JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: FINANCE, GROWTH AND DECAY

Solutions Manual

FINANCE, GROWTH AND DECAY

QUESTION 1

1.1 $i_{\text{eff}} = \left(1+\dfrac{0,15}{12}\right)^{12} - 1$

$\therefore i_{\text{eff}} = 0,1607545177$

1.2 $A = 65000\left(1+i_{\text{eff}}\right)^{3}$

$\therefore A = R101\ 656,35$

Alternatively:

$$A = 65000\left(1+\dfrac{0,15}{12}\right)^{36}$$

$\therefore A = R101\ 656,35$

1.3 $A = 65000\left(1+i_{\text{eff}}\right)^{3} + 10000\left(1+i_{\text{eff}}\right)^{2}$

$\therefore A = R115\ 129,86$

Alternatively:

$$A = 65000\left(1+\dfrac{0,15}{12}\right)^{36} + 10000\left(1+\dfrac{0,15}{12}\right)^{24}$$

$\therefore A = R115\ 129,86$

or

$$A = \left[65000\left(1+i_{\text{eff}}\right)^{1} + 10000\right]\left(1+i_{\text{eff}}\right)^{2}$$

$\therefore A = R115\ 129,86$

or

$$A = \left[65000\left(1+\dfrac{0,15}{12}\right)^{12} + 10000\right]\left(1+\dfrac{0,15}{12}\right)^{24}$$

$\therefore A = R115\ 129,86$

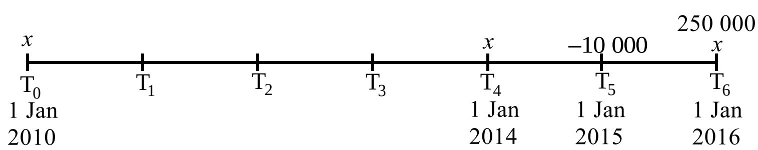
QUESTION 2

2.1 $3\ 200 = P(1 - 0,12\times 7)$

$\therefore 3\ 200 = P(0,16)$

$P = R20\ 000$

2.2.1



2.2.2 $i = \dfrac{0,06}{4} = 0,015$

$250\ 000 = x(1,015)^{24} + x(1,015)^{8} - 10\ 000(1,015)^{4} + x$

$\therefore x = R73\ 288,52$

QUESTION 3

3.1

$A = P(1+i)^n$

$23000 = 1570(1.12)^n$

$(1.12)^n = 14{,}64968153..$

$n\log(1{,}12) = \log 14{,}64968153..$

$n = 23{,}69$ years (23,68701...)

or n = 24 years

or n = 23 years 8 months

or n = 23,7 years

OR

$A = P(1+i)^n$

$23000 = 1570(1 + \frac{12}{100})^n$

$(1.12)^n = 14{,}64968153..$

$n\log(1{,}12) = \log 14{,}64968153..$

$n = 23{,}69$ years (23,68701...)

or n = 24 years

or n = 23 years 8 months

or n = 23,7 years

3.2.1

$A = P(1+i)^n$

$= 800000(1.08)^5$

$= R1175462{,}46$

$\therefore R1175462{,}46 - R200\ 000$

$= R975462{,}46$

Some calculators give R 975 462,50

3.2.2

$F = \frac{x[(1+i)^n - 1]}{i}$

$975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01}$

$\frac{975462{,}46 \times 0.01}{[1{,}01]^{60} - 1} = x$

$x = R\ 11944{,}00$

OR

$975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01}$

$975462{,}46 = 81{,}66966986x$

$x = R\ 11944{,}00$

3.2.3 $Service = [5000(1{,}01)^{48} + 5000(1{,}01)^{36} + 5000(1{,}01)^{24} + 5000(1{,}01)^{12} + 5000]$

