JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: FINANCE, GROWTH AND DECAY

CONTENT AND ACTIVITY MANUAL

LEARNER MANUAL

FINANCE, GROWTH AND DECAY

JENN MAREMATLOU TRAINING INSTITUTE

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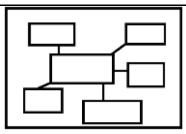
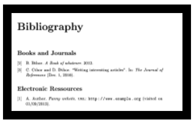
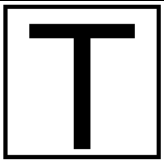
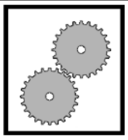
ICON DESCRIPTION

 **MIND MAP**	 **EXAMINATION GUIDELINE**	 **CONTENTS**	 **ACTIVITIES**
 **BIBLIOGRAPHY**	 **TERMINOLOGY**	 **WORKED EXAMPLES**	 **STEPS**

TOPIC 1: FINANCE, GROWTH AND DECAY

Outcomes: At the end of the session learners/ teachers must be able to:

- **Use the simple and compound growth formulae to solve problems**
- **Use the simple and compound decay formulae to solve problems**
- **Understand the effect of different periods of compound growth and decay, including nominal and effective interest rates**
- **Solve problems involving present value and future value annuities**
- **Make use of logarithms to calculate *n*, the time period in the compound growth and decay equations**
- **Critically analyse investment and loan options and make informed decisions as to best option(s)**

Important terminology and /or notes

T

Simple Growth

Formula: $A = P(1 + n.i)$

A = The final amount
P = The initial amount
n = number of years / period
i = interest rate in decimals

Compound Growth

Formula: $A = P(1 + i)^n$

A = The final amount
P = The initial amount
n = number of years / period
i = interest rate in decimals

Simple Decay (Straight line depreciation)

Formula: $A = P(1 - n.i)$

A = The final amount
P = The initial amount
n = number of years / period
i = interest rate in decimals

N.B the initial amount is bigger than the final amount

Compound Decay (reducing balance depreciation)

Formula: $A = P(1-i)^n$

A = The final amount
P = The initial amount
n = number of years / period
i = interest rate in decimals

N.B the initial amount is bigger than the final amount

Effect of different periods of compound growth(and Decay)

- **Annually:** interest is caluculated and added once in a year
- **Semi-annually (half yearly):** interest is calculated and added twice in year (every six months)
    - **Divide i by 2 and multiply n by 2**
- **Quarterly:** interest is calculated and added 4 times in a year (every 3 months)
    - **Divide i by 4 and multiply n by 4**
- **Monthly:** interest is calculated and added 12 times in a year (every month)
    - **Divide i by 12 and multiply n by 12**

Inflation

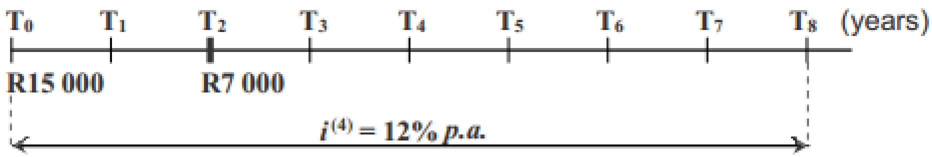
N.B inflation is always calculated using compound growth formula.

Timelines

Timeline is a useful tool to use more especially when there are deposits or withdrawals made. It is also useful when there are changes in interest rates.

Below is an example that illustrates the use of a timeline to calculate the final amount:

1. A deposit of R15000 is invested at 12% p.a. compounded quarterly. Two years later another deposit of R7000 is invested. The total period for the investment is 8 years. Calculate the accumulated amount after 8 years.



$$\text{Final Amount} = \left[15000\left(1+\frac{0.12}{4}\right)^{2\times4} + 7000\right]\times\left(1+\frac{0.12}{4}\right)^{6\times4} = R52855.80$$

Nominal and Effective nterest rates

Nominal Interest
The interest rate quoted and compounding periods are different:
e.g. 10% **p.a**. compunded **quarterly**

Effective interest rate
The interest rate quoted and compounding periods are the same:
e.g. 10% **p.a.** compunded **annually**
5% **per month** compounded **monthly**

