JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

ACTIVITY MANUAL

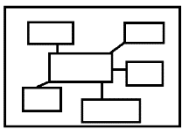
GRADE 12

2023

FUNCTIONS AND GRAPHS

CONTENTS **PAGE**

ICON DESCRIPTION



MIND MAP

EXAMINATION GUIDELINE

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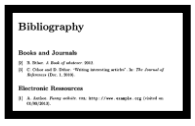
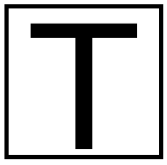
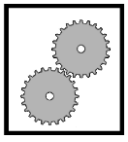
ACTIVITIES



BIBLIOGRAPHY



TERMINOLOGY



WORKED EXAMPLES

STEPS

FUNCTIONS AND GRAPHS

Outcomes:

- **Introduce a more formal definition of a function and extend Grade 11 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear, quadratic and some cubic polynomial functions, exponential and logarithmic functions, and some rational functions.**

- **The inverses of prescribed functions and be aware of the fact that, in the case of many-to-one functions, the domain has to be restricted if the inverse is to be a function.**

- **Problem solving and graph work involving the prescribed functions (including the logarithmic function)**

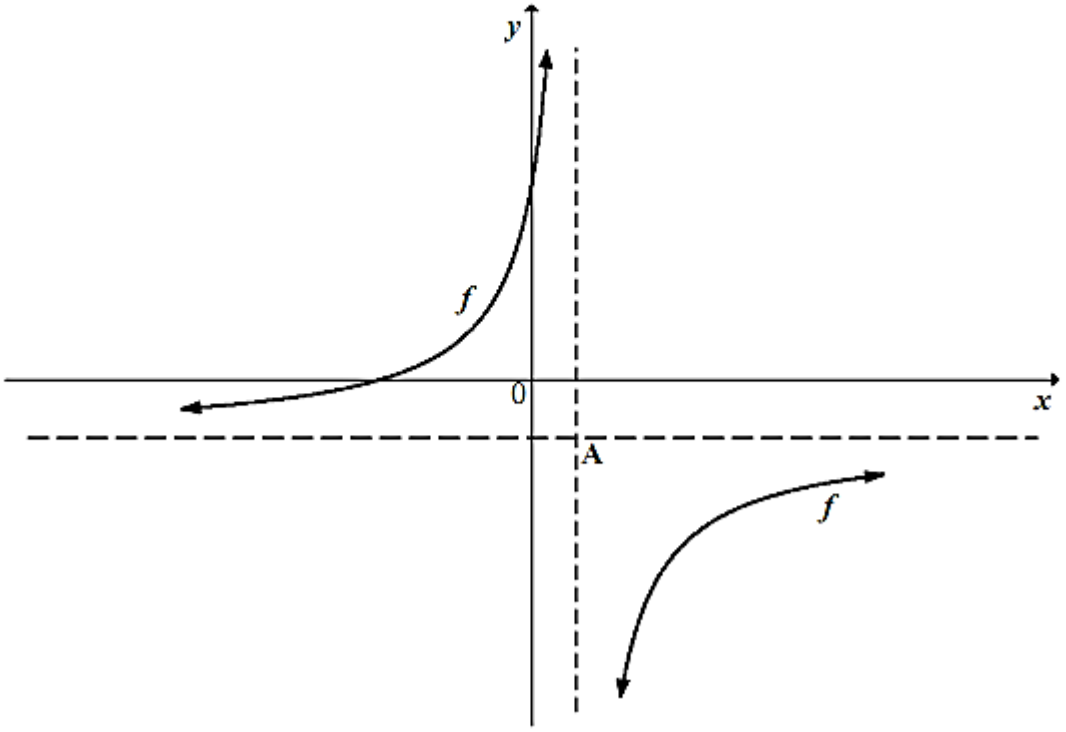
(SOURCE: CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS) FET PHASE GRADES (10 - 12) MATHEMATICS)

SECTION 1: Hyperbolic Function (Hyperbola)

Activities

QUESTION 1

1. The sketch below shows the graph of $f(x) = \frac{-9}{x-1} - 2$.
A is the point of intersection of the asymptotes of f.



1.1 Write down the coordinates of A.

1.2 Determine the coordinates of the x- and y-intercepts of f.

1.3 Write down an equation of the axis of symmetry of f that has a negative gradient.

1.4 Hence, or otherwise, determine the coordinates of a point that lies on f in the fourth quadrant, which is the closest to point A.

1.5 The graph of f is reflected about the x-axis to obtain the graph of g. Write down the equation of g in the form $y = ...$

QUESTION 2

Given: $f(x)=\frac{8}{x-2}+3$

2.1 Write down the equations of the asymptotes of f.

2.2 Calculate the x- and y-intercepts of f.

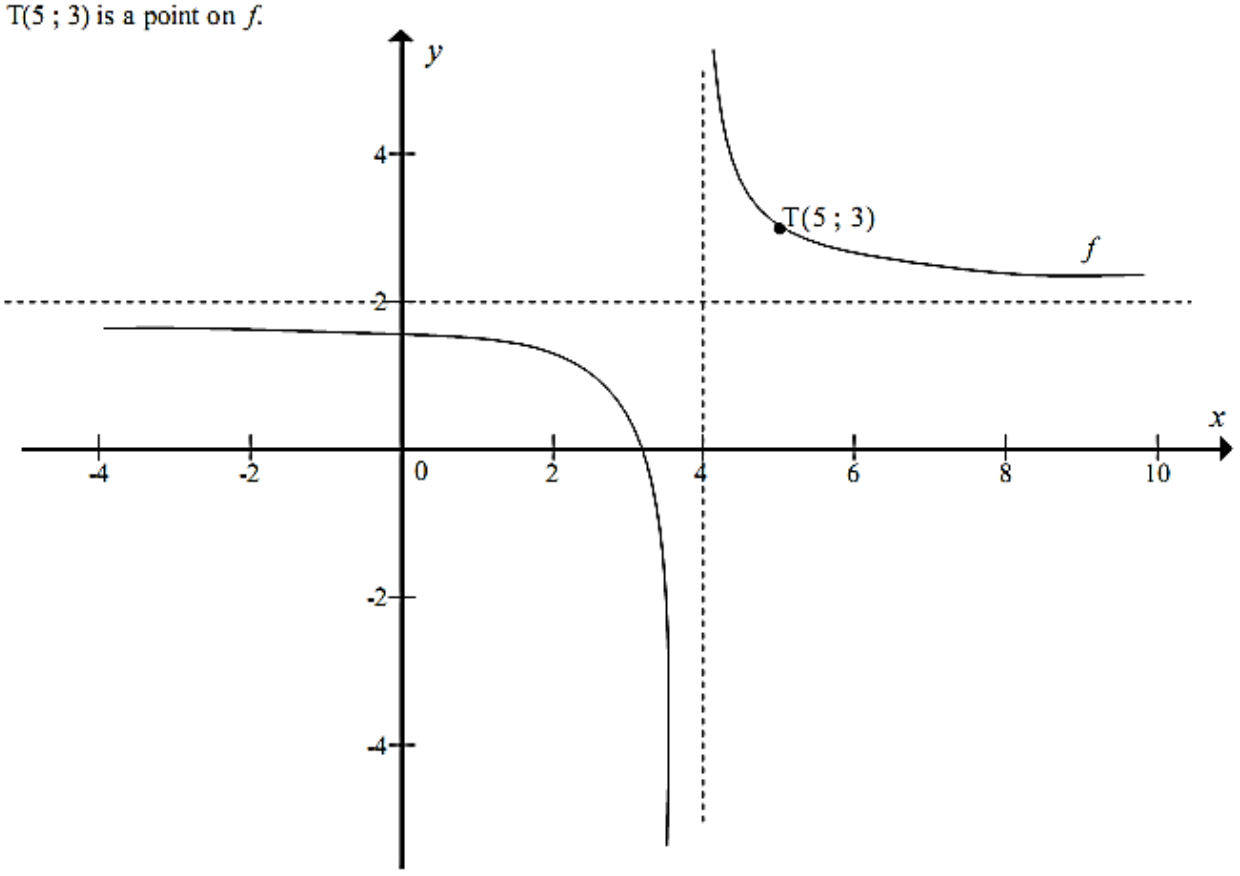
2.3 Sketch the graph of f. Show clearly the intercepts with the axes and the asymptotes.

2.4 If $y=x+k$ is an equation of the line of symmetry of f, calculate the value of k.

QUESTION 3

The diagram below represents the graph of $f(x)=\frac{a}{x-p}+q$.

T(5 ; 3) is a point on f.



3.1 Determine the values of a, p and q.

3.2 If the graph of f is reflected across the line having equation $y=-x+c$, the new graph coincides with the graph of $y=f(x)$. Determine the value of c.

QUESTION 4

Given: $f(x)=\frac{x+3}{x+1}$

4.1 Calculate the x- and y-intercepts of f.

4.2 Show that $f(x)=\frac{2}{x+1}+1$

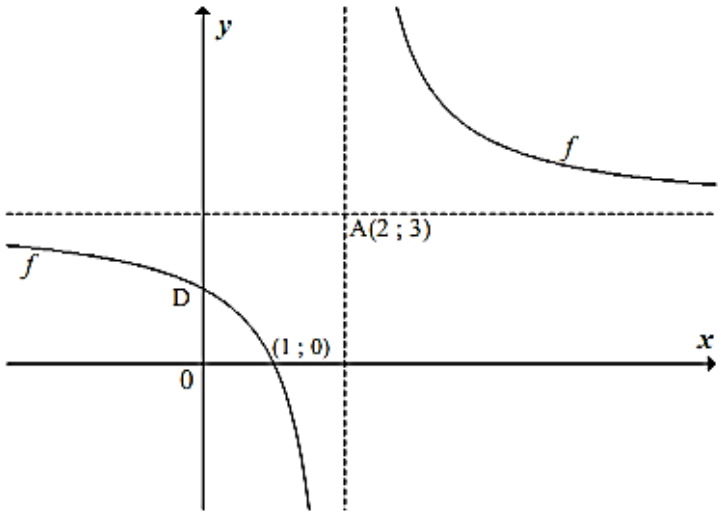
4.3 Write down the equations of the vertical and horizontal asymptotes of f.

4.4 Draw a sketch graph of $f(x)$ showing clearly the intercepts and asymptotes on the axes.

4.5 Use your graph to solve: $\frac{2}{x+1}\geq -1$

QUESTION 5

Given $f(x)=\frac{a}{x-p}+q$. The point A(2 ; 3) is the point of intersection of the asymptotes of f.
The graph of f intersects the x-axis at (1 ; 0).
D is the y-intercept of f.



5.1 Write down the equations of the asymptotes of f.

5.2 Determine an equation of f.

5.3 Write down the coordinates of D.

5.4 Write down an equation of g if g is the straight line joining A and D.

5.5 Write down the coordinates of the other point of intersection of f and g.

QUESTION 6

6.1 Consider the function: $f(x)=\frac{-6}{x-3}-1$

6.1.1 Calculate the coordinates of the y-intercept of f.

6.1.2 Calculate the coordinates of the x-intercept of f.

6.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes.

6.1.4 For which values of x is $f(x)>0$?

6.1.5 Calculate the average gradient of f between $x=-2$ and $x=0$.

QUESTION 7

The graph of a hyperbola with equation $y=f(x)$ has the following properties:

- Domain: $x\in\mathbf{R},\ x\neq 5$
- Range: $y\in\mathbf{R},\ y\neq 1$
- Passes through the point (2 ; 0)

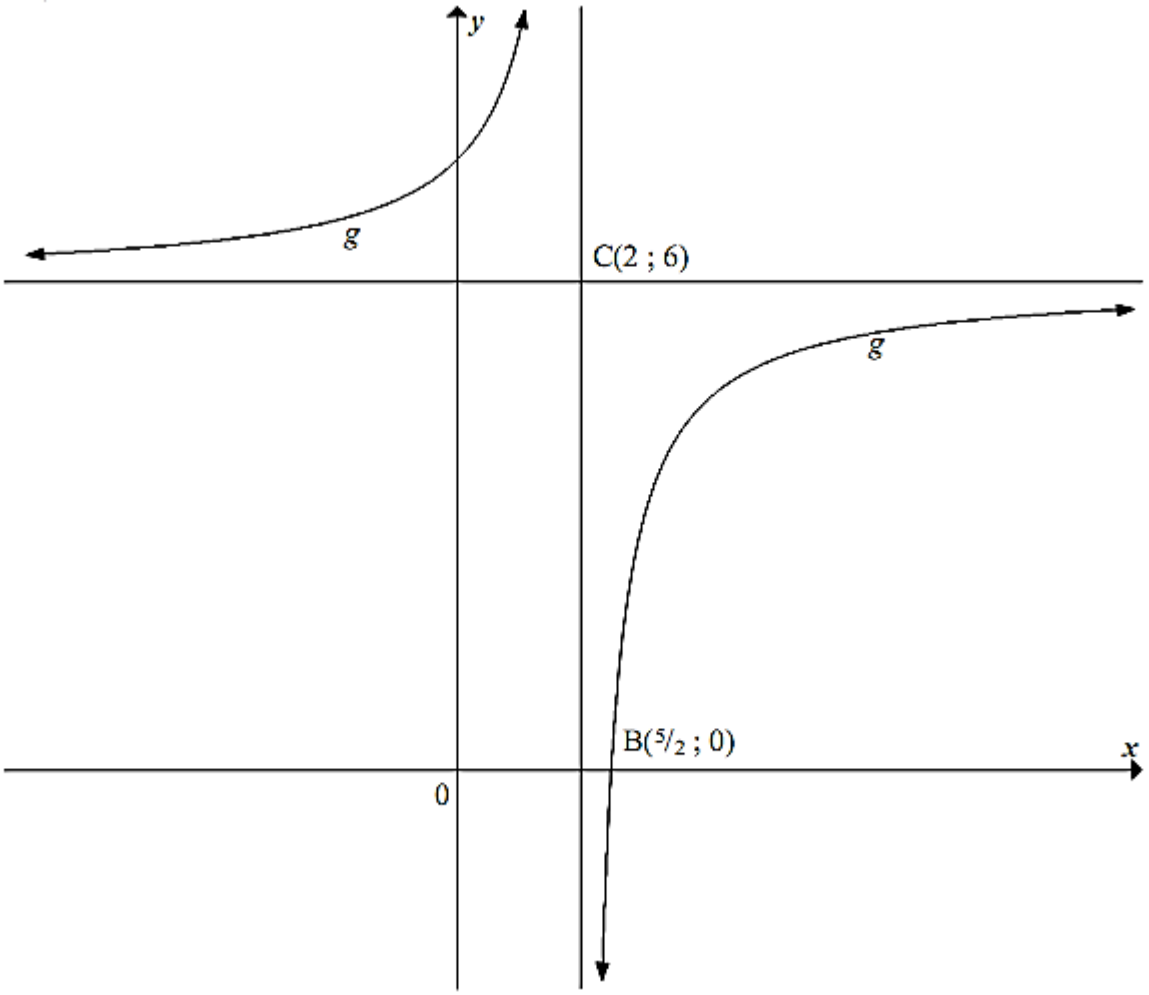
Determine $f(x)$.