$= 32197{,}77$

$$975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$$

$975462{,}46 = 81{,}66966986x - 32197{,}77$

$x = R\,12338{,}24$

OR

$$Service = \frac{5000[1{,}01^{60} - 1]}{1{,}01^{12} - 1}$$

$= 32197{,}77$

$$975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$$

$975462{,}46 = 81{,}66966986x - 32197{,}77$

$x = R\,12338{,}24$

OR

Present Value payment of R 5000

$= 5000\{(1{,}01)^{-12} + (1{,}01)^{-24} + (1{,}01)^{-36} + (1{,}01)^{-48} + (1{,}01)^{-60}\}$

$$= 5000(1{,}01)^{-12}\left\{\frac{1-(1{,}01)^{-60}}{1-(1{,}01)^{-12}}\right\}$$

= R 17 723,25

Present Value of the sinking fund

$= 975462{,}46(1{,}01)^{-60}$

= R 536 942,94

Total Value of sinking fund

= R 17 723,25 + R 536 942,94

= R 554 666,19

$$\therefore\ 554666{,}19 = x\left\{\frac{1-(1{,}01)^{-60}}{0{,}01}\right\}$$

x = R 12 338,24

OR

$(1 + i_{eff}) = (1 + 0{,}01)^{12}$

$i_{eff} = 0{,}12682503......$

$P(1+i)^n$

$$= 5000\frac{(1{,}12682503)^5 - 1}{0{,}12682503}$$

$= 32197{,}77$

$$975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - 32197{,}77$$

$975462{,}46 = 81{,}66966986x - 32197{,}77$

$x = R\,12338{,}24$

OR

$$5000 = \frac{x[(1{,}01)^{12} - 1)}{0{,}01}$$

$$x = \frac{5000 \times 0{,}01}{1{,}01^{12} - 1}$$

$x = 394{,}24$

So monthly deposit must be increased by R 394,24

New monthly deposit

= R 11 944 + R 394,24

= R 12 338,24

QUESTION 4

4.1 Depreciation value = $7\ 200(1-0{,}25)^3$

= R3 037,50

4.2.1

$$300\ 000 = \frac{5\ 000\left[1-(1{,}015)^{-n}\right]}{0{,}015}$$

$$4\ 500 = 5\ 000 - 5\ 000(1{,}015)^{-n}$$

$$5\ 000(1{,}015)^{-n} = 500$$

$$(1{,}015)^{-n} = 0{,}1 \quad \text{or} \quad (1{,}015)^{n} = 10$$

$$-n = \frac{\log 0{,}1}{\log 1{,}015}$$

$$n = 154{,}65$$

Number of payments = 155

4.2.2 Balance outstanding

$$= 300\ 000\left(1+\frac{0.18}{12}\right)^{154} - \frac{5\ 000\left[\left(1+\frac{0.18}{12}\right)^{154} - 1\right]}{\frac{0.18}{12}}$$

= R3 230,50

4.2.3 Amount paid in last month

$$= 3\ 230{,}50\left(1+\frac{0.18}{12}\right)$$

= R3 278,96

4.2.4 Total repaid = (154 × 5 000) + 3 278,96 = R773 278,96

QUESTION 5

5.1

$$2000\left(1+\frac{i}{12}\right)^{18} = 2860$$

$$\left(1+\frac{i}{12}\right)^{18} = 1{,}43$$

$$1+\frac{i}{12} = 1{,}020069541...$$

$$\frac{i}{12} = 0{,}020069541...$$

$$i = 0{,}24083...$$

$$i = 24{,}08\%$$

5.2

$$F_v = \frac{100\left[\left(1+\frac{0{,}08}{12}\right)^{12} - 1\right]}{\frac{0{,}08}{12}}$$

$$= R1\,244{,}99$$

The accumulated amount is less than R1 300 required to buy the bike. Farouk will not be able to buy the bike on 1 January 2009.

QUESTION 6

6.1 Loan = $125000 - \frac{15}{100} \times 125000$

Loan = R 106 250

OR

Loan = 0,85 × 125 000

Loan = R 106 250

6.2

$$106250 = x\left[\frac{1-\left(1+\frac{0{,}125}{12}\right)^{-6\times 12}}{\frac{0{,}125}{12}}\right]$$

$$1106{,}770833 = x\left(1-\left(1+\frac{0{,}125}{12}\right)^{-6\times 12}\right)$$

x = R 2104,94

QUESTION 7

7.1 R450 000

7.2

$$A = P(1-i)^n$$

$$f(x) = 450000(1-i)^x$$

$$243\ 736{,}90 = 450000(1-i)^4$$

$$i = 1 - \sqrt[4]{\frac{243\ 736{,}90}{450000}}$$

$$i = 0{,}1421$$

The rate of depreciation is 14,21 % p.a.

