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Training and Consultancy

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SUBJECT: MATHEMATICS

ACTIVITY MANUAL

GRADE 12

2022

EUCLID'S GEOMETRY

JENN MAREMATLOU TRAINING INSTITUTE

A Division of Marematlou Group Holdings

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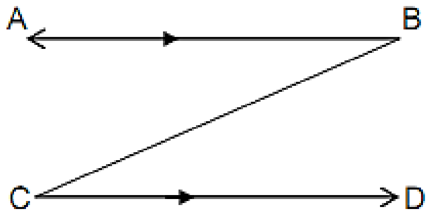
ACTIVITIES

N.B GIVE REASONS FOR YOUR STATEMENTS

<u>Lines, Angles and Triangles</u>

Question 1

1. A *Z*-letter shape is drawn below (AB // CD) showing a pair of alternate angles.

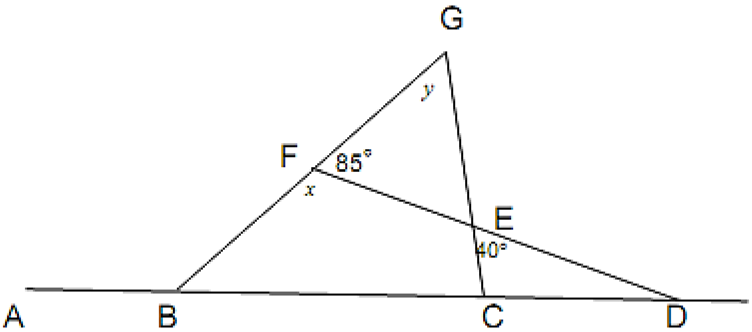


In the shape above identify the:

1.1 Angle of elevation

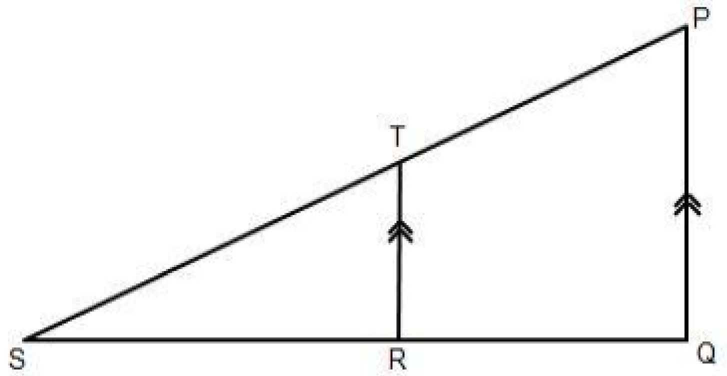
1.2 Angle of depression

2. Study the figure below:



AD; BG; FD and GC are straight lines. Calculate the value of y.

3 In the diagram below, $TR//PQ$, $\hat{S} = 28°$, $T\hat{R}S = x + 70°$ and $\hat{P} = x + 10°$

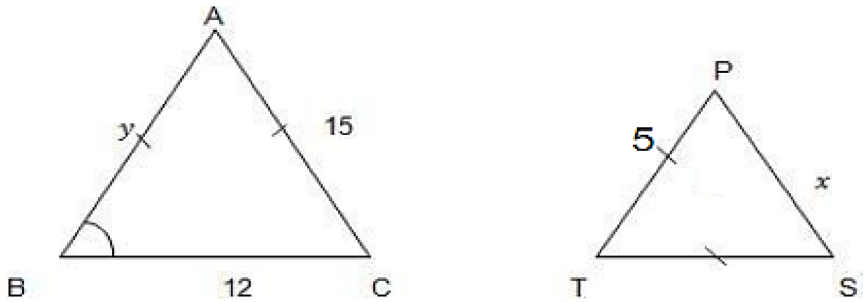


3.1 Calculate the value of x, giving reasons.

3.2 Calculate the value of $S\hat{T}R$, giving reasons.

3.3 Is ΔPQS a right angled triangle? Justify your answer by means of calculations.

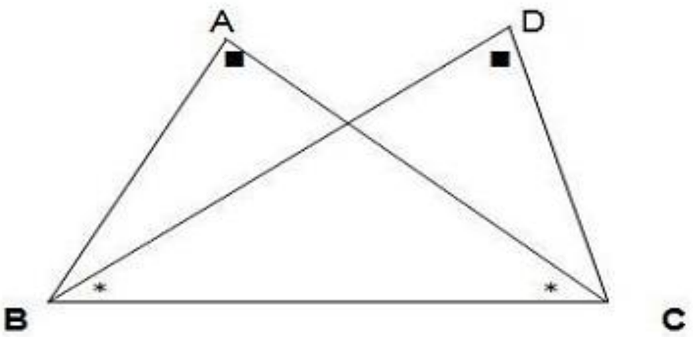
4 In ΔABC and ΔPTS $B = 70°\ and\ P = 70°$



4.1 Prove with reasons that $\Delta ABC///\Delta TSP$

4.2 Determine y and x.

5 Study the figure below and answer the questions that follow.

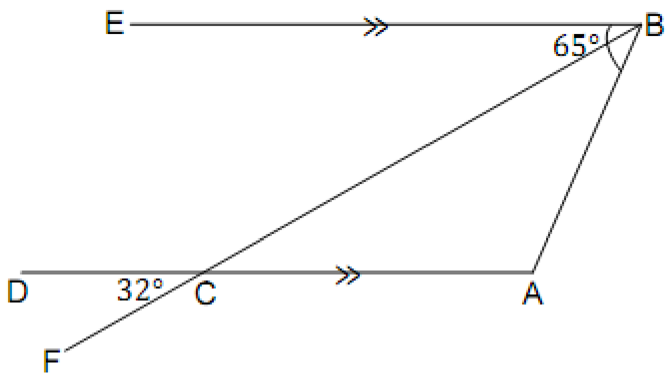


5.1 Prove with reasons that $\Delta ABC \equiv \Delta DCB$

5.2 If AB = 4 units, what is the length of DC?

Question 2

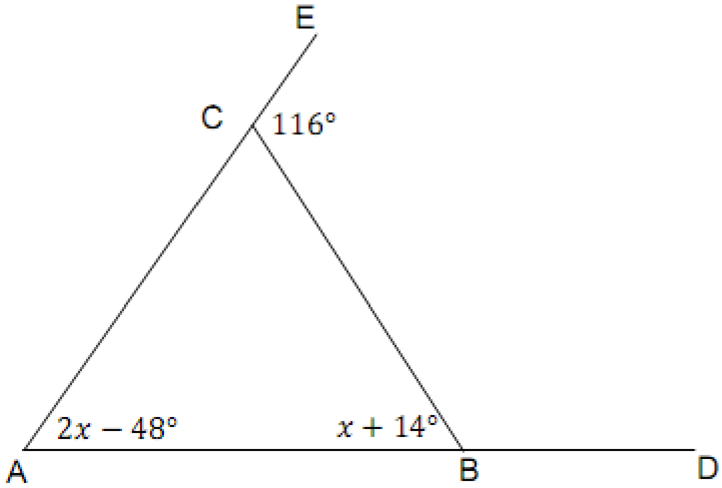
2.1 In the diagram below $A\hat{B}E = 65°$ and $D\hat{C}F = 32°$.



2.1.1 Calculate the size of $E\hat{B}C$. Give reasons for your answer.

2.1.2 Calculate the size of $C\hat{A}B$ Give reasons for your answer.

2.2 In the diagram below, $C\hat{A}B = 2x - 48°$, $A\hat{B}C = x + 14°$ and $B\hat{C}E = 116°$.

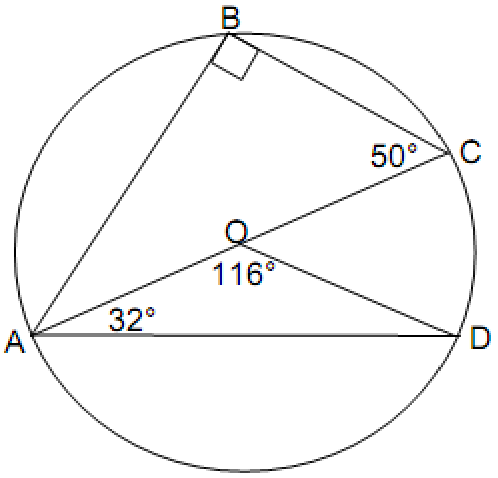


2.2.1 Calculate the value of x. Give reasons for your answer.

2.2.2 Calculate the actual size of $C\hat{A}B$.

2.2.3 What type of Δ is ΔABC? Give reasons for your answer.

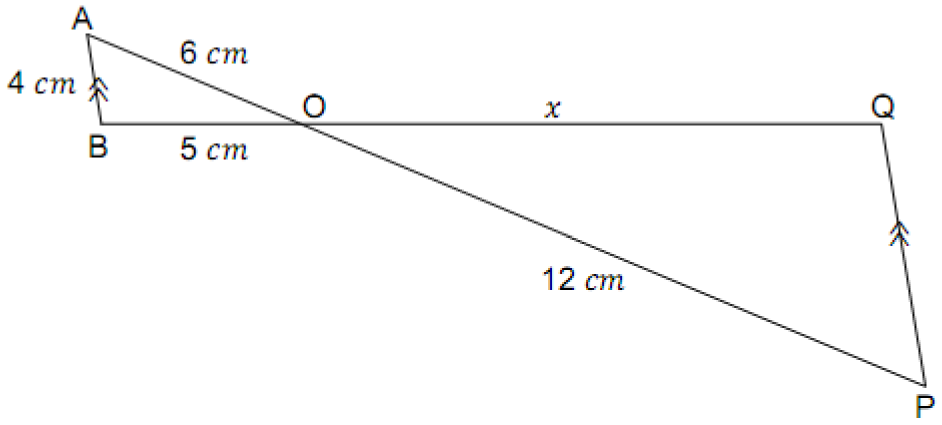
2.3 In the figure below, O is the centre of the circle.



2.3.1 Calculate the size of $C\hat{A}B$. Give a reason for your answer.

2.3.2 Calculate the size of $A\hat{D}O$. Give a reason for your answer.

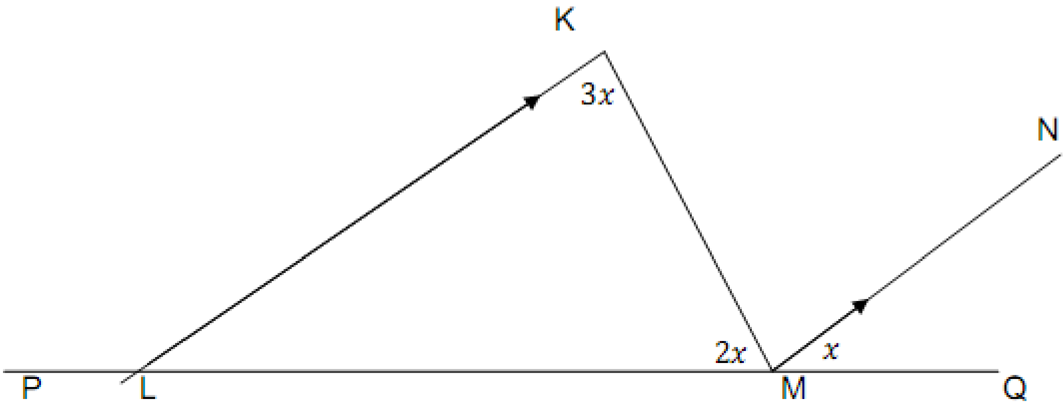
2.4 In the diagram below, AB ∥ PQ.



