (2)
4.1.4	$y = x + t$ $2 = 1 + t$ $t = 1$	✓subst (1 ; 2) ✓ $t = 1$ (2)
4.1.5	$-2 \le \frac{1}{x-1}$ Answer only: full marks $\frac{1}{x-1} + 2 \ge 0$ $\therefore x \le \frac{1}{2}$ or $x > 1$ **OR/*OF*** $x \in \left(-\infty\ ;\ \frac{1}{2}\right]$ or $(1\ ;\ \infty)$	✓ $\frac{1}{x-1} + 2 \ge 0$ ✓ $x \le \frac{1}{2}$ ✓ $x > 1$ (3)
4.2.1	$y = -5$	✓ answer (1)
4.2.2	$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2\ ;-9\ )$ **OR/*OF*** $f'(x) = 2x - 4$ $2x - 4 = 0$ $x = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2\ ;-9\ )$	✓ $x = 2$ ✓ $y = -9$ (2) **OR/*OF*** ✓ $x = 2$ ✓ $y = -9$ (2)

4.2.3	$q=-5$ $-9=a(2)^2-5$ $-4=4a$ $a=-1$ $\therefore g(x)=-2^x-5$	✓ $q=-5$ ✓ substitution of (2 ; – 9) ✓ $a=-1$ (3)
4.2.4	$y\in(-\infty;-5)$ OR $y<-5; y\in R$	✓answer (1)
4.2.5	$k<-9$	✓– 9 ✓ $k<-9$ (2)
		[20]

QUESTION 5/*VRAAG* 5

5.1	$g(x) = 2x + 6$ $y = 6$	✓ $y = 6$ (1)
5.2	$y = 2x + 6$ $x = 2y + 6$ $y = \frac{1}{2}x - 3$ Answer only: Full marks	✓ swop x and y ✓ equation (2)
5.3	$\frac{1}{2}x - 3 = 2x + 6$ $x - 6 = 4x + 12$ $3x = -18$ $x = -6$ $A(-6\,;-6)$ **OR/*OF*** $2x + 6 = x$ $x = -6$ $y = -6$	✓equating ✓$x = -6$ ✓ $y = -6$ (3) **OR/*OF*** ✓equating ✓$x = -6$ ✓$y = -6$ (3)
5.4	$AB = \sqrt{(6)^2 + (12)^2}$ $= \sqrt{180} = 6\sqrt{5} = 13{,}42$	✓substitution ✓answer (2)
		

5.5	$BC=\sqrt{6^2+6^2}=\sqrt{72}=6\sqrt{2}$ AB = AC = $\sqrt{180}$ symmetry of g and g^{-1} $\perp h=(\sqrt{180})^2-\left(\frac{\sqrt{72}}{2}\right)^2$ $=\sqrt{162}=9\sqrt{2}$ area of $\Delta ABC=\frac{1}{2}BC\times h$ $=\frac{1}{2}\times\sqrt{72}\times\sqrt{162}=54$ units^2	✓ BC ✓ AB = AC /midpoint (3 ; 3) ✓ $\perp h$ (A) ✓ substitution ✓ answer (A) (5)
	OR/*OF* $\tan B\hat{D}C=2$ $\therefore B\hat{D}C=63{,}43°$ $\tan D\hat{C}A=\frac{1}{2}$ $\therefore D\hat{C}A=26{,}57°$ $\therefore D\hat{A}C=36{,}86°$ (ext angle triangle) Area of $\Delta ABC=\frac{1}{2}\left(\sqrt{180}\right)\left(\sqrt{180}\right)\sin 36{,}86°$ $=53{,}99$ units^2	**OR/*OF*** ✓ $B\hat{D}C=63{,}43°$ ✓ $D\hat{A}C=36{,}86°$ ✓AC = $\sqrt{180}$ ✓ substitution into the correct formula ✓ answer (A) (5)
	OR/OF Area of ΔABC = Area of ΔBDC + Area of ΔADC $=\frac{1}{2}DC.BO+\frac{1}{2}DC.height$ $=\frac{1}{2}(9)(6)+\frac{1}{2}(9)(6)$ $=54$ units^2	**OR/OF** ✓ Areas (ΔBDC+ΔADC) ✓ $\frac{1}{2}DC.BO$ ✓ $\frac{1}{2}DC.height$ ✓ substitution ✓ answer (A) (5)
		[13]

QUESTION 6/*VRAAG* 6

6.1	$A = P(1+i)^n$ $13\,459 = 12\,000\left(1+\frac{m}{400}\right)^8$ $\left(1+\frac{m}{400}\right)^8 = 1{,}121...$ $1+\frac{m}{400} = \sqrt[8]{1{,}121..}$ $\frac{m}{400} = 0{,}0144...$ $\therefore m = 5{,}78\%$	✓ 8 ✓ subst into correct formula ✓ $1+\frac{m}{400} = \sqrt[8]{1{,}121..}$ ✓ 5,78 % (4)
6.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $F = \frac{1\,000\left[\left(1+\frac{0{,}075}{12}\right)^{12} - 1\right]}{\frac{0{,}075}{12}}$ $= R12\,421{,}22$ He won't be able to buy the computer because R13 000 – R12 421,22 = R578,78 **OR/OF** He won't be able to buy the computer because R12 421,22 < R13 000	✓ $\frac{0{,}075}{12}$ ✓ 12 ✓ answer ✓ conclusion (4)
6.3.1	Loan amount = 85% × R250 000 = R212 500 **OR/OF** Loan amount = R250 000 – (15% × R250 000) = R212 500	✓ answer (1) **OR/OF** ✓ answer (1)
6.3.2	$A = 212\,500\left(1+\frac{0{,}13}{12}\right)^5$ $A = 224\,262{,}53$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $224\,262{,}53 = \frac{x\left[1-\left(1+\frac{0{,}13}{12}\right)^{-67}\right]}{\frac{0{,}13}{12}}$ $\therefore x = R4\,724{,}96$	✓ $A = 212\,500\left(1+\frac{0{,}13}{12}\right)^5$ ✓answer ✓ substitution into correct formula ✓ – 67 ✓ answer (5)
		[14]

QUESTION 7/*VRAAG 7*

7.1	$f(x)=x^2+x$ $f^{/}(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f^{/}(x)=\lim\limits_{h\to 0}\dfrac{(x+h)^2+(x+h)-(x^2+x)}{h}$ $f^{/}(x)=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2+x+h-x^2-x}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2+h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h+1)}{h}$ $\therefore f^{/}(x)=2x+1$	✓substitution into the formula ✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓common factor ✓answer (5)
	OR/*OF*	**OR/*OF***
	$f(x)=x^2+x$ $f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h$ $f(x+h)-f(x)=x^2+2xh+h^2+x+h-x^2-x$ $=2xh+h^2+h$ $f^{/}(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2+h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h+1)}{h}$ $\therefore f^{/}(x)=2x+1$	✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓substitution into the formula ✓common factor ✓answer (5)
7.2	$f(x)=2x^5-3x^4+8x$ $f^{/}(x)=10x^4-12x^3+8$	✓$10x^4$ ✓$-12x^3$ ✓8 (3)

7.3	$g(x)=ax^3+3x^2+bx+c$
	$g^{/}(x)=3ax^2+6x+b$
	$g^{//}(x)=6ax+6$
	$g^{//}(-1)=6a(-1)+6=0$
	$\therefore a=1$
	For concave up $g^{//}(x)>0$
	$6x+6>0$
	$x>-1$
		✓ $g^{/}(x)=3ax^2+6x+b$
		✓ $g^{//}(-1)=6a(-1)+6=0$
		✓ $a=1$
		✓ $x>-1$ (4)

OR/OF

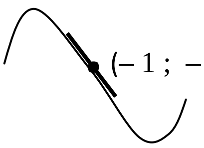
Min gradient at (– 1 ; – 7) implies:

at $x = -1$ - point of inflection
and g will be positive cubic
hence $a > 0$

Since g is concave up

$x > -1$

OR/OF



(– 1 ; – 7)

Since g is concave up
$x > -1$

(– 1 ; y)

Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

OR/OF

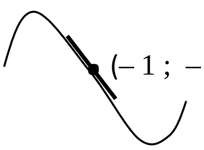
Min gradient at (– 1 ; – 7) implies:

at $x = -1$ - point of inflection
and g will be positive cubic
hence $a > 0$

Since g is concave up

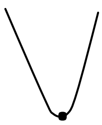
$x > -1$

OR/OF



(– 1 ; – 7)

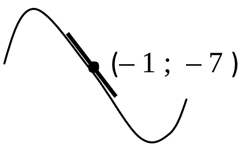
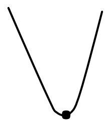
Since g is concave up
$x > -1$



(– 1 ; y)

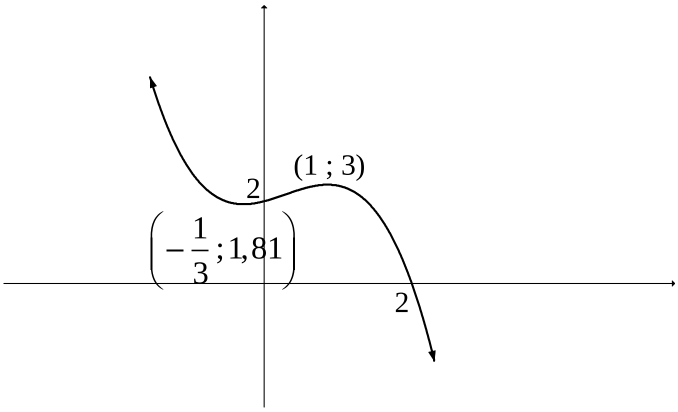
Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