7.3 At T :

$$A = P(1+i)^n$$

$$g(x) = 450000(1+i)^x$$

$$a = 450000(1+0{,}081)^4$$

$$= R614490{,}66$$

7.4 Future Value = R614 490, 66 – R243 736 ,90

= R370 753,76

Let x be the value of monthly payment

$$F_v = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$370753{,}76 = \frac{x\left[\left(1+\frac{0.062}{12}\right)^{36} - 1\right]}{\frac{0.062}{12}}$$

$$x = R9397{,}11$$

QUESTION 8

8.1 $A = P(1+i)^n$

$$1711{,}41 = 1430{,}77\left(1+\frac{i}{12}\right)^{18}$$

$$\left(1+\frac{i}{12}\right)^{18} = 1{,}196146131\ldots$$

$$1+\frac{i}{12} = 1{,}009999937\ldots$$

$$i = 0{,}1199992431\ldots$$

OR

$$\left[\frac{1711{,}41}{1430{,}77}\right]^{\frac{1}{18}}$$

$$= 1{,}00999\ldots$$

$$\therefore i = 12(1{,}01-1)$$

$$= 0{,}12$$

$$= 12\%$$

Rate = 12, 00% p.a. compounded monthly.

8.2.1 $P_v = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$

$$800000 = \frac{10000\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\frac{0{,}14}{12}}$$

$$1-\left(1+\frac{0{,}14}{12}\right)^{-n} = \frac{14}{15} \quad (= 0{,}933333)$$

$$\left(1+\frac{0{,}14}{12}\right)^{-n} = \frac{1}{15} \quad (= 0{,}06666666)$$

$$\log\left(1+\frac{0{,}14}{12}\right)^{-n} = \log\frac{1}{15}$$

$$-n\log\left(1+\frac{0{,}14}{12}\right) = \log\frac{1}{15} \qquad \left(-n = \frac{\log\frac{1}{15}}{\log\left(1+\frac{0{,}14}{12}\right)} = -233{,}47\right)$$

$n = 233{,}47$

$\therefore$ the loan will be paid off at the end of the 234^{th} month

OR

Balance outstanding after 233^{rd} month

$$= 800000\left(1+\frac{0{,}14}{12}\right)^{233} - \frac{10000\left[\left(1+\frac{0{,}14}{12}\right)^{233}-1\right]}{\frac{0{,}14}{12}}$$

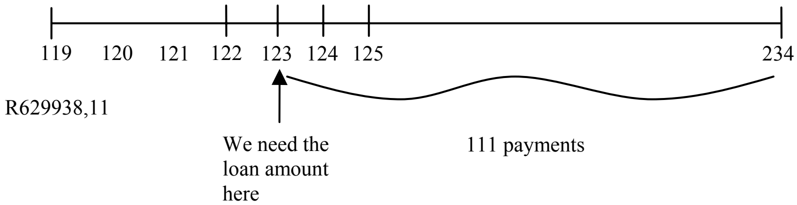
= R4 660,04 which is less than R10 000

Therefore the loan will be paid off after 234 months.

8.2.2 Balance Outstanding after 119 months

$$= 800000\left(1+\frac{0,14}{12}\right)^{119} - \frac{10000\left[\left(1+\frac{0,14}{12}\right)^{119} - 1\right]}{\frac{0,14}{12}}$$

= R629 938,11



Total Payable at the end of the 123rd month

$$= 629\,938{,}11\left(1+\frac{0,14}{12}\right)^{4}$$

= R659 853,68

New instalment:

$$659\,853{,}68 = \frac{x\left[1-\left(1+\frac{0,14}{12}\right)^{-111}\right]}{\frac{0,14}{12}}$$

x = R10 632,39

QUESTION 9

9.1 $A = P(1-i)^n$

$15000 = 24000(1-0{,}18)^n$

$0{,}625 = (0{,}82)^n$

$$n = \frac{\log 0{,}625}{\log 0{,}82}$$

= 2,37 years

9.2.1

$$130\,000\left(1+\frac{0,18}{12}\right)^{2}$$

$= 130000\,(1{,}015)^2$

= R 133 929,25

9.2.2(a)

$$133\,929{,}25 = \frac{x[1-(1{,}015)^{-54}]}{0{,}015}$$

$$2008{,}93875 = x[1-(1{,}015)^{-54}]$$

$$x = \text{R } 3636{,}36$$

OR

$$133\,929{,}25\left(1+\frac{0{,}18}{12}\right)^{54} = \frac{x\left[\left(1+\frac{0{,}18}{12}\right)^{54}-1\right]}{\frac{0{,}18}{12}}$$