2.4.1 Prove that ΔABO ||| ΔPQO.

2.4.2 Calculate the value of x.

2.5 The figure below shows that LK // MN; $L\hat{K}M = 3x$; $K\hat{M}L = 2x$ and $N\hat{M}Q = x$.



2.5.1 Calculate with reasons the value of x.

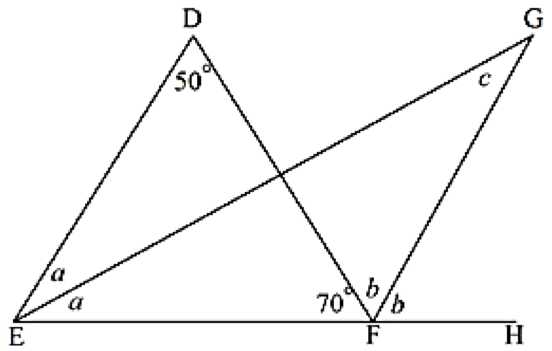
2.5.2 What is the size of $L\hat{K}M$?

2.5.3 Which type of triangle is ΔMKL?

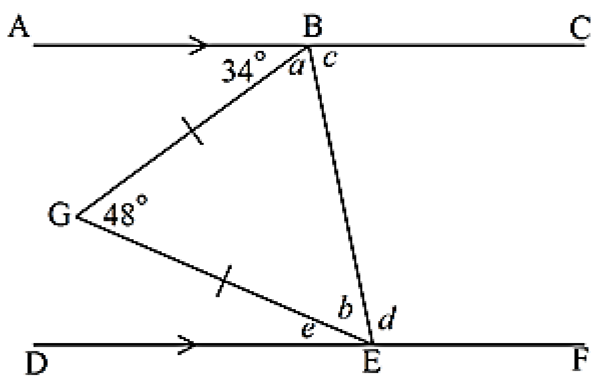
Question 3

3.1 In the given sketches angles that are marked with the same letter are equal to each other. Find the size of each of the following angles:

3.1.1 a, b, and c

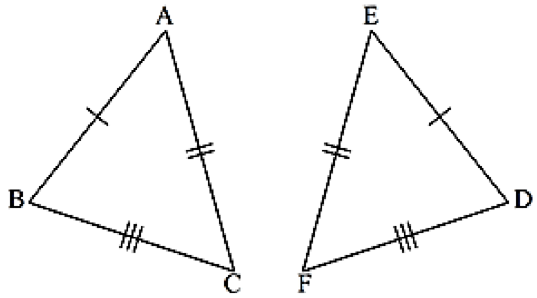


3.1.2 a, b, c, d and e

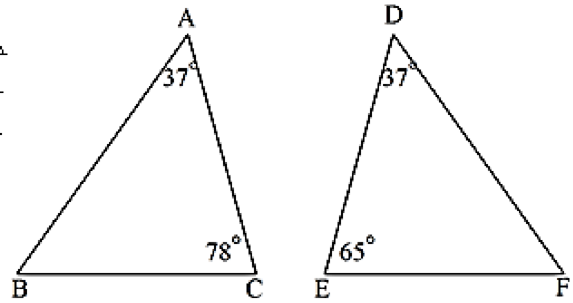


3.3 In each of the following, circle the correct answer from the options given that matches the statement to the given sketch:

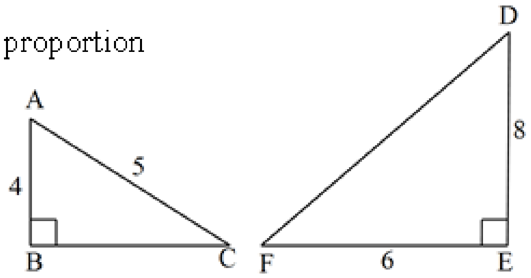
3.3.1
A) $\Delta ABC \equiv \Delta DEF$ S, S, S
B) $\Delta ABC \equiv \Delta EDF$ S, S, S
C) $\Delta ABC \equiv \Delta FED$ S, S, S
D) None of the above



3.3.2
A) $\Delta ABC \equiv \Delta DEF$ A, A, A
B) $\Delta ABC \,|||\, \Delta DEF$ A, A, A
C) $\Delta ABC \equiv \Delta DEF$ A, S, A
D) None of the above

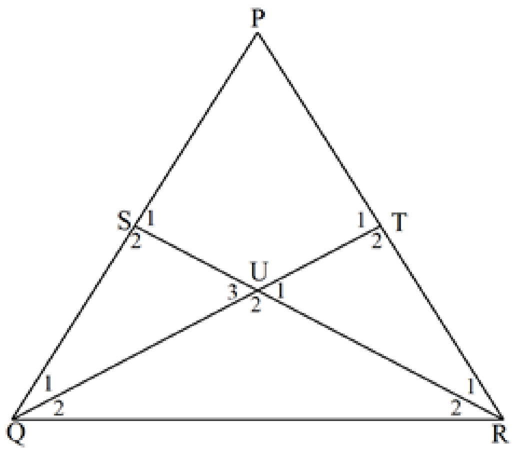


3.3.3
A) $\Delta ABC \,|||\, \Delta DEF$ sides are in proportion
B) $\Delta ABC \,|||\, \Delta DEF$ S, S, S
C) $\Delta ABC \equiv \Delta DEF$ R, H, S
D) None of the above



3.4 In the given sketch, ΔPQR is isosceles with $PQ = PR$ and $\hat{Q}_2 = \hat{R}_2$

Prove $\Delta QTP \equiv \Delta RSP$



Circle Geometry

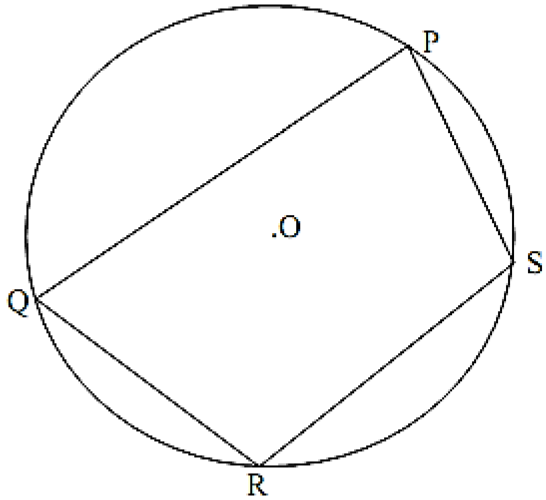
QUESTION 1

1.1 Complete the statements below by filling in the missing word(s) to make the statements correct.

1.1.1 The angle between a tangent and a chord is …

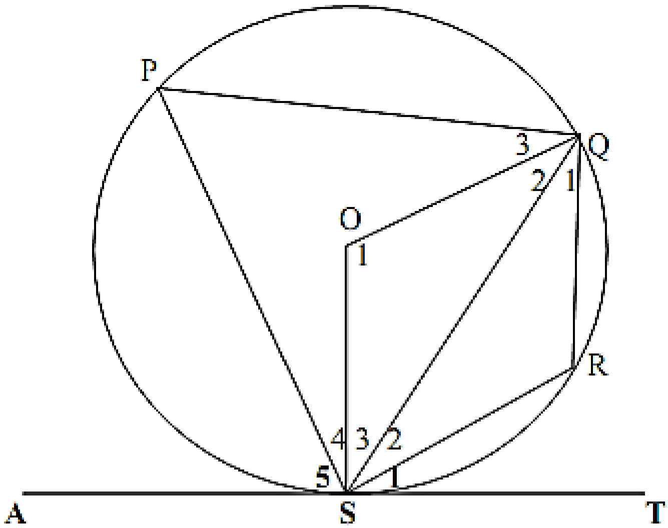
1.1.2 The exterior angle of a cyclic quadrilateral is equal to …

1.2 In the diagram below O is the centre of the circle. PQRS is cyclic quadrilateral.



Redraw the diagram or use the diagram on DIAGRAM SHEET 2 to prove the theorem which states that $\widehat{P} + \widehat{R} = 180°$.

1.3 In the diagram below, AST is a tangent to a circle O at S.
$R\hat{S}T = \hat{S}_1 = 23°$ and QR = RS.



Calculate, with reasons, the sizes of:

1.3.1 $Q\hat{S}R$

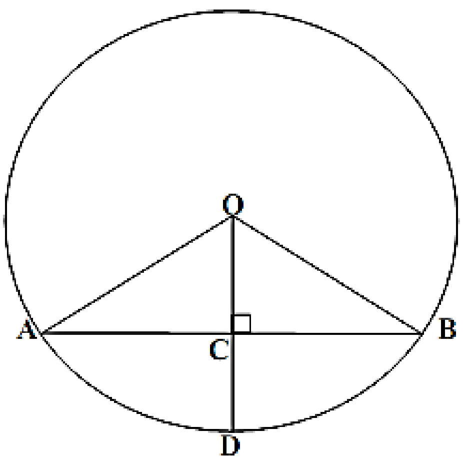
1.3.2 $\hat{R}$

1.3.3 $\hat{P}$

1.3.4 $\hat{O}_1$

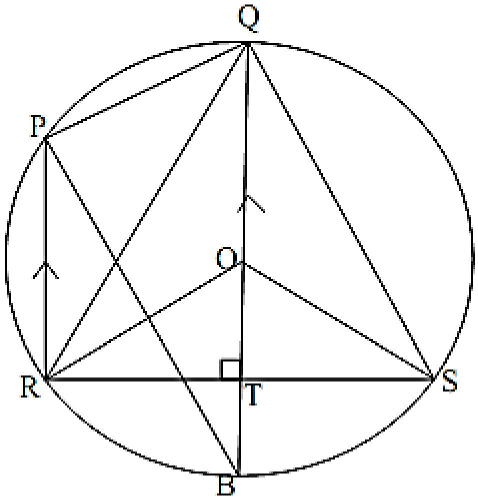
QUESTION 2

2.1 In the diagram below, AB is a chord of a circle with centre O.
D is a point on the circle. OD is perpendicular to AB. OA = 25 cm and CD = 18 cm.



Calculate, with reasons, the length of AB.

2.2 In the diagram below, QOB is the diameter of the circle with centre O. PR||QB, QB⊥RS and $P\hat{B}Q = 25°$. P, R and S are points on the circle.



2.2.1 Determine, with reasons, three other angles each equal to 25°.

2.2.2 Determine, with reasons:

(a) $R\hat{O}B$

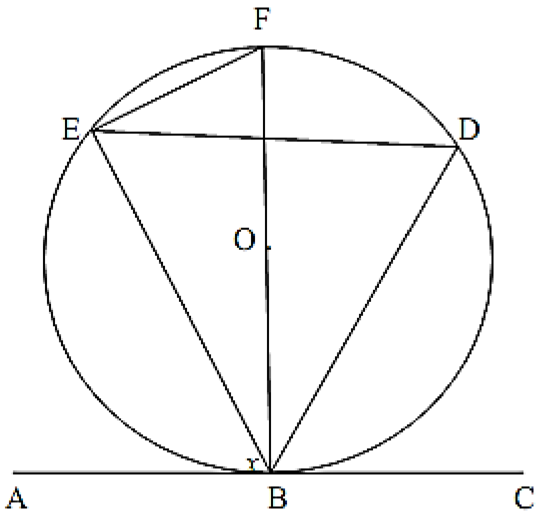
(b) $O\hat{R}T$

(c) $R\hat{O}S$

(d) $R\hat{P}Q$

QUESTION 3

3.1 In the diagram below, ABC is a tangent at B to the circle with centre O. D and E are points on this circle. $A\hat{B}E = x$.