Formula to convert from nominal interest rate to effective annual interest rate (and vice versa)

$$1+i_{eff}=\left(1+\frac{i_{nom}}{m}\right)^{m}$$

i_{eff} = effective annual interest rate
i_{nom} = nominal interest rate
m = number of compounding periods

Solving for n

$$A=P(1\pm i)^{n}$$
$$(1\pm i)^{n}=\frac{A}{P}$$
$$\log\ (1\pm i)^{n}=\log\ \frac{A}{P}$$
$$n\ \log(1\pm i)=\log\ A-\log\ P$$
$$\therefore n=\frac{\log A-\log P}{\log(1\pm i)}$$

N.B

- **Always round up**

  2,3456 years = 2 years 5 months
  How to get 5 months: $[2,3456-2]\times 12=5\text{months}$

Future Value annuty

Formula: $F = \frac{x\left[(1+i)^n - 1\right]}{i}$ **(Proof of the formula is available in your textbook)**

$F =$ Future value

$x =$ fixed regular payments

$n =$ number of payments

$i =$ interest rate in decimals

When there is "x" immediate payment made and the last payment is made at the end of the period:

Use the following formula: $F = \frac{x\left[(1+i)^{n+1} - 1\right]}{i}$

When there is an immediate payment made of an amount that is not x, say t, and the last payment is made at the end of the period:

Use the following formula: $F = t(1+i)^n + \frac{x\left[(1+i)^n - 1\right]}{i}$

When payments are made at the beginning of each period or when payments are made at the end of each period and the last payment is made, for an example 1 month before the end of the period if interest is compounded monthly:

Use the following formula: $F = \frac{x\left[(1+i)^n - 1\right]}{i} \times (1+i)^n$

Sinking Fund

Sinking fund is an amount that is invested to replace something (e.g. Vehicle, machenry) in future. We use future value annuity to save money in regular intervals for the money to be used in future.

N.B Sinking fund = New price after inflation − Book value

Present Value annuty

Formula: $P = \frac{x\left[1-\left(1+i\right)^{-n}\right]}{i}$ **(Proof of the formula is available in your textbook)**

$P =$ Present value (loan amount)

$x =$ fixed regular payments

$n =$ number of payments

$i =$ interest rate in decimals

Interest paid

Interest amount paid = All payments made $-$ loan amount

Balance on the loan

Balance = $P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$ *where P is the loan amount*

Guidline to solving problems

1. Read the question more than once, at least three times, before answering.
2. Know the formula for converting between effective interest and nominal interest rate by heart as it is not given in the information sheet.
3. Only round off the final answer.

Worked example(s)

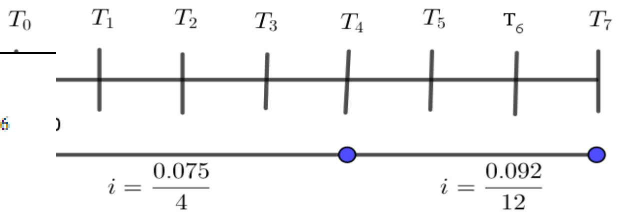
1. (Compound growth)

Mrs Pillay invested R80 000 in an account which offers the following:

- 7,5 % p.a., compounded quarterly, for the first 4 years and thereafter
- 9,2% p.a., compounded monthly, for the next 3 years

Calculate the total amount of money that will be in the account at the end of 7 years if no further transactions happen on the account.

Solution



$$A = P(1+i_1)^{n_1}(1+i_2)^{n_2}$$

$$= 80000\left(1+\frac{0.075}{4}\right)^{16}\left(1+\frac{0,092}{12}\right)^{36}$$

$$= R141768,60$$

OR

$$A_1 = 80000\left(1+\frac{0,075}{4}\right)^{16}$$

$$= 107689,1465..$$

$$A_2 = 107689,1465\left(1+\frac{0,092}{12}\right)^{36}$$

$$= \text{R}141768,60$$

2. (Compound growth and timelines)

Exactly 8 years ago Tashil invested R30 000 in an account earning 6,5% per annum, compounded monthly.