	OR/OF Min gradient at (– 1 ; – 7) implies: at $x = -1$ - point of inflection and g will be positive cubic hence $a > 0$ Since g is concave up $x > -1$ **OR/OF**  Since g is concave up $x > -1$  (– 1 ; y) Since g is concave up $x > -1$ Answer only: $\frac{1}{4}$	**OR/OF** ✓ point of inflection ✓✓ $a > 0$ ✓ $x > -1$ (4) **OR/OF** ✓✓pos graph ✓point of inflection ✓ $x > -1$ (4)
		[12]

QUESTION 8/*VRAAG* 8

8.1	$f'(x)=mx^2+nx+k$ $f'(x)=m\left(x+\frac{1}{3}\right)(x-1)$ $1=m\left(0+\frac{1}{3}\right)(0-1)$ $1=-\frac{1}{3}m$ $\therefore m=-3$ $f'(x)=-3\left(x+\frac{1}{3}\right)(x-1)$ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ $f'(x)=-3x^2+2x+1$ $\therefore n=2$ $\therefore k=1$ **OR/OF** $k=1$ $0=m+n+1$ and $\frac{1}{9}m-\frac{1}{3}n+1=0$ $m+n=-1$ (1) $m-3n=-9$ (2) (1) – (2) $4n=8$ $\therefore n=2$ $m+2=-1$ $\therefore m=-3$	✓ substitution of $\left(-\frac{1}{3}\,;0\right)$ and (1 ; 0) ✓ substitution of (0 ; 1) ✓ $m=-3$ ✓ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ ✓ $n=2$ ✓ $k=1$ (6) **OR/OF** ✓ $k=1$ ✓ $m+n=-1$ ✓ $m-3n=-9$ ✓ $4n=8$ ✓ $n=2$ ✓ $m=-3$ (6)
8.2.1	$f(x)=-x^3+x^2+x+2$ $f\left(-\frac{1}{3}\right)=\frac{49}{27}=1{,}81$ T.P$\left(-\frac{1}{3}\,;\frac{49}{27}\right)$ $f(1)=3$ T.P$(1\,;3)$	✓ *x*-coordinates of the TP ✓ T.P$\left(-\frac{1}{3}\,;\frac{49}{27}\right)$ ✓ T.P$(1\,;3)$ (3)

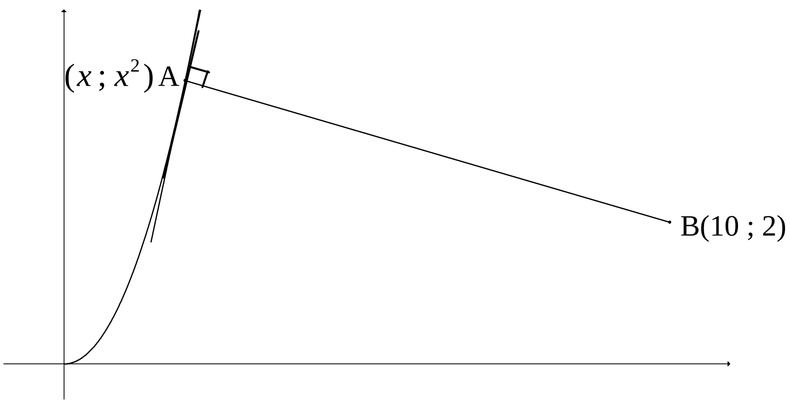
<table>
<tr><td>8.2.2</td><td>$f(x)=-x^3+x^2+x+2$
$-x^3+x^2+x+2=0$
$(x-2)(-x^2-x-1)=0$
$x=2$ or no solution
</td><td>✓ $x=2$

✓one x-intercept
✓ two turning points
✓y-intercept
✓ shape: neg cubic
(5)</td></tr>
<tr><td>8.3.1</td><td>$a=\frac{-\frac{1}{3}+1}{2}$
$=\frac{1}{3}$
OR/OF
$f^{/}(x)=-3x^2+2x+1$
$f^{//}(x)=-6x+2$
$f^{//}(a)=-6a+2=0$
$-6a=-2$
$a=\frac{1}{3}$</td><td>✓ answer (1)
OR/OF

✓ answer (1)</td></tr>
<tr><td>8.3.2</td><td>$b<\frac{4}{3}$ units</td><td>✓ $\frac{4}{3}$
✓ $b<\frac{4}{3}$ (2)</td></tr>
<tr><td></td><td></td><td>[17]</td></tr>
</table>

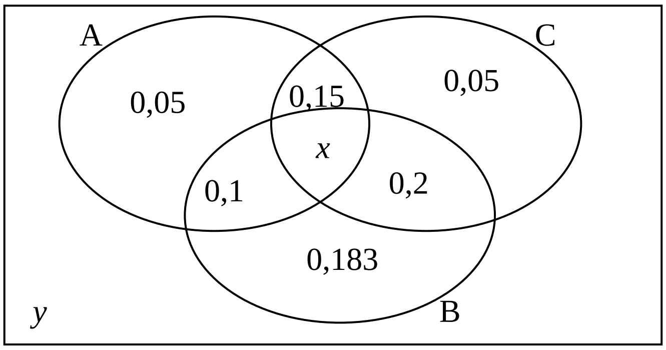
QUESTION9/*VRAAG* 9

9.1	Any point on f : $(x\,;\,x^2)$ $\text{distance} = \sqrt{(x-10)^2 + (x^2-2)^2}$ $= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$ $= \sqrt{x^4 - 3x^2 - 20x + 104}$ For min distance $\frac{d}{dx}\left(x^4 - 3x^2 - 20x + 104\right) = 0$ $4x^3 - 6x - 20 = 0$ $(x-2)(4x^2 + 8x + 10) = 0$ $\Delta = 8^2 - 4(4)(10) = -96 \quad \therefore$ no roots $\therefore x = 2$ $d = \sqrt{2^4 - 3(2)^2 - 20(2) + 104} = 2\sqrt{17} = 8{,}25$	✓ $(x\,;\,x^2)$ ✓ substitution ✓ simplification ✓ answer ✓ $4x^3 - 6x - 20$ ✓ derivative = 0 ✓ $x = 2$ ✓ answer (A) (8)



9.2	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2 - 2}{x - 10}$ $\therefore m_{\tan g} = -\frac{x-10}{x^2-2}$ $f'(x) = 2x$ $\therefore 2x = -\frac{x-10}{x^2-2}$ $-2x^3 + 4x = x - 10$ $2x^3 - 3x - 10 = 0$ $x = 2$ $y = (2)^2 = 4$ $\therefore AB = \sqrt{(2-10)^2 + (4-2)^2}$ $= 2\sqrt{17} = 8{,}25$	✓ m_{AB} ✓ $m_{\tan g} = -\frac{x-10}{x^2-2}$ ✓ $f'(x) = 2x$ ✓ equating ✓ standard form ✓ $x = 2$ ✓ substitute into distance ✓ answer (A) (8)
		[8]

QUESTION 10/*VRAAG* 10



10.1.1(a)	$y = 1 - 0{,}893 = 0{,}107$ (0,11)	✓ $y = 1 - 0{,}893$ (1)
10.1.1(b)	$x = 0{,}893 - 0{,}733$ $= 0{,}16$	✓ $x = 0{,}893 - 0{,}733$ (1)
10.1.2	P(at least 2 events) = 0,1 + 0,15 + 0,16 + 0,2 = 0,61 Answer only: Full Marks	✓ values ✓ answer (2)
10.1.3	P(B) = 0,643 P(C) = 0,56 P(B and C) = 0,36 P(B) × P(C) = 0,643 × 0,56 = 0,36 $\therefore$ P(B and C) = P(B) × P(C) $\therefore$ B and C are independent	✓ P(B) = 0,643 ✓ P(C) = 0,56 ✓ P(B and C) = 0,36 ✓ P(B) × P(C) = 0,36 ✓ independent because P(B and C) = P(B) × P(C) (5)
10.2.1	$7 \times 6 \times 5 \times 4 = 840$	✓ 4 ✓ 7 ✓ $7 \times 6 \times 5 \times 4 = 840$ (3)
10.2.2	start with 5, 7, 9 or start with 6 or start with 8 $(3\times5\times1\times3)+(1\times5\times1\times2)+(1\times5\times1\times2)$ $= 45+10+10$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0{,}08$ **OR/*OF*** ends in 4 or ends in 6 or ends in 8 $(5\times5\times1\times1)+(4\times5\times1\times1)+(4\times5\times1\times1)$ $= 25+20+20$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0{,}08$	✓ $(3\times5\times1\times3) = 45$ ✓ $(1\times5\times1\times2) = 10$ ✓ $(1\times5\times1\times2) = 10$ ✓ 65 ✓ answer (5) **OR/*OF*** ✓ $(5\times5\times1\times1) = 25$ ✓ $(4\times5\times1\times1) = 20$ ✓ $(4\times5\times1\times1) = 20$ ✓ 65 ✓ answer (5)
		[17]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2022

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

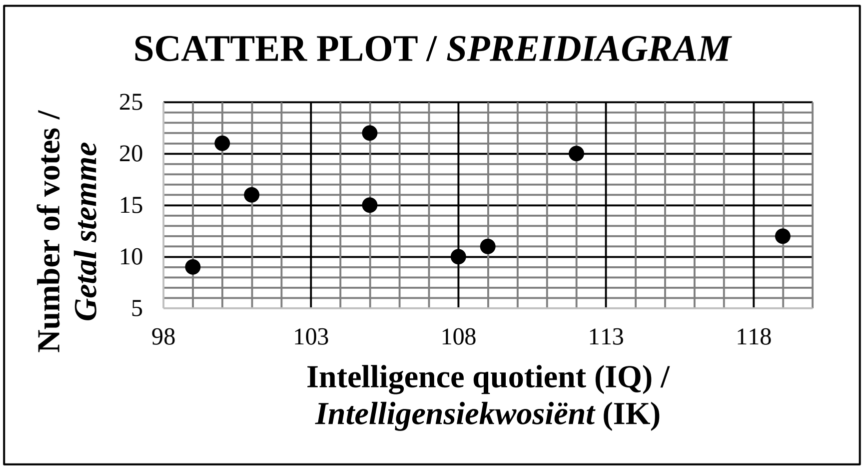
- **If a candidate answers a question TWICE, only mark the FIRST attempt.**
- **If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.**
- **Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.**
- **Assuming answers/values in order to solve a problem is NOT acceptable.**

