JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

EUCLID'S GEOMETRY

MEMORANDUM/ANSWER BOOKLET

LEARNER/TEACHER

2022

EUCLID'S GEOMETRY

JENN MAREMATLOU TRAINING INSTITUTE

A Division of Marematlou Group Holdings

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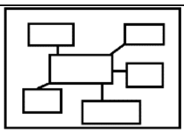
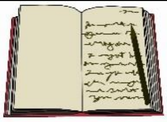
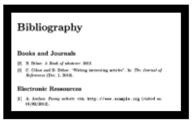
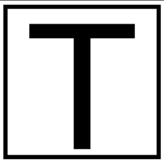
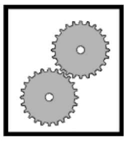
ICON DESCRIPTION

 **MIND MAP**	 **EXAMINATION GUIDELINE**	 **CONTENTS**	 **ACTIVITIES**
 **BIBLIOGRAPHY**	 **TERMINOLOGY**	 **WORKED EXAMPLES**	 **STEPS**

SOLUTIONS

Lines, Angles and Triangles

Question 1

1. 1.1 $\angle BCD$ or $\angle DCB$ √

   1.2 $\angle ABC$ or $\angle CBA$ √

2. $\angle FEG = \angle DEC$ vertically opp $\angle s$ √

   $\angle FEG = 40°$

   $\angle y + 85° + 40° = 180°$ sum $\angle s$ of a Δ √

   $\therefore \angle y = 55°$ √

3 3.1 $S\widehat{R}T = \hat{Q} = x + 70°$ (corr. $\angle s$, $RT//QP$) ✓A

$\hat{S} + T\hat{R}S + \hat{P} = 180°$ (sum of f $\angle s$ of Δ)

$x + 10° + 28° + 70° = 180°$

$2x + 108° = 180°$

$2x = 72°$ ✓A

$x = 36°$ ✓A

3.2 $S\hat{T}R = \hat{P} = x + 10°$ ✓A (corr. $\angle s$, $RT//QP$) ✓

A $S\hat{T}R = 36° + 10°$

$= 46°$ ✓A

3.3 $S\widehat{R}T = \hat{Q} = x + 70°$ (corr. $\angle s$, $RT//QP$) $x + 70° = 36° + 70°$ ✓A

$= 106°$

$106° \neq 90°$

$\therefore PQS$ is not a right angled triangle ✓A

4 4.1 $In\ \ \Delta ABC\ and\ \Delta TSP$

$\hat{B} = \hat{P} = 70°$ (given) ✓

$\hat{C} = \hat{S} = 70°$ (base $\angle s$ of is os. Δ) ✓A

$\hat{A} = \hat{T} = 40°$ (sum of $\angle s$ of Δ) ✓A

$\therefore \Delta ABC /// \ \Delta TSP$ ($\angle\angle\angle$) ✓A

	4.2	$y = AC = 15$	(given) ✓A
		$\frac{PS}{BC} = \frac{TS}{AB} = \frac{PT}{AC}$	(Sides are proportional)✓A
		$\frac{x}{12} = \frac{5\times12}{15}$	
		$\therefore x = 4$ units	✓A
5	5.1	In ΔABC and ΔDCB	
		1. $\hat{A} = \hat{D}$	(given) ✓A
		2. $A\hat{C}B = D\hat{B}C$	(given) ✓A
		3. $BC = BC$	(Common) ✓A
		4. $\Delta ABC \equiv \Delta DCB$	$(\angle\angle S)$ ✓A
	5.2	$AB = DC$	(From congruency) ✓A
		$\therefore BC = 4$ units ✓A	

Question 2

2.1.1 $\angle ACB = \angle DCF = 32°$ (Vert. opp. $\angle's$)

$\angle EBC = \angle ACB = 32°$ (Alt. $\angle's$, EB ∥ DA)

2.1.2 $\angle CAB + \angle ABE = 180°$ (Co int. $\angle's : EB||DA$)✓**S/R**

$\angle CAB = 180° - 65°$ ✓**M**

$\angle CAB = 115°$✓**A**

OR

$\angle CAB + \angle ACB + \angle ABC = 180°$ ($\angle's$ of a Δ)✓**S/R**

$\angle CAB = 180° - (32° + 33°)$ [$\angle ABC = 65° - 32°$]✓**M**

$\angle CAB = 180° - 65°$

$\angle CAB = 115°$ ✓**A**

2.2.1 $\angle A + \angle ABC = \angle BCE$ (Ext $\angle$ of a Δ) ✓**S/R**

$(2x - 48°) + (x + 14°) = 116°$ ✓**M**

$3x - 34° = 116°$

$3x = 150°$ ✓**A**

$x = 50°$

OR ✓**S/R**

$\angle A + \angle ABC + + \angle ACB = 180°$ ($\angle's$ of a Δ)

$(2x - 48°) + (x + 14°) + 64° = 180°$ ✓**M**

$3x + 30° = 180°$

$3x = 150°$

$x = 50°$ ✓**A**

2.2.2 $\angle A = 2x - 48°$

$= 2(50°) - 48°$ ✓**M**

$= 100° - 48°$

$= 52°$ ✓**A**

2.2.3 $\angle ABC = 50° + 14° = 64°$

$\angle ACB = 180° - 116° = 64°$

✓**S** ✓**R**

ΔABC is an isosceles triangle ($\angle ABC = \angle ACB$)

2.3.1 ✓**S** ✓**R**

$\angle ABC = 40°$ (Complementary $\angle's$)

2.3.2 ✓**S** ✓**R**

$\angle AD\hat{O} = 32°$ (AO =OD / radii)

2.4.1

STATEMENT	REASON
$\hat{A} = \hat{P}$	Alt .∠'s, AB ∥ PQ ✓
$\hat{B} = \hat{Q}$	Alt .∠'s, AB ∥ PQ ✓
$A\hat{O}B = P\hat{O}Q$	Vert. opp. ∠'s ✓ **A**
∴ Δ ABO ∣∣∣ ΔPQO.	AAA ✓ **A**

2.4.2 $\frac{OQ}{OB} = \frac{OP}{AO}$ (Corr. sides are proportional) ✓**S/R**

$$\frac{x}{5\ cm} = \frac{12\ cm}{6\ cm}$$ ✓**A**

$x = OQ = 10\ cm$ ✓**CA**

2.5 2.5.1

$\angle KMN = 3x$ (alt. ∠s LK// MN) √

$2x + 3x + x = 180°$ (∠s on straight line)√

$6x = 180°$

$$\frac{6x}{6} = \frac{180°}{6}$$

$x = 30°$ √

2.5.2

$\angle LKM = 3x$

$= 3(30)$

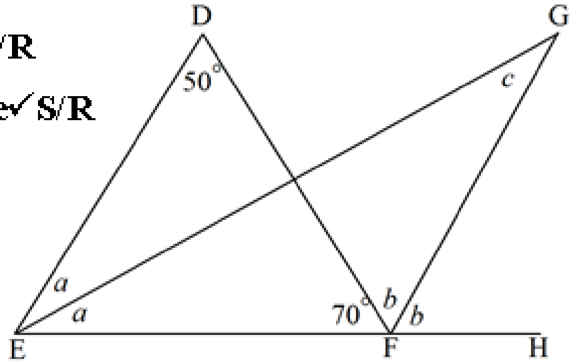
$= 90°$ √

2.5.3 Triangle MKL is a right-angled triangle √

Question 3

3.1 In the given sketches angles that are marked with the same letter are equal to each other.
Find the size of each of the following angles.