$$299255{,}2087 = 82{,}29517136...x$$

$$x = \text{R } 3636{,}36$$

OR

$$130000\left(1+\frac{0{,}18}{12}\right)^{56} = \frac{x\left[\left(1+\frac{0{,}18}{12}\right)^{54}-1\right]}{\frac{0{,}18}{12}}$$

$$299255{,}2087 = 82{,}29517136...x$$

$$x = \text{R } 3636{,}36$$

9.2.2(b)

$$\text{Total} = 3636{,}36 \times 54$$

$$= \text{R}196\,363{,}66$$

9.2.3

$$130\,000 = \frac{x[1-(1{,}015)^{-54}]}{0{,}015}$$

$$1950 = x[1-(1{,}015)^{-54}]$$

$$x = \text{R } 3529{,}68$$

Total payments = R 3529,68 × 54
= R 190 602,72

OR

9.2.3

$$130\,000\left(1+\frac{0,18}{12}\right)^{54} = \frac{x\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$$

$$290475,5842 = 82,29517136...x$$

$$x = \text{R } 3529,68$$

Total payments = R 3529,68 × 54
= R 190 602,72

OR

$$130\,000\left(1+\frac{0,18}{12}\right)^{55} = \frac{x\left(1+\frac{0,18}{12}\right)\left[\left(1+\frac{0,18}{12}\right)^{54}-1\right]}{\frac{0,18}{12}}$$

$$290475,5842 = 82,29517136...x$$

$$x = \text{R } 3529,68$$

Total payments = R 3529,68 × 54
= R 190 602,72

9.2.4 R196 363,66 – R190 602,72
=R5 760,96

QUESTION 10

10.1 $F = P(1+i)^n$

$$= 4\,000\,000(1+0,06)^3$$

$$= \text{R}4\ 764\ 064$$

10.2.1

$$4\,000\,000 = \frac{30\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$$

$$\frac{4000000\times\left(\frac{0,06}{12}\right)}{30000} = 1-\left(1+\frac{0,06}{12}\right)^{-n}$$

$$\frac{1}{3} = \left(1+\frac{0,06}{12}\right)^{-n}$$

$$\log_{\left(1+\frac{0,06}{12}\right)}\frac{1}{3} = -n$$

$$n = 220,27$$

Therefore she will make 220 withdrawals of R30 000.

OR

$$4\,000\,000 = \frac{30\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$$

$$\frac{4000000\times\left(\frac{0,06}{12}\right)}{30000} = 1-\left(1+\frac{0,06}{12}\right)^{-n}$$

$$\frac{1}{3} = \left(1+\frac{0,06}{12}\right)^{-n}$$

$$\log\frac{1}{3} = -n\log\left(1+\frac{0,06}{12}\right)$$

$$n = 220,27$$

Therefore she will make 220 withdrawals of R30 000.

10.2.2

$$4\,000\,000 = \frac{20\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$$

$$0 = \left(1+\frac{0,06}{12}\right)^{-n}$$

She can make as many withdrawals as she pleases.

QUESTION 11

$$\left(1+\frac{0,08}{12}\right)^{12} = \left(1+\frac{r}{2}\right)^{2}$$

$$\frac{r}{2} = 0,040672622$$

$$r = 8,13452446\%$$

$$r = 8,13\%$$

QUESTION 12

12.1

$$A = P(1+i)^n$$

$$2P = P\left(1+\frac{r}{4}\right)^{6\times 4}$$

$$2 = \left(1+\frac{r}{4}\right)^{24}$$

$$1+\frac{r}{4} = 2^{\frac{1}{24}}$$

$$r = 4\left(2^{\frac{1}{24}}-1\right)$$

$$r = 4\left(2^{\frac{1}{24}}\right)-4$$

$r = 0,1172\ \ldots$

rate = 11,72% p.a. compounded quarterly

12.2.1

$$A = 10000\left(1+\frac{0,095}{12}\right)^{5}$$

$$= \text{R } 10\ 402,15$$

12.2.2

$$10402{,}15=\frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-n}\right]}{\frac{0{,}095}{12}}$$

$$0{,}183000787=1-\left(1+\frac{0{,}095}{12}\right)^{-n}$$

$$\left(1+\frac{0{,}095}{12}\right)^{-n}=0{,}816999213$$

$$\log\left(1+\frac{0{,}095}{12}\right)^{-n}=\log 0{,}816999213$$

$$-n\log\left(1+\frac{0{,}095}{12}\right)=\log 0{,}816999213...$$

$$n=25{,}63151282....$$

$$n=25{,}63\ months$$

$$n=26$$

Accept: $n = 31$ (because of first 5 months)