QUESTION 8

Sketched below is the graph of $g(x)=\frac{a}{x-p}+q$.

C(2 ; 6) is the point of intersection of the asymptotes of g.

$B\left(\frac{5}{2};0\right)$ is the x-intercept of g.



8.1 Determine the equation for g in the form $g(x)=\frac{a}{x-p}+q$

8.2 F is the reflection of B across C. Determine the coordinates of F.

QUESTION 9

Consider the function $f(x)=\frac{3}{x-1}-2$.

9.1 Write down the equations of the asymptotes of f.

9.2 Calculate the intercepts of the graph of f with the axes.

9.3 Sketch the graph of f on DIAGRAM SHEET 1.

9.4 Write down the range of $y = -f(x)$.

9.5 Describe, in words, the transformation of f to g if $g(x)=\frac{-3}{x+1}-2$.

QUESTION 10

Consider: $f(x)=\frac{6}{x-2}+3$

10.1 Write down the equations of the asymptotes of the graph of f.

10.2 Write down the domain of f.

10.3 Draw a sketch graph of f in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes.

10.4 The graph of f is translated to g. Describe the transformation in the form $(x\,;y)\rightarrow \ldots$ if the axes of symmetry of g are $y=x+3$ and $y=-x+1$.

Past Papers

NOVEMBER 2018

QUESTION 5

Given: $f(x)=\frac{-1}{x-1}$

5.1 Write down the domain of f. (1)

5.2 Write down the asymptotes of f. (2)

5.3 Sketch the graph of f, clearly showing all intercepts with the axes and any asymptotes. (3)

5.4 For which values of x will $x.f'(x)\geq 0$? (2)

[8]

MARCH 2015

QUESTION 4

Given: $g(x)=\frac{6}{x+2}-1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y=....$ (3)

4.5 Determine the value(s) of x for which $\frac{6}{x+2}-1\geq -x-3$. (2)

[13]

MARCH 2014

QUESTION 6

Consider: $f(x)=\frac{6}{x-2}+3$

6.1 Write down the equations of the asymptotes of the graph of f. (2)

6.2 Write down the domain of f. (1)

6.3 Draw a sketch graph of f in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes. (4)

6.4 The graph of f is translated to g. Describe the transformation in the form $(x\,;y)\rightarrow\ldots$ if the axes of symmetry of g are $y=x+3$ and $y=-x+1$. (4)

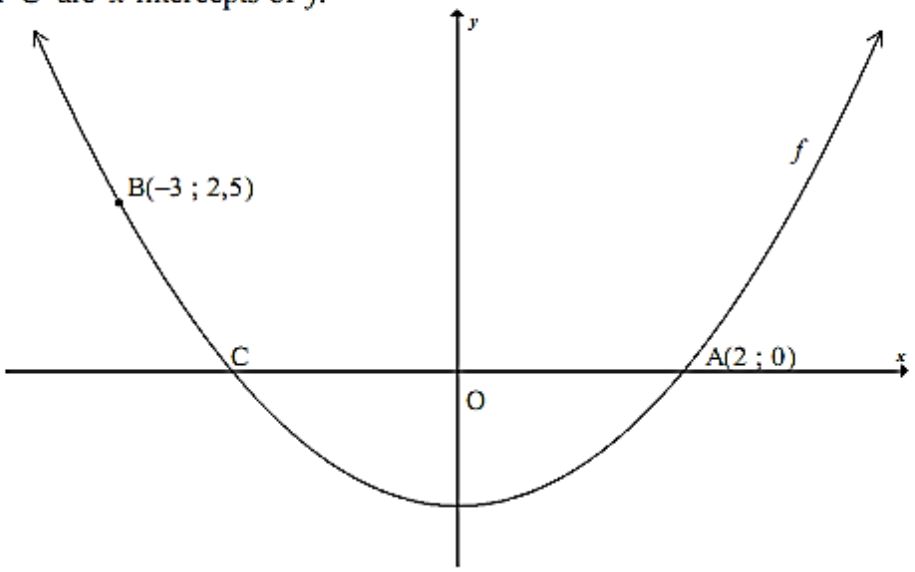
[11]

Section 2: Quadratic Function (Parabola)

Activities

QUESTION 1

The graph of $f(x)=ax^2+q$ is sketched below.
Points A(2 ; 0) and B(– 3 ; 2,5) lie on the graph of f.
Points A and C are x-intercepts of f.



1.1 Write down the coordinates of C.

1.2 Determine the equation of f.

1.3 Write down the range of f.

1.4 Write down the range of h, where $h(x)=-f(x)-2$.

1.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and which passes through the point A.

QUESTION 2

Given: $f(x)=ax^2+c$
f passes through the x-axis at $(d-5)$ and $(d-1)$, where $d \in R$.

2.1 Determine the value of d.

2.2 Determine the values of a and c if it is also given that $f(1)=-9$.

QUESTION 3

3.1 Give the equation of the quadratic function if it is given that:

- The range of f is: $y \geq -4$
- Domain: $x \in \mathbb{R}$
- Zero points are (3 ; 0), (-1 ; 0) and (0 ; -3)

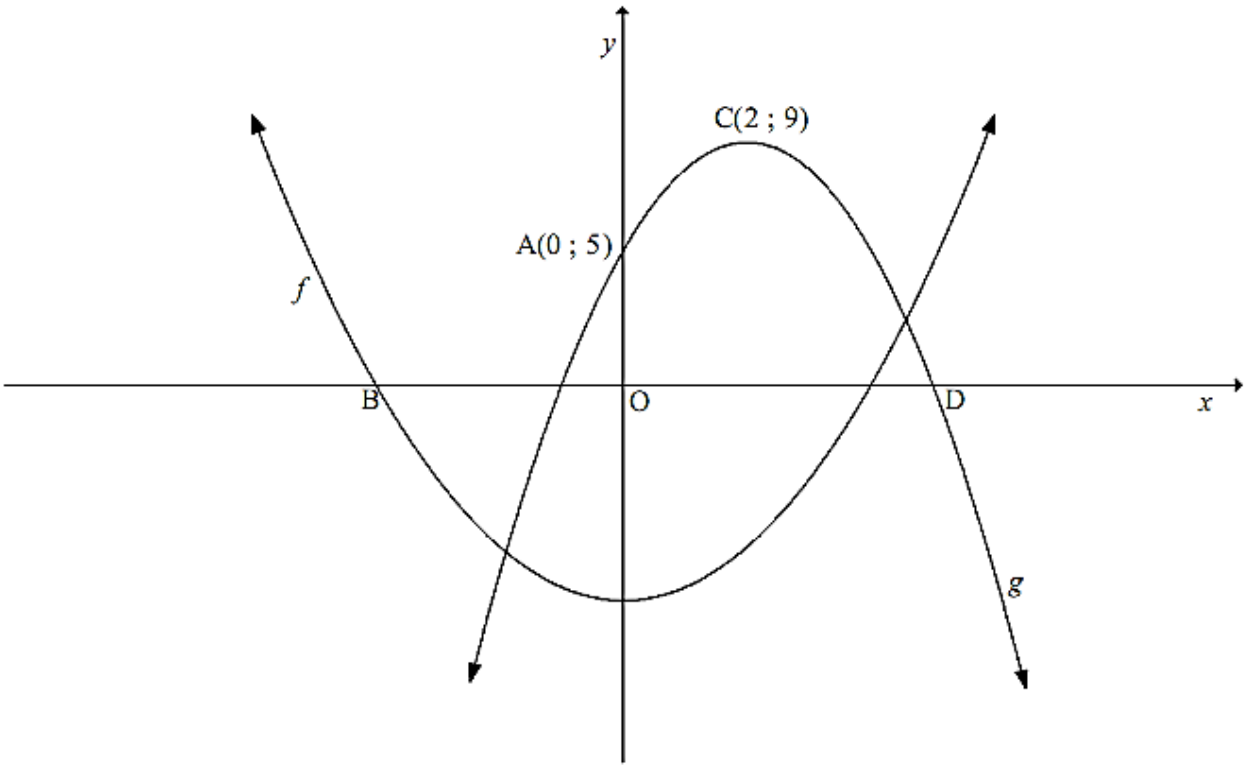
3.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0$, $b < 0$, $c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution.

QUESTION 4

The sketch below represents the graphs of two parabolas, f and g.

$f(x) = \frac{1}{2}x^2 - 8$

The turning point of g is C (2 ; 9) and the y-intercept of g is A (0 ; 5).
B and D are the x-intercepts of f and g respectively.



4.1 Show that $g(x) = -x^2 + 4x + 5$.

4.2 Calculate the average gradient of g between A and C.

4.3 Calculate the length of BD.

4.4 Use the graphs to solve for x, if:

4.4.1 $f(x) \geq 0$

4.4.2 f and g are both strictly increasing

QUESTION 5

Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- The range of f is $(-\infty;7]$
- $a \neq 0$
- $b < 0$
- One root of f is positive and the other root of f is negative.

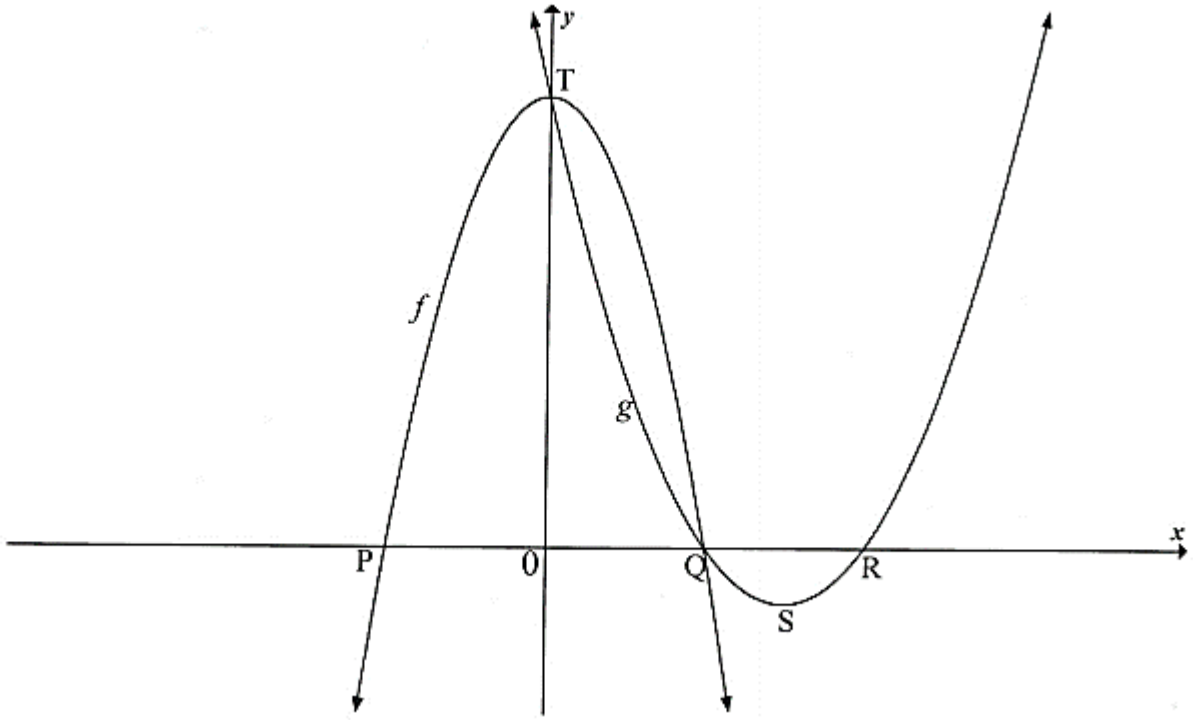
Past Papers

NOVEMBER 2015

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

[8]

NOVEMBER 2017

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\,;\,\frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