3.1.1 a, b, and c

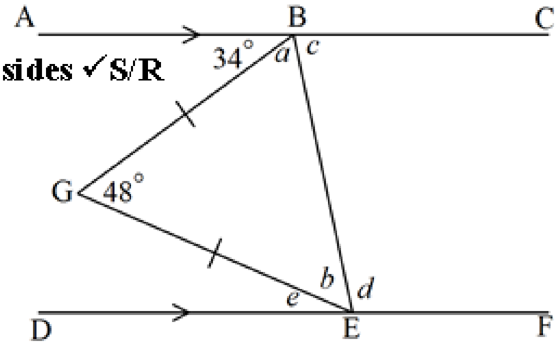


$a = 30^0$....int $\angle$'s $\Delta = 180^0$ ✓ S/R

$b = 55^0$....$\angle$'s on a straight line✓S/R

$c = 25^0$....int $\angle$'s $\Delta = 180^0$ ✓S/R

3.1.2 a, b, c, d and e



$a = 66^0$....int $\angle$'s $\Delta = 180^0$ ...$\angle$'s opp equal sides ✓S/R

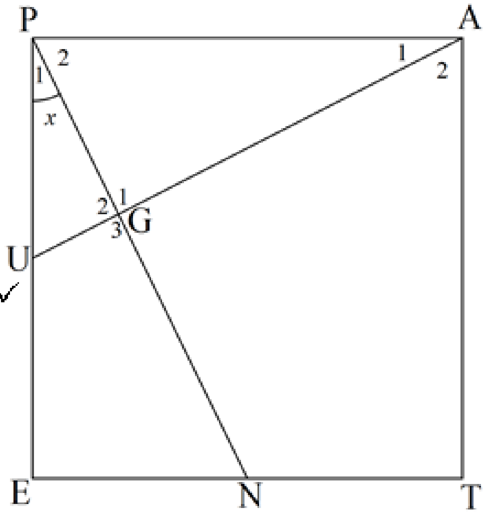
$b = 66^0$....int $\angle$'s $\Delta = 180^0$ ✓S/R

$c = 80^0$....$\angle$'s on a straight line✓S/R

$d = 100^0$....co– int $\angle$'s || lines✓S/R

$e = 14^0$....$\angle$'s on a straight line✓S/R

3.2 In the diagram, PATE is a square with $\hat{P}_1 = \hat{A}_1$. Prove that $PG \perp AU$.

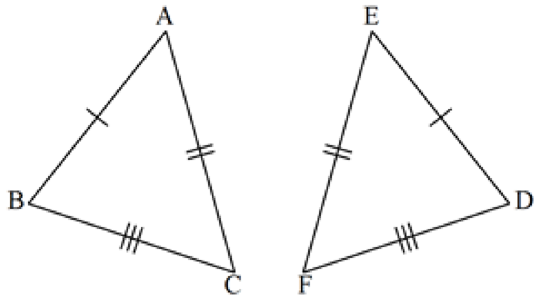


$\hat{P}_1 = \hat{A}_1 = x$....given

$\hat{P}_2 = 90° - x$✓ .....$A\hat{P}E = 90°$....PATE is a square✓

$\therefore \hat{G}_1 = 90°$✓ ....int $\angle$'s of $\Delta = 180°$

3.3 In each of the following, circle the correct answer from the options given that matches the statement to the given sketch:

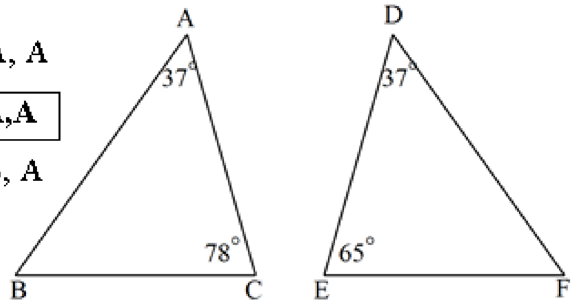


3.3.1
A) $\Delta ABC \equiv \Delta DEF$ S, S, S

B) $\Delta ABC \equiv \Delta EDF$ S,S,S

C) $\Delta ABC \equiv \Delta FED$ S, S, S

D) None of the above

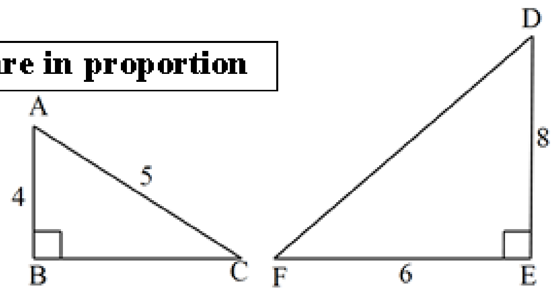


3.3.2
A) $\Delta ABC \equiv \Delta DEF$ A, A, A

B) $\Delta ABC \,|||\, \Delta DEF$ A,A,A

C) $\Delta ABC \equiv \Delta DEF$ A, S, A

D) None of the above



3.3.3
A) $\Delta ABC \,|||\, \Delta DEF$ sides are in proportion

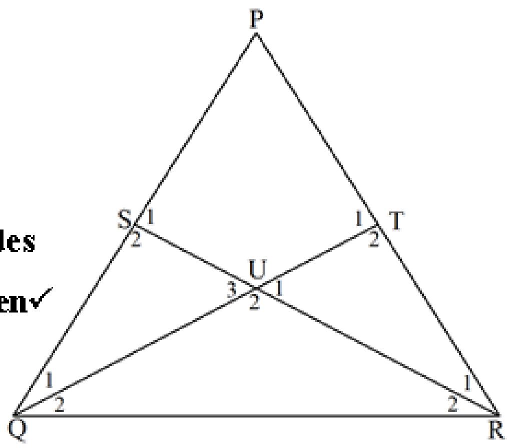
B) $\Delta ABC \,|||\, \Delta DEF$ S, S, S