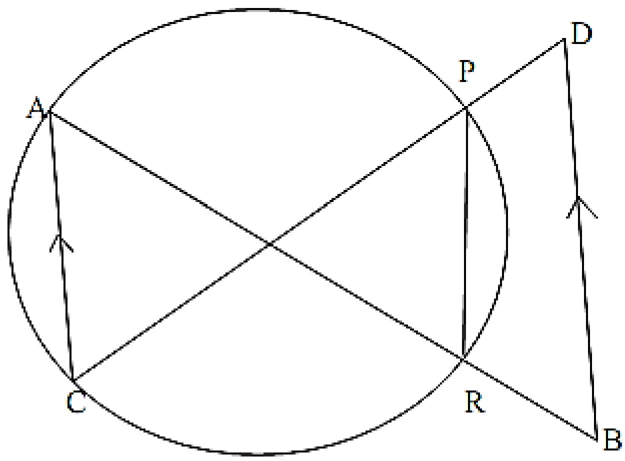
3.1.1 Express in terms of x:

(a) $F\hat{B}E$

(b) $\hat{F}$

3.1.2 Prove that AB is NOT a tangent to circle OEB.

3.2 In the diagram, chords AR and CP intersect inside the circle.
AR and CP are respectively produced to B and D such that AC || DB.



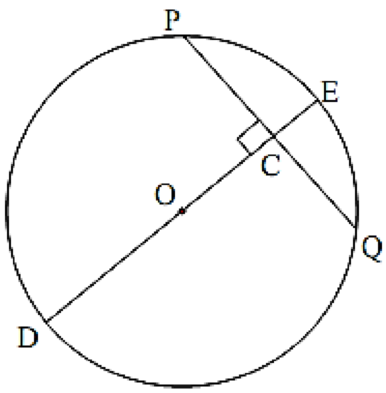
Prove that PDBR is a cyclic quadrilateral.

QUESTION 4

4.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord …

4.2 In the diagram, O is the centre of the circle. The diameter DE is perpendicular to the chord PQ at C. DE = 20 cm and CE = 2 cm.



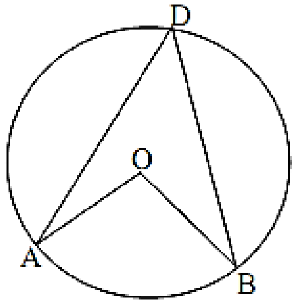
Calculate the length of the following with reasons:

4.2.1 OC

4.2.2 PQ

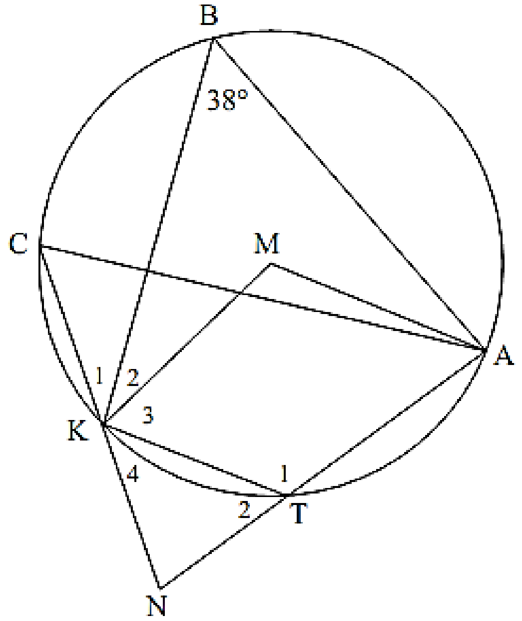
QUESTION 5

5.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$.



5.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and $\hat{B} = 38°$.

5.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and $\hat{B} = 38°$.



5.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$

(b) $\hat{T}_2$

(c) $\hat{C}$

(d) $\hat{K}_4$

5.2.2 Show that NK = NT.

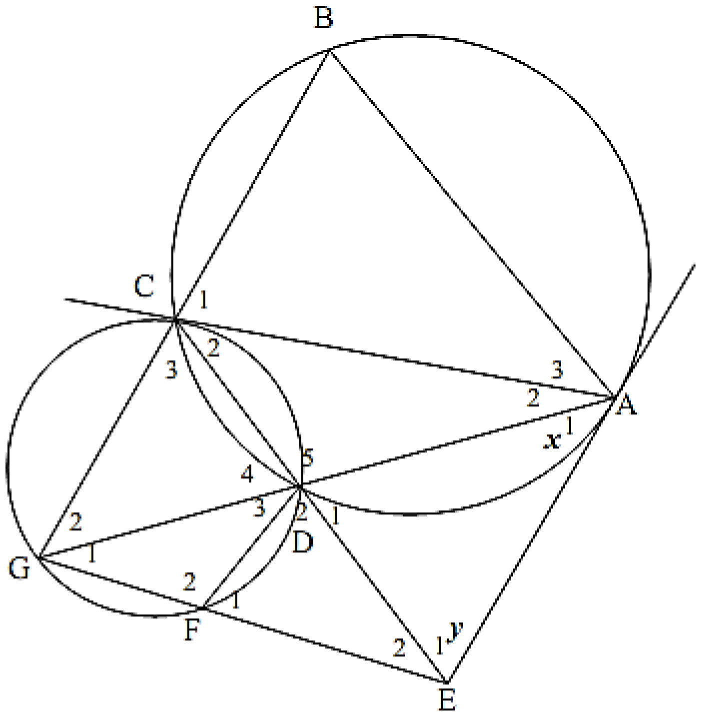
5.2.3 Prove that AMKN is a cyclic quadrilateral.

QUESTION 6

6.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is …

6.2 In the diagram, EA is a tangent to circle ABCD at A.
AC is a tangent to circle CDFG at C.
CE and AG intersect in D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

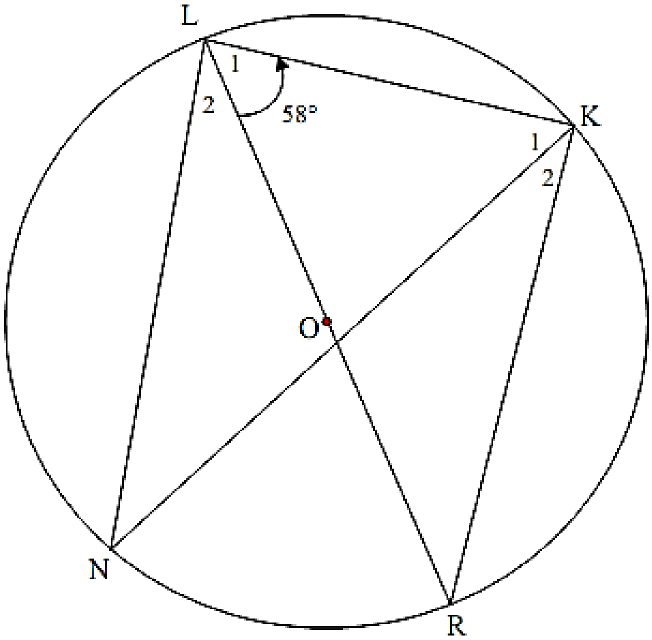
6.2.1 BCG || AE

6.2.2 AE is a tangent to circle FED

6.2.3 AB = AC

QUESTION 7

In the diagram, O is the centre of the circle. Diameter LR subtends $L\hat{K}R$ at the circumference of the circle. N is another point on the circumference and chords LN and KN are drawn. $\hat{L}_1 = 58°$.



Calculate, giving reasons, the size of:

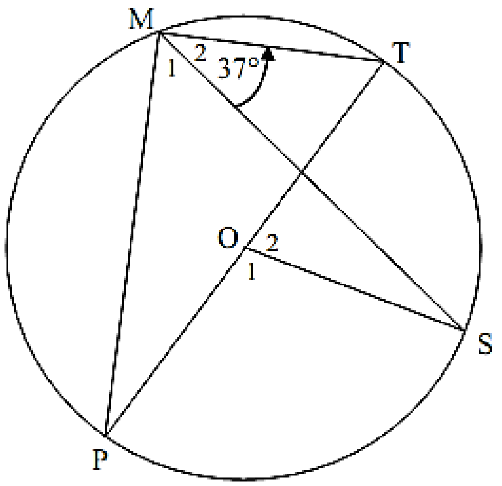
7.1 $L\hat{K}R$

7.2 $\hat{R}$

7.3 $\hat{N}$

QUESTION 8

8.1 In the diagram below, PT is a diameter of the circle with centre O. M and S are points on the circle on either side of PT.
MP, MT, MS and OS are drawn.
$\hat{M}_2 = 37°$

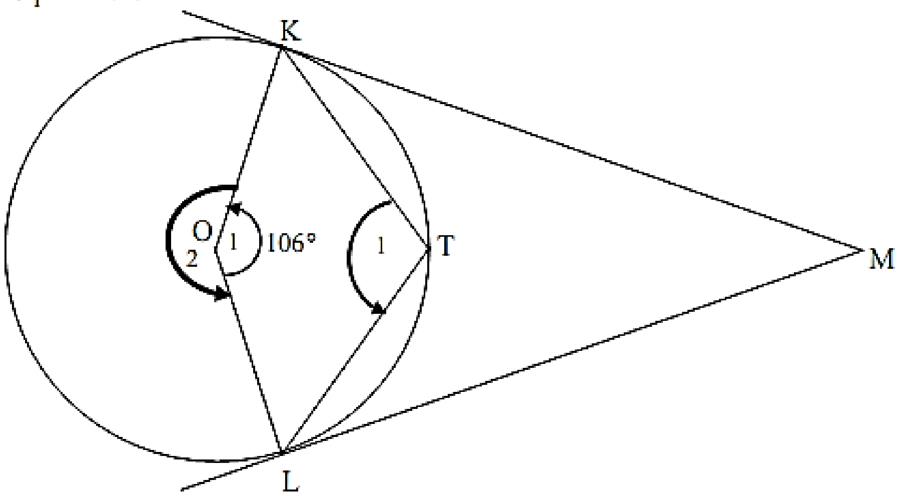


Calculate, with reasons, the size of:

8.1.1 $\hat{M}_1$

8.1.2 $\hat{O}_1$

8.2 In the diagram O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and TL are joined. $\hat{O}_1 = 106°$.



8.2.1 Calculate, with reasons, the size of $\hat{T}_1$.

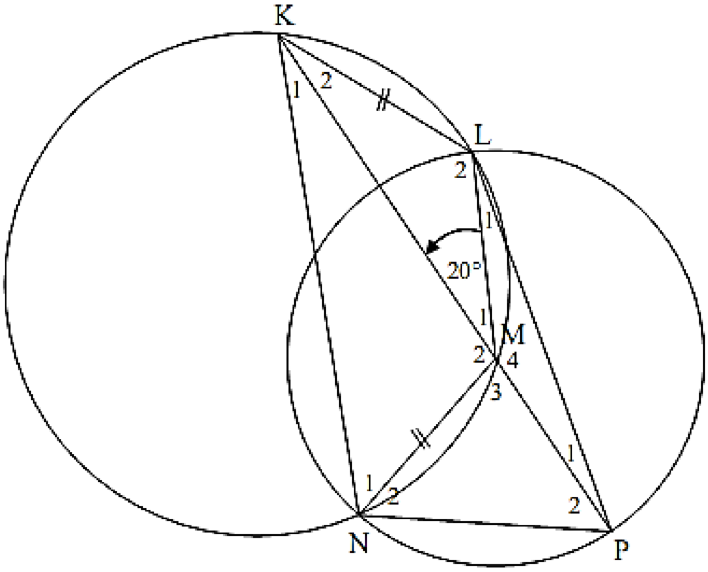
8.2.2 Prove that quadrilateral OKML is a kite.

8.2.3 Prove that quadrilateral OKML is a cyclic quadrilateral.

8.2.4 Calculate, with reasons, the size of $\hat{M}$.

QUESTION 9

In the diagram M is the centre of the circle passing through points L, N and P. PM is produced to K. KLMN is a cyclic quadrilateral in the larger circle having KL = MN. LP is joined. $K\hat{M}L = 20°$.