OR

$$10402{,}15\left(1+\frac{0{,}095}{12}\right)^{n}=\frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{n}-1\right]}{\frac{0{,}095}{12}}$$

$$10402{,}15\left(1+\frac{0{,}095}{12}\right)^{n}=56842{,}10526\left[\left(1+\frac{0{,}095}{12}\right)^{n}-1\right]$$

$$56842{,}10526=46439{,}95526\left(1+\frac{0{,}095}{12}\right)^{n}$$

$$\log 1{,}223991387=n\log\left(1+\frac{0{,}095}{12}\right)$$

$$n=\frac{\log 1{,}223991387}{\log\left(1+\frac{0{,}095}{12}\right)}$$

$$n=25{,}63\quad \text{months}$$

$$n=26$$

Accept: $n = 31$ (because of first 5 months)

12.2.3 Balance outstanding after 25 months

$$= 10402,15\left(1+\frac{0,095}{12}\right)^{25} - \frac{450\left[\left(1+\frac{0,095}{12}\right)^{25}-1\right]}{\frac{0,095}{12}}$$

$= R\ 282,36$

OR

Balance Outstanding after 25 months

$$= 10000\left(1+\frac{0,095}{12}\right)^{30} - \frac{450\left[\left(1+\frac{0,095}{12}\right)^{25}-1\right]}{\frac{0,095}{12}}$$

$= R\ 282,36$

OR

$n = 25,6315128204.... - 25$

$= 0,6315128204\ ...$

Balance Outstanding after 25 months

$$= \frac{450\left[1-\left(1+\frac{0,095}{12}\right)^{-0,631512804}\right]}{\frac{0,095}{12}}$$

$= R\ 282,36$

OR

Present value at beginning of 25 months

$$= 10402,15 - \frac{450\left[1-\left(1+\frac{0,095}{12}\right)^{-25}\right]}{\frac{0,095}{12}}$$

$= R\ 231,84$

Balance Outstanding

$$= 231,84\left(1+\frac{0,095}{12}\right)^{25}$$

$= R\ 282,36$

QUESTION 13

13.1

$$A = P(1-i)^n$$
$$\frac{P}{2} = P(1-0{,}07)^n$$
$$\frac{1}{2} = 0{,}93^n$$
$$\log\frac{1}{2} = n\log 0{,}93$$
$$n = \frac{\log\frac{1}{2}}{\log 0{,}93}$$
$$= 9{,}55 \text{ years}$$

OR

$$A = P(1-i)^n$$
$$\frac{P}{2} = P(1-0{,}07)^n$$
$$\frac{1}{2} = 0{,}93^n$$
$$\log_{0{,}93}\frac{1}{2} = n$$
$$n = 9{,}55 \text{ years}$$

13.2 **Radesh:**

$$A = P(1+in)$$
$$= 6\ 000(1+0{,}085\times 5)$$
$$= 8\ 550$$

OR

$$A = 6\ 000 + 8{,}5\% \text{ of } 6000\times 5$$
$$= 6000 + 510\times 5$$
$$= 6000 + 2550$$
$$= 8\ 550$$

$$\text{Bonus} = 0{,}05\times 6\ 000$$
$$= 300$$

$$\text{Received} = 8\ 550 + 300$$
$$= \text{R}8\ 850$$

Thandi:

$$A = P(1+i)^n$$
$$= 6\ 000\left(1+\frac{0{,}08}{4}\right)^{20}$$
$$= \text{R}8\ 915{,}68$$

Thandi's investment is bigger.

13.3 F_v = initial deposit with interest + annuity

$$=1\,000\left(1+\frac{0{,}15}{12}\right)^{18}+700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$$

$$=1\,250{,}58+14\,032{,}33$$

$$=\text{R}15\,282{,}91$$

OR

F_v = initial deposit with interest + annuity

$$=1\,000\left(1+\frac{0{,}15}{12}\right)^{18}+700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$$

$$=1\,250{,}58+11220{,}68\left(1+\frac{0.15}{12}\right)^{18}$$

$$=1\,250{,}58+14\,032{,}33$$

$$=\text{R}15\,282{,}91$$

QUESTION 14

14.1

Term	Income	Expenses	Savings
1	120 000	90 000	30 000
2	132 000	105 000	27 000
3	144 000	120 000	24 000

30 000 + 27 000 + 24 000 + …+ 0.