C) $\Delta ABC \equiv \Delta DEF$ R, H, S

D) None of the above

3.4 In the given sketch, ΔPQR is isosceles with $PQ = PR$ and $Q_2 = R_2$

Prove $\Delta QTP \equiv \Delta RSP$



In ΔQTP and ΔRSP

1) $\hat{Q}_1 = \hat{R}_1$......$P\hat{Q}R = P\hat{R}Q$...$\angle$'s opp equal sides

and $\hat{Q}_2 = \hat{R}_2$...given✓

2) PQ=PR...given✓

3) $\hat{P} = \hat{P}$........common angle✓

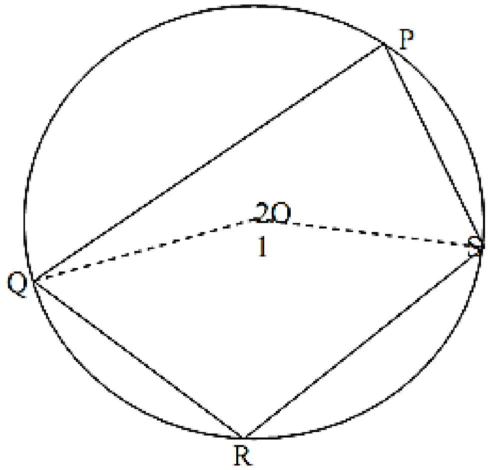
or $\hat{T}_1 = \hat{S}_1$........$\hat{T}_1 = \hat{Q}_2 + \hat{R} = \hat{R}_2 + \hat{Q} = \hat{S}_1$

$\therefore$ $\Delta QTP \equiv \Delta RSP$.........S,A,A✓

Circle Geometry

QUESTION 1

1.1	1.1.1	Equal to angle in the alternate segment
	1.1.2	Interior opposite angle
1.2		



Constr.: Join OQ and OS

Proof :

$\widehat{O_1} = 2\,\hat{P}$ (angle at centre)

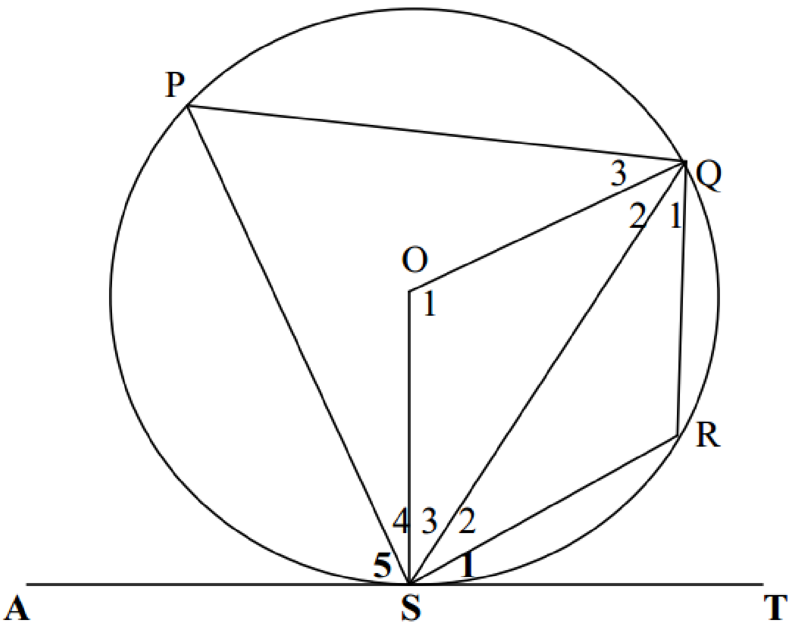
$\widehat{O_2} = 2\,\hat{R}$ (angle at centre)

But $\widehat{O_1} + \widehat{O_2} = 360°$

$2\,\hat{P} + 2\,\hat{R} = 360°$

Hence $\hat{P} + \hat{R} = 180°$

1.3 In the diagram below, AST is a tangent to a circle O at S.
$R\hat{S}T = \hat{S}_1 = 23°$ and QR = RS.



1.3.1 $\hat{S}_1 = \hat{Q}_1 = 23°$ …(angle between tangent and chord)

$\therefore\ \hat{S}_2 = \hat{Q}_1 = 23°$ …(RS = RQ)

1.3.2 $\therefore\ \hat{R} = 180° - (23° + 23°)$
$= 134°$….(angle sum of triangle)

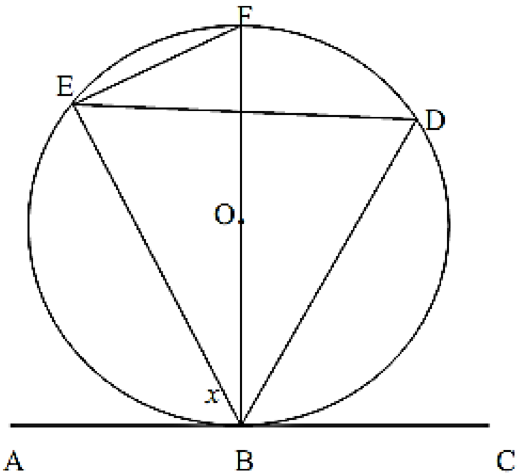
1.3.3 $\hat{P} = 180° - 134°$
$= 46°$ ……… (Opposite angles of a cyclic quad are supp.)

1.3.4 $\hat{O}_1 = 2\hat{P} = 92°$ …. (angle at the centre is twice…)

QUESTION 2

2.1	OD = 25 cm ∴ OC = 25 cm – 18cm = 7 cm $AC^2 + OC^2 = OA^2$ $AC^2 + (7)^2 = (25)^2$ $AC^2 = 576$ $\therefore AC = 24$ cm $AB = 2 \times AC$ (OD⊥AB) $\therefore AB = 48$ cm	
2.2	2.2.1	$B\hat{P}R = 25°$ (PR‖QB, alt angles) $R\hat{Q}B = 25°$ (Subtended by RB) $P\hat{R}Q = 25°$ (Subtended by PQ) *OF* alt angles)
	2.2.2 (a)	$R\hat{O}B = 2 \times R\hat{Q}B$ (angle at centre) $\therefore R\hat{O}B = 50°$
	2.2. 2 (b)	$O\hat{R}T + R\hat{O}T + R\hat{T}O = 180°$(angles of triangle) $O\hat{R}T + 50° + 90° = 180°$ $\therefore O\hat{R}T = 40°$
	2.2. 2 (c)	$R\hat{O}S = 100°$ ($\Delta ROT \equiv \Delta SOT$)
	2.2. 2 (d)	$R\hat{P}Q = 115°$($B\hat{P}Q = 90°$, angle in semi-circle)

QUESTION 3

3.1	3.1.1 (a)	 $\hat{FBE} = 90° - x$ (tangent is perp. to a diameter)
	3.1.1 (b)	$\hat{F} = x$
	3.1.2	$\hat{EOB} = 2x$ (angle at centre) $\therefore \hat{ABE} \neq \hat{EOB}$
3.2	$\hat{CAR} = \hat{ABD}$ (alt angles , $AC \parallel DB$) $\hat{CAR} = \hat{CPR}$ (subtended by CR) $\therefore \hat{RBD} = \hat{CPR}$ (both $= \hat{CAR}$) Hence PDBR is a cyclic quadrilateral (Ext. angle = int. opp. angle	

QUESTION 4

4.1 …bisects the chord.