9.1 Write down, with a reason, the size of $N\hat{K}M$.

9.2 Give a reason why $KN \,||\, LM$.

9.3 Prove that $KL = LM$.

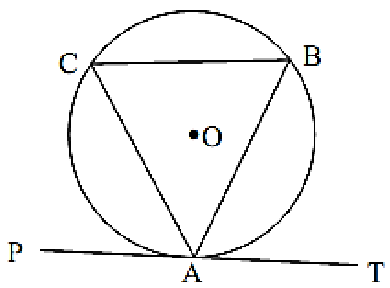
9.4 Calculate, with reasons, the size of:

9.4.1 $K\hat{N}M$

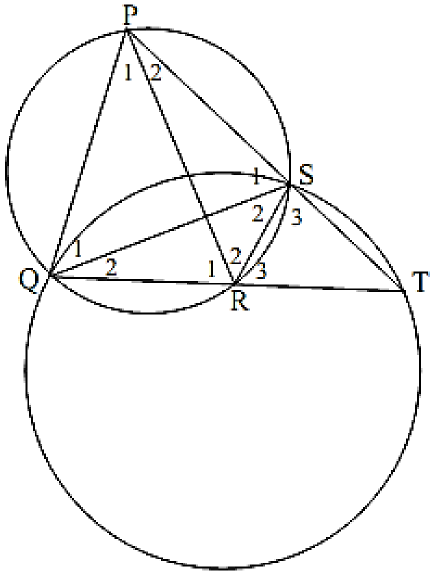
9.4.2 $L\hat{P}N$

QUESTION 10

10.1 Use the sketch in the SPECIAL ANSWER BOOK to prove the theorem which states that $B\hat{A}T = \hat{C}$.



10.2 In the diagram PQ is a tangent to the circle QST at Q such that QT is a chord of the circle and TS produced meets the tangent at P. R is a point on QT such that PQRS is a cyclic quadrilateral in another circle. PR, QS and RS are joined.



10.2.1 Give a reason for each statement. Write down only the reason next to the question number in the SPECIAL ANSWER BOOK.

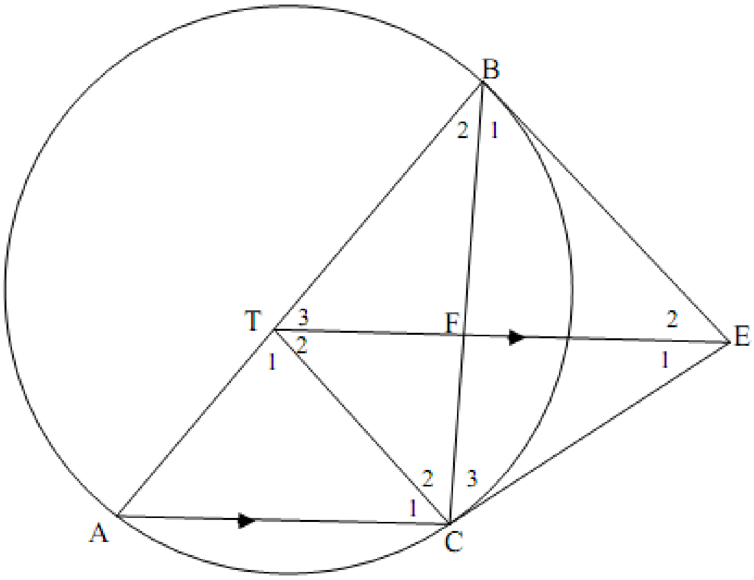
Statement	**Reason**
$\hat{Q}_1 = \hat{T}$	10.2.1 (a)
$\hat{Q}_2 = \hat{P}_2$	10.2.1 (b)

10.2.2 Prove that PQR is an isosceles triangle.

10.2.3 Prove that PR is a tangent to the circle RST at point R.

QUESTION 11

In the diagram, the vertices A, B and C of ΔABC are concyclic. EB and EC are tangents to the circle at B and C respectively. T is a point on AB such that TE || AC. BC cuts TE in F.



11.1 Prove that $\hat{B}_1 = \hat{T}_3$.

11.2 Prove that TBEC is a cyclic quadrilateral.

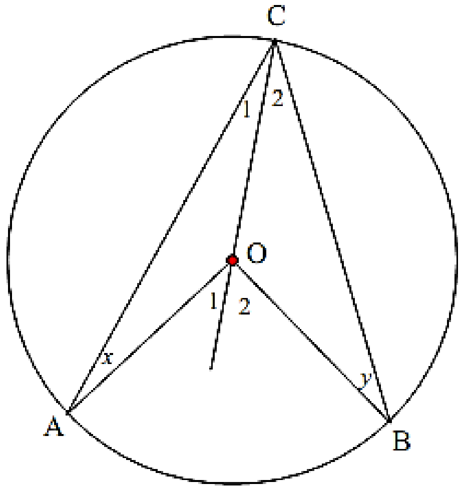
11.3 Prove that ET bisects $B\hat{T}C$.

11.4 If it is given that TB is a tangent to the circle through B, F and E, prove that TB = TC.

11.5 Hence, prove that T is the centre of the circle through A, B and C.

QUESTION 12

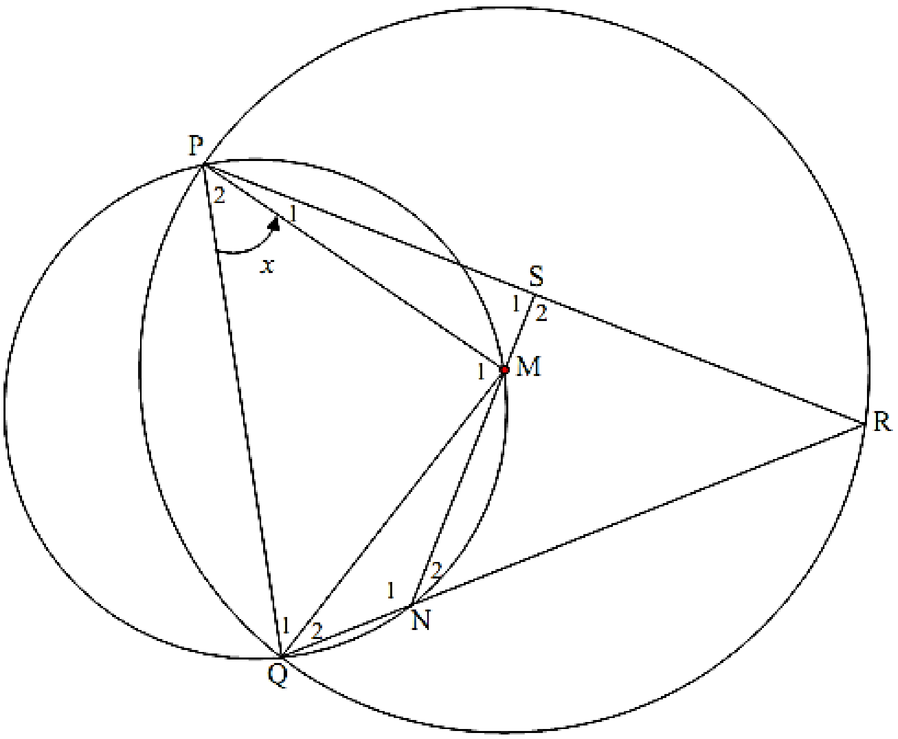
12.1 In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chords AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.



12.1.1 Determine the size of $\hat{O}_1$ in terms of x.

12.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $A\hat{O}B = 2A\hat{C}B$.

12.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



12.2.1 Give a reason why $\hat{N}_2 = x$.

12.2.2 Write down another angle equal in size to x. Give a reason.

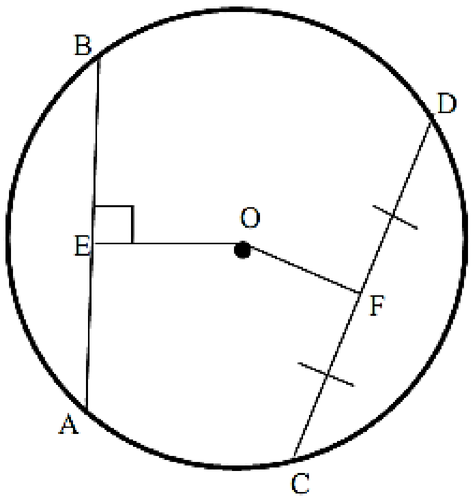
12.2.3 Determine the size of $\hat{R}$ in terms of x.

12.2.4 Prove that PS = SR.

QUESTION 13

13.1 Complete: The line drawn from the centre of the circle perpendicular to the chord …

13.2 In the figure below, AB and CD are chords of the circle with centre O. OE $\perp$ AB.
CF = FD. OE = 4 cm, OF = 3 cm and CD = 8 cm.

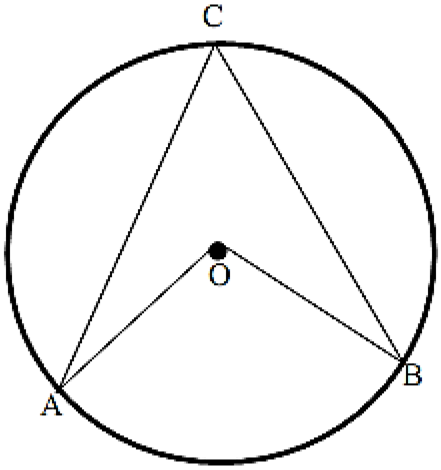


13.2.1 Calculate the length of OD.

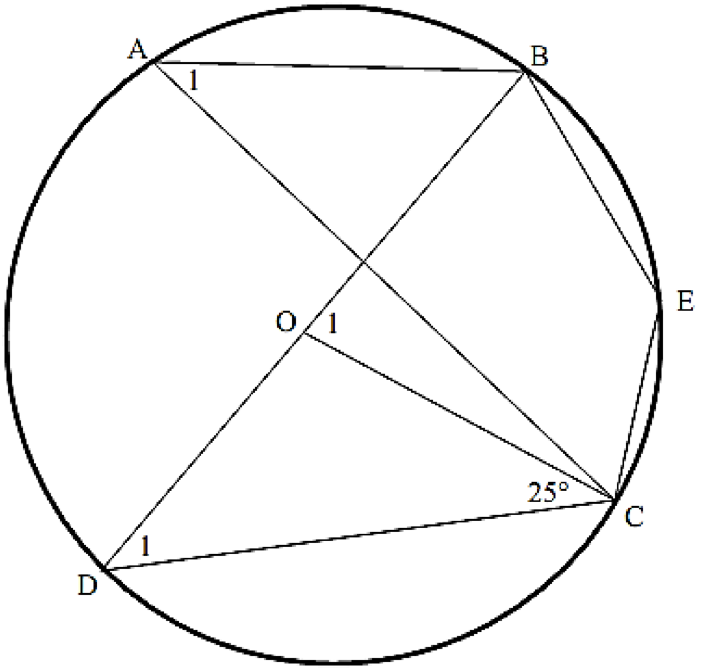
13.2.2 Hence calculate the length of AB.

QUESTION 14

14.1 In the diagram O is the centre of the circle and ABC are points on the circle. Use the diagram in your SPECIAL ANSWER BOOK to prove that: $A\hat{O}B = 2A\hat{C}B$.