14.2 Savings = Income – Expenses
Income in year n = 120 000 + 12 000(n – 1)
Expenses in year n = 90 000 + 15 000(n – 1)

$$120000+12000(n-1)=90000+15000(n-1)$$
$$30000+12000n-12000=15000n-15000$$
$$33000=3000n$$
$$n=11$$

$\therefore$ After 11 years.

OR

a = 30 000 d = –3000

$$T_n = 30000 + (n-1)(-3000)$$
$$0 = 30000 - 3000n + 3000$$
$$3000n = 33000$$
$$\therefore n = 11$$

∴ After 11 years

14.3 $120000 + 12000(25-1) = 90000 + x(25-1)$

$$x = 13250$$

QUESTION/*VRAAG* 15

15.1

$$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$$

$$1 + i_{eff} = \left(1 + \frac{0{,}07}{12}\right)^{12}$$

$$i_{eff} = 0{,}07229008$$

$$i_{eff} = 7{,}23\%$$

15.2

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$350\,000 = \frac{6300[1 - \left(1 + \frac{0.07}{12}\right)^{-n}}{\frac{0{,}07}{12}}$$

$$\frac{73}{108} = \left(1 + \frac{0.07}{12}\right)^{-n}$$

$$\log\frac{73}{108} = -n\log\left(1 + \frac{0.07}{12}\right)$$

$$n = 67{,}33938079$$

$$n = 67{,}34 \text{ months}$$

15.3

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}(1+i)$$

$$P_v = \frac{6\,300[1 - (1 + \frac{0{,}07}{12})^{-0{,}3393....}]}{\frac{0{,}07}{12}}\left(1 + \frac{0{,}07}{12}\right)$$

$$P_v = \text{R } 2\,142{,}21$$

OR

Balance outstanding:

$$= \left[350\,000\left(1+\frac{0{,}07}{12}\right)^{67} - \frac{6\,300[(1+\frac{0{,}07}{12})^{67.} - 1]}{\frac{0{,}07}{12}} \right]\left(1+\frac{0{,}07}{12}\right)$$

$= \text{R } 2\,142{,}21$

15.4

$$252\,000 = 350\,000(1-i)^3$$

$$(1-i)^3 = \frac{252\,000}{350\,000}$$

$$i = 1 - \sqrt[3]{\frac{252}{350}}$$

$$i = 10{,}37\%$$

QUESTION 16

16.1 $\text{S} = 450\,000(1-0{,}20)^5 = \text{R}147\,456$

16.2 $\text{A} = 450\,000(1+0{,}12)^5 = \text{R}793\,053{,}76$

16.3

$$793\,053{,}76 = \frac{x\left[\left(1+\frac{0{,}13}{12}\right)^{61} - 1\right]}{\frac{0{,}13}{12}}$$

$$\therefore x = \text{R}9\,242{,}69$$

QUESTION 17

17.1

$$\text{B} = 1\,275\,000\left(1+\frac{0{,}092}{12}\right)^{84} - \frac{11\,636{,}02\left[\left(1+\frac{0{,}092}{12}\right)^{84} - 1\right]}{\frac{0{,}092}{12}}$$

$$= \text{R}1\,056\,675{,}39$$

or

$$\text{B} = \frac{11\,636{,}02\left[1-\left(1+\frac{0{,}092}{12}\right)^{-156}\right]}{\frac{0{,}092}{12}}$$

$$= \text{R}1\,056\,676{,}21$$

17.2

$$1\ 056\ 675,39\left(1+\frac{0,092}{12}\right)^{5}=\frac{x\left[1-\left(1+\frac{0,092}{12}\right)^{-151}\right]}{\frac{0,092}{12}}$$