4.2.1 $OE = 10$ cm … O midpoint of DE

$OC = OE - CE$

$= 10 - 2$

$= 8$ cm

4.2.2 In ΔCOQ:

$QC^2 = OQ^2 - OC^2$ … Theorem of Pythagoras

$= (10)^2 - (8)^2$

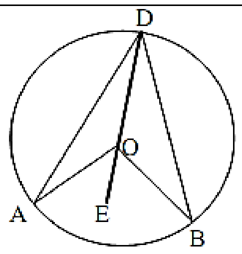
$= 36$

$QC = 6$ cm

$\therefore PQ = 2QC$ … line drawn from centre $\perp$ to chord bisects chord

$PQ = 12$ cm

QUESTION 5

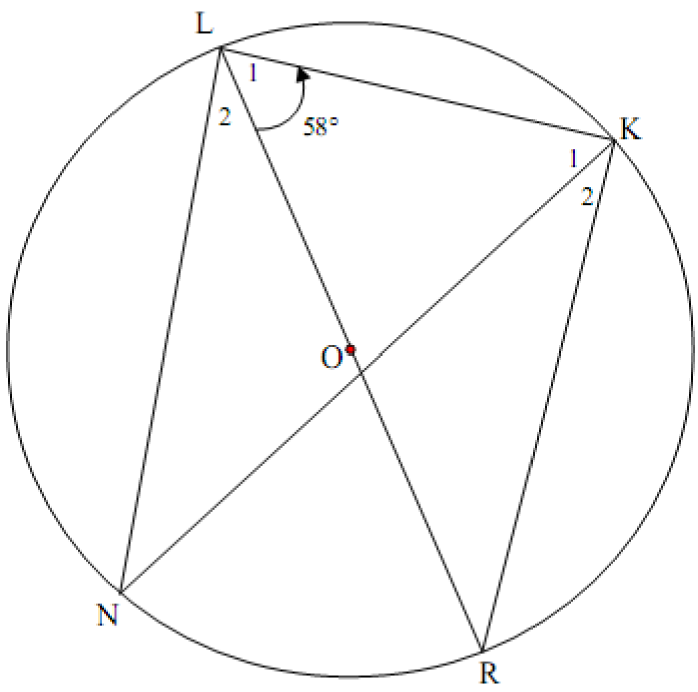
5.1	 Construction: Produce DO to E Proof: In ΔOBD: $O\hat{B}D = O\hat{D}B$ ...OD = OB = r $E\hat{O}B = 2 \times O\hat{D}B$ ...exterior angle of triangle In ΔAOD: $O\hat{A}D = O\hat{D}A$ ...OA = OD = r $E\hat{O}A = 2 \times O\hat{D}A$ ...exterior angle of triangle $A\hat{O}B = E\hat{O}B + E\hat{O}A$ $= 2 \times O\hat{D}B + 2 \times O\hat{D}A$ $= 2(O\hat{D}B + O\hat{D}A)$ $= 2A\hat{D}B$

5.2.1(a)	$\hat{M} = 76°$	... $\angle$ at centre = 2($\angle$ at circumference)
5.2.1(b)	$\hat{T}_2 = 38°$	...ext$\angle$ of cyc quad KTAB
5.2.1(c)	$\hat{C} = 38°$	... ext$\angle$ of cyclic quad or $\angle^s$ in same segment
5.2. 1(d)	$C\hat{A}N = \hat{C} = 38°$ $\hat{K}_4 = 38°$	...NA = NC ...ext $\angle$ of cyclic quad CATK
5.2.2	$\therefore \hat{K}_4 = \hat{T}_2$ $\therefore$NK = NT	 ...base $\angle^s$ equal
5.2.3	$\hat{N} = 180° - (38° + 38°)$ $= 104°$ $\hat{N} + K\hat{M}A = 104° + 76° = 180°$ $\therefore$ AMKN is cyclic quad	...$\angle^s$ of ΔKNT ...opposite $\angle^s$ = 180°

QUESTION 6

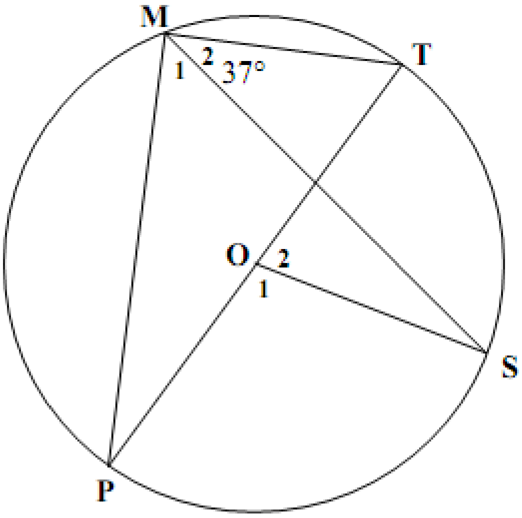
6.1	..... equal to the angle subtended by the same chord in the alternate segment.
6.2.1	$\hat{A}_1 = \hat{C}_2 = x$ ...tangent chord theorem $\hat{C}_2 = \hat{G}_2 = x$ ...tangent chord theorem $\therefore \hat{A}_1 = \hat{G}_2 = x$ $\therefore$ BCG $\|$ EA ...alternate $\angle^s =$
6.2.2	$\hat{E}_1 = \hat{C}_3 = y$ ...alternate $\angle^s$; BG $\|$ EA $\hat{F}_1 = \hat{C}_3 = y$ ...ext$\angle$ of cyclic quad CDFG $\therefore \hat{E}_1 = \hat{F}_1 = y$ $\therefore$ EA is a tangent ...converse tangent-chord theorem
6.2.3	$\hat{B} = C\hat{A}E$ ...tangent-chord theorem $\hat{C}_1 = C\hat{A}E$ ... alternate $\angle^s$; BG $\|$ EA $\hat{C}_1 = \hat{B}$ $\therefore$ AB = AC ...base $\angle^s =$

QUESTION 7



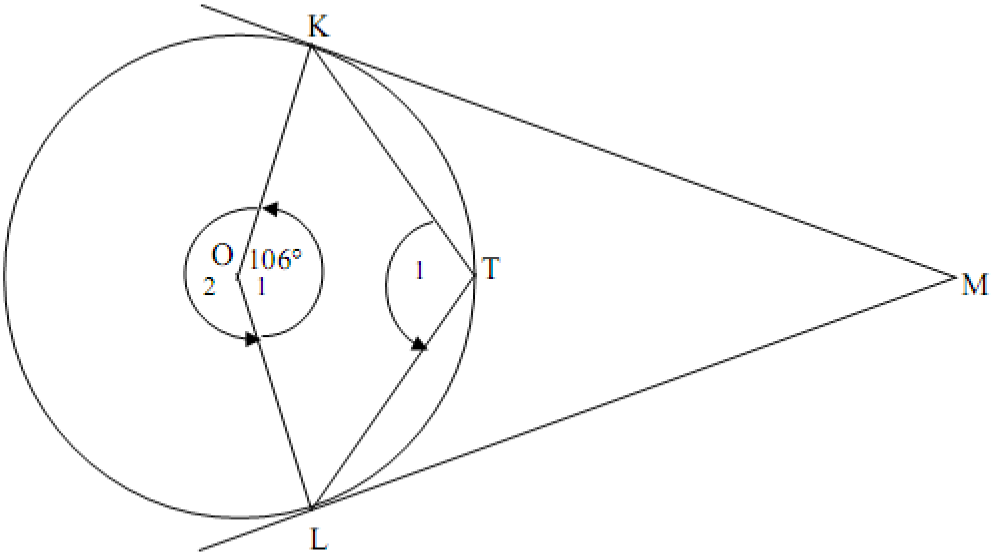
7.1	$L\hat{K}R = 90°$ [$\angle$ in semi-circle]	
7.2	$\hat{R} = 180° - (90° + 58°) = 32°$ [$\angle$s of triangle]	
9.3	$\hat{N} = 32°$ [$\angle$ in same segment]	