14.2 In the figure below, $D\hat{C}O = 25°$ and O is the centre of the circle. A, B, E C and D are points on the circumference. Calculate, giving reasons, the sizes of:



14.2.1 $\hat{D}_1$

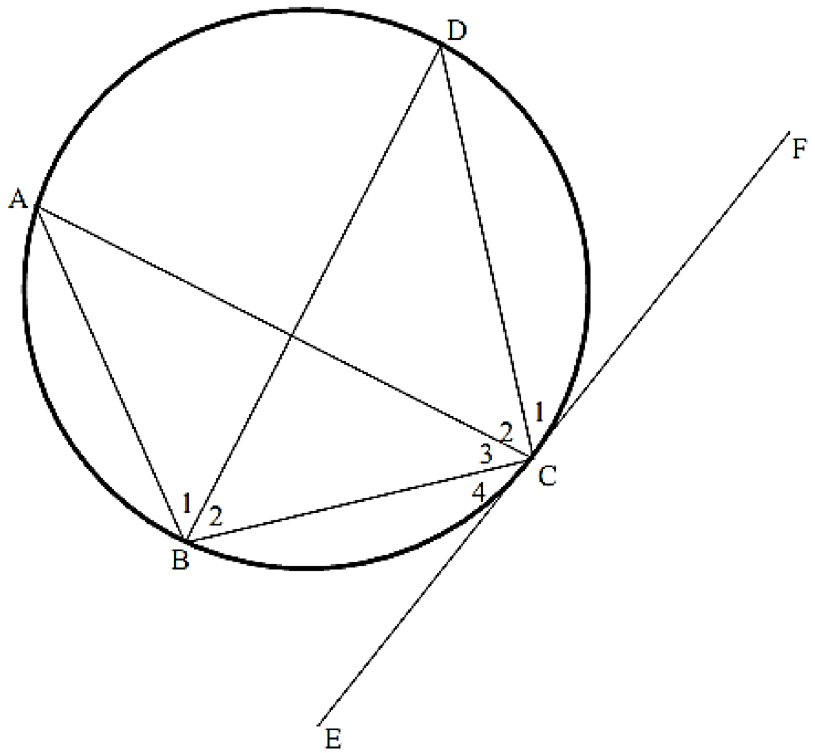
14.2.2 $\hat{O}_1$

14.2.3 $\hat{A}_1$

14.2.4 $\hat{E}$

QUESTION 15

A, B, C and D are points on the circumference of the circle in the diagram below. ECF is a tangent at C, $B_1 = B_2$.



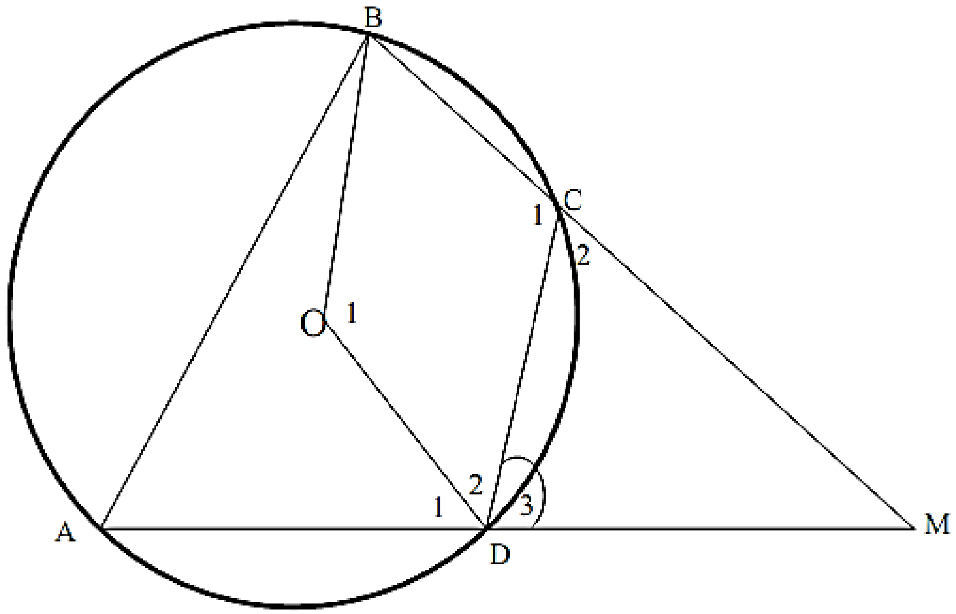
15.1 If $B_1 = x$, find, with reasons, TWO other angles equal to x.

15.2 Hence, show that DC bisects $A\hat{C}F$.

QUESTION 16

16.1 Complete: Opposite angles of a cyclic quadrilateral …

16.2 In the figure, ABCD is a cyclic quadrilateral. AB ∥ DC in circle with centre O. BC and AD produced meet at M. $D_3 = x$



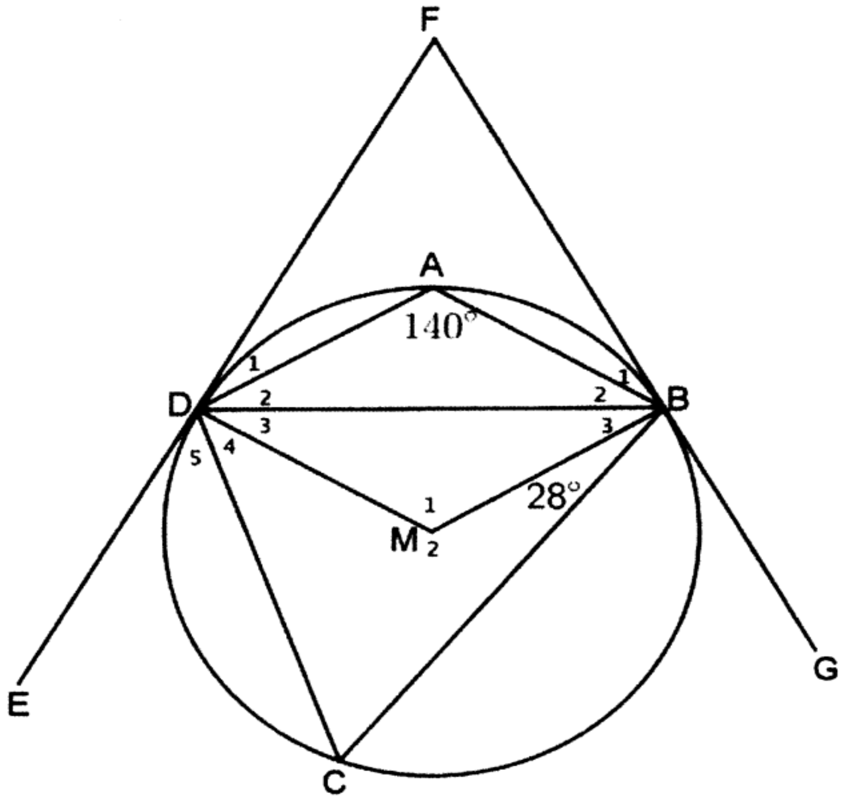
16.2.1 Show that MC = MD.

16.2.2 If $D_3 = x$, determine the value of $\hat{M}$, in terms of x.

16.2.3 Hence, show that BODM is a cyclic quadrilateral.

QUESTION 17

In the diagram below, M is the centre of the circle DABC. EDF is a tangent to the circle at D and FBG is another tangent to the circle at B.



Calculate the following angles, with reasons:

17.1 $\hat{C}$ (2)

17.2 $\hat{M}_1$ (3)

17.3 $\hat{B}_3$ (3)

17.4 $\hat{D}_5$ (2)

[10]

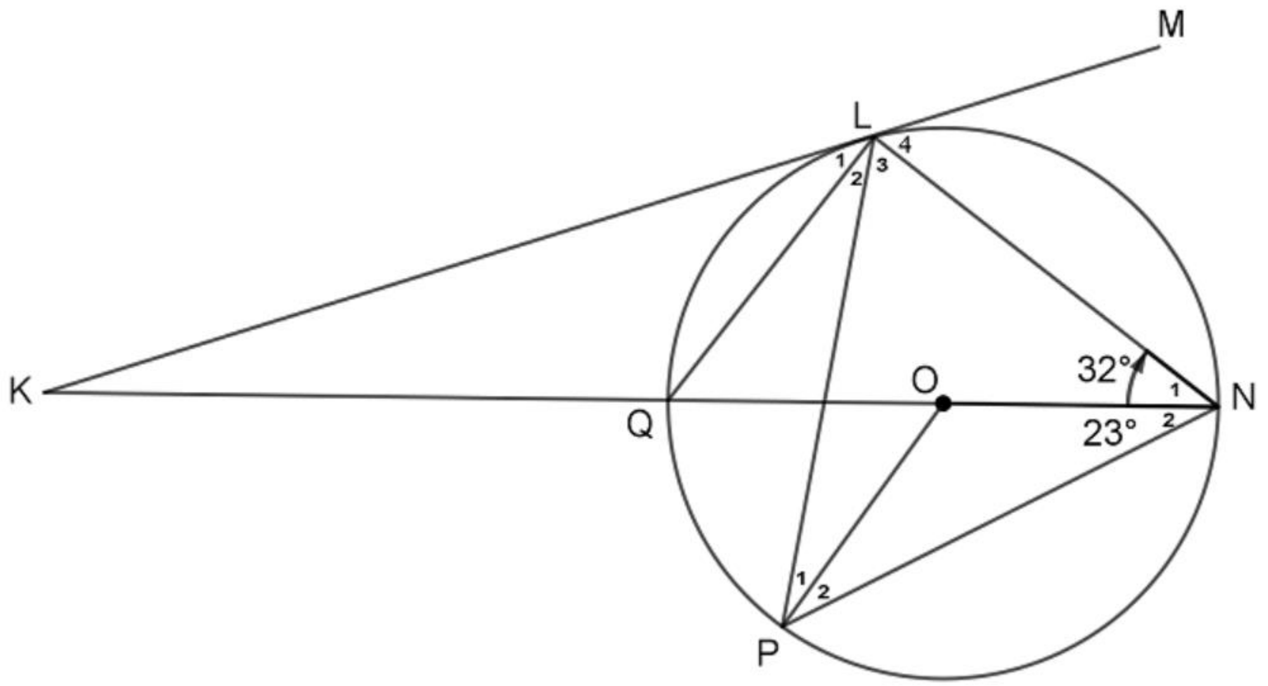
QUESTION 18

18.1 The angle at the point of contact between a tangent to a circle and a chord is --------------- . (1)

18.2 In the sketch below, circle centre O has a tangent KLM.

Diameter NQ produced meet the tangent in K.

$\hat{N}_1 = 32°$ and $\hat{N}_2 = 23^0$.