2.1 How much will he receive if he withdrew his money today?

2.2 Tashil withdrew R10 000 three years after making the initial deposit and re-invested R10 000 five years after making the initial deposit.

Calculate the difference between the final amount Tashil will now receive after eight years and the amount he would have received had there not been any transactions on the account after the initial deposit.

Solutions

2.1 Investment : end of third year :

$$A = P(1+i)^n$$

$$= 30\,000\left(1+\frac{0{,}065}{12}\right)^{96}$$

$$= R50390{,}07$$

2.2

T_0	T_3	T_5	T_8
30000	-10000	$+10000$	

$$A = 30000\left(1+\frac{0{,}65}{12}\right)^{96} - 10000\left(1+\frac{0{,}65}{12}\right)^{60} + 10000\left(1+\frac{0{,}65}{12}\right)^{36}$$

$$A = \text{R}48708{,}61$$

$$\therefore \text{difference} = 48708{,}61 - 50390{,}07$$

$$= -\text{R}1681{,}46$$

3. (Compound decay / deucing balance depreciation)

A tractor bought for R120 000 depreciates to R11 090,41 after 12 years by using the reducing balance method. Calculate the rate of depreciation per annum. (The rate was fixed over the 12 years.)

Solution

$$A = P(1-i)^n$$

$$11090{,}41 = 120000(1-i)^{12}$$

$$\therefore i = 1 - \sqrt[12]{\frac{11090{,}41}{120000}}$$

Thus $i = 0{,}179999...$

Rate of Depreciation $= 18\,\%$

4. (Nominal and effective interest rates)

Calculate the effective interest rate if interest is 9,8% p.a., compounded monthly.

Solution

$$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$$

$$= \left(1+\frac{0{,}098}{12}\right)^{12} - 1$$

$$= 0{,}10252.....$$

rate $= 10{,}25\%$

5. (Solving for n in compound decay/growth)

A company bought office furniture that cost R120 000. After how many years will the furniture depreciate to a value of R41 611,57 according to the reducing-balance method, if the rate of depreciation is 12,4% p.a.?

Solution

$$A = P(1-i)^n$$

$$41\ 611,57 = 120\,000(1-0,124)^n$$

$$\frac{41\ 611,57}{120\,000} = (0,876)^n$$

$$n = \log_{(0,876)} \frac{41611,57}{120\,000}$$

$$= 8 \text{ years}$$

OR/*OF*

$$A = P(1-i)^n$$

$$41611,57 = 120000(1-0,124)^n$$

$$\frac{41611,57}{120000} = (0,876)^n$$

$$\log\frac{41611,57}{120000} = n\log(0,876)^n$$

$$n = \frac{\log\frac{41611,57}{120000}}{\log 0,876}$$

$$= 8 \text{ years}$$

6. (Future value annuity)

Tebogo opened a savings account with a single deposit of R5 000 at the beginning of June 2015. He then made 24 monthly deposits of R800 at the end of every month, starting at the end of June 2015. The account earned interest at 15% p.a. compounded monthly.

Calculate the amount that should be in his savings account immediately after he makes the last deposit.

Solution

final amount

$$= P(1+i)^n \quad + \quad \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$= 5000\left(1+\frac{0,15}{12}\right)^{24} \quad + \quad \frac{800\left[\left(1+\frac{0,15}{12}\right)^{24} - 1\right]}{\frac{0,15}{12}}$$

$$= \quad 6736,755 \quad + \quad 22\ 230,467$$

$$= R28\ 967,22$$

7. (Present Value annuity)

Musa takes a personal loan from a bank to buy a motorcycle that costs R46 000. The bank charges interest at 24% per annum, compounded monthly.

How many months will it take Musa to repay the loan, if the monthly instalment is R1 900?

Solution

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$46\ 000 = \frac{1900\left[1-\left(1+\frac{0,24}{12}\right)^{-n}\right]}{\frac{0,24}{12}}$$

$$\frac{46}{95} = 1-\left(1+\frac{0,24}{12}\right)^{-n}$$

$$\left(1+\frac{0,24}{12}\right)^{-n} = \frac{49}{95}$$

$$n = -\log_{\left(1+\frac{0,24}{12}\right)}\frac{49}{95} \quad \text{OR/OF} \quad -n\log\left(1+\frac{0,24}{12}\right) = \log\frac{49}{95}$$

$$= 33,43276544\ldots \text{ months}$$

It will take him 34 months to pay back the loan.