QUESTION 8



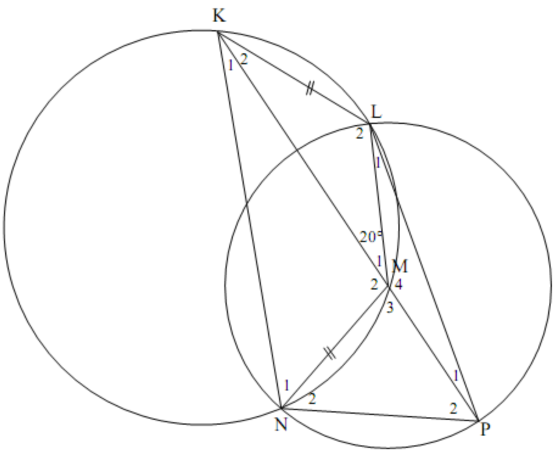
8.1.1	$\hat{M}_1 + \hat{M}_2 = 90°$ ($\angle$ in semi circle/ **or**/***of*** $\angle \frac{1}{2} \odot$) $\hat{M}_1 = 53°$ **OR/*OF*** $\hat{O}_2 = 74°$ ($\angle$ at centre/*midpt* = 2 × $\angle$ at circum) $\hat{O}_1 = 106°$ ($\angle$s on a str line) $\hat{M}_1 = 53°$ ($\angle$ at centre/*midpt* = 2 × $\angle$ at circum)
8.1.2	$\hat{O}_1 = 2 \times \hat{M}_1$ ($\angle$ at centre = 2 × $\angle$ at circum) $\hat{O}_1 = 106°$ **OR/*OF*** $\hat{O}_2 = 74°$ ($\angle$ at centre = 2 × $\angle$ at circum) $\hat{O}_1 = 106°$ ($\angle$s on a str line)

8.2



8.2.1	$\hat{O}_2 = 360° - 106° = 254°$ (∠s round a pt **or** ∠s in a rev) $\hat{T}_1 = \frac{1}{2} \times \hat{O}_2$ (∠ at centre = 2 ×∠ at circum) $= 127°$
8.2.2	KO = OL (radii equal) KM = ML (Tans from common/same pt) ∴ KOLM is a kite (adj sides of quad are =)
8.2.3	$O\hat{K}M = 90°$ (tan ⊥ radius **or**/ tan ⊥ diam) $O\hat{L}M = 90°$ (tan ⊥ radius **or**/ tan ⊥ diam) $O\hat{K}M + O\hat{L}M = 180°$ OKML = cyc quad (opp ∠s quad supp **or** converse opp ∠s of cyclic quad)/
8.2.4	$\hat{M} + \hat{O}_1 = 180°$ (opp ∠s of cyclic quad) $\hat{M} = 74°$

QUESTION 9

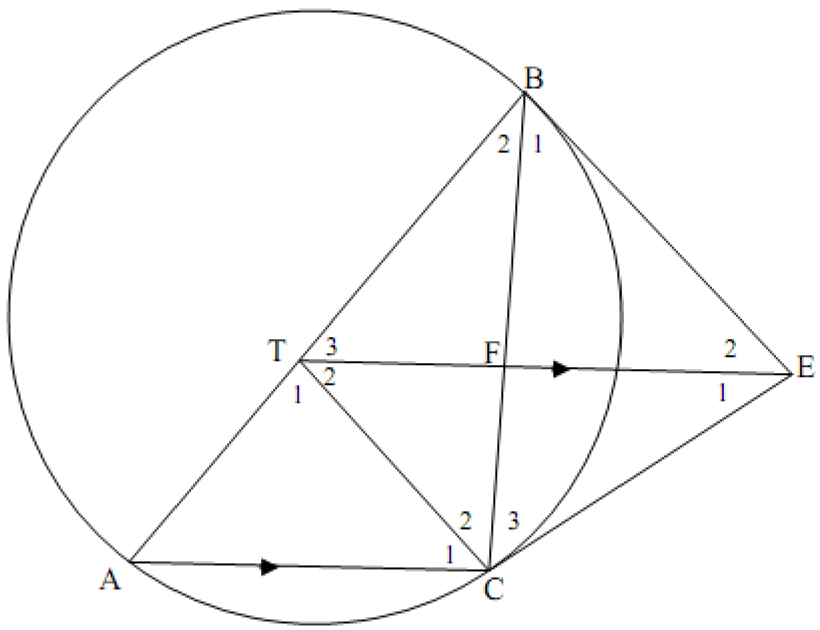


9.1	$N\hat{K}M = \hat{K}_1 = 20°$ (equal chords; equal $\angle$s)
9.2	Alternate $\angle$s are equal
9.3	$NM = LM$ (radii) $NM = KL$ (given) $\therefore\ KL = LM$
9.4.1	$M\hat{K}L = \hat{K}_2 = 20°$ ($\angle$s/*e* opp equal sides) $K\hat{L}M = \hat{L}_2 = 140°$ ($\angle$s sum in Δ) $K\hat{N}M = \hat{N}_1 = 180° - 140° = 40°$ (opp $\angle$s of cyclic quad)
9.4.2	$K\hat{M}N = \hat{M}_2 = 180° - (20° + 40°) = 120°$ ($\angle$s sum in Δ) $L\hat{M}N = \hat{M}_1 + \hat{M}_2 = 20° + 120° = 140°$ $L\hat{P}N = \hat{P}_1 + \hat{P}_2 = 70°$ ($\angle$ at centre = 2 $\times \angle$ at circumference)