NOTA:

- ***As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.***
- ***As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.***
- ***Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.***
- ***Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.***

GEOMETRY/*MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

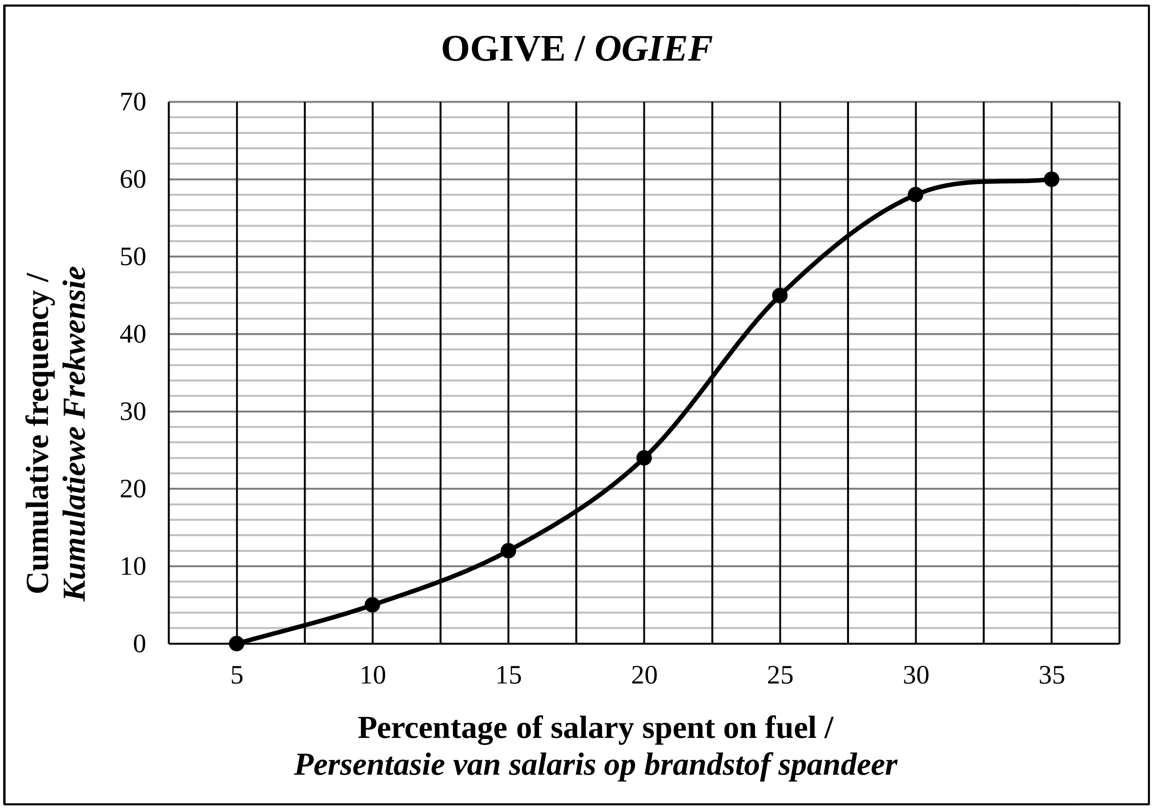
QUESTION/*VRAAG* 1



Popularity score (x) ***Gewildheidspunt* (x)**	32	89	35	82	50	59	81	40	79	65
Number of votes (y) ***Getal stemme* (y)**	9	22	10	21	11	15	20	12	19	16

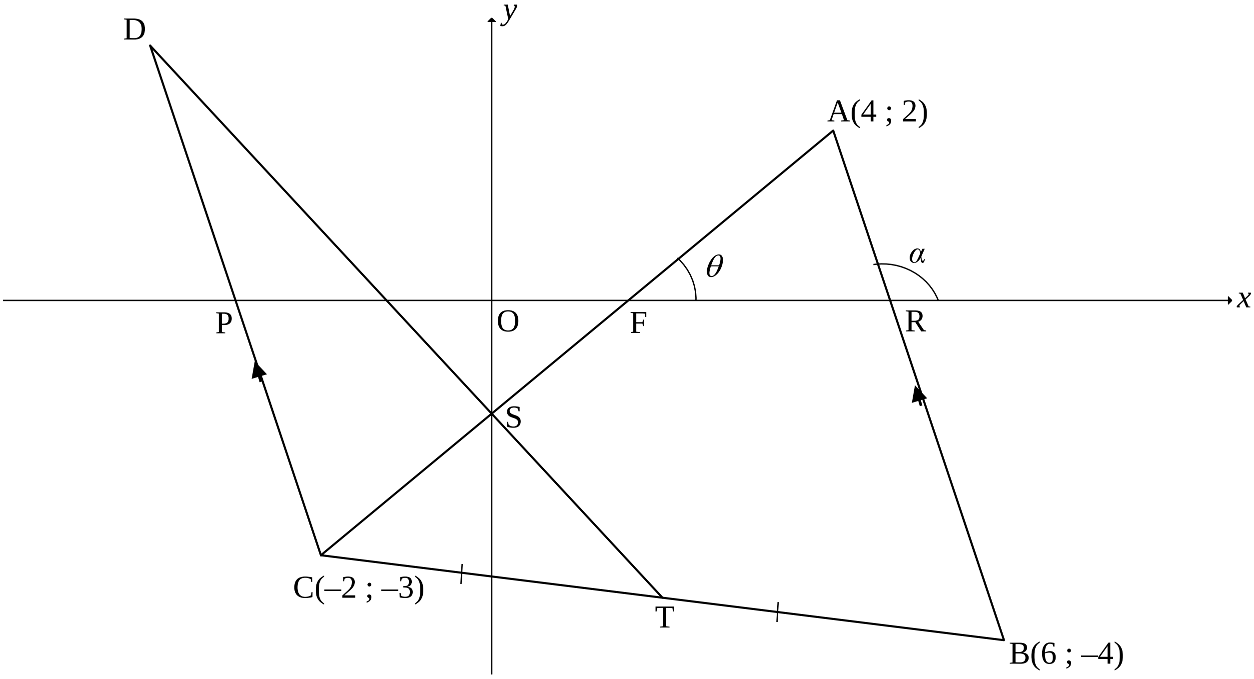
1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$ ANSWER ONLY: Full marks	✓155 ✓answer (2)
1.1.2	SD $= 4,59$	✓ answer (1)
1.2	$\bar{y} - \text{SD}$ $= 15,5 - 4,59$ $= 10,91$ $\therefore$ 10 – 2 = 8 learners	✓ value of $\bar{y} - \text{SD}$ ✓ answer (2)
1.3	$a = 1,7709\ldots$ $b = 0,2243\ldots$ $\hat{y} = 1,77 + 0,22x$	✓ a ✓ b ✓ equation (3)
1.4	$\hat{y} = 1,77 + 0,22(72)$ $= 17,61$ ≈ 18 votes **OR/*OF*** $\hat{y} = 17,92 \approx 18$ votes	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*	✓ R (1)
1.5.2	$r = 0,98$/correlation very strong/*korrelasie baie sterk* $\therefore$ a reliable prediction/*'n betroubare voorspelling*	✓ S (1)
		[12]

QUESTION/*VRAAG* 2



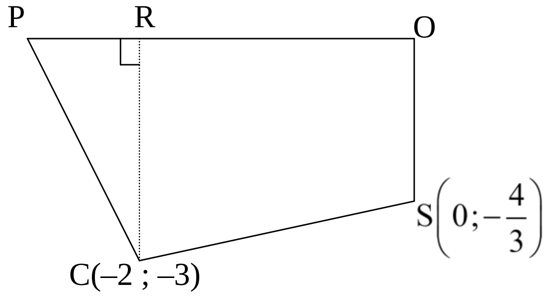
2.1	60 employees	✓ answer (A) (1)
2.2	$20 < x \leq 25$	✓ answer (1)
2.3	60 – 34 = 26 employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
2.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
2.5	∴ Ogive/Cumulative frequency graph will shift to the right/will become steeper. ∴ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 3



3.1.1	$m_{AB}=\frac{2-(-4)}{4-6}$ **OR** $m_{AB}=\frac{-4-2}{6-4}$ $m_{AB}=-3$ ANSWER ONLY: Full marks	✓ substitution ✓ answer (2)
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108,43°$ ANSWER ONLY: Full marks	✓ $\tan\alpha=m_{AB}=-3$ ✓ answer (2)
3.1.3	$T\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $T\left(\frac{-2+6}{2};\frac{-3-4}{2}\right)$ $T\left(2;\frac{-7}{2}\right)$	✓ $x_T=2$ ✓ $y_T=\frac{-7}{2}$ (2)
3.1.4	$5(0)-6y=8$ $y=-\frac{4}{3}$ $S\left(0;\frac{-4}{3}\right)$	✓ $x_S=0$ ✓ $y_S=\frac{-4}{3}$ (2)
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $y=-3x-9$ $y=-3x-9$	✓ gradient ✓ substitution of C(–2 ; –3) ✓ equation (3)