$\therefore x = \text{R}12\ 297,82$

QUESTION 18

18.1 $3\ 488,45 = \text{P}(1-0,13)^{5}$

$\therefore \text{P} = \text{R}6\ 999$

18.2 Working to T_0:

$$x = 80\ 312,55\left(1+\frac{0,14}{4}\right)^{-20}.\left(1+\frac{0,12}{12}\right)^{-24}$$

$$-10\ 000\left(1+\frac{0,14}{4}\right)^{-12}.\left(1+\frac{0,12}{12}\right)^{-24}$$

$$= \text{R}26\ 576,04$$

Working to T_2:

$$x\left(1+\frac{0,12}{12}\right)^{24} = 80\ 312,55\left(1+\frac{0,14}{4}\right)^{-20}$$

$$-10\ 000\left(1+\frac{0,14}{4}\right)^{-12}$$

$\therefore x = \text{R}26\ 576,04$

Working to T_7:

$$x\left(1+\frac{0,12}{12}\right)^{24}.\left(1+\frac{0,14}{4}\right)^{20} = 80\ 312,55$$

$$-10\ 000\left(1+\frac{0,14}{4}\right)^{8}$$

$\therefore x = \text{R}26\ 576,04$

18.3 $x = 2x(1-0,08)^{n}$

$\therefore 0,5 = (0,92)^{n}$

$\therefore \log_{0,92} 0,5 = n$

$\therefore n = 8,31$

QUESTION 19

19.1.1

$$500\ 000 = \frac{x\left[1-\left(1+\frac{0,12}{12}\right)^{-240}\right]}{\frac{0,12}{12}}$$

$x = \text{R}5\ 505,43$

19.1.2

$$\text{B}_{10} = \frac{5\ 505,430668\left[1-\left(1+\frac{0,12}{12}\right)^{-230}\right]}{\frac{0,12}{12}}$$

$= \text{R}494\ 712,08$

Alternatively:

$$\text{B}_{10} = 500\ 000\left(1+\frac{0,12}{12}\right)^{10} - \frac{5\ 505,430668\left[\left(1+\frac{0,12}{12}\right)^{10}-1\right]}{\frac{0,12}{12}}$$

$= \text{R}494\ 712,08$

19.1.3

$$500\ 000 = \frac{6\ 000\left[1-\left(1+\frac{0,12}{12}\right)^{-n}\right]}{\frac{0,12}{12}}$$

$n = 180,0703409$

It will take Mary 181 months to pay off the loan (180 payments of R6 000 and a smaller final payment).

19.1.4 Jeremy's total payments:

$5\ 505,43 \times 240 = \text{R}1\ 321\ 303,20$

Interest paid: $1\ 321\ 303,20 - 500\ 000 = \text{R}821\ 303$

Mary's total payments:

$6\ 000 \times 180 = 1\ 080\ 000$ plus one smaller payment.

Interest paid: $1\ 080\ 000 - 500\ 000 = \text{R}580\ 000$

Therefore, Jeremy paid more interest.

19.2

$$614\ 490,66 = \frac{x\left[\left(1+\frac{0,062}{12}\right)^{49}-1\right]}{\frac{0,062}{12}}$$

$\therefore x = \text{R}11\ 052,30$

QUESTION 20

20.1 $150\ 000 = 210\ 000(1-0,165)^n$

$\therefore \frac{150\ 000}{210\ 000} = (0,835)^n$

$\therefore \frac{5}{7} = (0,835)^n$

$\therefore \log_{0,835}\left(\frac{5}{7}\right) = n$

$\therefore n = 1,865\ldots$

$\therefore n = 2$ years

20.2

$$350\ 000 = \frac{x\left[\left(1+\frac{0,12}{2}\right)^{11} - 1\right]}{\frac{0,12}{2}}$$

$\therefore x = \text{R}23\ 377,53$

20.3.1 Deposit is R45 000.
Loan is R405 000.

20.3.2

$$405\ 000 = \frac{x\left[1-\left(1+\frac{0,08}{12}\right)^{-240}\right]}{\frac{0,08}{12}}$$

$x = \text{R}3\ 387,58$

20.3.3

$$B_{17} = 405\ 000\left(1+\frac{0,08}{12}\right)^{204} - \frac{3\ 387,58\left[\left(1+\frac{0,08}{12}\right)^{204} - 1\right]}{\frac{0,08}{12}}$$

$$= \text{R}108\ 104,85$$

OR

$$B_{17} = \frac{3\ 387,58\left[1-\left(1+\frac{0,08}{12}\right)^{-36}\right]}{\frac{0,08}{12}}$$

$$= \text{R}108\ 103,79$$

Note:

If $x = 3\ 387,582279$ is used, then both methods will yield an answer of R108 103,87.