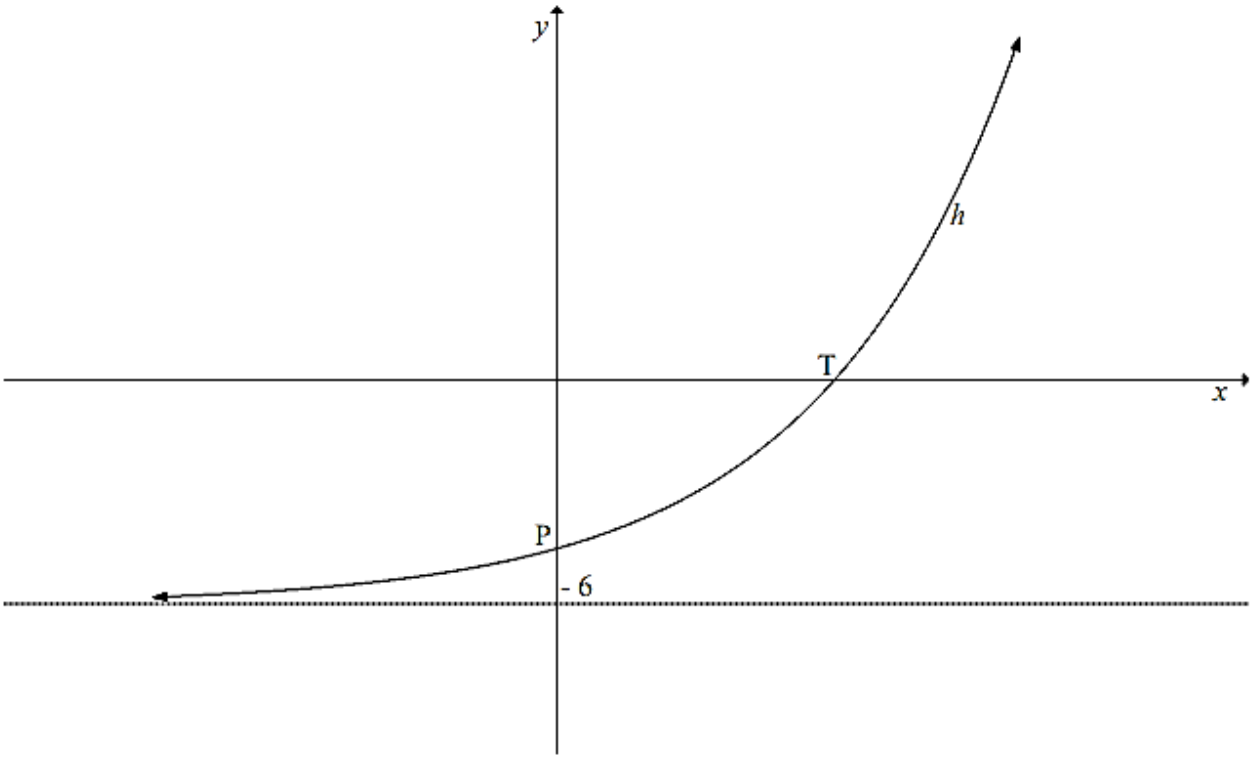
[23]

Section 3: Exponential Function

Activities

QUESTION 1

Given: $h(x) = a.2^{x-1} + q$. The line $y = -6$ is an asymptote to the graph of h. P is the y-intercept of h and T is the x-intercept of h.



1.1 Write down the value of q.

1.2 If the graph of h passes through the point $\left(-1; -5\frac{1}{4}\right)$, calculate the value of a.

1.3 Calculate the average gradient between the x-intercept and the y-intercept of h.

1.4 Determine the equation of p if $p(x) = h(x-2)$ in the form $p(x) = a.2^{x-1} + q$.

QUESTION 2

Given: $h(x)=4\left(2^{-x}\right)+1$

2.1 Determine the coordinates of the y-intercept of h.

2.2 Explain why h does not have an x-intercept.

2.3 Draw a sketch graph of h, clearly showing all asymptotes, intercepts with the axes and at least one other point on h.

2.4 Describe the transformation from h to g if $g(x)=4\left(2^{-x}+2\right)$.

QUESTION 3

Consider the function $f(x)=4^{-x}-2$.

3.1 Calculate the coordinates of the intercepts of f with the axes.

3.2 Write down the equation of the asymptote of f.

3.3 Sketch the graph of f.

3.4 Write down the equation of g if g is the graph of f shifted 2 units upwards.

3.5 Solve for x if $f(x)=3$. (You need not simplify your answer.)

QUESTION 4

4.1 Consider the function $f(x)=3.2^{x}-6$.

4.1.1 Calculate the coordinates of the y-intercept of the graph of f.

4.1.2 Calculate the coordinates of the x-intercept of the graph of f.

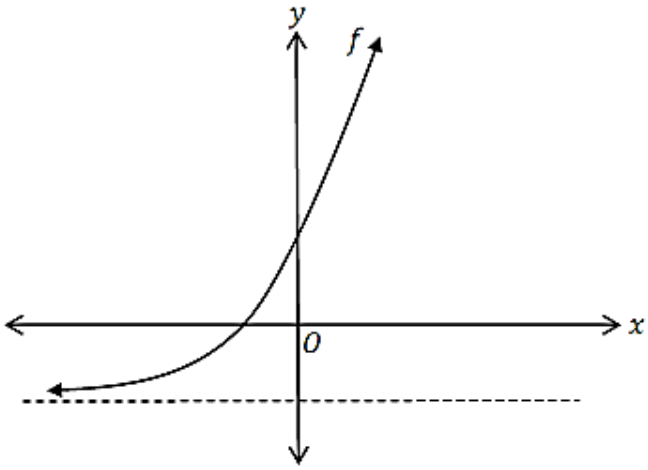
4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes.

4.1.4 Write down the range of f.

QUESTION 5

The sketch below shows the graph of $f(x) = 2 \times a^x - 1$. The point $A(1;5)$ is a point on the graph.



5.1 Show that $a = 3$.

5.2 Determine the y-intercept of f.

5.3 Write down the range of f.

5.4 Determine $f(0{,}23)$, rounded off to three decimal places.

5.5 Write down the equation if f is reflected about the x-axis followed by a translation of 2 units to the left.

Past Papers

MARCH 2016

QUESTION 4

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the y-intercept of f. (1)

4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3)

4.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h. (1)

[8]

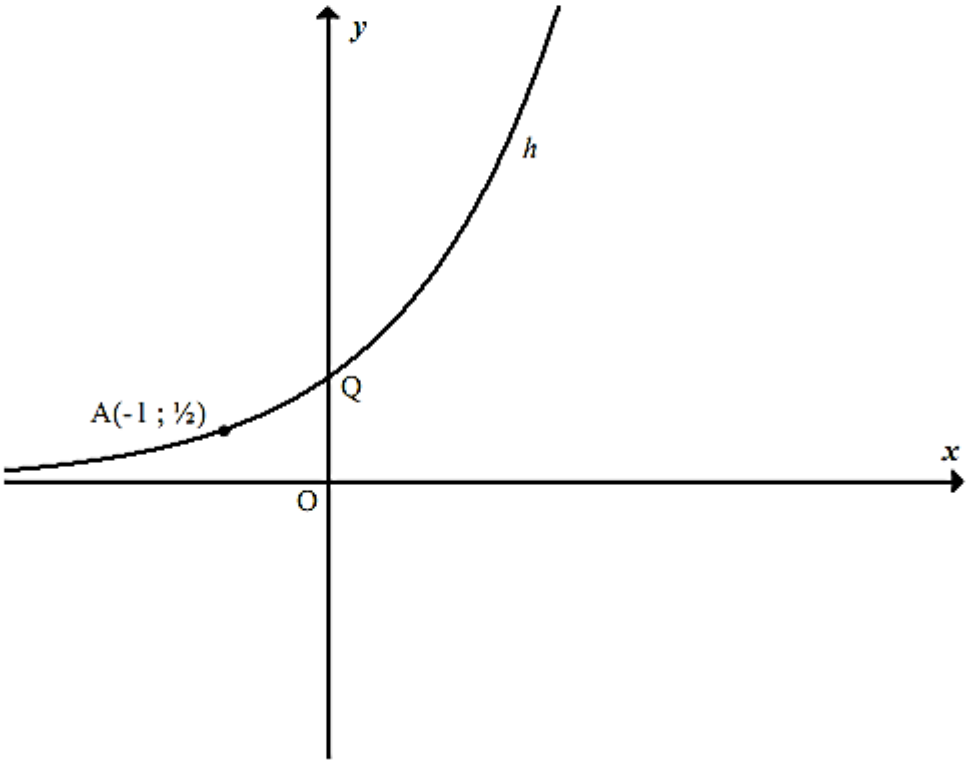
Section 4: Inverse Functions

Activities

QUESTION 1

The graph of $h(x) = a^x$ is sketched below.

$A\left(-1\,;\frac{1}{2}\right)$ is a point on the graph of h.



1.1 Explain why the coordinates of Q are (0 ; 1).

1.2 Calculate the value of a.

1.3 Write down the equation for the inverse function, h^{-1}, in the form $y = ...$

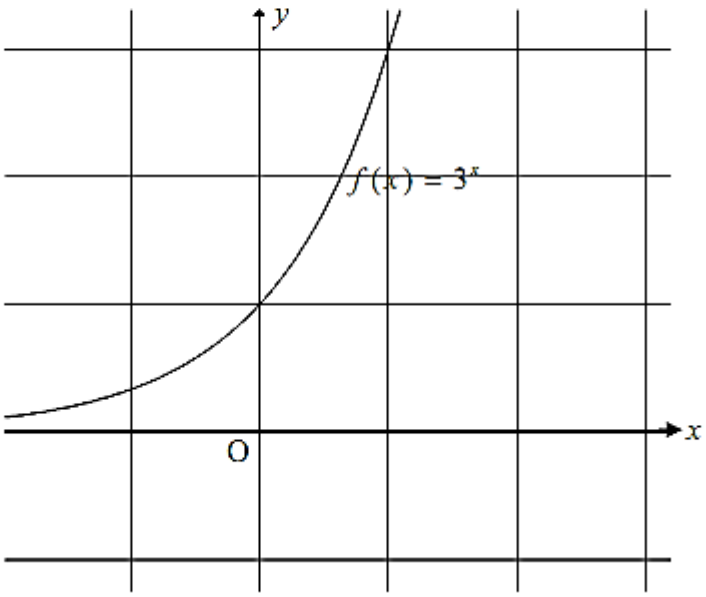
1.4 Draw a sketch graph, on ANSWER BOOK, of h^{-1}. Indicate on this graph the coordinates of two points that lie on this graph.

1.5 Read off from your graph the values of x for which $\log_2 x > -1$.

1.6 If $g(x) = (100).3^x$, determine the value of x for which $h(x) = g(x)$.

QUESTION 2

The graph of $f(x) = 3^x$ is drawn below.



2.1 Write f^{-1} in the form $y = \ldots$

2.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$

2.3 Use your graphs to solve for x if $\log_3(x-2) < 1$.

QUESTION 3

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$.

3.1 Is f an increasing or decreasing function? Give a reason for your answer.

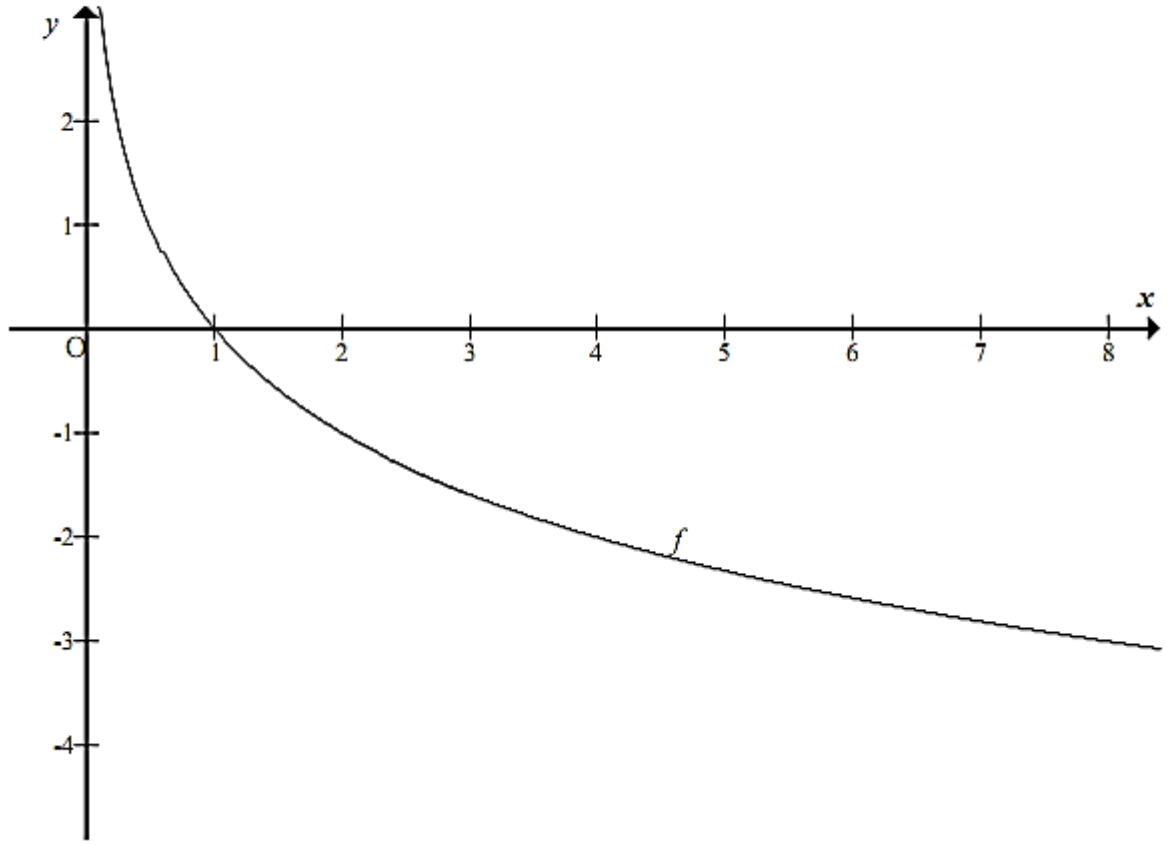
3.2 Determine $f^{-1}(x)$ in the form $y = \ldots$

3.3 Write down the equation of the asymptote of $f(x) - 5$.

3.4 Describe the transformation from f to g if $g(x) = \log_3 x$.

QUESTION 4

Sketched below is the graph of $f(x) = -\log_2 x$.