Calculate, with reasons, the size of:

18.2.1 $\hat{P}_2$ (1)

18.2.2 $P\hat{O}Q$ (2)

18.2.3 $\hat{L}_2$ (2)

18.2.4 $N\hat{L}Q$ (1)

18.2.5 $\hat{L}_3$ (2)

18.2.6 $P\hat{L}K$ (2)

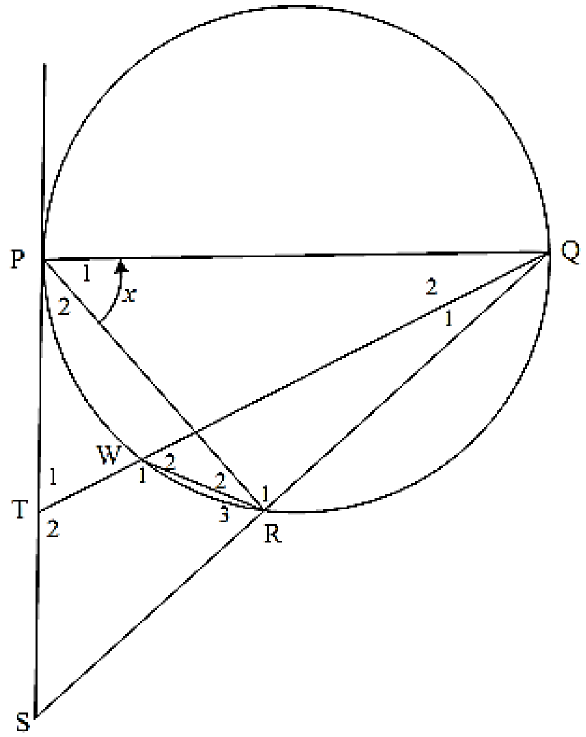
[11]

Circle Geometry and, Similarity and Proportionality

QUESTION 1

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

Let $\hat{P}_1 = x$



1.1 Why is $P\hat{R}Q = 90°$?

1.2 Prove that $\hat{P}_1 = \hat{S}$.

1.3 Prove that SRWT is a cylic quadrilateral.

1.4 Prove that $\Delta QWR \ /// \ \Delta QST$.

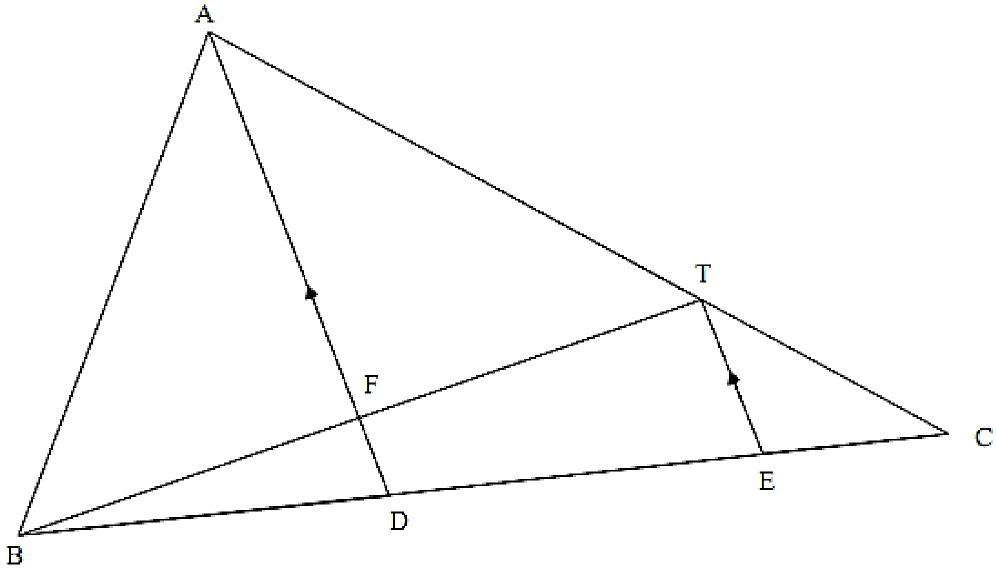
1.5 If $QW = 5$ cm, $TW = 3$ cm, $QR = 4$ cm and $WR = 2$ cm, calculate the length of:

1.5.1 TS

1.5.2 SR

QUESTION 2

In the figure below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD || TE.



2.1 Write down the numerical value of $\frac{CE}{ED}$

2.2 Show that D is the midpoint of BE.

2.3 If FD = 2 cm, calculate the length of TE.

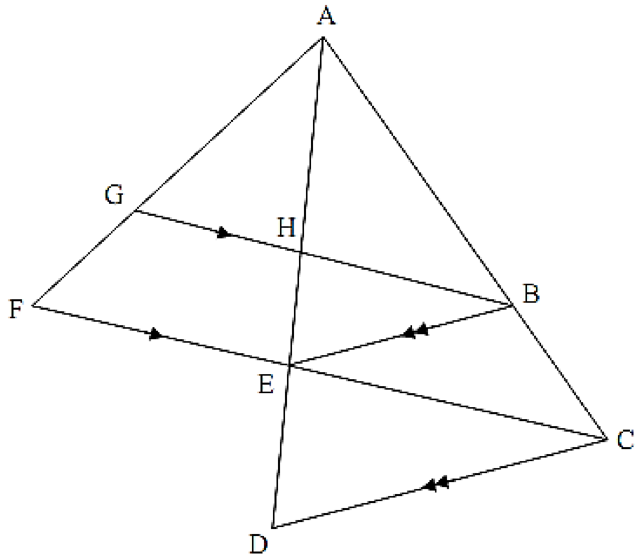
2.4 Calculate the numerical value of:

2.4.1 $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$

2.4.2 $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$

QUESTION 3

In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



3.1 Calculate with reasons:

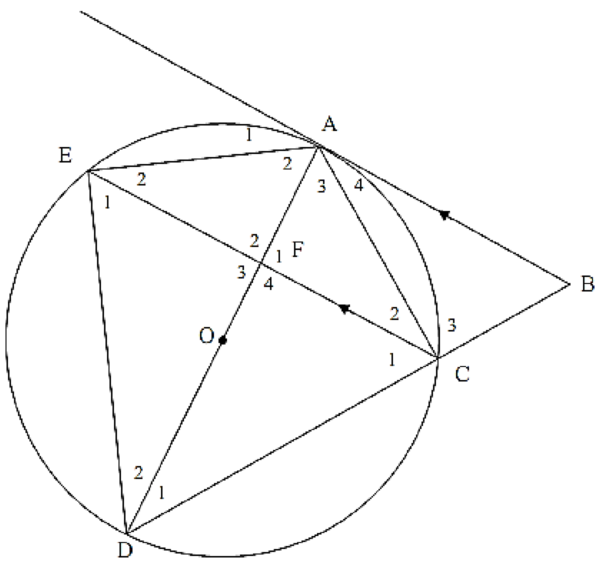
3.1.1 AH : ED

3.1.2 $\frac{BE}{CD}$

3.2 If HE = 2 cm, calculate the value of AD × HE.

QUESTION 4

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



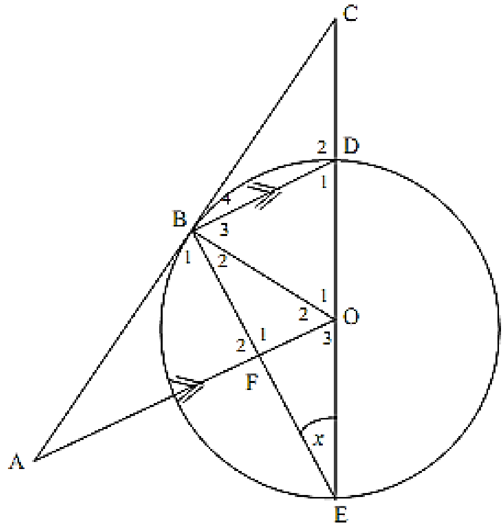
4.1 Show $\hat{C}_2 = \hat{D}_1$.

4.2 Prove that $\Delta ACF \parallel\!| \Delta ADC$.

4.3 Prove that AD = 4AF.

QUESTION 5

5.1 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F. BD || AO. $\hat{E} = x$.



5.1.1 Write down, with reasons, THREE other angles equal to x.

5.1.2 Determine, with reasons, $C\hat{B}E$ in terms of x.

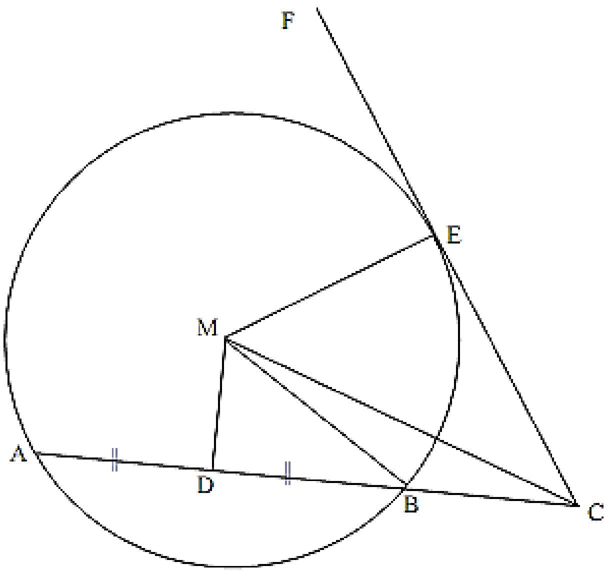
5.1.3 Prove that F is the midpoint of BE.

5.1.4 Prove that $\Delta CBD \parallel\!| \Delta CEB$.

5.1.5 Prove that 2EF.CB = CE.BD.

QUESTION 6

In the diagram below M is the centre of the circle. FEC is a tangent to the circle at E. D is the midpoint of AB.



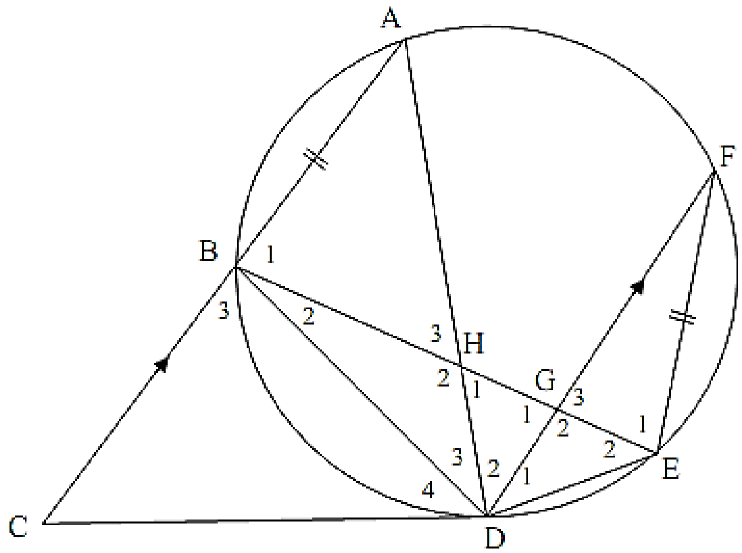
6.1 Prove MDCE is a cyclic quadrilateral.

6.2 Prove that $MC^2 = MB^2 + DC^2 - DB^2$.

6.3 Calculate CE if AB = 60 mm, ME = 40 mm and BC = 20 mm.

QUESTION 7

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. AC || FD and FE = AB. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



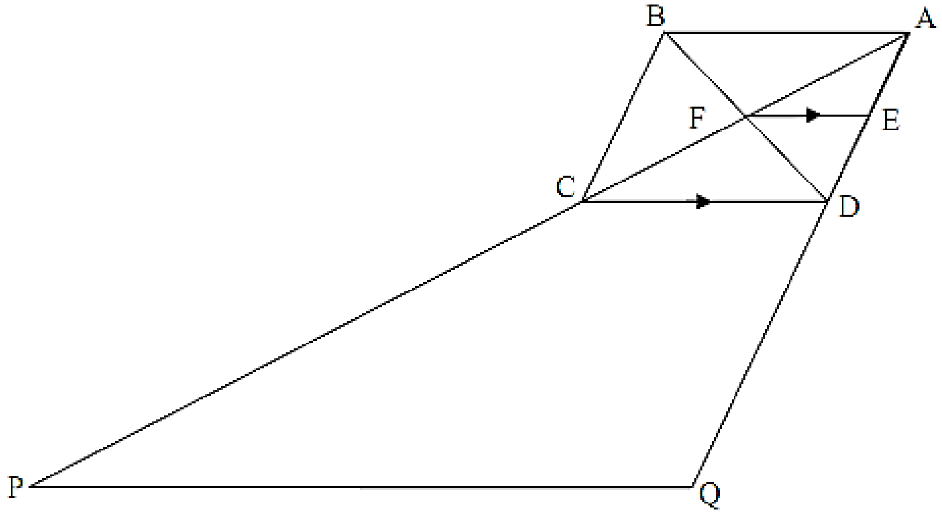
7.1 Determine THREE other angles that are each equal to x.

7.2 Prove that $\Delta BHD \,|||\, \Delta FED$.

7.3 Hence, or otherwise, prove that AB.BD = FD.BH.

QUESTION 8

ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that PC = 2AC and AD is produced to Q such that DQ = 2AD.