ACTIVITIES

QUESTION 1

David deposits R65 000 into a savings account paying 15% per annum compounded monthly. He wants to buy a car in three years' time.

1.1 Convert the nominal rate of 15% per annum compounded monthly to the equivalent effective rate (annual).

1.2 Calculate how much money will be available to him in three years' time.

1.3 If he makes a further deposit of R10 000 into the account one year after his first deposit of R65 000, how much money will he have then saved in three years' time.

QUESTION 2

2.1 Calculate the original price of a laptop if its depreciated value after 7 years is R3200 and the rate of depreciation was 12% per annum based on the linear depreciation method.

2.2 On the 1st January 2010, Kevin deposited Rx into a savings account. On the 1st January 2014, he deposited the same amount, Rx, into the account. On the 1st January 2015, he withdrew R10 000 from the account. On the 1st January 2016, he deposited Rx into the account. On this same day, after his deposit of Rx, the accumulated value of his savings was R250 000. The interest received was 6% per annum compounded quarterly.

2.2.1 Draw a timeline to represent the above situation.

2.2.2 Calculate the value of x.

QUESTION 3

3.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000?

3.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

3.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay?

3.2.2
- One month after purchasing his present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a., compounded monthly.
- He continued to deposit the same amount at the end of each month for a total of 60 months.
- At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.

Calculate the value of x

3.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be?

QUESTION 4

4.1 A new cellphone was purchased for R7 200. Determine the depreciation value after 3 years if the cellphone depreciates at 25% p.a. on the reducing-balance method.

4.2 Jill negotiates a loan of R300 000 with a bank which has to be repaid by means of monthly payments of R5 000 and a final payment which is less than R5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% per annum, compounded monthly.

4.2.1 Determine the number of payments required to settle the loan.

4.2.2 Calculate the balance outstanding after Jill has paid the last R5 000.

4.2.3 Calculate the value of the final payment made by Jill to settle the loan.

4.2.4 Calculate the total amount that Jill repaid to the bank.

QUESTION 5

5.1 R2 000 was invested in a fund paying i% interest compounded monthly. After 18 months the value of the fund was R2 860,00. Calculate i, the interest rate.

5.2 On 31 January 2008 Farouk banked R100 in an account that paid 8% interest per annum, compounded monthly. He continued to deposit R100 on the last day of every month until 31 December. He was hoping to have enough money on 1 January 2009 to buy a bike for R1 300. Determine whether he will be able to do so, or not.

QUESTION 6

Rowan plans to buy a car for R125 000,00. He pays a deposit of 15% and takes out a bank loan for the balance. The bank charges 12,5% p.a. compounded monthly.

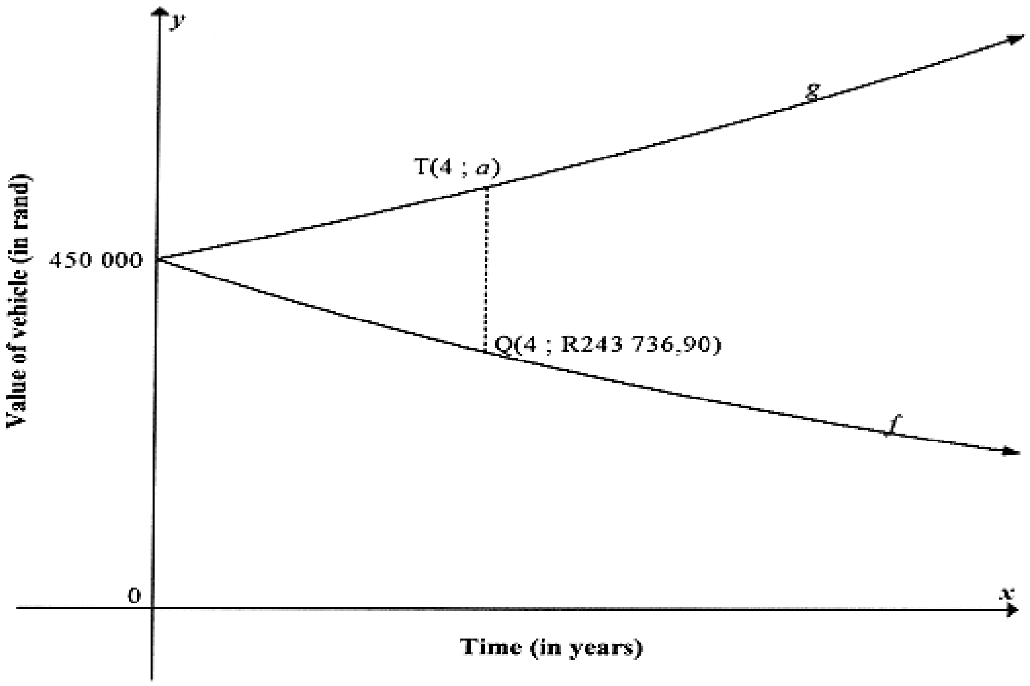
Calculate:

6.1 The value of the loan borrowed from the bank

6.2 The monthly repayment on the car if the loan is repaid over 6 years

QUESTION 7

The graph of f shows the book value of a vehicle x years after the time Joe bought it.
The graph of g shows the cost price of a similar new vehicle x years later.



7.1 How much did Joe pay for the vehicle?

7.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought.

7.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a.

7.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund.

QUESTION 8

8.1 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i.

8.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

8.2.1 Show that the loan would be paid off in 234 months.

8.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120^{th}, 121^{st}, 122^{nd} and 123^{rd} months. At the end of the 124^{th} month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments.
Calculate the value of this new instalment.

QUESTION 9

9.1 A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000?

9.2 A car that costs R130 000 is advertised in the following way: 'No deposit necessary and first payment due three months after date of purchase.' The interest rate quoted is 18% p.a. compounded monthly.

9.2.1 Calculate the amount owing two months after the purchase date, which is one month before the first monthly payment is due.

9.2.2 Herschel bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.

(a) Calculate his monthly repayments.

(b) Calculate the total of all Herschel's repayments.

9.2.3 Hashim also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car. Calculate the total of all Hashim's repayments.

9.2.4 Calculate the difference between Herschel's and Hashim's total repayments.

QUESTION 10

10.1 Raeesa invests R4 million into an account earning interest of 6% per annum, compounded annually. How much will her investment be worth at the end of 3 years?

10.2 Joanne invests R4 million into an account earning interest of 6% per annum, compounded monthly.

10.2.1 She withdraws an allowance of R30 000 per month. The first withdrawal is exactly one month after she has deposited the R4 million. How many such withdrawals will Joanne be able to make?

10.2.2 If Joanne withdraws R20 000 per month, how many withdrawals will she be able to make?

QUESTION 11

Jeffrey invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded semi-annually (that is every six months) at r% per annum. Jeffrey and his friend's investments are worth the same at the end of 12 months. Calculate r.

QUESTION 12

12.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years?

12.2 Timothy buys furniture to the value of R10 000. He borrows the money on 1 February 2010 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Timothy agrees to pay monthly instalments of R450. The agreement of the loan allows Timothy to start paying these equal monthly instalments from 1 August 2010.

12.2.1 Calculate the total amount owing to the financial institution on 1 July 2010.

12.2.2 How many months will it take Timothy to pay back the loan?

12.2.3 What is the balance of the loan immediately after Timothy has made the 25^{th} payment?

QUESTION 13

13.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum?

13.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations.

13.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012).

QUESTION 14

Matli's annual salary is R120 000 and his expenses total R90 000. His salary increases by R12 000 each year while his expenses increase by R15 000 each year. Each year he saves the excess of his income.

14.1 Represent his total savings as a series.

14.2 If Matli continues to manage his finances this way, after how many years will he have nothing left to save?

14.3 Matli calculates that if his expenses increase by x rand every year (instead of R15 000 each year), he will spend as much as he earns in the 25^{th} year. Determine x.