4.1 Write down the domain of f.

4.2 Write down the equation of f^{-1} in the form $y = \ldots$

4.3 Write down the equation of the asymptote of f^{-1}.

4.4 Explain how, using the graph of f, you would sketch the graphs of:

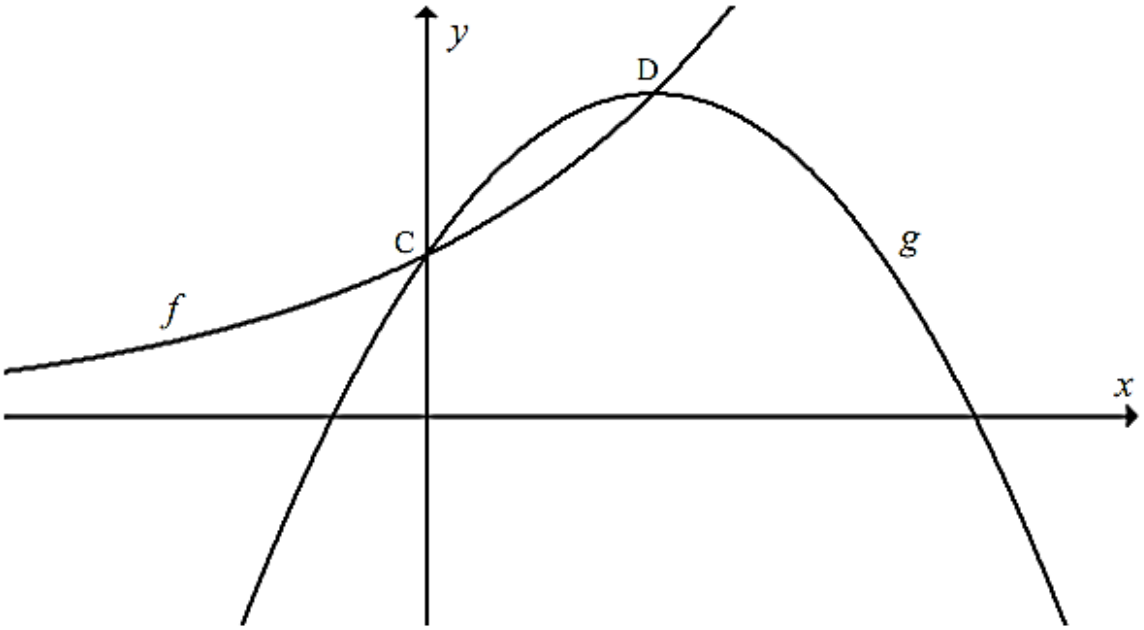
4.4.1 $g(x) = \log_2 x$

4.4.2 $h(x) = 2^{-x} - 5$

4.5 Use the graph of f to solve for x where $\log_2 x < 3$.

QUESTION 5

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + b$, where b is a constant.
The graphs of f and g intersect the y-axis at C. D is the turning point of g.



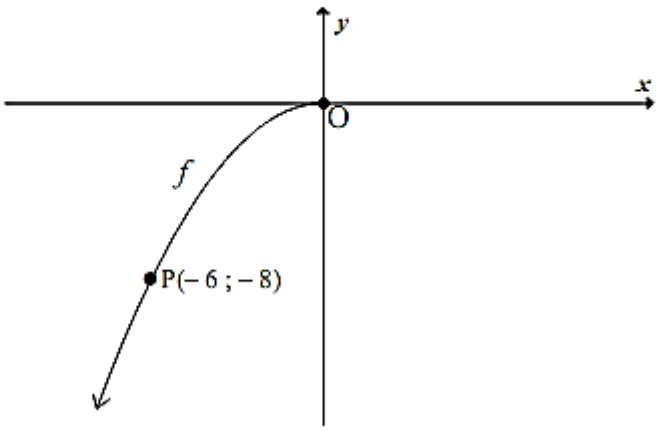
5.1 Write down the coordinates of the turning point of g.

5.2 Write down the equation of $f^{-1}(x)$ in the form $y = ...$

5.3 Sketch the graph of f^{-1} .
Indicate the x-intercept and the coordinates of one other point on your graph.

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P$(-6\,;-8)$ lies on the graph of f.



6.1 Calculate the value of a.

6.2 Determine the equation of f^{-1}, in the form $y = \ldots$

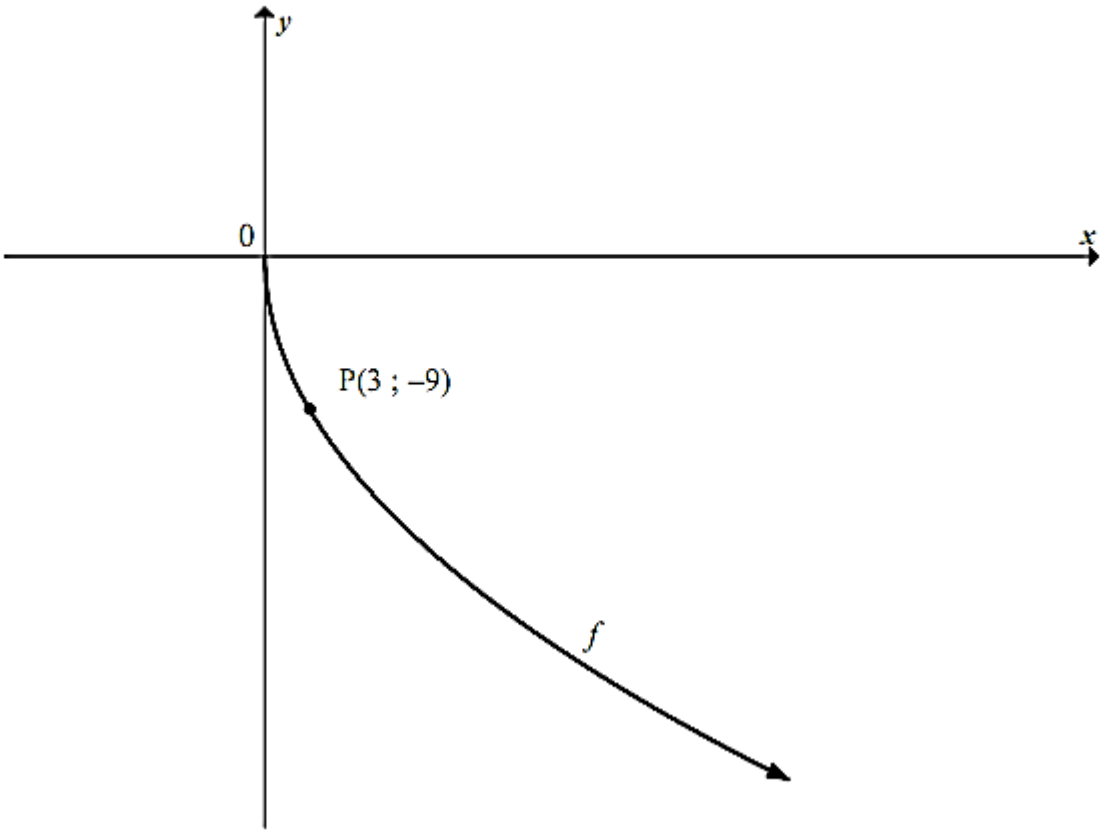
6.3 Write down the range of f^{-1}.

6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from $(0\ ;\ 0)$.

6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axis. Determine the equation of the new function in the form $y = \ldots$

QUESTION 7

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



7.1 Use your graph to determine the values of x for which $f(x) \geq -9$.

7.2 Write down the equation of f^{-1} in the form $y = \ldots$ Include ALL restrictions.

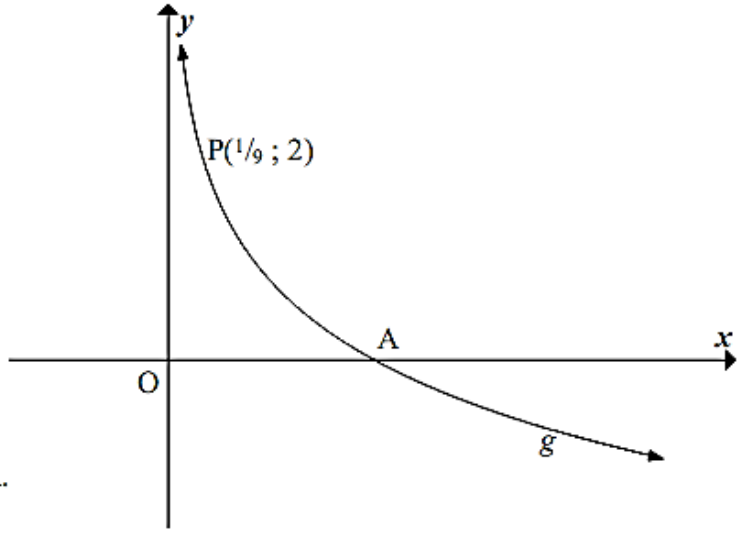
7.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point.

7.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$.

QUESTION 8

Given the graph of $g(x) = \log_{\frac{1}{3}} x$

- A is the x-intercept of g.
- P $\left(\frac{1}{9}; 2\right)$ is a point on g.



8.1 Write down the coordinates of A.

8.2 Sketch the graph of g^{-1} indicating an intercept with the axes and ONE other point on the graph.

8.3 Write down the domain of g^{-1}.

QUESTION 9

Given: $f(x) = 3^x$

9.1 Determine an equation for f^{-1} in the form $f^{-1}(x) = \ldots$

9.2 Sketch, in your ANSWER BOOK, the graphs of f and f^{-1}, showing clearly ALL intercepts with the axes.

9.3 Write down the domain of f^{-1}.

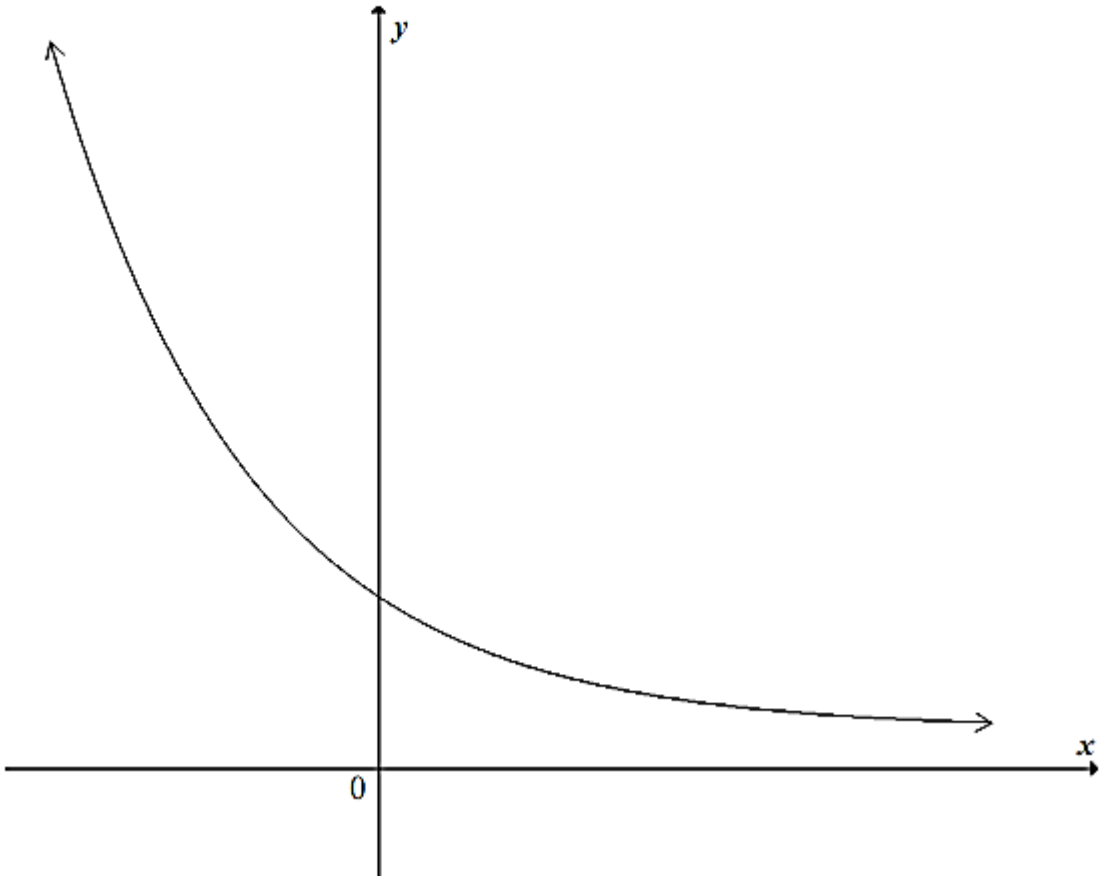
9.4 For which values of x will $f(x).f^{-1}(x) \leq 0$?

9.5 Write down the range of $h(x) = 3^{-x} - 4$

9.6 Write down an equation for g if the graph of g is the image of the graph of f after f has been translated two units to the right and reflected about the x-axis.

QUESTION 10

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



10.1 Write down the domain of f.

10.2 Write down the equation of the asymptote of f.

10.3 Write down the equation of f^{-1} in the form $y=...$

10.4 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate the x-intercept and ONE other point.