QUESTION 10

10.2.1(a)	Tan chord theorem/*rklyn-koordstelling*
10.2.1(b)	$\angle$s in same segment/ $\angle$*e in dieselfde segment*
10.2.2	$\hat{R}_1 = \hat{P}_2 + \hat{T}$ (ext $\angle$ of Δ/ *buite* $\angle$ *v* Δ) $\hat{P}_2 = \hat{Q}_2$ (from/*vanaf* 10.2.1(b)) $\hat{Q}_1 = \hat{T}$ (from/*vanaf* 10.2.1(a)) $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P}_2 + \hat{T}$ $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{R}_1$ $\therefore PQ = PR$ (sides opp $= \angle$s/*sye to* $= \angle$*e*) $\therefore \Delta PQR$ = isosceles triangle/*gelykbenige driehoek*
10.2.3	$\hat{R}_2 = \hat{Q}_1$ ($\angle$s in same segment/$\angle$*e in dies segment*) $\hat{T} = \hat{Q}_1$ (from/*vanaf* 10.2.1(a)) $\hat{R}_2 = \hat{T}$ PR is a tangent to circle RST at R (converse tan chord th) *PR is 'n rklyn aan sirkel RST by R (omgekeerde rkl-kdst)* **OR/*OF*** $\hat{P}_1 = 180° - (\hat{Q}_1 + \hat{Q}_2 + \hat{R}_1)$ ($\angle$s/*e* of/*van* Δ) $\hat{R}_2 = \hat{Q}_1$ ($\angle$s in same segment/$\angle$*e in dies segment*) $\hat{Q}_1 = \hat{T}$ (from/*vanaf* 10.2.1(a)) $\therefore \hat{R}_2 = \hat{T}$

QUESTION 11



11.1	$\hat{B}_1 = \hat{A}$ [tangent-chord theorem] $\hat{A} = \hat{T}_3$ [corresp $\angle$s ; TE || AC] $\therefore \hat{B}_1 = \hat{T}_3$
11.2	BE = CE [tangents from same point] $\hat{B}_1 = \hat{C}_3$ [$\angle$s opp equal sides] $\hat{C}_3 = \hat{T}_3$ [$\hat{B}_1 = \hat{T}_3$] $\therefore$ TBEC a cyclic quad [converse $\angle$s in the same segment]
11.3	$\hat{B}_1 = \hat{T}_2$ [$\angle$s in the same segment] $\hat{B}_1 = \hat{T}_3$ [proven in 11.1] $\therefore \hat{T}_2 = \hat{T}_3$ $\therefore$ ET bisects $B\hat{T}C$
11.4	$\hat{B}_2 = \hat{E}_2$ [tangent-chord theorem] $\hat{C}_2 = \hat{E}_2$ [$\angle$s in the same segment] $\therefore$ TB = TC [sides opposite equal $\angle$s]
11.5	$\hat{C}_1 = \hat{T}_2$ [alternate $\angle$s ; TE || AC] $\therefore \hat{C}_1 = \hat{A}$ $\therefore$ AT =TC [sides opposite equal $\angle$s T is a point that is equidistant from A, B and C on the circle $\therefore$ T is the centre of the circle

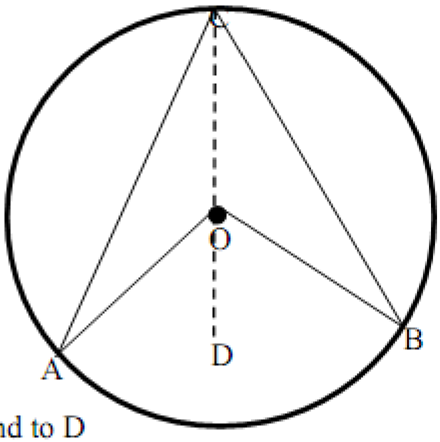
QUESTION 12

12.1.1	AO and/*en* CO are radii/*is radiusse* $\hat{A} = \hat{C}_1 = x$ [∠s opp equal sides/∠*e to gelyke sye*] $\hat{O}_1 = \hat{A} + \hat{C}_1 = 2x$ [ext/*buite* ∠ of/*van* Δ]
12.1.2	$\hat{B} = \hat{C}_2 = y$ [∠s opp equal sides/∠*e to gelyke sye*] $\hat{O}_2 = \hat{B} + \hat{C}_2 = 2y$ [ext/*buite* ∠ of/*van* Δ] $A\hat{O}B = 2x + 2y$ $= 2(x + y)$ $= 2(\hat{C}_1 + \hat{C}_2)$ $= 2A\hat{C}B$
12.2.1	ext ∠ of cyc quad/*buite* ∠*v koordevh*
12.2.2	MP = QM [radii] $\hat{Q}_1 = x$ [∠s opp equal sides/∠*e to gelyke sye*]
12.2.3	$\hat{M}_1 = 180° - 2x$ [∠s/*e* of/*van* Δ] $\hat{R} = 90° - x$ [∠ at centre = 2× ∠ at circumference/ *midpts∠ = 2 × omtreks∠*]
12.2.4	In ΔNSR: $\hat{R} = 90° - x$ and $\hat{N}_2 = x$ $\hat{S}_2 = 180° - (90° - x + x)$ [∠s/*e* of/*van* Δ] $= 90°$ PS = SR [line from centre ⊥ chord/*lyn v midpt ⊥ kd*]

QUESTION 13

13.1	bisects the chord.
13.2.1	$OD^2 = OF^2 + DF^2$ (Pythagoras) $= 3^2 + 4^2$ (substitution/*vervang*) $= 25$ $OD = 5\ cm$
13.2.2	$AE^2 = AO^2 - OE^2$ (Pythagoras) $AE^2 = 5^2 - 4^2$ (substitution/*vervang*) $AE^2 = 9$ AE = 3 cm But AB = 2AE (OE ⊥ AB) AB = 2(3) = 6 cm

QUESTION 14

14.1	 CONSTR: Join CO, extend to D PROOF: In ΔAOC i) $\hat{O}_1 = \hat{A}_1 + \hat{C}_1$ (ext $\angle$ of Δ/*buitehoek van* Δ) ii) $\hat{O}_2 = \hat{B}_2 + \hat{C}_2$ (ext $\angle$ of Δ/*buitehoek van* Δ) iii) $\hat{O}_1 = 2\hat{C}_1$ (AO = OC) iv) $\hat{O}_2 = 2\hat{C}_2$ (BO = OC) $\therefore$ $\hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$ $\therefore$ $A\hat{O}C = 2(\hat{C}_1 + \hat{C}_2)$ $= 2A\hat{C}B$
14.2.1	$\hat{D}_1 = 25°$ (radii equal/*radiusse gelyk*)
14.2.2	$\hat{O}_1 = 50°$ (ext $\angle$ of Δ/buitehoek van Δ)
14.2.3	$\hat{A}_1 = 25°$ (angles in same segment/ *hoeke in dieselfde segment)*
14.2.4	$\hat{E} = 155°$ (opp angles of cyclic quad/ *teenoorstaande hoeke van 'n koordevierhoek)*

QUESTION 15

15.1	$\hat{B}_1 = \hat{C}_2 = x$ (angles in the same segment/ *hoeke in dieselfde segment*) $\hat{B}_2 = \hat{C}_1 = x$ (tan chord/*tan koord)*
15.2	$\hat{C}_1 = \hat{C}_2$ (both equal to *x*/*albei gelyk aan x)* $\therefore$ DC *bisects/halveer* $A\hat{C}F$