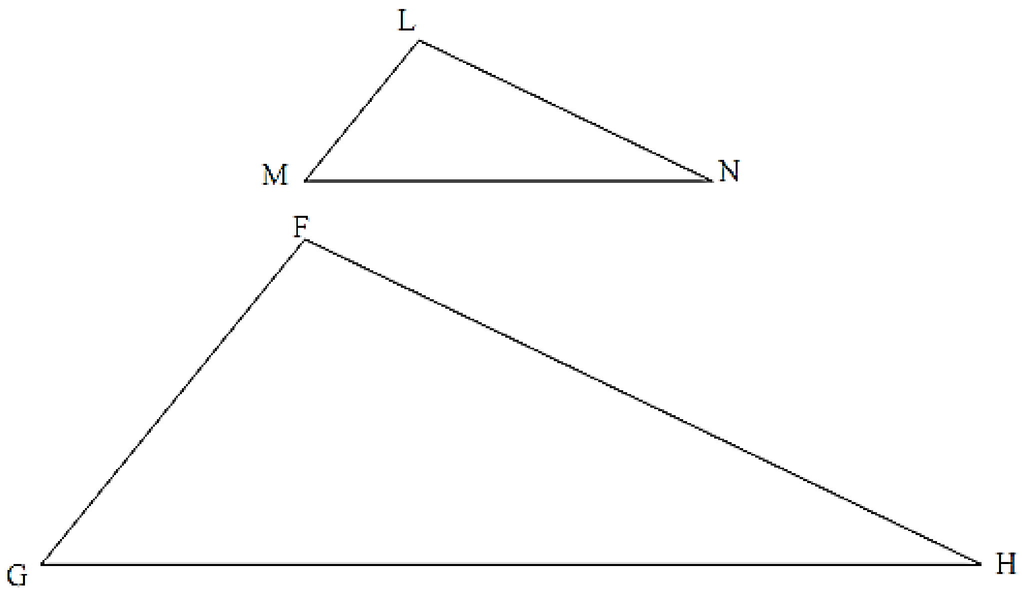
8.1 Show that E is the midpoint of AD.

8.2 Prove PQ || FE.

8.3 If PQ is 60 cm, calculate the length of FE.

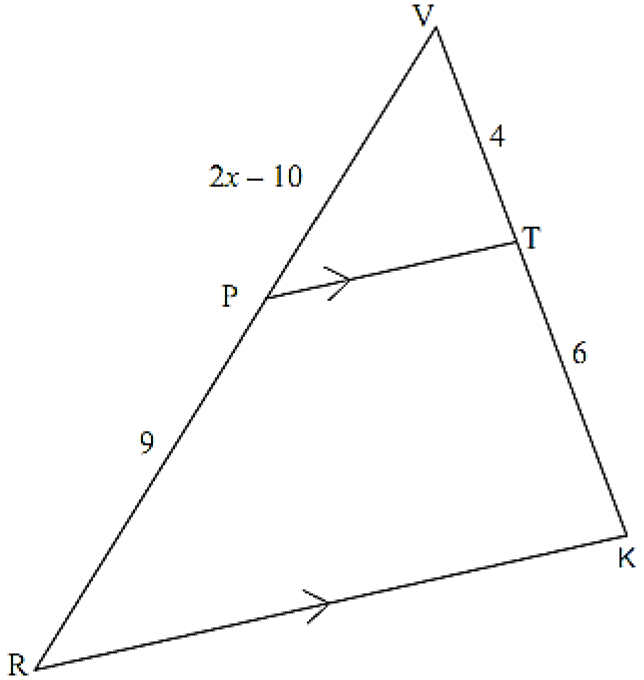
QUESTION 9

9.1 If in Δ LMN and Δ FGH it is given that $\hat{L} = \hat{F}$ and $\hat{M} = \hat{G}$, prove the theorem that states $\frac{LM}{FG} = \frac{LN}{FH}$.



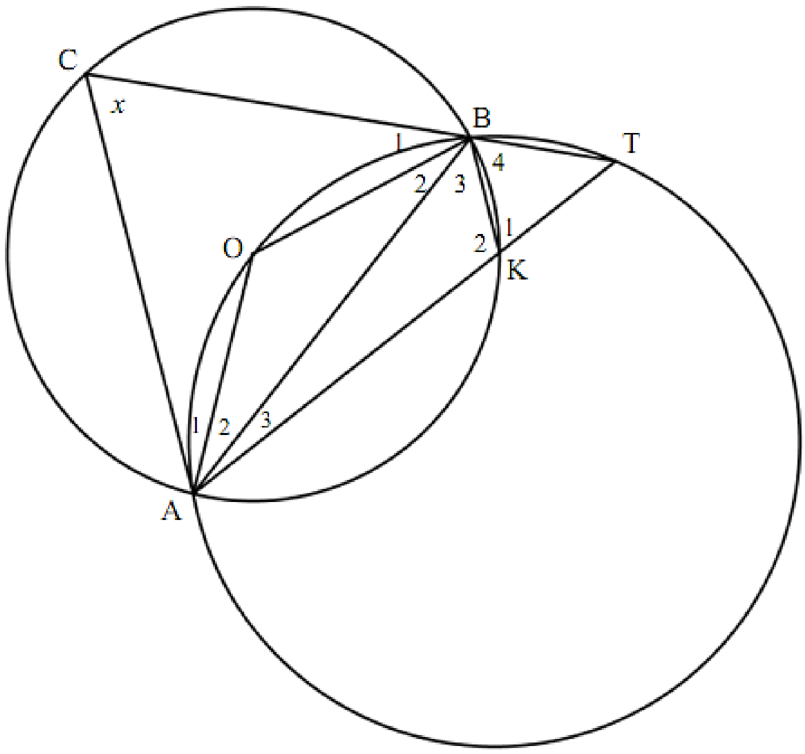
9.2 In the diagram below, ΔVRK has P on VR and T on VK such that PT || RK. VT = 4 units, PR = 9 units, TK = 6 units and VP = $2x - 10$ units.

Calculate the value of x.



QUESTION 10

O is the centre of the circle CAKB.
AK produced intersects circle AOBT at T.
$A\hat{C}B = x$



10.1 Prove that $\hat{T} = 180° - 2x$.

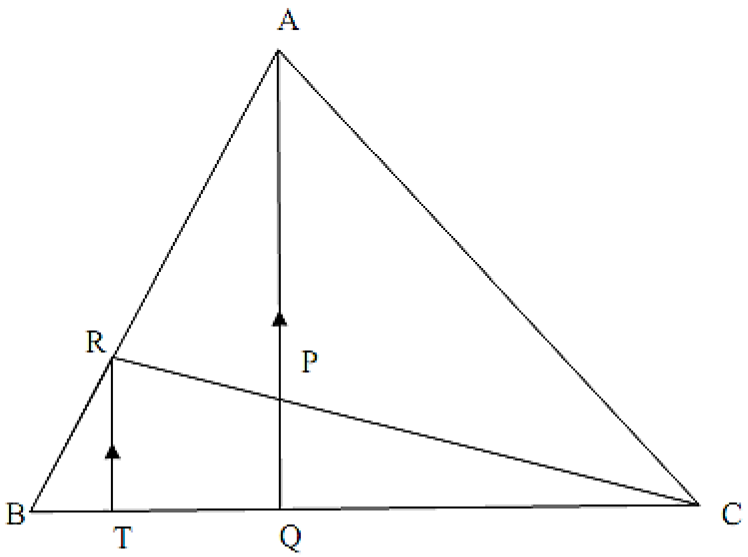
10.2 Prove AC || KB.

10.3 Prove $\Delta BKT \parallel\!| \Delta CAT$

10.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$

QUESTION 11

In the figure AQ || RT, $\frac{BQ}{QC} = \frac{3}{5}$ and $\frac{BR}{RA} = \frac{1}{2}$.



11.1 If BT = k, calculate TQ in terms of k.

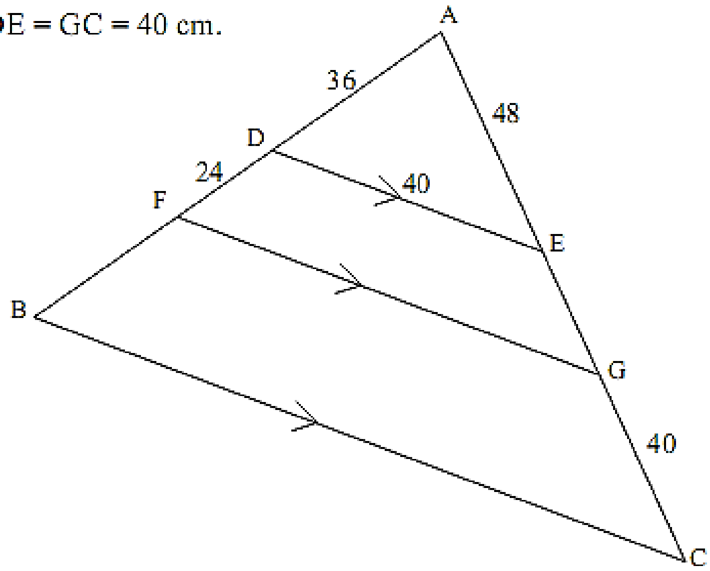
11.2 Hence, or otherwise, calculate the numerical value of:

11.2.1 $\frac{CP}{PR}$

11.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC}$

QUESTION 12

In the figure below DE || FG || BC.
AD = 36 cm, DF = 24 cm, AE = 48 cm and DE = GC = 40 cm.



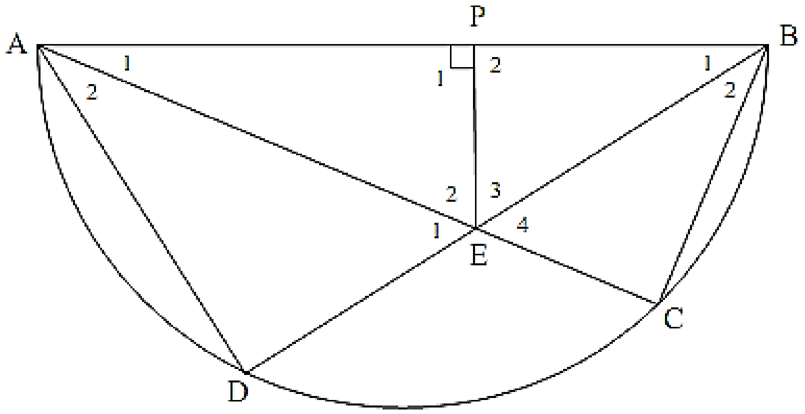
Determine, with reasons, the lengths of:

12.1 EG

12.2 BC

QUESTION 13

In the accompanying figure, AB is the diameter of circle ADCB. Chords AC and BD intersect at E. EP is perpendicular to AB.



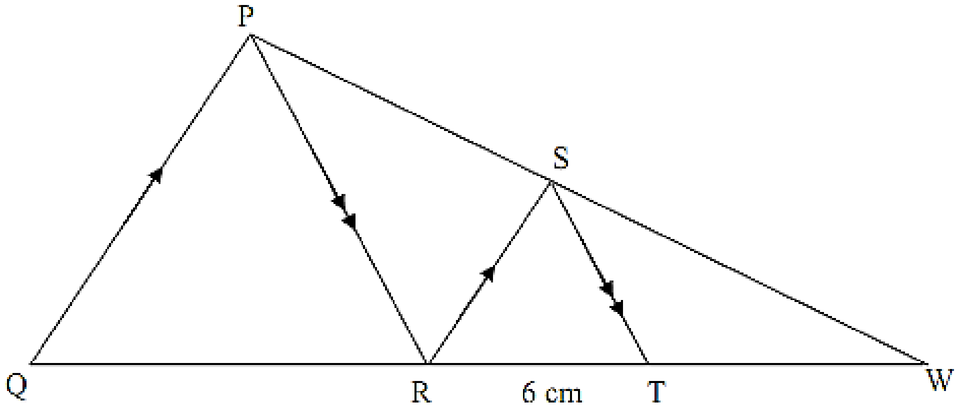
13.1 Prove that $\Delta BPE \;|||\; \Delta BDA$.