10.5 Write down the equation of the asymptote of $f^{-1}(x+2)$.

10.6 Prove that: $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$ for all values of x.

QUESTION 11

11.1 Consider the function f where $f(x)=3x-1$.

11.1.1 Write down the domain and range of f.

11.1.2 Determine the inverse function f^{-1}.

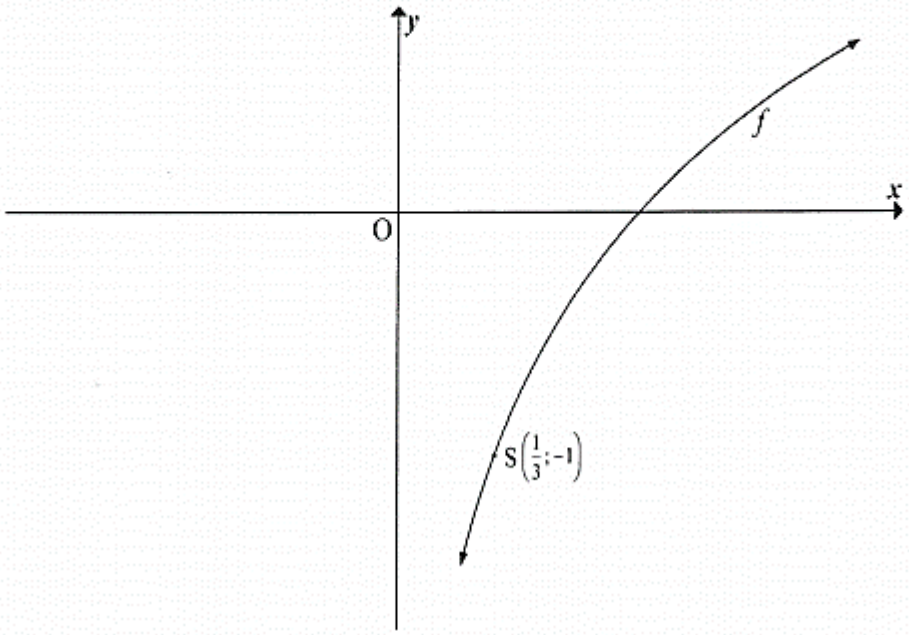
11.1.3 Sketch the graphs of the functions f, f^{-1} and $y=x$ line on the same set of axes.

Past Papers

NOVEMBER 2014

QUESTION 5

Given: $f(x)=\log_a x$ where $a>0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of f.



5.1 Prove that $a=3$. (2)

5.2 Write down the equation of h, the inverse of f, in the form $y=...$ (2)

5.3 If $g(x)=-f(x)$, determine the equation of g. (1)

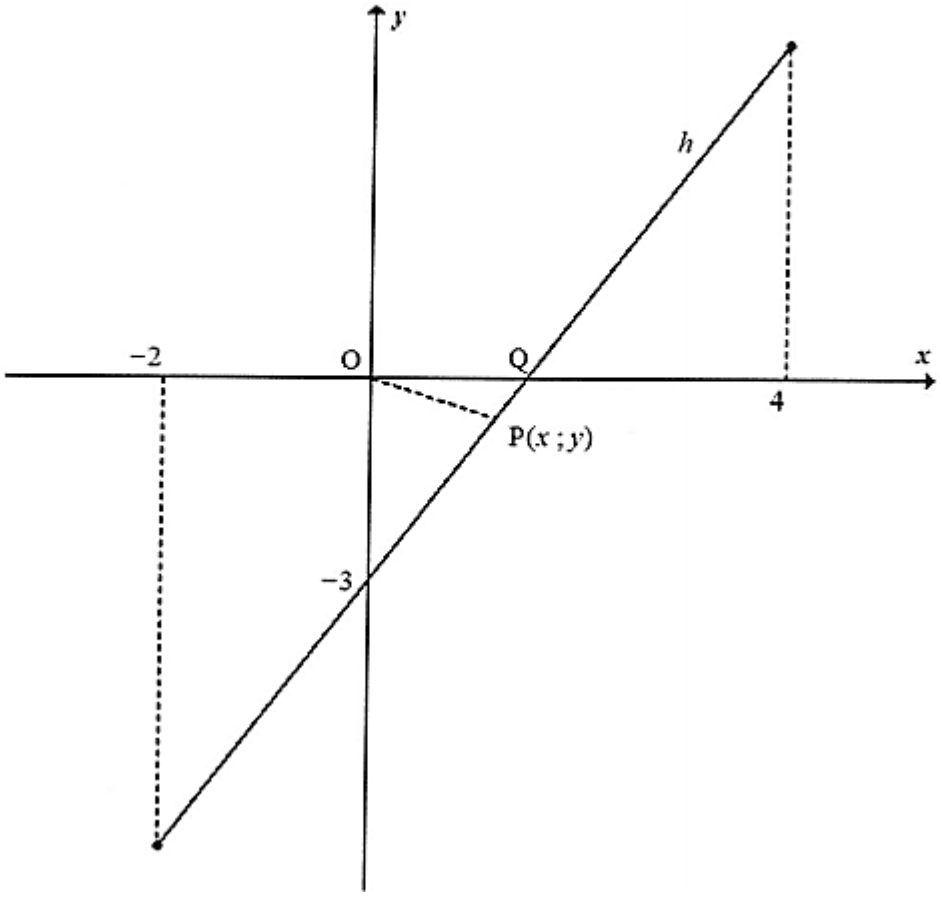
5.4 Write down the domain of g. (1)

5.5 Determine the values of x for which $f(x)\geq -3$. (3)

[9]

NOVEMBER 2015
QUESTION 5

Given: $h(x) = 2x - 3$ for $-2 \le x \le 4$. The x-intercept of h is Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 P$(x ; y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \le x \le 4$.

5.6.1 Explain why f has a local minimum. (2)

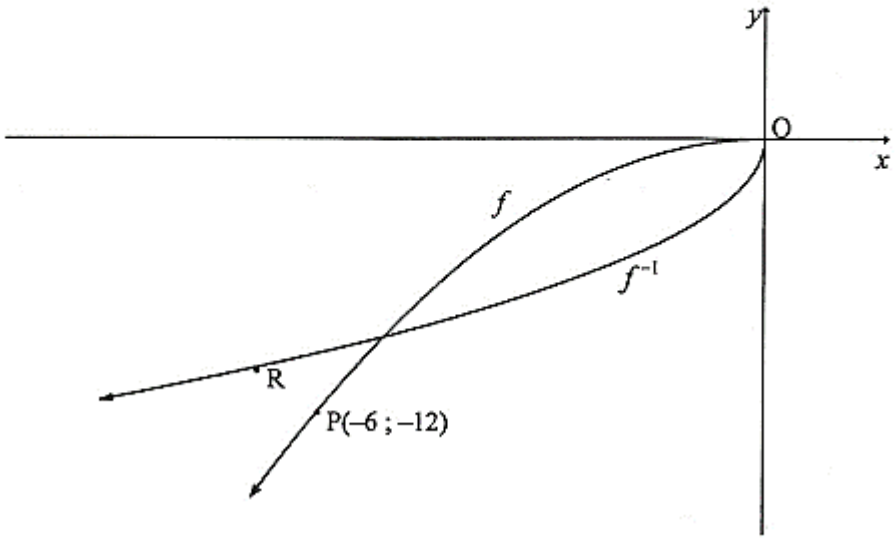
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

[19]

NOVEMBER 2018

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1 Is f^{-1} a function? Motivate your answer. (2)

4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 Write down the equation of f^{-1} in the form $y = ...$ (3)
[8]

MARCH 2016

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = \ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)
[8]

Section 5: Combinations

Activities

QUESTION 1

Given the following two functions:

$h(x) = \frac{1}{x} + 5$ and $g(x) = x + 5$

1.1 Determine the x-intercept of h.

1.2 Sketch neat graphs of h and g on the same set of axes. Clearly show all intercepts with the axes as well as asymptotes.

1.3 Write down the equation of the vertical asymptote of h.

1.4 Determine the coordinates of the points of intersection of h and g. Show all calculations.

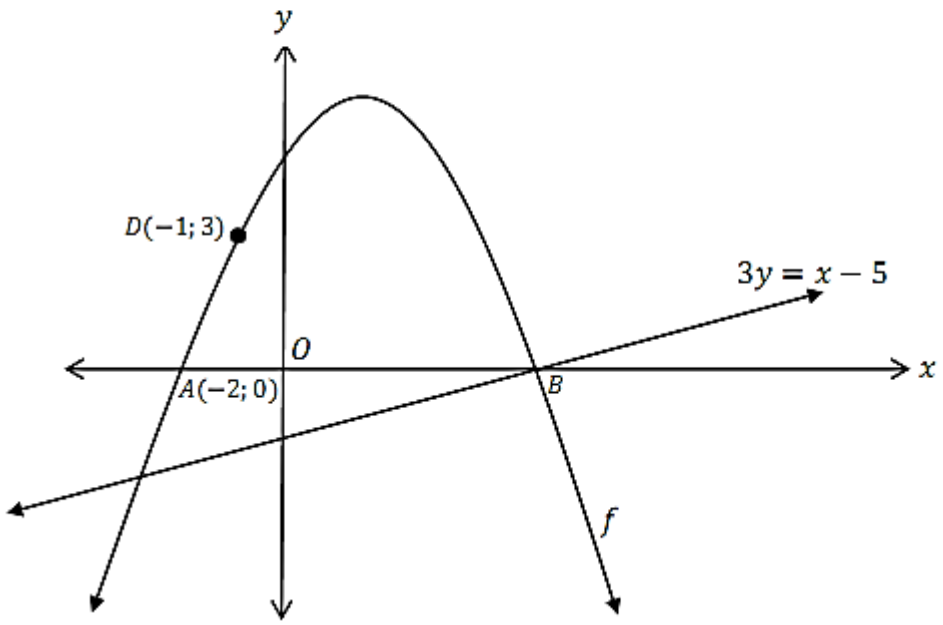
1.5 Write down the equation of f if f is the reflection of g about the line $y = 4$.

1.6 Write down the equation if h is translated so that $(-2; 3)$ is the new point of intersection of the asymptotes.

QUESTION 2

The sketch below shows the graph of the function $f(x) = ax^2 + bx + c$.

The straight line with equation $3y = x - 5$ intersects f at B. The points $A(-2; 0)$ and B are the x-intercepts of f. Point $D(-1; 3)$ is a point on f.



2.1 Determine the coordinates of B.

2.2 Determine the equation of f.

2.3 Determine the coordinates of the turning point of f.

2.4 Point E is a point on the straight line so that DE is parallel to the y-axis. Determine the length of DE.

2.5 Calculate the average gradient between B and D.

2.6 For which value(s) of x is $x \times f(x) \geq 0$?

QUESTION 3

Given $f(x) = 2.3^x - 1$ and $g(x) = \frac{4}{x+3}$

3.1 Write down the equation of the asymptote of f.

3.2 Determine the y-intercept of f. (Give your answer in co-ordinate form)

3.3 Find ONE other point on the graph of f.

3.4 Sketch the graph of f.

3.5 What is the range of f?

3.6 Write down the equations of the asymptotes of g.

3.7 Determine the y-intercept of g.

3.8 Write down the equation of the axis of symmetry of g.

3.9 Sketch the graph of g on its own set of axes.

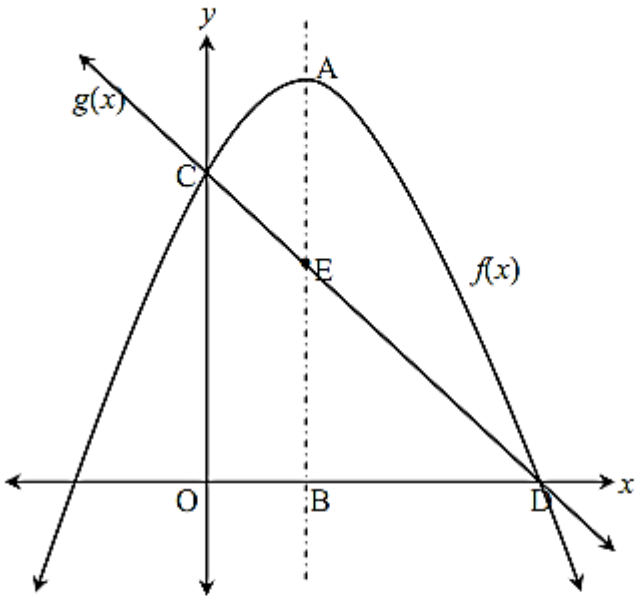
3.10 Determine the average gradient of g between the points x = -2 and x = 1.

QUESTION 4

4.1 4.1.1 Find the equation of the parabola that cuts the x-axis at -2 and 3, and the y-axis at the point, (0 ; -12).

4.1.2 Write your answer in QUESTION 4.1.1 in the form $y = a(x-p)^2 + q$.

4.2 In the diagram below, $f(x) = -x^2 + x + 12$ and $g(x) = mx + c$



4.2.1 Determine the coordinates of C and D.

4.2.2 Determine the values of m and c and hence determine the equation of $g(x)$.

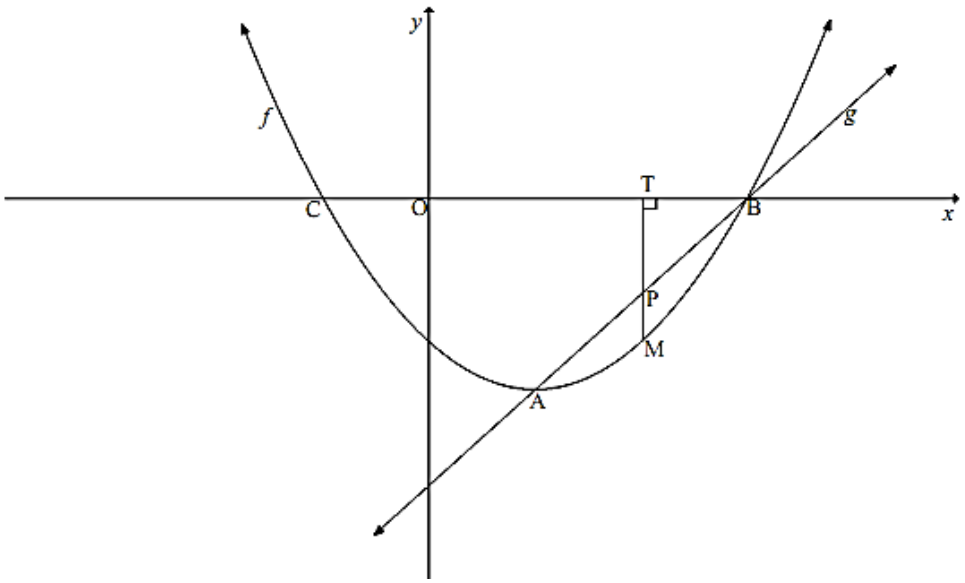
4.2.3 If OB $= \frac{1}{2}$, find the length of AE.