QUESTION 16

16.1	Are supplementary **OR** add to 180°.	
16.2.1	$\hat{C}_2 = \hat{A}$ $\hat{A} = \hat{D}_3 = x$ $\therefore MC = MD$	(Ext ∠ of cyclic quad/*buitehoek van koordevhk*) (corresponding angles, AB ∥ DC/ *Ooreenkomstige hoeke AB ∥ DC)* (base angles of Δ equal/*basis hoeke van Δ gelyk)*
16.2.2	$\hat{M} = 180° - 2x$	(angles of Δ/*hoeke van* Δ*)*
16.2.3	$\hat{O}_1 = 2x$ $\hat{M} + \hat{O}_1 = 180°$ $\therefore$ *BODM is a cyclic quad.* $\therefore$ *BODM is koordevierhoek*	(∠ at centre = 2∠ at circumference/ *∠ by middle = 2∠ by omtreks)*

QUESTION 17

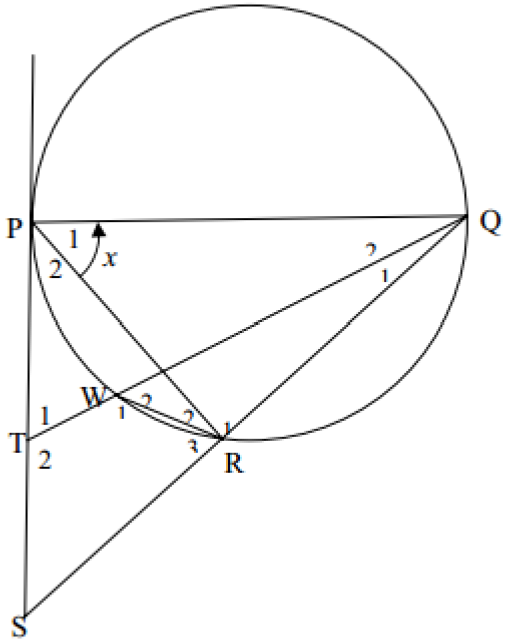
17.1	$\hat{C} + 140° = 180°$ opp ∠s of cyclic quad $\therefore \hat{C} = 40°$	✓ S/R ✓ A Answer only with reason 2/2	(2)
17.2	$\hat{M}_1 = 2\hat{C}$ ∠ at centre is twice ∠ at circum. $= 2(40°)$ $= 80°$	✓ S✓R 80° ✓ CA Answer only with reason 3/3	(3)
17.3	$\hat{B}_3 = \frac{1}{2}(180° - 80°)$ ∠s opp = sides $= 50°$	✓ S✓R ✓ A Answer only with reason 3/3	(3)
17.4	$\hat{D}_5 = \hat{B}_3 + 28°$ tan – chord theorem $= 50° + 28°$ $= 78°$	✓ S/R ✓ CA Answer Answer only with reason 2/2	(2)

QUESTION 18

18.1	…….. equal to the angle subtended by the chord in the opposite circle segment.	A✓S	(1)
18.2.1	$\hat{P}_2 = 23°$ …… (ON = OP; radii)	A✓ S/R	(1)
18.2.2	$P\hat{O}Q = 2\hat{N}_2 = 46°$ …… (∠at centre) / (Ext ∠of∆)	A✓ S A✓R	(2)
18.2.3	$N\hat{L}Q = 90°$ …… (subt. by diameter NQ)	A✓S/R	(1)
18.2.4	$\hat{L}_3 = 90° - 23°$ $= 67°$ **OR** $P\hat{O}N = 134°$ …..(angles of triangle) $P\hat{O}N = 2\hat{L}$ …..(angle at centre theorem) $= 67°$	CA✓ S CA✓ answer **OR** CA✓ S CA✓ answer	(2) (2)
18.2.5	$P\hat{L}K = L\hat{N}P$ …… (tan-chord theorem) $= 32° + 23°$ $= 55°$	A✓ S/R A✓ answer	(2)

Circles, Similarity and proportionality

QUESTION 1



1.1 $\hat{R}_1 = 90°$ …(angle in a semi-circle)

1.2 $\hat{P}_2 = 90° - x$ …(angle between radius and tangent)

$\hat{S} = 90° - \hat{P}_2$ …(ext. angle of Triangle)(sum of angles of triangle)

$= 90° - (90° - x) = x$

$\therefore \hat{P}_1 = \hat{S} = x$

1.3 $\hat{W}_2 = \hat{P}_1 = x$ …(angles in the same segment)

Also $\hat{S} = x$ …(proved 9.2)

$\hat{W}_2 = \hat{S}$

$\therefore$ SRWT is a cyclic quad…(ext angle = int. opposite angle)

1.4 In Δ QWR ; Δ QST

$\hat{W}_2 = \hat{S}$ ….(proved 9.3)

$\hat{Q}_1$ is common

$W\hat{R}Q = \hat{T}_2$ ….(remaining angles)

Δ QWR ||| Δ QST (AAA) or ($\angle\angle\angle$) or equiangular

1.5.1 $\frac{TS}{RW} = \frac{QT}{QR}$ ..... $\Delta QWR \mid\mid\mid \Delta QST$

$\therefore \frac{TS}{2} = \frac{8}{4}$

$4TS = 16$

$\therefore TS = 4\ cm$

1.5.2

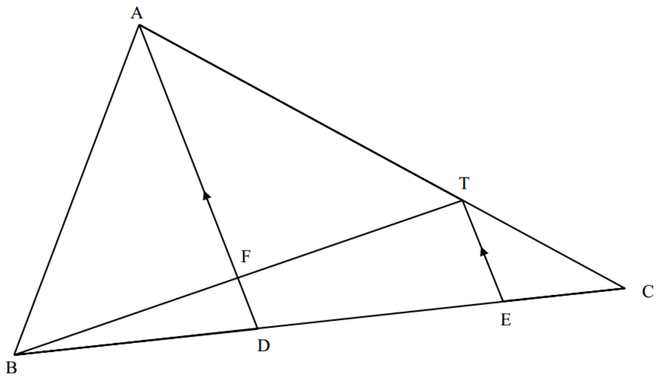
$\frac{SQ}{WQ} = \frac{TS}{RW}$

$SQ = \frac{4 \times 5}{2} = 10\ cm$

$\therefore SR = SQ - RQ$

$= 6\ cm$

QUESTION 2



2.1

$$\frac{CE}{ED} = \frac{CT}{TA} = \frac{1}{2}$$

2.2 From 10.1 $\frac{CE}{ED} = \frac{1}{2}$

But DC = 9 cm
$\therefore$ DE = 6 cm
= BD.
$\therefore$ D is the midpoint of BE.