QUESTION 15

Susan buys a car for R350 000. She secures a loan at an interest rate of 7% p.a., compounded monthly. The monthly instalment is R6 300. She pays the first instalment one month after the loan was secured.

15.1 Calculate the effective annual interest rate on the loan. Leave your answer correct to TWO decimal places.

15.2 How many months will it take to repay the loan?

15.3 Calculate the value of the final instalment.

15.4 The value of the car depreciates at i % p.a. After 3 years its value is R252 000. Calculate i.

QUESTION 16

Your school buys computer at a cost of R450 000. Computers depreciate at 20% per annum on the reducing balance method.

16.1 Calculate the scrap value of the computers after 5 years. (2)

16.2 Computer appreciates at 12% per annum. Calculate the replacement cost of the computers after 5 years. (2)

16.3 The school sets up a sinking fund to make provision for the purchase of new computers after 5 years. The interest rate is 13% per annum compounded monthly. If the old computers are sold at scrap value, calculate the monthly instalment. The first instalment is made immediately. (3)

QUESTION 17

17.1 Mark buys a property for R1 500 000. After paying a deposit, he takes out a loan for the remaining amount of R1 275 000, at an interest rate of 9,2% per annum, compounded monthly over a period of 20 years. The monthly instalment on the loan is R11 636,02. The first repayment is made one month after the granting of the loan.

Calculate the outstanding balance of the loan after 7 years.

17.2 After 7 years, due to financial difficulty, Mark misses 5 consecutive payments. Thereafter, he continues making monthly payments into the loan account until the end of the 20-year period.

Calculate the value of the new monthly instalment to settle the loan.

QUESTION 18

18.1 Calculate the original price of an iPad if the depreciated value after 5 years is R3 488,45. The rate of depreciation is 13% per annum based on the reducing balance method.

18.2 A man invests a certain amount of money into an account. Interest is 12% per annum compounded monthly for the first two years and 14% per annum compounded quarterly for the remaining time. Five years after the initial investment, he needs to withdraw R10 000 to do some renovations on his house. Seven years after the initial investment, he had R80 312,55. Calculate the amount of money he invested initially.

18.3 Determine how many years it would take for the value of a car to depreciate to 50% of its original value, if the rate of depreciation, based on the reducing-balance method, is 8% per annum.

QUESTION 19

19.1 Jeremy took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

19.1.1 Calculate the value of Jeremy's monthly instalment.

19.1.2 Calculate Jeremy’s balance outstanding after his 10^{th} payment.

19.1.3 Mary took out a loan for the same amount and at the same interest rate as Jeremy. Mary decided to pay R6 000 at the end of every month. Calculate how many months it took for Mary to settle the loan.

19.1.4 Who pays more interest, Jeremy or Mary? Justify your answer.

19.2 A vehicle is bought at the beginning of the month. It is to be replaced at the end of 4 years from the time of purchase. This vehicle will be used as a trade-in. The cost of a new vehicle is estimated to be R614 490,66. A sinking fund is created to cover the replacement cost of this vehicle. Monthly payments will be made at the end of each month. The first payment will be made on the day of purchase of the vehicle and the last payment will be made at the end of the four-year period. The sinking fund earns interest at a rate of 6,2% per annum compounded monthly.

Calculate the monthly payment to the fund.

QUESTION 20

20.1 A man buys a new car for R210 000. How long will it take for this car to depreciate to R150 000 at the rate of 16,5% per annum calculated on a reducing-balance. Give your answer to the nearest year.

20.2 A company starts to invest money into a **sinking fund** to have R350 000 available in five years’ time to purchase new computers. The first payment is made immediately and future payments are made every six months from the first payment. If the interest is 12% per annum compounded semi-annually, determine the value of each payment.

20.3 Lionel buys a small house on auction for R450 000. He pays a deposit of 10% and takes out a bank loan for the balance.

20.3.1 Calculate the value of the loan.

20.3.2 Lionel pays back the loan by means of equal monthly payments over a period of 20 years. The first repayment is made one month after the granting of the loan. Interest is calculated at 8% per annum compounded monthly. Calculate the value of his monthly repayment.