4.2.4 For which values of x is $f(x)$ decreasing?

4.2.5 Write down the range of $f(x)$.

QUESTION 5

The graph of $f(x) = x^2 + bx + c$ and the straight line g are sketched below. A and B are the points of intersection of f and g. A is also the turning point of f. The graph of f intersects the x-axis at B(3 ; 0) and C. The axis of symmetry of f is $x = 1$.



5.1 Write down the coordinates of C.

5.2 Determine the equation of f in the form $y = x^2 + bx + c$.

5.3 Determine the range of f.

5.4 Calculate the equation of g in the form $y = mx + c$.

5.5 For which values of x will:

5.5.1 $f(x) \geq 0$

5.5.2 $\frac{f(x)}{g(x)} > 0$

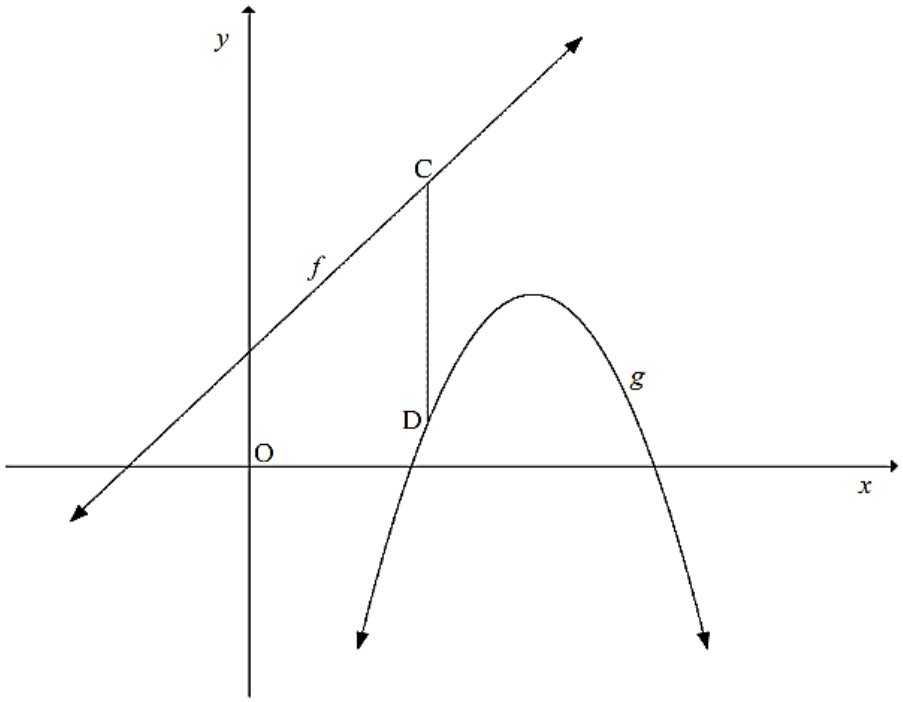
5.5.3 $x.f(x) > 0$

5.6 For what values of p will $x^2 - 2x = p$ have non-real roots?

5.7 T is a point on the x-axis and M is a point on f such that TM $\perp$ x-axis. TM intersects g at P. Calculate the maximum length of PM.

QUESTION 6

The sketch below shows the graphs of $f(x) = 2x + 3$ and $g(x) = -2x^2 + 14x + k$.
C is any point on f and D any point on g, such that CD is parallel to the y-axis.
k is a value such that C lies above D.



6.1 Write down a simplified expression for the length of CD in terms of x and k.

6.2 If the minimum length of CD is 5, calculate the value of k.

QUESTION 7

$f(x) = -2x^2 + 2$ and $g(x) = 2^x + 1$ are the defining equations of graphs f and g respectively.

7.1 Write down an equation for the asymptote of g.

7.2 Sketch the graphs of f and g on the same set of axes, clearly showing ALL intercepts with the axes, turning points and asymptotes.

7.3 Write down the range of f.

7.4 Determine the maximum value of h if $h(x) = 3^{f(x)}$.

7.5 What transformation does the graph of $y = f(x)$ undergo in order to obtain the graph of $y = 2x^2 - 2$?

QUESTION 8

Given: $h(x) = 4^x$ and $f(x) = 2(x-1)^2 - 8$.

8.1 Sketch the graphs of h and f on the diagram sheet provided. Indicate ALL intercepts with the axes and any turning points.

8.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes.

8.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph.

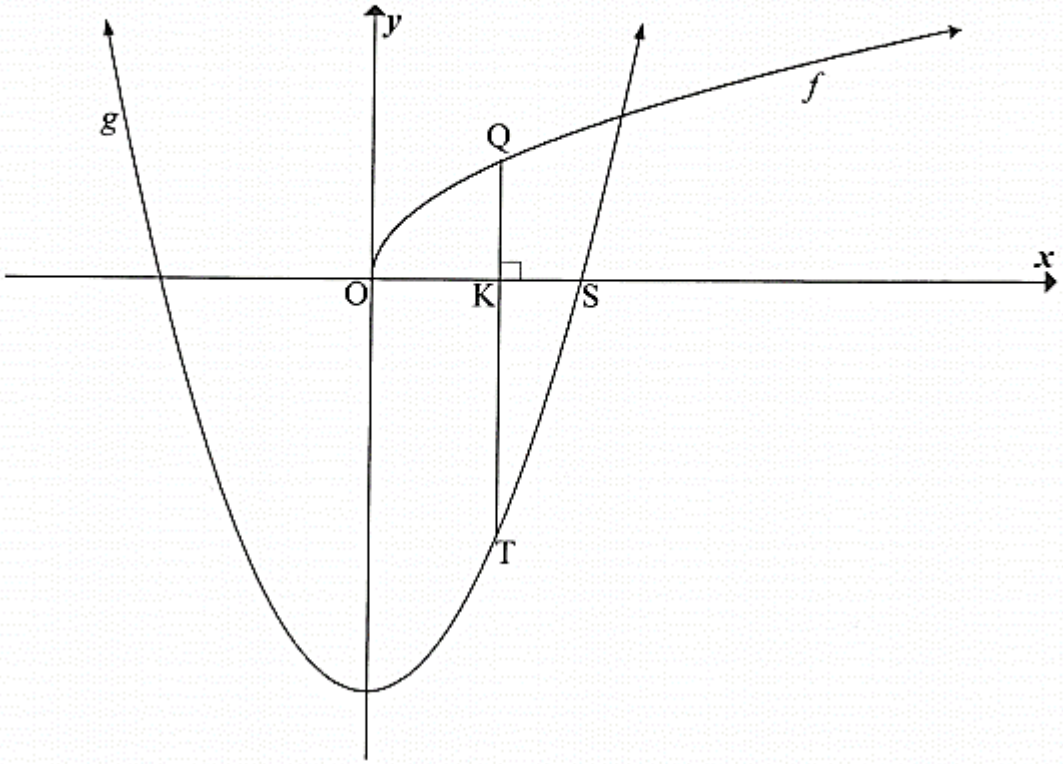
8.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$.

Past Papers

NOVEMBER 2014

QUESTION 6

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



6.1 Determine the x-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3 6.3.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K. (3)

6.3.2 Calculate the maximum length of QT. (6)

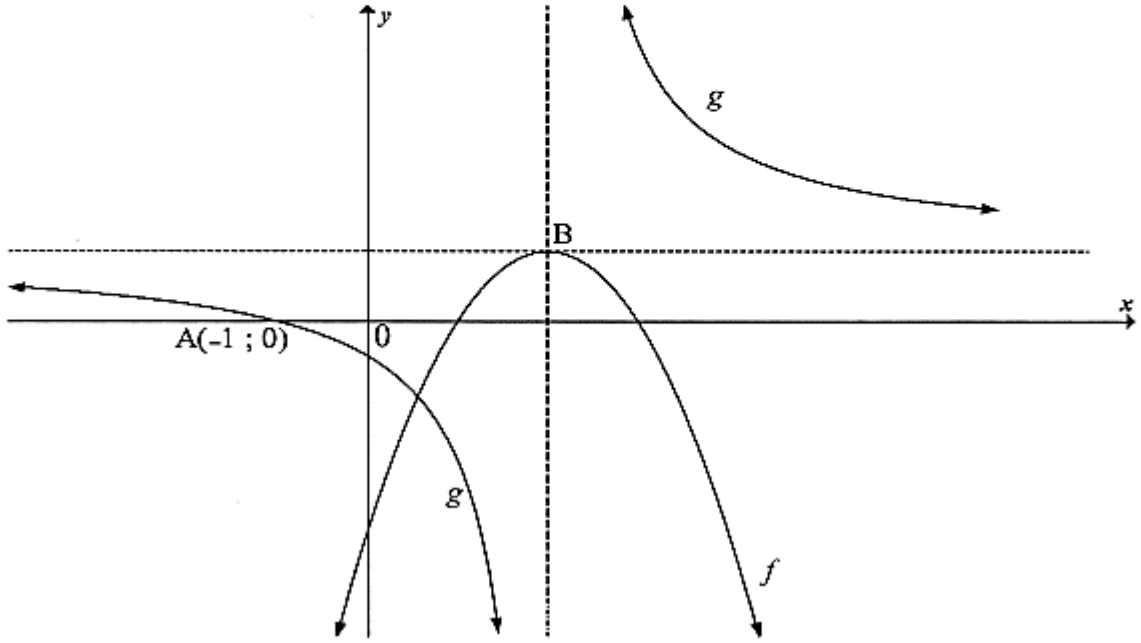
[13]

NOVEMBER 2016

QUESTION 5

Sketched below is the parabola f with equation $f(x) = -x^2 + 4x - 3$ and a hyperbola g, with equation $(x - p)(y + t) = 3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- A$(-1\,;\,0)$ is the x-intercept of g.



5.1 Show that the coordinates of B are $(2\,;\,1)$ (2)

5.2 Write down the range of f. (1)

5.3 For which value(s) of x will $g(x) \geq 0$? (2)

5.4 Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x + 4)$ (1)

5.5 Determine the values of p and t. (4)

5.6 Write down the values of x for which $f(x).g'(x) \geq 0$ (4)

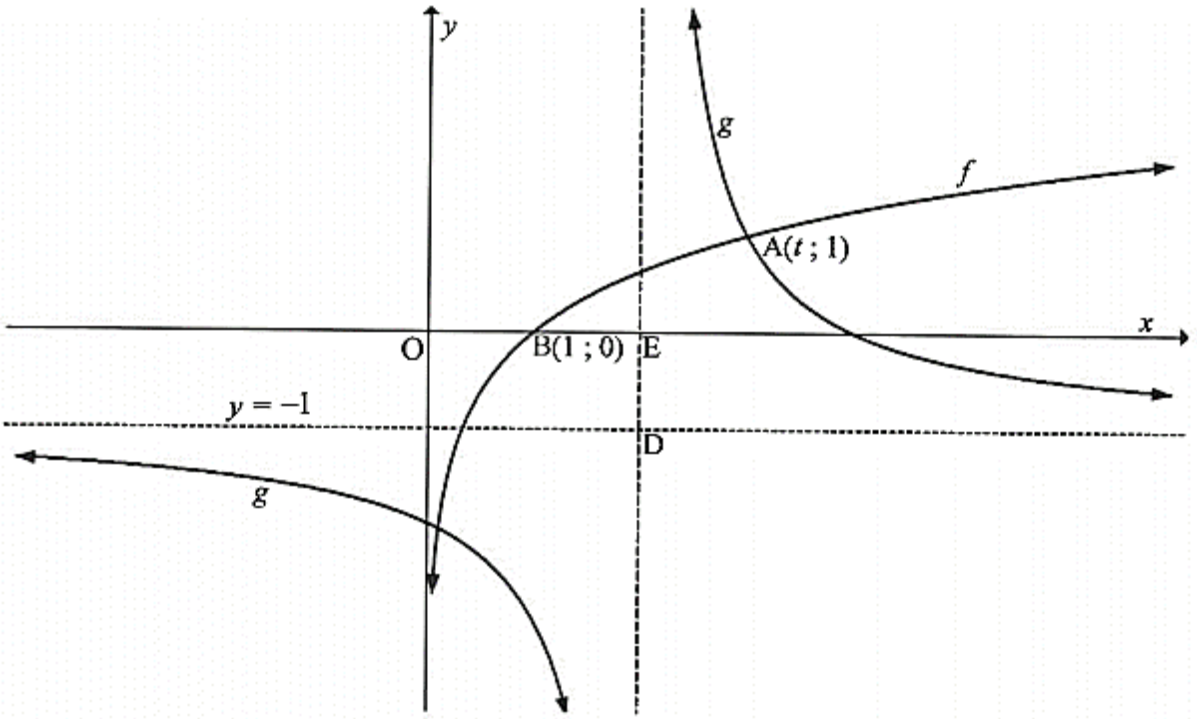
[14]

NOVEMBER 2017

QUESTION 5

The diagram below shows the graphs of $g(x)=\frac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$ (2)