3.3.1	$5x-6y=8$ $y=\frac{5}{6}x-\frac{8}{6}$ $\tan\theta=m_{\text{AC}}=\frac{5}{6}$ $\theta=39{,}81°$ $\hat{\text{A}}=108{,}43°-39{,}81°$ $=68{,}62°$ $\text{D}\hat{\text{C}}\text{A}=68{,}62°$ [alt ∠s ; DC‖AB]	✓ $\tan\theta=m_{\text{AC}}=\frac{5}{6}$ ✓ $\theta=39{,}81°$ ✓ $\hat{\text{A}}=68{,}62°$ ✓ answer (4)
3.3.2	$\text{P}(-3;0)$ and F(1,6 ; 0) Area POSC = Area ΔFPC – Area ΔOFS $=\frac{1}{2}(4{,}6)(3)-\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ = 6,9 – 1,07 = 5,83 units^2 **OR/*OF*** $\text{P}(-3;0)$ $\text{FC}=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$ $\text{Area }\Delta\text{PFC}=\frac{1}{2}(\text{PF})(\text{FC})\sin\text{O}\hat{\text{F}}\text{S}$ $=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ $=6{,}90$ $\text{Area }\Delta\text{OFS}=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ = 1,07 Area POSC = 6,90 – 1,07 = 5,83 units^2 **OR/*OF***	✓ $\text{P}(-3;0)$ ✓ method ✓ $\frac{1}{2}(4{,}6)(3)$ ✓ $\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ ✓ answer (5) ✓ $\text{P}(-3;0)$ ✓ $\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ ✓ $\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)



$P(-3;0)$

Area of POSC = Area of OSCR + Area of ΔPRC

$$=\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$$
$$=\frac{35}{6}$$
$$=5,83\,\text{units}^2$$

✓ $P(-3;0)$

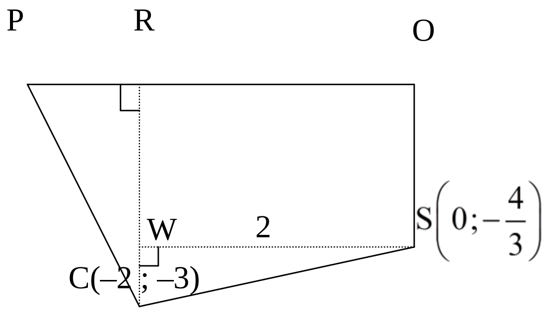
✓ method

✓ $\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2$ ✓ $\frac{1}{2}(1\times 3)$

✓ answer

(5)

OR/
OF



$P(-3;0)$

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC

$$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$
$$=\frac{35}{6}$$
$$=5,83\text{units}^2$$

✓ $P(-3;0)$

✓ method

✓ $\frac{1}{2}(1)(3)$

✓ $\left(\frac{4}{3}\right)(2)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$

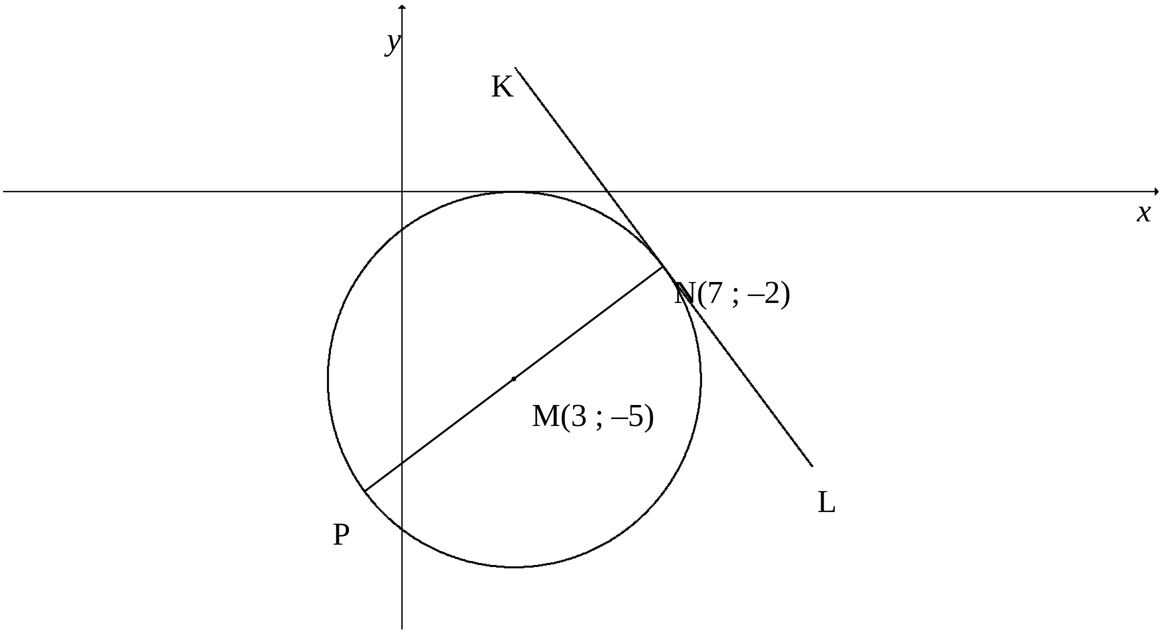
✓ answer

(5)

OR/*OF*

	$P(-3;0)$ Area of $\Delta PSC = \frac{1}{2}(PC)(CS)\sin D\hat{C}A$ $= \frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ $= 3,833..$ Area of $\Delta POS = \frac{1}{2}(PO)(OS)$ $= \frac{1}{2}(3)\left(\frac{4}{3}\right)$ $= 2$ Area POSC = 3,833… + 2 $= 5,83 \text{units}^2$	✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ ✓ $\frac{1}{2}(3)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)
		[20]

QUESTION/*VRAAG* 4



4.1	$P(x;y);\ N(7;-2)\ ;\ M(3;-5)$ $\frac{x+7}{2}=3 \qquad \frac{y-2}{2}=-5$ $x=-1 \qquad y=-8$ $P(-1;-8)$	✓ $x_P=-1$ ✓ $y_P=-8$ (2)
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$	✓ substitution into distance formula ✓ $(x-3)^2+(y+5)^2$ ✓ $r^2=25$ (3)
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius ⊥ tangent/*raaklyn* ⊥ *radius*] $-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\qquad y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$	✓ substitution ✓ $m_{\text{radius}}=\frac{-3}{-4}=\frac{3}{4}$ ✓ $m_{\text{tangent}}=-\frac{4}{3}$ ✓ substitution of *m* and N(7 ; –2) ✓ equation (5)
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$	✓ subst *m* and P ✓ value of *c* ✓✓ answer (4)

4.4.1	$AB^2 = AM^2 - MB^2$ $AB^2 = \left[(t-3)^2 + (t+5)^2\right] - 5^2$ $= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$ $AB = \sqrt{2t^2 + 4t + 9}$	✓ substitution into Pythagoras ✓ simplification (A) (2)
4.4.2	$t = \dfrac{-4}{2(2)}$ $= -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** $4t + 4 = 0$ $t = -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** Length of $AB = \sqrt{2t^2 + 4t + 9}$ $= \sqrt{2\left(t^2 + 2t + \frac{9}{2}\right)}$ $= \sqrt{2\left[(t+1)^2 + \frac{7}{2}\right]}$ $= \sqrt{2(t+1)^2 + 7}$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ substitution into correct formula ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ derivative = 0 ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ completing of the square ✓ $t = -1$ ✓substitution ✓answer (4)
		[20]

QUESTION/*VRAAG* 5

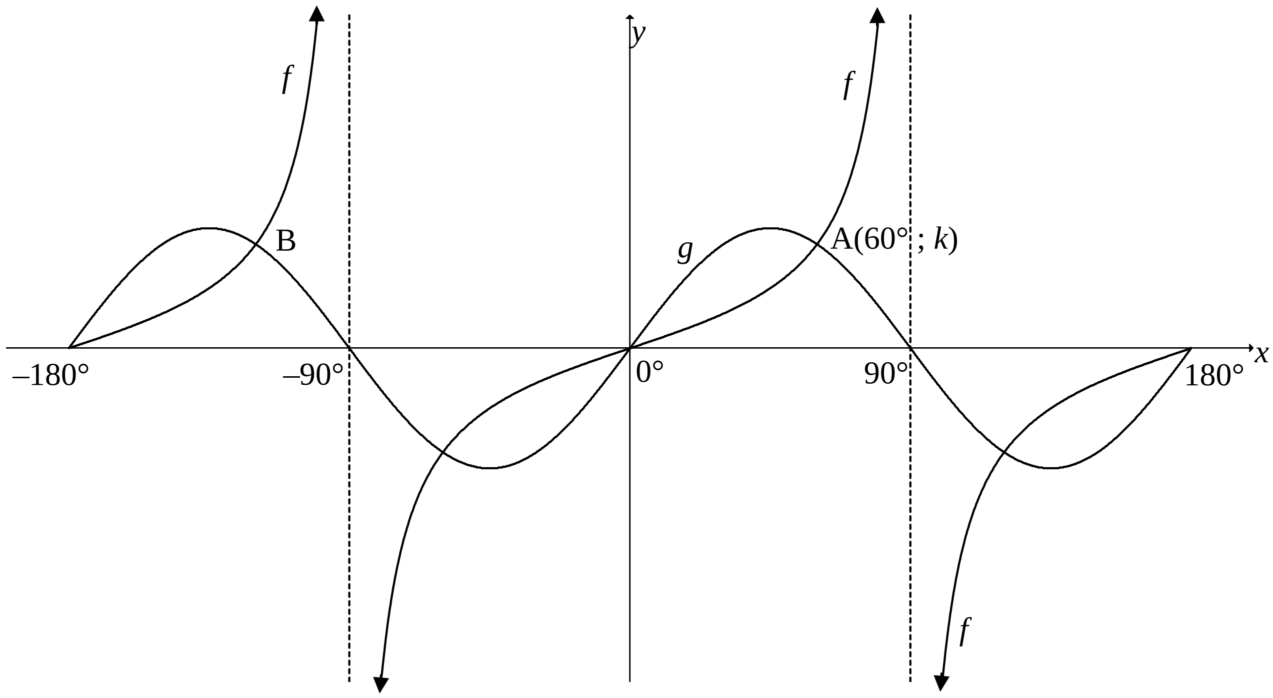
5.1.1	$\sin(360°+x)$ $=\sin x$	✓ + ✓ $\sin x$ (2)
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\frac{-3}{-2}$ $=\frac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\frac{3}{2}$	✓✓ substitution ✓ method (3) ✓✓ substitution ✓ method (3)
5.1.3	$\cos(180°+x)$ $=-\cos x$	✓ – ✓ $\cos x$ (2)
5.2	$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\frac{-\sin\theta}{\sin\left(-\left(180°-\theta\right)\right)-3\sin\theta}$ $=\frac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\frac{-\sin\theta}{-4\sin\theta}$ $=\frac{1}{4}$	✓ $-\sin\theta$ ✓ $-3\sin\theta$ ✓ $-\sin\theta$ ✓ simplification ✓ answer (5)