13.2 Hence show that $\frac{BP}{BD} = \frac{PE}{AD}$.

13.3 Prove that $AB^2 = BD^2 + \frac{BD^2.PE^2}{BP^2}$.

QUESTION 14

In ΔPQW, S is a point on PW and R is a point on QW such that $SR \parallel PQ$.
T is a point on QW such that $ST \parallel PR$.
RT = 6 cm
WS : SP = 3 : 2



Calculate:

14.1 WT

14.2 WQ

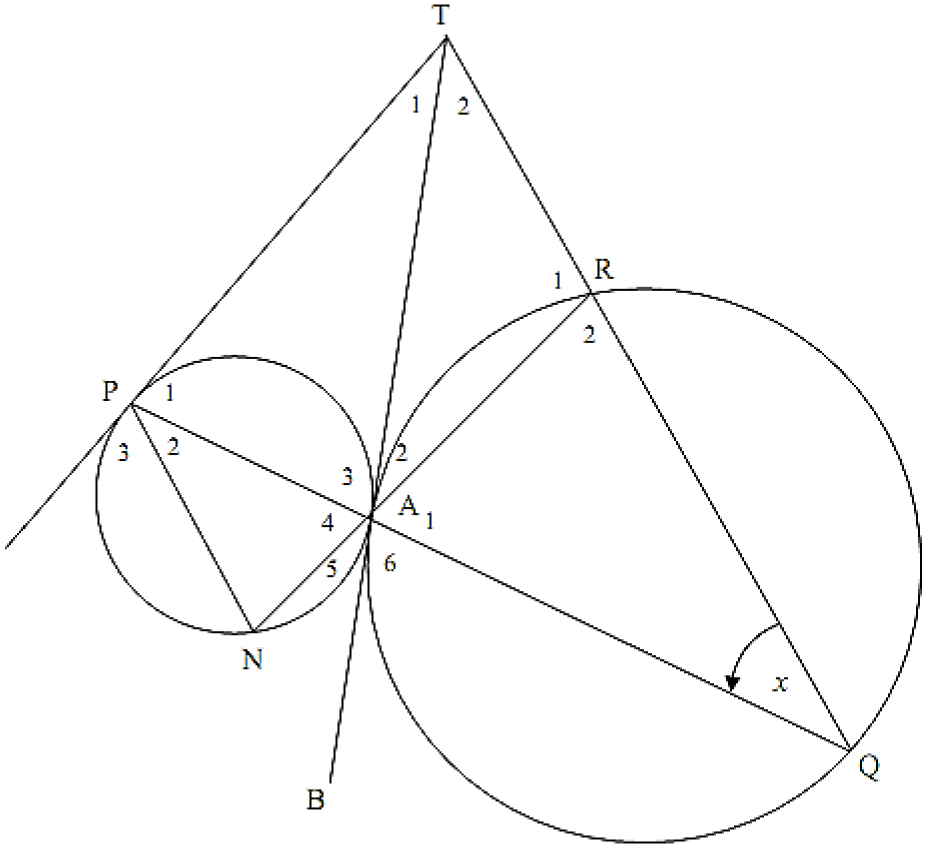
QUESTION 15

15.1 Complete the following statement:

The angle between the tangent and the chord …

15.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines.

Let $\hat{Q} = x$.

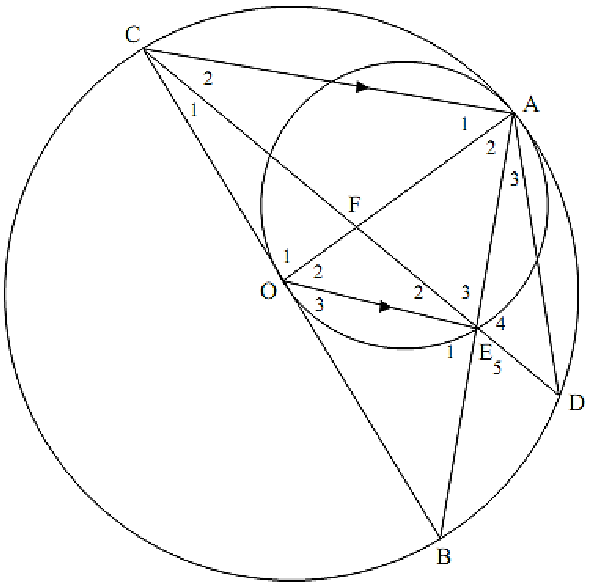


15.2.1 Name, with reasons, THREE other angles equal to x.

15.2.2 Prove that APTR is a cyclic quadrilateral.

QUESTION 16

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle.
OE || CA.



16.1 Prove, with reasons, that AE = BE.

16.2 Prove that $\Delta AED\ |||\ \Delta CEB$.

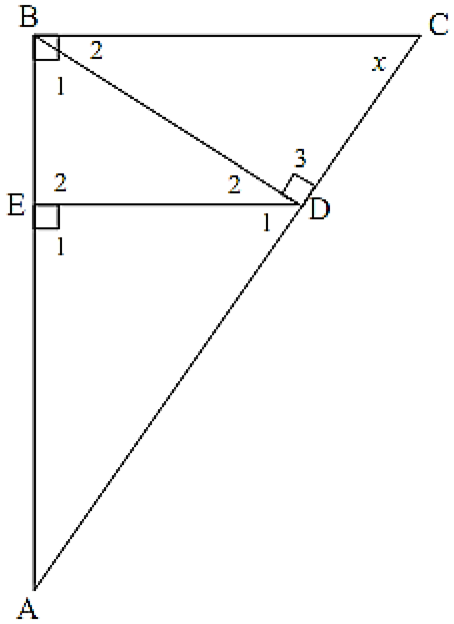
16.3 Hence, or otherwise, show that $AE^2 = DE.CE$.

16.4 If $AE.EB = EF.EC$, show that E is the midpoint of DF.

QUESTION 17

ΔABC is a right-angled triangle with $\hat{B} = 90°$. D is a point on AC such that $BD \perp AC$ and E is a point on AB such that $DE \perp AB$. E and D are joined.

AD : DC = 3 : 2.
AD = 15 cm.



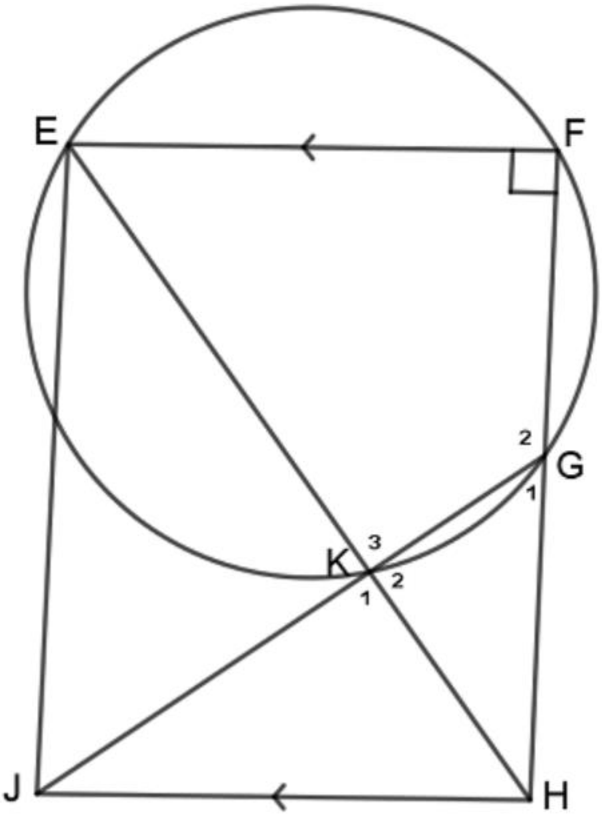
17.1 Prove that $\Delta BDA \parallel\!| \Delta CDB$.

17.2 Calculate BD (Leave your answer in surd form).

17.3 Calculate AE (Leave your answer in surd form).

QUESTION 18

In the diagram below, EFGK is a cyclic quadrilateral with $\hat{F} = 90^0$.
EK and FG are produced to meet at H. HJ is drawn parallel to FE. GK produced meets HJ at J.



18.1 Prove that:

18.1.1 $J\hat{H}F = 90^0$ (2)

18.1.2 $\hat{K}_2 = 90^0$ (2)

18.1.3 Δ HKG ||| Δ JHG (3)

18.2 Calculate JG and KG if HG = 5cm and JH = 10cm. (4)

[11]

Appendix A: Examination Guidelines

Elaboration of Content/Topics

1. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
   - The opposite angles of a cyclic quadrilateral are supplementary;
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
   - that a line drawn parallel to one side of a triangle divides the other two sides proportionally;
   - equiangular triangles are similar.
2. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circle
   - Equal chords subtend equal angles at the circumference
   - Equal chords subtend equal angles at the centre
   - In equal circles, equal chords subtend equal angles at the circumference
   - In equal circles, equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
   - Tangents drawn from a common point outside the circle are equal in length.
3. The theory of quadrilaterals will be integrated into questions in the examination.
4. Concurrency theory is excluded.

ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	$\angle$s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle$s supp
The adjacent angles in a revolution add up to 360°.	$\angle$s round a pt **OR** $\angle$s in a rev
Vertically opposite angles are equal.	vert opp $\angle$s =
If AB || CD, then the alternate angles are equal.	alt $\angle$s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp $\angle$s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int $\angle$s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle$s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle$s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint $\angle$s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle$s in Δ **OR** Int $\angle$s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle$s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle$s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S$\angle$S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** $\angle\angle$S
If in. two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem

The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan $\perp$ radius tan $\perp$ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line $\perp$ radius **OR** converse tan $\perp$ radius **OR** converse tan $\perp$ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre $\perp$ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	$\angle$ at centre = 2 $\times\angle$ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	$\angle$s in semi circle **OR** diameter subtends right angle **OR** $\angle \frac{1}{2} \odot$
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse $\angle$s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	$\angle$s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal $\angle$s **OR** converse $\angle$s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal $\angle$s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal $\angle$s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal $\angle$s

Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad sup **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common ptOR Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ||
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

Appendix B: Information Sheet

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

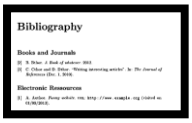
$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

Bibliography



1. MATHEMATICS QUESTION PAPER GRADE 9 2013 EASTERN CAPE
2. MATHEMATICS QUESTION PAPER GRADE 9 2014 EASTERN CAPE
3. MATHEMATICS QUESTION PAPER GRADE 9 2016 EASTERN CAPE
4. MATHEMATICS QUESTION PAPER GRADE 9 2011 EASTERN CAPE
5. BISHOPS DIOCESAN COLLEGE GRADE 9 PAPER 2 2017
6. MATHEMATICS EXAMINATION GUIDELINES
7. MATHEMATICS QUESTION PAPERS GRADE 12 (2008 – 2017) NATIONAL

Outcomes reached

	YES	NO
Lines. Angles and Triangles		
Angle relations		
Classifying 2D shapes		
Similar and congruent 2D shapes		
Circle Geometry		
Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles.		
Solve circle geometry problems, providing reasons for statements when required.		
Prove riders		
Circle Geometry and, Similarity and Proportionality		
Prove that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Midpoint Theorem as a special case of this theorem);		
Prove that equiangular triangles are similar;		
Prove riders		