5.5 For which values of x will $f^{-1}(x)<3$? (2)

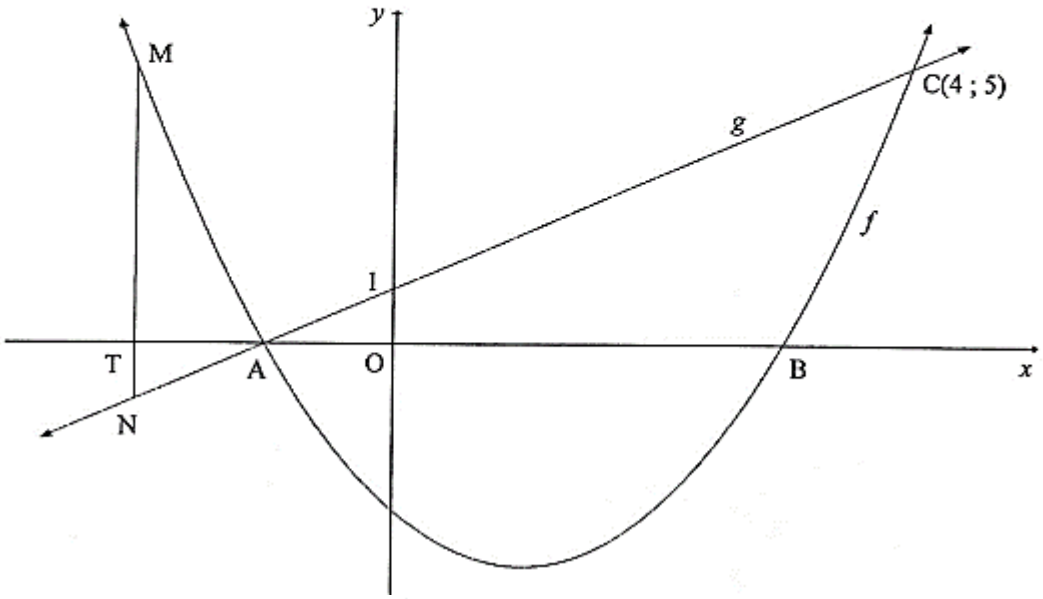
5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

NOVEMBER 2018

QUESTION 6

In the diagram below, A and B are the x-intercepts of the graph of $f(x)=x^2-2x-3$.
A straight line, g, through A cuts f at C(4 ; 5) and the y-axis at (0 ; 1).
M is a point on f and N is a point on g such that MN is parallel to the y-axis.
MN cuts the x-axis at T.



6.1 Show that $g(x)=x+1$. (2)

6.2 Calculate the coordinates of A and B. (3)

6.3 Determine the range of f. (3)

6.4 If MN = 6:

6.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working. (4)

6.4.2 Hence, write down the coordinates of N. (2)

6.5 Determine the equation of the tangent to f drawn parallel to g. (5)

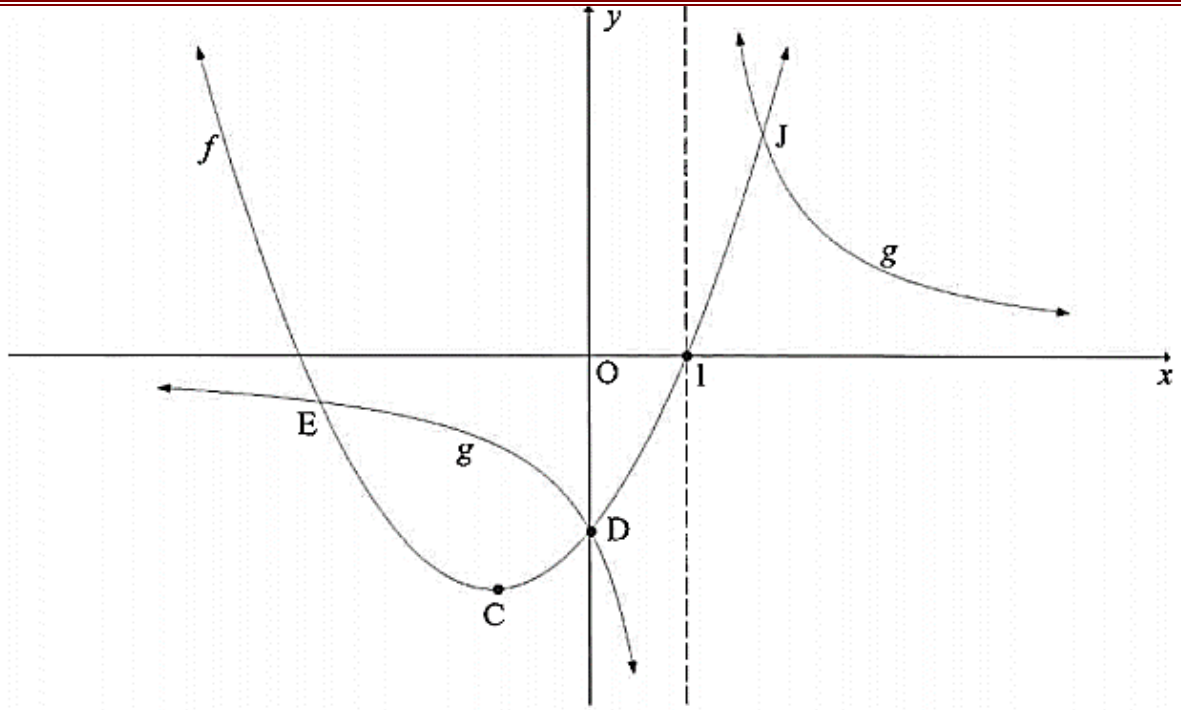
6.6 For which value(s) of k will $f(x)=x^2-2x-3$ and $h(x)=x+k$ NOT intersect? (1)
[20]

NOVEMBER 2019

QUESTION 4

Below are the graphs of $f(x)=x^2+bx-3$ and $g(x)=\dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at (1 ; 0).
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



4.1 Write down the value of p. (1)

4.2 Show that $a = 3$ and $b = 2$. (3)

4.3 Calculate the coordinates of C. (4)

4.4 Write down the range of f. (2)

4.5 Determine the equation of the line through C that makes an angle of 45° with the positive x-axis. Write your answer in the form $y = \ldots$ (3)

4.6 Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer. (2)

4.7 The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q. (4)

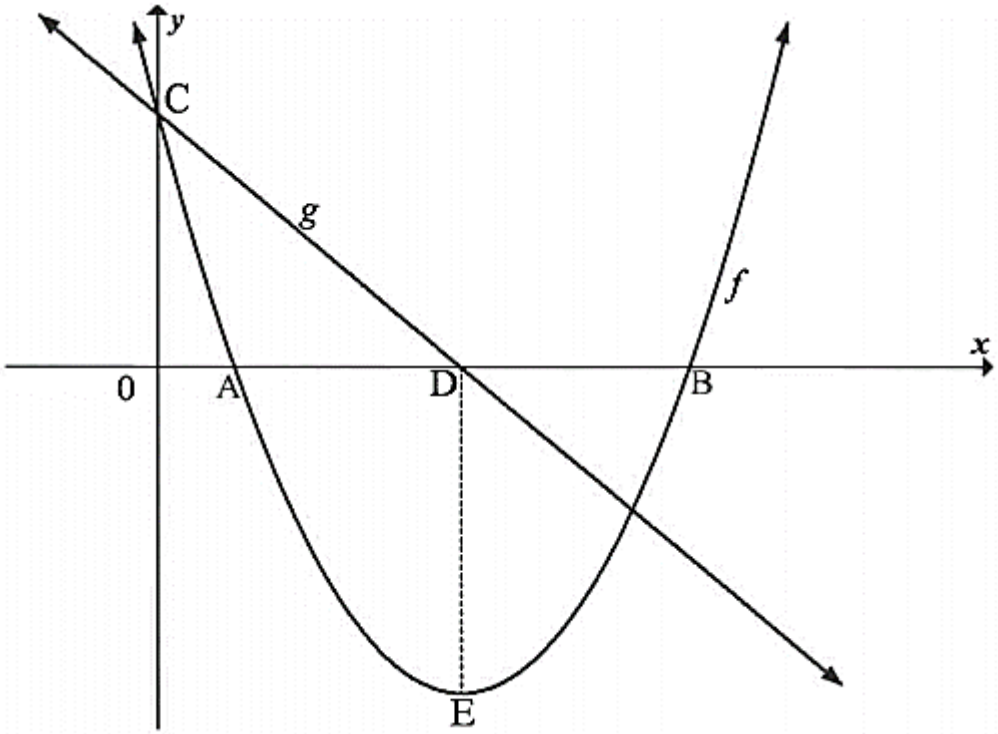
[19]

MARCH 2018

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



4.1 Write down the coordinates of E. (2)

4.2 Calculate the coordinates of A. (3)

4.3 M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M. (3)

4.4 Determine the equation of g in the form $y = mx + c$. (3)

4.5 Write down the equation of g^{-1} in the form $y = \ldots$ (3)

4.6 For which values of x will $x(f(x)) \leq 0$? (4)

[18]

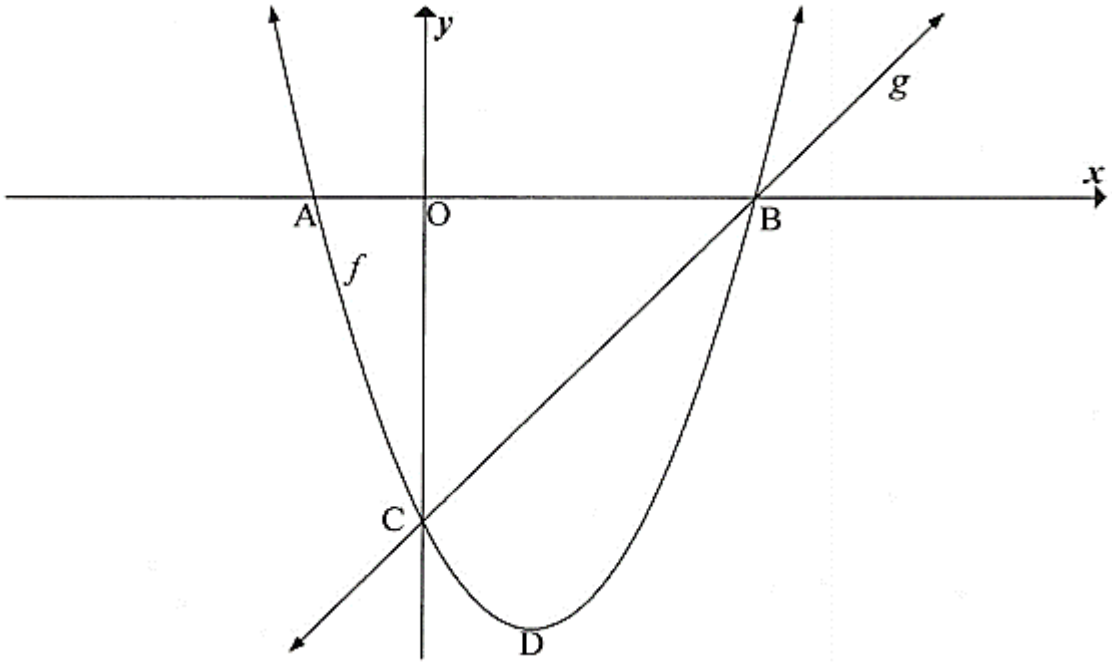
MARCH 2017

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1 Determine the coordinates of C. (1)

5.1.2 Calculate the length of AB. (4)

5.1.3 Determine the coordinates of D. (2)

5.1.4 Calculate the average gradient of f between C and D. (2)

5.1.5 Calculate the size of $O\hat{C}B$ (2)

5.1.6 Determine the values of k for which $f(x)=k$ will have two unequal positive real roots. (3)

5.1.7 For which values of x will $f'(x).f''(x)>0$? (3)

5.2 The graph of a parabola f has x-intercepts at $x=1$ and $x=5$. $g(x)=4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

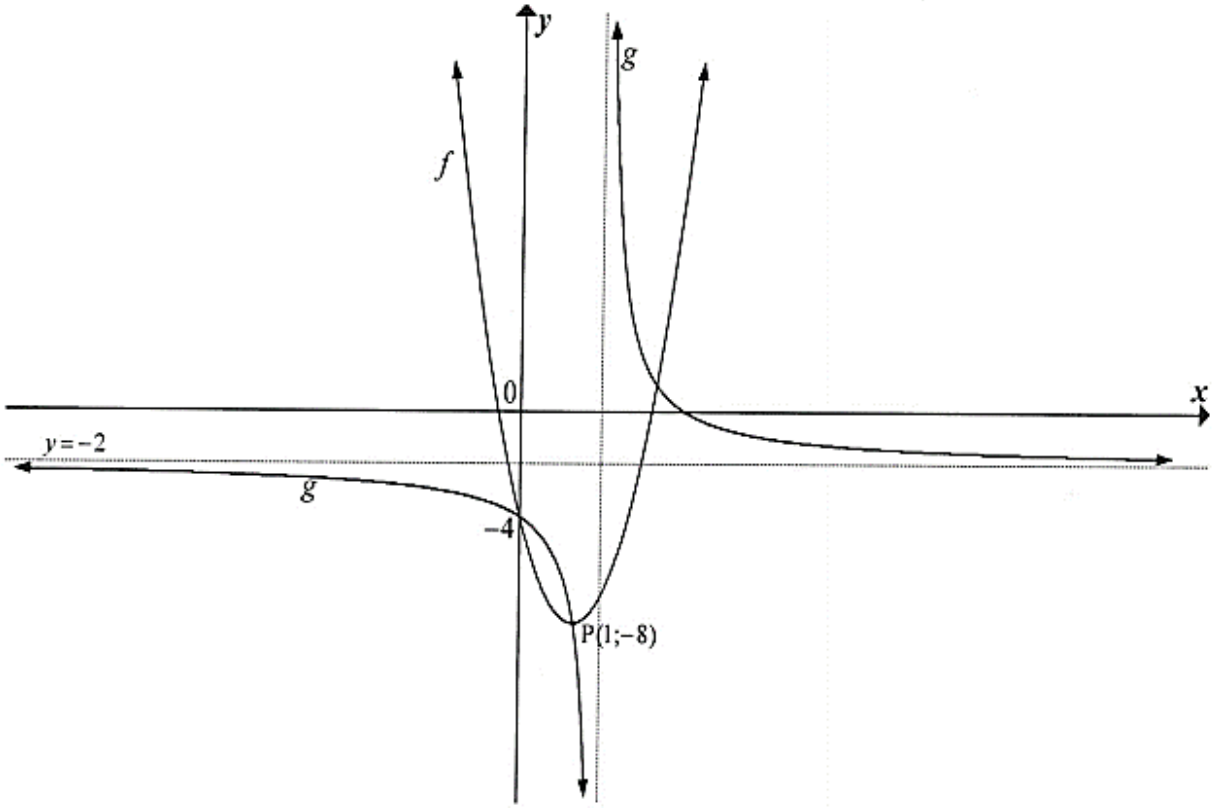
[22]

MARCH 2016

QUESTION 5

The graphs of the functions $f(x)=a(x+p)^2+q$ and $g(x)=\frac{k}{x+r}+d$ are sketched below.

Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of f. The horizontal asymptote of g is $y=-2$.



5.1 Calculate the values of a, p and q. (4)

5.2 Calculate the values of k, r and d. (6)

5.3 Determine the value(s) of x in the interval $x \leq 1$ for which $g(x) \geq f(x)$. (2)

5.4 Determine the value(s) of k for which $f(x) = k$ has two, unequal positive roots. (2)

5.5 Write down an equation for the axis of symmetry of g that has a negative gradient. (3)

5.6 The point P is reflected in the line determined in QUESTION 5.5 to give the point Q. Write down the coordinates of Q. (2)

[19]

SECTION D: Functions - Selected Questions

NOVEMBER 2020

QUESTION 4

4.1 Given: $h(x)=\frac{-3}{x-1}+2$

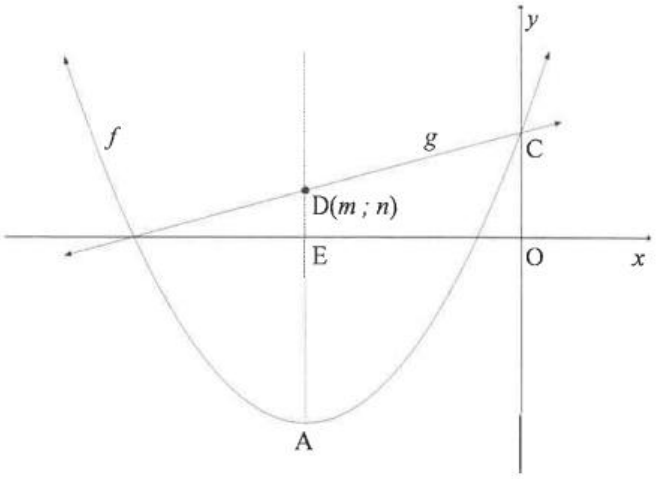
4.1.1 Write down the equations of the asymptotes of h. (2)

4.1.2 Determine the equation of the axis of symmetry of h that has a negative gradient. (2)

4.1.3 Sketch the graph of h, showing the asymptotes and the intercepts with the axes. (4)

4.2 The graphs of $f(x)=\frac{1}{2}(x+5)^2-8$ and $g(x)=\frac{1}{2}x+\frac{9}{2}$ are sketched below.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at D$(m\,;n)$.
- C is the y-intercept of f and g.



4.2.1 Write down the coordinates of A. (2)

4.2.2 Write down the range of f. (1)

4.2.3 Calculate the values of m and n. (3)

4.2.4 Calculate the area of OCDE. (3)

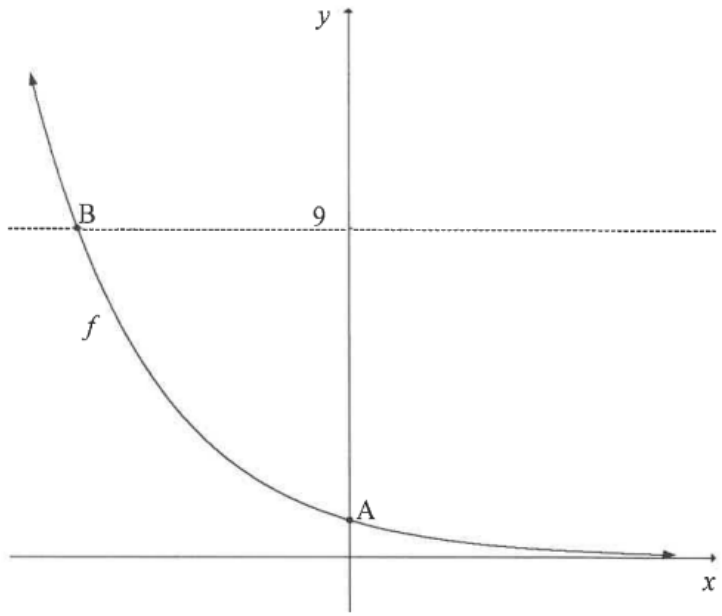
4.2.5 Determine the equation of g^{-1}, the inverse of g, in the form $y=\ldots$ (2)

4.2.6 If $h(x)=g^{-1}(x)+k$ is a tangent to f, determine the coordinates of the point of contact between h and f. (4)

[23]

QUESTION 5

The graph of $f(x)=3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y=9$.



5.1 Write down the coordinates of A. (1)

5.2 Determine the coordinates of B. (3)

5.3 Write down the domain of f^{-1}. (2)

5.4 Describe the translation from f to $h(x)=\frac{27}{3^x}$. (3)

5.5 Determine the values of x for which $h(x)<1$. (3)
[12]

JUNE 2021

QUESTION 4

The lines $y=x+1$ and $y=-x-7$ are the axes of symmetry of the function $f(x)=\frac{-2}{x+p}+q$.

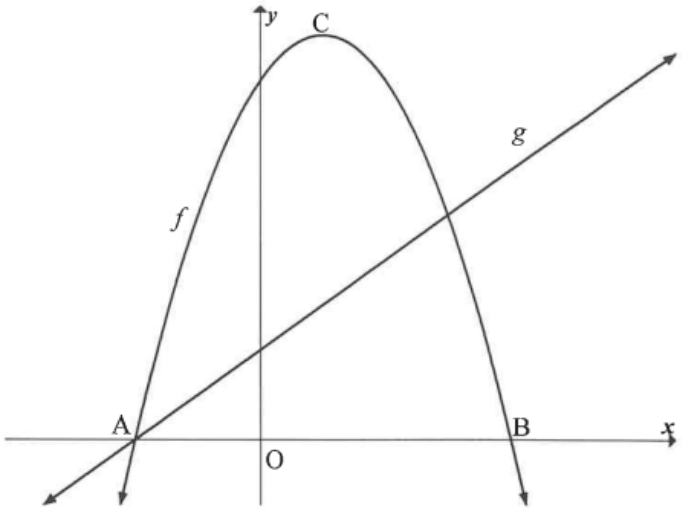
4.1 Show that $p=4$ and $q=-3$. (4)

4.2 Calculate the x-intercept of f. (2)

4.3 Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes. (4)
[10]

QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B. (3)

5.2 Determine the coordinates of C, the turning point of f. (2)

5.3 Write down the range of f. (1)

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$.
Determine the values of p and q. (3)

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$? (2)

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)
[18]

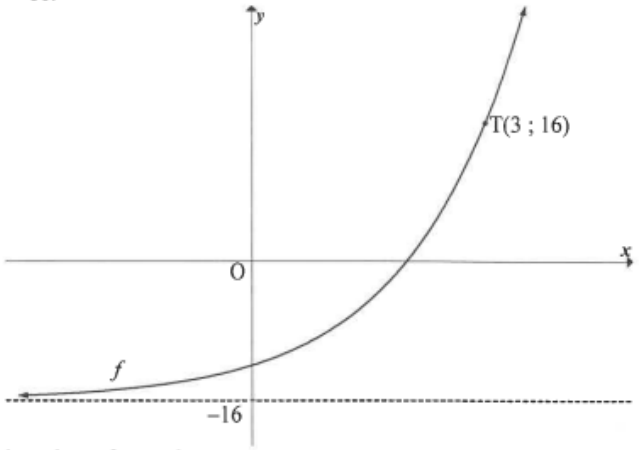
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q. (4)
[8]

NOVEMBER 2021

QUESTION 5

Given: $f(x) = \dfrac{-1}{x-3} + 2$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Write down the domain of f. (1)

5.3 Determine the coordinates of the x-intercept of f. (2)

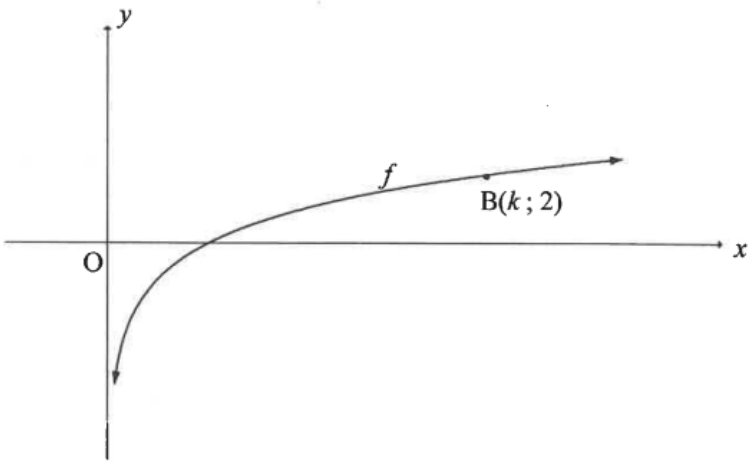
5.4 Write down the coordinates of the y-intercept of f. (2)

5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)
[10]

QUESTION 6

The graph of $f(x) = \log_4 x$ is drawn below.

B(k ; 2) is a point on f.



6.1 Calculate the value of k. (2)

6.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

6.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

6.4 For which values of x will $x.f^{-1}(x) < 0$? (2)

[8]

JUNE 2022

QUESTION 4

The graph of $g(x) = a\left(\frac{1}{3}\right)^x + 7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x) = \left(\frac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y = \ldots$ (2)

[9]

QUESTION 5

Consider: $g(x)=\frac{a}{x+p}+q$

The following information of g is given:

- Domain: $x \in \mathrm{R};\ x \neq -2$
- x-intercept at K(1 ; 0)
- y-intercept at $\mathrm{N}\left(0\ ;-\frac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x)=\frac{-3}{x+2}+1$ (6)

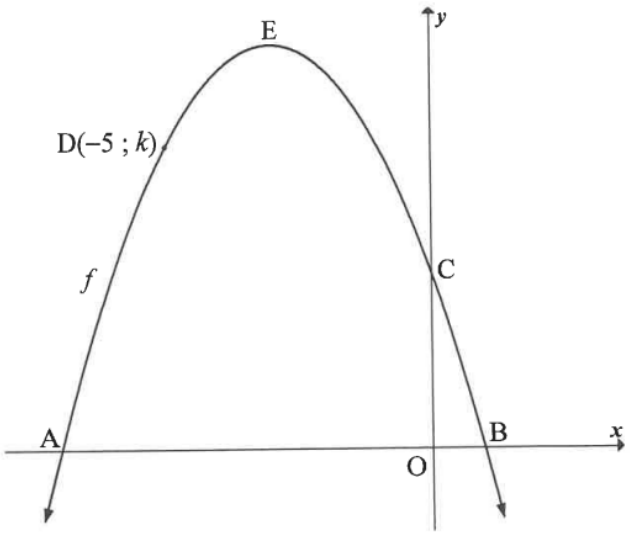
5.2 Write down the range of g. (1)

5.3 Determine the equation of h, the axis of symmetry of g, in the form $y=mx+c$, where $m>0$. (3)

5.4 Write down the coordinates of K^{I}, the image of K reflected over h. (2)
[12]

QUESTION 6

The sketch below shows the graph of $f(x)=-x^2-6x+7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D(–5 ; k) is a point on f.



6.1 Calculate the coordinates of E, the turning point of f. (3)

6.2 Write down the value of k. (1)

6.3 Determine the equation of the straight line passing through C and D. (4)

6.4 A tangent, parallel to CD, touches f at P. Determine the coordinates of P. (4)

6.5 For which values of x will $f(x)-12>0$? (2)
[14]

Appendix A: Examination Guidelines

ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as learners should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $af(x)$ and $x=f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in Paper 2.

Appendix B: Information Sheet

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

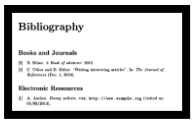
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$


Bibliography

Books and Journals

Electronic Resources

1. **CURRICULUM AND ASSESSMENT POLICY STATEMENT(CAPS) FET PHASE GRADES (10 – 12) MATHEMATICS**
2. **MATHEMATICS GR12 EXAM GUIDELINES**
3. **NOVEMBER 2014 – 2015 GR 11 NATIONAL AND EASTERN CAPE**
4. **MATHEMATICS NOVEMBER AND MAR/JUN NATIONAL PAPERS GRADE 12 (2008 – 2022)**

OUTCOMES REACHED	YES	NO
• **Introduce a more formal definition of a function and extend Grade 11 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear, quadratic and some cubic polynomial functions, exponential and logarithmic functions, and some rational functions.**		
• **The inverses of prescribed functions and be aware of the fact that, in the case of many-to-one functions, the domain has to be restricted if the inverse is to be a function.**		
• **Problem solving and graph work involving the prescribed functions (including the logarithmic function)**		