5.3	$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ $\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$ $\tan x = -\frac{1}{2}$ $\sin 2x = \frac{1}{3}$ ref $\angle = 26,565...°$ ref $\angle = 19,471...°$ $x = 153,43° + k.180°$; $k \in Z$ $x = 9,74° + k.180°$; $k \in Z$ **OR/*OF*** or $x = 153,43° + k.360°$; $k \in Z$ $x = 80,26° + k.180°$; $k \in Z$ or $x = 333,43° + k.360°$; $k \in Z$	✓ both equations ✓ $\tan x = -\frac{1}{2}$ ✓ $\sin 2x = \frac{1}{3}$ ✓ $x = 153,43°$ **OR** $x = 153,43°$ &333,43° ✓ $x = 9,74°$ & $80,26°$ ✓ $+ k.180°$; $k \in$ Z (6)
5.4.1	LHS $= \cos(x+y).\cos(x-y)$ $= [\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$ $= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$ $= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$ $= 1 + \sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$ $= 1 - \sin^2 x - \sin^2 y$ = RHS	✓ expansion ✓ simplification ✓ square identity ✓ product (4)
5.4.2	$1 - \sin^2 45° - \sin^2 15°$ $= \cos(45° + 15°).\cos(45° - 15°)$ $= \cos 60°.\cos 30°$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}$ **OR/*OF***	✓ identifying x and y ✓ substitution ✓ answer (3)

	$1-\sin^2 45°-\sin^2 15°$ $=\sin^2 15°+\cos^2 15°-\sin^2 45°-\sin^2 15°$ $=\cos^2 15°-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\cos^2 15°-\frac{1}{2}$ $=\frac{2\cos^2 15°-1}{2}$ $=\frac{\cos 30°}{2}$ $=\frac{\sqrt{3}}{2}\times\frac{1}{2}$ $=\frac{\sqrt{3}}{4}$	✓ identity ✓ substitution ✓ answer (3)
	OR $1-\sin^2 45°-\sin^2 15°$ $=\cos^2 45°-\sin^2(45°-30°)$ $=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45°\cos 30°-\cos 45°\sin 30°)^2$ $=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right)$ $=\frac{\sqrt{3}}{4}$	✓ expansion ✓ substitution ✓ answer (3)

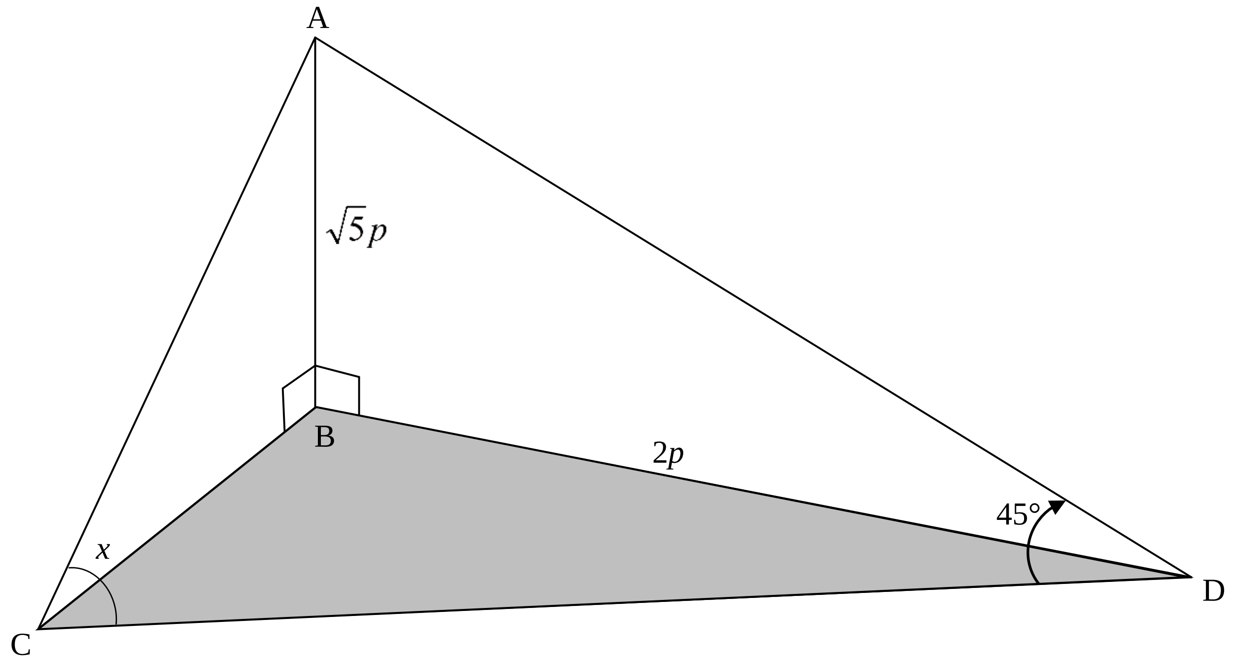
5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8(2\cos^2 x - 1)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$	✓ factorisation ✓ 4sin2x ✓ cos2x ✓ double angle (4) ✓ factorisation ✓ $4\sin 2x$ ✓ cos2x ✓ double angle (4)
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67{,}5°$	✓ answer (1)
		[30]

QUESTION/*VRAAG* 6



6.1	180°	✓ answer (1)
6.2.1	$k=\sqrt{3}=1{,}73$	✓ answer (1)
6.2.2	$B\left(-120°;\sqrt{3}\right)$	✓ $x=-120°$ (1)
6.3	Range of g: $y\in[-2;\ 2]$ Range of $2g(x)$: $y\in[-4;\ 4]$ **OR/*OF*** ANSWER ONLY: Full marks Range of g: $-2\leq y\leq 2$ Range of $2g(x)$: $-4\leq y\leq 4$	✓ $y\in[-2;\ 2]$ ✓ answer (2) ✓ $-2\leq y\leq 2$ ✓ answer (2)
6.4	$x\in[-65°;\ -5°]$ **OR/*OF*** $-65°\leq x\leq -5°$	✓✓ $x\in[-65°;\ -5°]$ (2) ✓✓ $-65°\leq x\leq -5°$ (2)
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p=\pm 2$ $\therefore\ p=-\frac{1}{2}$ or $\frac{1}{2}$ ANSWER ONLY: Full marks	✓ $2\sin 2x=4p$ ✓ $4p=\pm 2$ ✓ answers (3)
		[10]

QUESTION/*VRAAG* 7

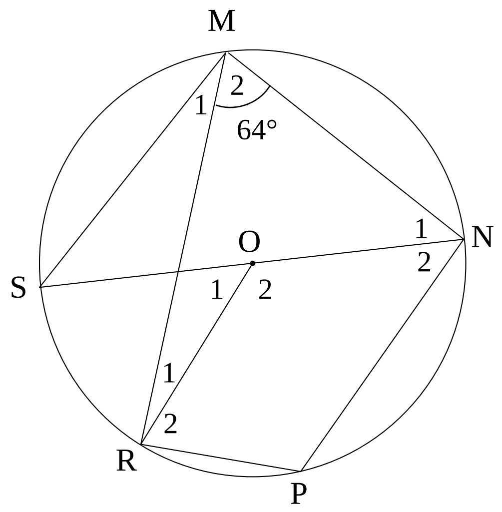


7.1	$AD^2 = AB^2 + BD^2$ $AD^2 = (\sqrt{5}p)^2 + (2p)^2$ $AD^2 = 9p^2$ $AD = 3p$	✓ substitution in Pythagoras ✓ answer (2)
7.2	$\dfrac{CD}{\sin(135° - x)} = \dfrac{3p}{\sin x}$ $CD = \dfrac{3p\sin(135° - x)}{\sin x}$ $CD = \dfrac{3p(\sin 135° \cos x - \cos 135° \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin 45° \cos x + \cos 45° \sin x)}{\sin x}$ $CD = \dfrac{3p\left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)}{\sin x}$ $CD = \dfrac{3p\left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$	✓ correct use of sine rule ✓ $135° - x$ ✓ compound angle ✓ special values ✓ factorisation (5)

7.3	Area $\Delta ADC = \frac{1}{2}(AD)(CD)\sin A\hat{D}C$ $= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$ $= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$ $= 143{,}11 m^2$	✓ correct use of area rule ✓ substitution in area rule ✓ answer (3)
		[10]