20.3.3 Lionel inherits money and is keen to settle the loan after 17 years. Calculate the outstanding balance on the loan if his last payment is made at the end of the 17^{th} year.

PAST PAPERS

QUESTION 3 **[1]**

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

3.1 What distance will he cover on day 3 of the trip? (2)

3.2 On what day of the trip will Themba pass the halfway point? (4)

3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)

[9]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85^{th} instalment. (3)

7.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4)

[16]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees`to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

QUESTION 6

6.1 Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing-balance method. The book value of Sandile's car is currently R79 866,96.

6.1.1 How many years ago did Sandile buy the car? (3)

6.1.2 At exactly the same time that Sandile bought the car, Anil deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Anil accumulated enough money in his savings account to buy Sandile's car now? (3)

6.2 Exactly 10 months ago, a bank granted Jane a loan of R800 000 at an interest rate of 10,25% p.a., compounded monthly.
The bank stipulated that the loan:

- Must be repaid over 20 years
- Must be repaid by means of monthly repayments of R7 853,15, starting one month after the loan was granted

6.2.1 How much did Jane owe immediately after making her 6^{th} repayment? (4)

6.2.2 Due to financial difficulties, Jane missed the 7^{th}, 8^{th} and 9^{th} payments. She was able to make payments from the end of the 10^{th} month onwards. Calculate Jane's increased monthly payment in order to settle the loan in the original 20 years. (5)

[15]

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.
The cost of petrol is $\left(1{,}2+\frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

QUESTION 6

6.1 On 31 January 2020, Tshepo made the first of his monthly deposits of R1 000 into a savings account. He continues to make monthly deposits of R1 000 at the end of each month up until 31 January 2032. The interest rate was fixed at 7,5% p.a., compounded monthly

6.1.1 What will the investment be worth immediately after the last deposit? (4)

6.1.2 If he makes no further payments but leaves the money in the account, how much money will be in the account on 31 January 2033? (2)

6.2 Jim bought a new car for R250 000. The value of the car depreciated at a rate of 22% p.a. annually according to the reducing-balance method. After how many years will its book value be R92 537,64? (3)

6.3 Mpho is granted a loan under the following conditions:

- The interest rate is 11,3% p.a., compounded monthly.
- The period of the loan is 6 years.
- The monthly repayment on the loan is R1 500.
- Her first repayment is made one month after the loan is granted.

6.3.1 Calculate the value of the loan. (3)

6.3.2 How much interest will Mpho pay in total over the first 5 years? (4)
[16]

ANNEXURE A: EXAMINATION GUIDELINE

- **Weighting of cognitive levels**

Papers 1 and 2 will include questions across four cognitive levels. The distribution of cognitive levels in the papers is given below.

Cognitive level	Description of skills to be demonstrated	Weighting	Approximate number of marks in a 150-mark paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding • Learners are expected to solve problems by integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks

- **Elaboration of content: Finance, Growth and Decay**

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating for i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will not be examined in the examination.

ANNEXURE B: INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ \ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$In\ \Delta ABC:\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

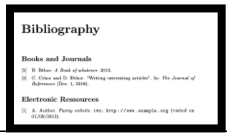
$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Bibliography	

1. **MATHEMATICS CAPS DOCUMENT**

2. **GRADE 12 MATHEMATICS EXAMINATION GUIDELINES**

3. **NOVEMBER 2008 – 2012 GRADE 12 PAPERS**

4. **NOVEMBER 2013 – 2015 GRADE 11 PAPERS**

5. **MAY 2015 – 2017 GRADE 12 PAPERS**

6. **FEB/MAR 2009 – 2013 GRADE 12 PAPERS**

Outcomes reached

	YES	**NO**
• **Use the simple and compound growth formulae to solve problems**		
• **Use the simple and compound decay formulae to solve problems**		
• **Understand the effect of different periods of compound growth and decay, including nominal and effective interest rates**		
• **Solve problems involving present value and future value annuities**		
• **Make use of logarithms to calculate *n*, the time period in the compound growth and decay equations**		
• **Critically analyse investment and loan options and make informed decisions as to best option(s)**		