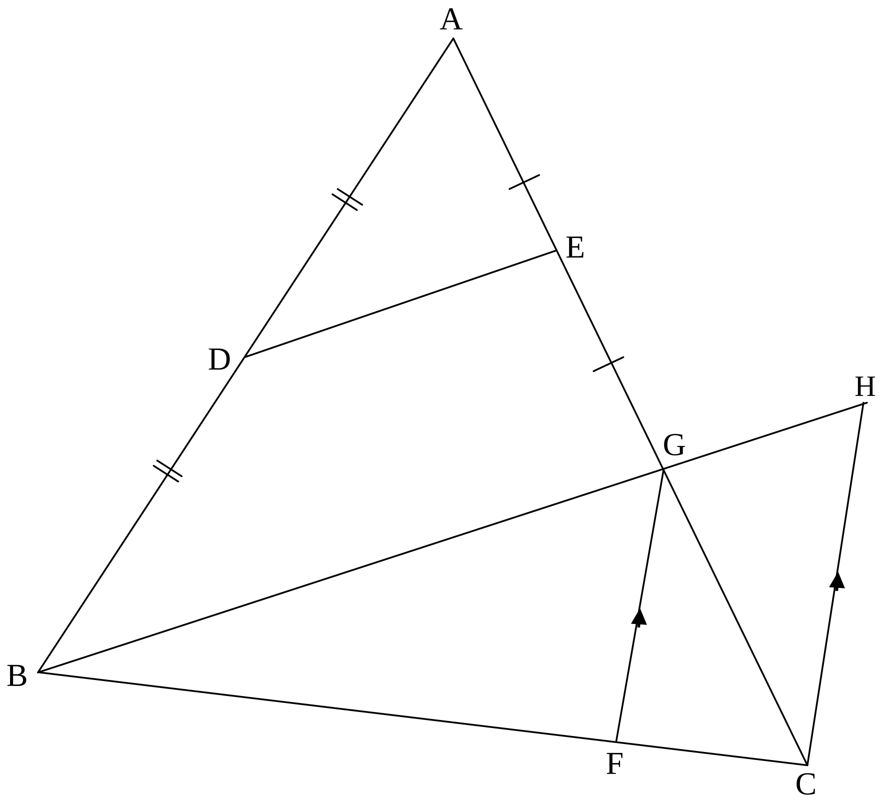
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 116°$	[opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]	✓ S ✓ R (2)
8.1.2	$\hat{M}_1 + 64° = 90°$ $\hat{M}_1 = 26°$	[∠ in semi-circle/∠ *in halwe sirkel*]	✓ R ✓ S (2)
8.1.3	$\hat{O}_1 = 52°$	[∠ at centre = 2 x ∠ at circumference/*midpts. ∠ = 2 x omtreks. ∠*]	✓ S ✓R (2)

8.2

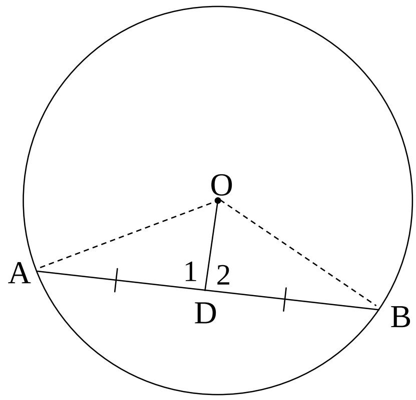


8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	✓ R (1) ✓ R (1)
8.2.2	BG = 2DE or $6x - 2$ [Midpt theorem/*Midpt. stelling*] BG = $6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ [line || one side of Δ **OR** prop theorem; FG || CH / *lyn* || *een sy v.* Δ] $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$ **OR/*OF***	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)

	$\frac{BF}{FC} = \frac{BG}{GH}$ [line ∥ one side of Δ **OR** prop theorem; FG ∥ CH / *lyn* ∥ *een sy v.* Δ] $\frac{AE}{AG} = \frac{DE}{BG}$ [ΔADE ∣∣∣ ΔABG] BG = 4x + 4 $\frac{1}{2} = \frac{3x-1}{4x+4}$ $\therefore 4x + 4 = 6x - 2$ $\therefore x = 3$	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)
		[13]

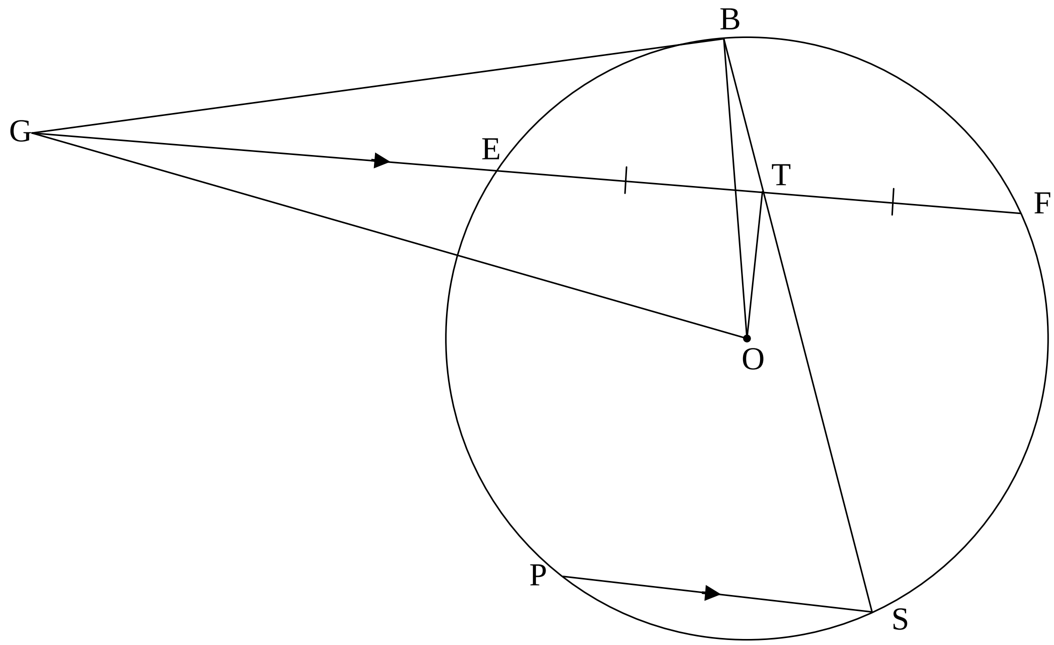
QUESTION/*VRAAG* 9

9.1



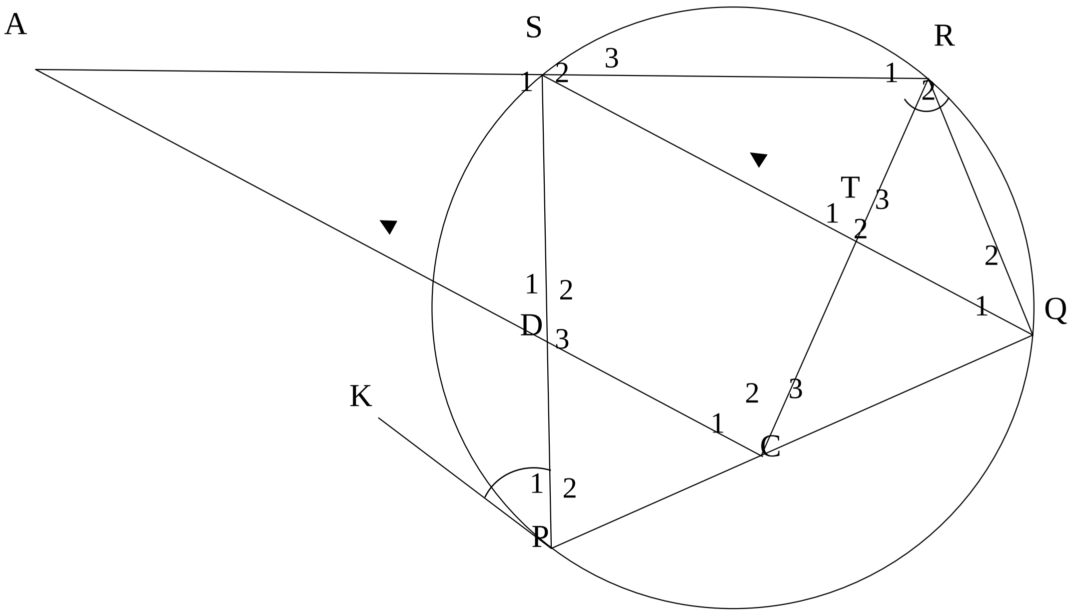
9.1.1	Construction: Draw OA and OB In ΔADO and ΔBDO OA = OB [radii/*radiusse*] OD = OD [common side/*gemeenskaplike sy*] AD = DB [given/*gegee*] $\therefore \Delta ADO \equiv \Delta BDO$ [S;S;S] ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\Delta ADO \equiv \Delta BDO$ $\therefore OD \perp AB$ [∠s on a str line/∠*e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)
	OR/*OF* Construction: Draw OA and OB In ΔADO and ΔBDO AD = DB [given/*gegee*] $\hat{A} = \hat{B}$ [∠s opp; ∠s sides /∠*e teenoor gelyke sye*] OA = OB [radii/*radiusse*] $\therefore \Delta ADO \equiv \Delta BDO$ [S;∠;S] ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\Delta ADO \equiv \Delta BDO$ $\therefore OD \perp AB$ [∠s on a str line/∠*e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)

9.2



9.2.1	$O\hat{T}G = 90°$ [line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*] $O\hat{B}G = 90°$ [tan ⊥ radius/*raaklyn* ⊥ *radius*] $\therefore O\hat{T}G = O\hat{B}G = 90°$ $\therefore$ OTBG is a cyclic quadrilateral [line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]	✓S ✓R ✓S ✓R ✓R (5)
9.2.2	$\hat{S} = B\hat{T}G$ [corresp ∠s; GF || PS / *ooreenk.* ∠s; GF || PS] But $B\hat{T}G = G\hat{O}B$ [∠s in the same segment/∠*e in dies. sirkelsegment*] $G\hat{O}B = \hat{S}$	✓S ✓R ✓S ✓R (4)
		[14]

QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = \hat{Q}_1$ [tan-chord theorem/∠ *tussen raaklyn en koord*] $\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ [ext ∠ of cyclic quad/*buite* ∠ *v. kvh*] $\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$ $\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$ [ext ∠ of Δ/*buite* ∠ *v.* Δ] but $\hat{P}_1 = \hat{R}_2$ [given/*gegee*] $\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	✓ S ✓ S / R ✓ S ✓ S (4)
10.2	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common ∠/*gemeenskaplike* ∠] $\hat{S}_1 = \hat{T}_2$ [proven/*reeds bewys*] $\hat{T}_2 = \hat{C}_2$ [alt ∠s; QS ‖ CA/verw. ∠*e*; QS ‖ CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\hat{D}_1 = \hat{R}_1$ [sum of ∠s in Δ/∠*e v.* Δ] ΔASD ||| ΔACR $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*] **OR/*OF***	✓ identifying Δ's ✓ S ✓ S / R ✓ S ✓ S (5)

	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*gegee*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\Delta ASD \parallel\!\mid \Delta ACR$ [$\angle;\angle;\angle$] $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	✓ identifying Δ's ✓ S ✓ S/R ✓ S ✓ R (5)
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ [$\Delta ASD \parallel\!\mid \Delta ACR$] $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ [line || one side of Δ OR prop theorem; TS || CA/*lyn* || *een sy v.* Δ] $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	✓ S ✓ S ✓ R ✓ equating (4)
		[13]

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P1/*WISKUNDE V1*

MARKING GUIDELINES/*NASIENRIGLYNE*

2022

MARKS: 150
PUNTE: 150

These marking guidelines consist of 16 pages.
Hierdie nasienriglyne bestaan uit 16 bladsye.

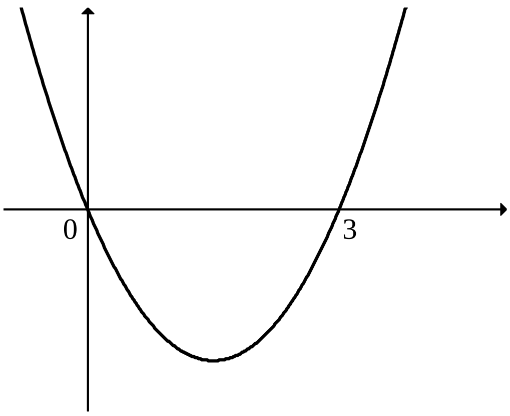
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking guidelines.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging*.
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2+2x-15=0$ $(x+5)(x-3)=0$ $x=-5$ or $x=3$	✓ factors ✓ $x=-5$ ✓ $x=3$ (3)
1.1.2	$5x^2-x-9=0$ $x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(5)(-9)}}{2(5)}$ $x=\dfrac{1\pm\sqrt{181}}{10}$ $x=1{,}45$ or $x=-1{,}25$	✓substitution into the correct formula ✓ $x=1{,}45$ ✓ $x=-1{,}25$ (3)
1.1.3	$x^2\le 3x$ $x^2-3x\le 0$ $x(x-3)\le 0$  $0\le x\le 3$ OR $x\in[0;3]$	✓ standard form ✓ factors ✓✓ answer (4)
1.2.1	$a+\dfrac{64}{a}=16$ $a^2-16a+64=0$ $(a-8)^2=0$ $a=8$	✓ standard form ✓ factors ✓ answer (3)

1.2.2	$2^x + 2^{6-x} = 16$ $2^x + \frac{64}{2^x} = 16$ $2^x = 8$ (from 1.2.1) $2^x = 2^3$ $x = 3$	✓ exp law ✓ $2^x = 8$ ✓ answer (3)
1.3	$\sqrt{\frac{2^{1002}(1+2^4)}{17(2)^{998}}}$ $= \sqrt{\frac{2^4(17)}{17}}$ $= \sqrt{2^4}$ $= 2^2$ $= 4$	✓common factor ✓second factor ✓simplification ✓answer (4)
1.4	$2x - y = 2 \quad \ldots(1)$ $\frac{1}{x} - 3y = 1 \quad \ldots(2)$ $y = 2x - 2$ $\frac{1}{x} - 3(2x-2) = 1$ $\frac{1}{x} - 6x + 6 - 1 = 0$ $1 - 6x^2 + 6x - x = 0$ $-6x^2 + 5x + 1 = 0$ $6x^2 - 5x - 1 = 0$ $(6x+1)(x-1) = 0$ $x = -\frac{1}{6}$ or $x = 1$ $y = 2\left(-\frac{1}{6}\right) - 2$ or $y = 2(1) - 2$ $y = -\frac{7}{3}$ or $y = 0$	✓ $y = 2x - 2$ ✓substitution ✓simplification ✓standard form ✓x-values ✓y-values (6)

OR/*OF*

$x=\dfrac{2+y}{2}\quad \ldots(1)$	**OR/*OF***
$\dfrac{1}{x}-3y=1\quad \ldots(2)$	✓ $x=\dfrac{2+y}{2}$
$\dfrac{1}{\frac{2+y}{2}}-3y=1$	
$\dfrac{2}{2+y}-3y=1$	✓substitution
$\dfrac{2-6y-3y^2}{2+y}=1$	
$2-6y-3y^2=2+y$	
$-3y^2-7y=0$	✓simplification
$-y(3y+7)=0$	✓standard form
$y=0$ or $y=-\dfrac{7}{3}$	
$x=1$ or $x=-\dfrac{1}{6}$	✓*y*-values
	✓*x*-values

(6)
[26]

QUESTION/*VRAAG* 2

2.1.1	$a+6d=35$ $-1+6d=35$ $6d=36$ $d=6$ **OR/OF** $\frac{35-(-1)}{7-1}=6$ ANSWER ONLY: FULL MARKS	✓ substitution ✓answer (2) **OR/OF** ✓ substitution ✓answer (2)
2.1.2	$T_n=a+(n-1)d$ $473=-1+(n-1)(6)$ $79=n-1$ $\therefore n=80$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{40}=\frac{40}{2}[2(-1)+(40-1)(6)]$ $\therefore S_{40}=4\,640$ **OR/*OF*** $T_{40}=6(40)-7$ $=233$ $S_n=\frac{n}{2}(a+l)$ $=\frac{40}{2}(-1+233)$ $=4\,640$	✓ substitution ✓answer (2) **OR/*OF*** ✓ substitution ✓answer (2)
2.2.1	75 53 35 21 –22 –18 –14 4 4 $T_5=11$	✓answer (A) (1)
2.2.2	$T_n=an^2+bn+c$ $2a=4$ $a=2$ $3a+b=-22$ $6+b=-22$ $b=-28$ $a+b+c=75$ $2-28+c=75$ $c=101$ $\therefore T_n=2n^2-28n+101$	✓ $T_n=an^2+bn+c$ ✓ $a=2$ ✓ $b=-28$ ✓ $c=101$ (4)

2.2.3 Minimum value of T_n

$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$

$n = 7$ ✓ $n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/*OF* **OR/*OF***

$T_n^{/} = 4n - 28$

$4n - 28 = 0$

$4n = 28$

$n = 7$ ✓ $n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$

(4)

OR/OF **OR/OF**

$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$ ✓✓ $T_n \div (-5)$

$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(\frac{-2}{5}\right)}$

$= 7$ ✓ $n = 7$

$T_7 = -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/OF **OR/OF**

$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$ $T'_n = -\frac{4}{5}n + \frac{28}{5}$ $-\frac{4}{5}n + \frac{28}{5} = 0$ $-4n = -28$ $n = 7$ Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. Maximum value of new pattern $= -\frac{3}{5}$	✓✓ $T_n \div (-5)$ ✓$n = 7$ ✓ max value $= -\frac{3}{5}$ (4)

[16]

QUESTION/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_{10} = 1\,024\left(\frac{1}{4}\right)^{10-1}$ $\therefore T_{10} = \frac{1}{256}$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ answer (2)
3.1.2	$\sum_{p=0}^{8} 256\left(4^{1-p}\right) = 1\,024 + 256 + 64 + \ldots$ $S_n = \frac{a\left[1-r^n\right]}{1-r}$ $S_9 = \frac{1024\left[1-\left(\frac{1}{4}\right)^9\right]}{1-\frac{1}{4}}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$ **OR/OF** $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ $= 1\,024 + 256 + 64 + 16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$	✓ 1024 ✓ $n = 9$ ✓substitution into the correct formula ✓answer (4) **OR/OF** ✓ 1024 ✓ rest of expansion ✓ n = 9 terms ✓ answer (4)
3.2	$-t^2 - 6t - 9\,;\ \frac{t^3 + 9t^2 + 27t + 27}{2}$ $-\left(t^2 + 6t + 9\right);\ \frac{1}{2}(t+3)\left(t^2 + 6t + 9\right)$ $-(t+3)^2\ ;\ \frac{1}{2}(t+3)^3$ $r = \frac{-(t+3)}{2}$ $-1 < \frac{-t-3}{2} < 1$ $-2 < -t - 3 < 2$ $1 < -t < 5$ $-5 < t < -1$	✓ $r = \frac{\frac{t^3 + 9t^2 + 27t + 27}{2}}{-t^2 - 6t - 9}$ ✓ $-(t^2 + 6t + 9)$ ✓ $\frac{1}{2}(t+3)(t^2 + 6t + 9)$ ✓ $-1 < \frac{-t-3}{2} < 1$ ✓ answer (5)
		[11]

QUESTION 4

4.1	$10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$	✓ subs (–2 ; 10) ✓ simplification ✓ answer (3)
4.2	$y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^{0} + 7$ $y = \frac{22}{3} = 7,33$ $\therefore \left(0\ ;\ \frac{22}{3}\right)$ ANSWER ONLY: FULL MARKS	✓ substitution of $x = 0$ ✓ answer (2)
4.3.1	Translation by 1 unit to the right and 7 units downwards	✓ 1 unit right ✓ 7 units downwards (2)
4.3.2	$h(x) = \left(\frac{1}{3}\right)^{x}$ ANSWER ONLY: FULL MARKS $h^{-1}:\quad x = \left(\frac{1}{3}\right)^{y}$ $y = \log_{\frac{1}{3}}(x)$ **OR/*OF*** $y = -\log_{3}(x)$	✓ swap x and y ✓ answer (2)
		[9]

QUESTION 5

5.1	$g(x)=\dfrac{a}{x+2}+q$ Subs (1 ; 0): $0=\dfrac{a}{1+2}+q$ $0=a+3q$ Subs $\left(0\ ;\ -\dfrac{1}{2}\right)$ $-\dfrac{1}{2}=\dfrac{a}{0+2}+q$ $-1=a+2q$ Solving simultaneously: $q=1$ $a=-3$ $\therefore g(x)=\dfrac{-3}{x+2}+1$	✓ $g(x)=\dfrac{a}{x+2}+q$ ✓ $0=a+3q$ ✓ $-1=a+2q$ ✓ solving simultaneously ✓ $q = 1$ ✓ $a = -3$ (6)
5.2	$y \in$ R; $y \neq 1$ **OR/*OF*** $(-\infty\,;1)$ or $(1\,;\infty)$ **OR/*OF*** $y<1$ or $y>1$	✓ answer (1)
5.3	$y-1=1(x+2)$ $y=x+3$ **OR/*OF*** $1=1(-2)+c$ $c = 3$ $y=x+3$ ANSWER ONLY: FULL MARKS	✓ $m = 1$ ✓ subs point (– 2 ; 1) ✓ answer (3)
5.4	K$^{/}$ (–3 ; 4)	✓ x-value ✓ y-value (2)
		[12]

QUESTION 6

6.1	$f(x) = -x^2 - 6x + 7$ $f^{/}(x) = -2x - 6$ $-2x - 6 = 0$ **OR/*OF*** $x = -\frac{(-6)}{2(-1)}$ $x = -3$ E(–3 ; 16) ANSWER ONLY: FULL MARKS	✓ method ✓ x-value ✓ y-value (3)
6.2	$k = f(-5)$ $k = -(-5)^2 - 6(-5) + 7$ $\therefore k = 12$	✓ answer (A) (1)
6.3	C(0 ; 7) D(–5 ; 12) $m_{CD} = \frac{12-7}{-5-0}$ $m_{CD} = -1$ Equation of CD: $y = -x + 7$	✓ coordinates of C ✓ substitution ✓ m ✓ answer (4)
6.4	$-2x - 6 = -1$ $-2x = 5$ $x = -\frac{5}{2}$ $y = f\left(\frac{-5}{2}\right) = -\left(\frac{-5}{2}\right)^2 - 6\left(\frac{-5}{2}\right) + 7 = \frac{63}{4} = 15,75$ $\therefore \text{P}\left(-\frac{5}{2}; \frac{63}{4}\right)$	✓ $f^{/}(x) = -2x - 6$ ✓ equating to -1 ✓ x-value ✓ y-value (A) (4)
6.5	Point by symmetry: (–1 ; 12) $-5 < x < -1$ **OR/*OF*** $-x^2 - 6x + 7 > 12$ $-x^2 - 6x - 5 > 0$ $x^2 + 6x + 5 < 0$ $(x+1)(x+5) < 0$ $-5 < x < -1$ ANSWER ONLY: FULL MARKS	✓ –1 ✓ answer (2) ✓ –1 ✓ answer (2)
		[14]

QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1+\dfrac{0{,}085}{4}\right)^{4n}$ $4n = \log_{\left(1+\frac{0{,}085}{4}\right)} 2$ $n = 8{,}24$ years	✓ 2 ✓ $\dfrac{0{,}085}{4}$ In correct formula ✓ use of logs ✓ answer in years (4)
7.2.1	$A = P(1-i)^n$ $180\ 000 = 500\ 000(1-i)^5$ $\dfrac{9}{25} = (1-i)^5$ $\sqrt[5]{\dfrac{9}{25}} = 1-i$ $i = 0{,}1848068...$ $r = 18{,}48\%$	✓ subs into correct formula ✓ simplification ✓ $i = 0{,}1848\ldots$ ✓ answer (4)
7.2.2	$A = P(1+i)^n$ $\text{A} = 500\ 000(1+0{,}063)^5$ A = R678 635,11	✓ subs into correct formula ✓ answer (2)
7.2.3	Sinking Fund = 678 635,11 – 180 000 = R 498 635,11 $498\ 635{,}11 = \dfrac{x\left[\left(1+\dfrac{0{,}1025}{12}\right)^{58} - 1\right]\left(1+\dfrac{0{,}1025}{12}\right)^{3}}{\dfrac{0{,}1025}{12}}$ $x = \text{R}6\,510{,}36$	✓ value of sinking fund ✓ $\dfrac{0{,}1025}{12}$ ✓ $n = 58$ (A) ✓ $\left(1+\dfrac{0{,}1025}{12}\right)^{3}$ ✓ answer (A) (5)
		[15]

QUESTION/*VRAAG* 8

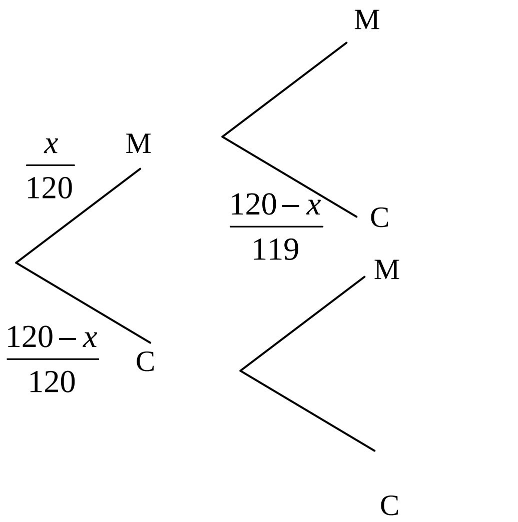
8.1	$f(x)=-x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-(x+h)^2+x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{-x^2-2xh-h^2+x^2}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$ **OR/*OF*** $f(x)=-x^2$ $f(x+h)=-(x+h)^2=-x^2-2xh-h^2$ $f(x+h)-f(x)=-x^2-2xh-h^2-(-x^2)=-2xh-h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	✓ substitution into formula ✓ $-\left(x^2+2xh+h^2\right)$ ✓ $-2xh-h^2$ ✓ $-2x-h$ ✓answer (5) **OR/*OF*** ✓ $-x^2-2xh-h^2$ ✓ $-2xh-h^2$ ✓substitution into the formula ✓ $-2x-h$ ✓answer (5)
8.2.1	$f(x)=4x^3-5x^2$ $f'(x)=12x^2-10x$	✓ $12x^2$ (A) ✓ $-10x$ (A) (2)
8.2.2	$D_x\left[\frac{-6\sqrt[3]{x}+2}{x^4}\right]$ $=D_x\left[\frac{-6(x)^{\frac{1}{3}}}{x^4}+\frac{2}{x^4}\right]$ $=D_x\left[-6x^{-\frac{11}{3}}+2x^{-4}\right]$ $=22x^{-\frac{14}{3}}-8x^{-5}$	✓ $x^{\frac{1}{3}}$ ✓ $-6x^{-\frac{11}{3}}+2x^{-4}$ ✓ $22x^{-\frac{14}{3}}$ ✓ $-8x^{-5}$ (4)
		[11]

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f'(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $\text{N}\left(\frac{5}{3}\,;\,-\frac{256}{27}\right)=(1{,}67\,;-9{,}48)$	✓ $f'(x)=3x^2-2x-5$ ✓ = 0 ✓ factors ✓ x-value ($x > 0$) ✓ y-value (A) (5)
9.3.1	$x<3$; $x\neq-1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty\,;-1)$ or $(-1\,;3)$	✓ $x<3$ ✓ $x\neq-1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty\,;-1)$ ✓ $(-1\,;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq-1$ or $x\geq\frac{5}{3}$ **OR/*OF*** $(-\infty\,;-1)$ or $\left(\frac{5}{3}\,;\infty\right)$ OR/OF $(-\infty\,;-1]$ or $\left[\frac{5}{3}\,;\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty\,;-1)$ ✓ $\left(\frac{5}{3}\,;\infty\right)$ (2)
9.3.3	$f''(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3}\,;\infty\right)$ ANSWER ONLY: FULL MARKS **OR/OF** $\dfrac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3}\,;\infty\right)$	✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x+1)(x-3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

QUESTION/*VRAAG* 10

10.1.1	7! = 5 040	✓✓ answer (2)
10.1.2	$4! \times 4!$ = 576 P(African flags together) = $\dfrac{576}{5040}$ $\left(=\dfrac{4}{35}=0{,}11\right)$	✓ 4! ✓ $4! \times 4!$ ✓ answer (A) (3)
10.2	P(A or B) = P(A) + P(B) – P(A and B) 0,88 = 0,4 + P(B) – P(A and B) 0,88 = 0,4 + P(B) – 0,4P(B) 0,48 = 0,6P(B) P(B) = 0,8	✓subs into rule ✓P(A and B) = 0,4P(B) ✓ answer (3)
10.3	First Passenger Second Passenger  Probability of first passenger choosing meat $=\dfrac{x}{120}$ Probability of second passenger choosing cheese $=\dfrac{120-x}{119}$ $\dfrac{x}{120}\times\dfrac{120-x}{119}=\dfrac{18}{85}$ $120x-x^2=3\ 024$ $x^2-120x+3\,024=0$ $(x-84)(x-36)=0$ $x=84$ or $x=36$ $\therefore$ P(1st cheese) $=\dfrac{36}{120}=\dfrac{3}{10}$	✓ $\dfrac{x}{120}$ ✓ $\dfrac{120-x}{119}$ ✓ $\dfrac{x}{120}\times\dfrac{120-x}{119}=\dfrac{18}{85}$ ✓ $x=84$ or $x=36$ ✓ $\dfrac{3}{10}$ (5)
		[13]

TOTAL/*TOTAAL*: 150