2.3

$$\frac{FD}{TE} = \frac{BD}{BE}$$
$$\frac{2}{TE} = \frac{6}{12}$$
$$6 \times TE = 24$$
$$TE = 4\ cm$$

ALTERNATIVE

D is the midpoint of BE. (from 10.2)
Then F is the midpoint of BT. … (sides in proportion)

$\therefore$ TE = 2FD (midpoint theorem)
= 4 cm

2.4.1 $\frac{\Delta ADC}{\Delta ABD}=\frac{3}{2}$

2.4.2

$$\frac{\Delta TEC}{\Delta ABC}=\frac{\Delta TEC}{\Delta TBC}\times\frac{\Delta TBC}{\Delta ABC}$$
$$=\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$$
$$=\frac{1}{15}$$

OR

$$\frac{\text{area } \Delta TEC}{\text{area } \Delta ABC}=\frac{\frac{1}{2}.TC.EC.\sin\hat{C}}{\frac{1}{2}.AC.BC.\sin\hat{C}}$$
$$=\frac{TC.EC}{AC.BC}$$
$$=\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$$
$$=\frac{1}{15}$$

QUESTION 3

3.1.1	$\frac{AH}{HE}=\frac{2}{1}$ $AH = 2y$ $HE = y$ $\frac{AE}{ED}=\frac{2}{1}$ $ED = 1,5\,y$ $\frac{AH}{ED}=\frac{2}{1,5}$ $\frac{AH}{ED}=\frac{4}{3}$	(GHB || FEC) (BE || CD)
3.1.2	$\frac{BE}{CD}=\frac{4}{6}$ $=\frac{2}{3}$	(ΔAEB ||| ΔADC)
3.2	HE = 2 cm AH = 4 cm ED = 3 cm AD.HE = (AH + HE + ED).HE = (4 + 2 + 3).(2) = 18	(given)

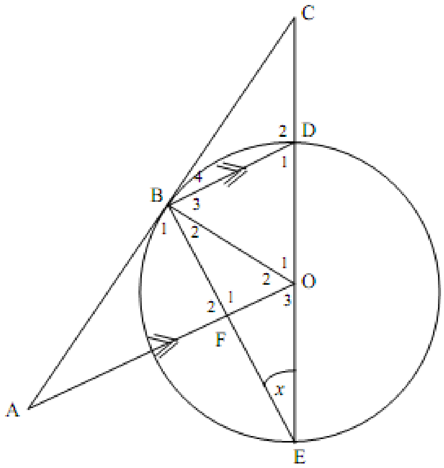
QUESTION 4

4.1	$\hat{D}_1 = \hat{A}_4$ $= \hat{C}_2$	(tan-chord theorem) (alt $\angle$'s , BA || CE)

4.2	In ΔACF and ΔADC 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) $\Delta ACF \parallel\!| \Delta ADC$ ($\angle\angle\angle$) **OR** In ΔACF and ΔADC 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) 3. $\hat{F}_1 = A\hat{C}D$ (remaining $\angle$s in triangles) $\Delta ACF \parallel\!| \Delta ADC$

4.3	$\frac{AF}{AC} = \frac{AC}{AD}$ (sim Δ's $\therefore$ sides in proportion) $AF = \frac{AC.AC}{AD}$ $AC = AO = \frac{1}{2}AD$ (2radius = diameter) $AF = \frac{\frac{1}{2}AD.\frac{1}{2}AD}{AD}$ $AF = \frac{AD}{4}$ $4AF = AD$ **OR** ΔAOC is equilateral $\therefore A\hat{O}C = \hat{A}_3 = 60°$ $\cos 60° = \frac{AF}{AC} = \frac{1}{2}$ $AF = \frac{1}{2}AC = \frac{1}{2}AO$ $AF = \frac{1}{2}(\frac{1}{2}AD)$ (2radius = diameter) $AF = \frac{1}{4}AD$ $AD = 4AF$

QUESTION 5



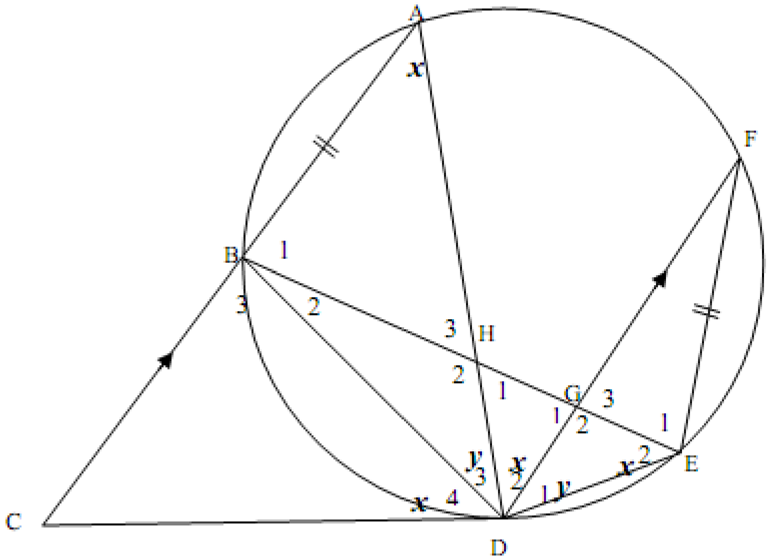
5.1.1	$\hat{B}_4 = x$ (tan chord theorem) $\hat{A} = \hat{B}_4 = x$ (corres ∠; BD ∥ AO) $\hat{B}_2 = x$ (BO = EO = radii) **Note:** If start with $\hat{A} = x$ and do not use tan ch th: max 2 marks	✓ $\hat{B}_4 = x$ ✓ tan chord theorem ✓ $\hat{A} = \hat{B}_4 = x$ with reason ✓ $\hat{B}_2 = x$ (4)
5.1.2	$D\hat{B}E = 90°$ (∠ in semi-circle) $C\hat{B}E = 90° + x$ **OR** $C\hat{B}O = 90°$ (rad ⊥ tan) $C\hat{B}E = 90° + x$ **OR** $\hat{O}_1 = 2x$ (∠ circ cent) $\hat{B}_3 = \hat{D}_1 = 90° - x$ (radii) $C\hat{B}E = x + (90° - x) + x$ $= 90° + x$	✓ $D\hat{B}E = 90°$ ✓ ∠ in semi-circle ✓ $C\hat{B}E = 90° + x$ (3) ✓ $C\hat{B}O = 90°$ ✓ rad ⊥ tan ✓ $C\hat{B}E = 90° + x$ (3) ✓ $\hat{O}_1 = 2x$ ✓ ∠ circ cent ✓ $C\hat{B}E = 90° + x$ (3)
5.1.3	$D\hat{B}E = 90°$ (proved in 8.2.2) $B\hat{F}O = 90°$ (co-int angles supp; BD ∥ AO) BF = FE (line from circ cent ⊥ ch bisect ch) F is the midpoint of EB	✓ $D\hat{B}E = 90°$ ✓ $B\hat{F}O = 90°$ and reason ✓ BF = FE ✓ line from circ cent ⊥ ch bisect ch) (4)

	OR OD = OE (radii) BF = FE (BD ‖ AO) F is the midpoint of EB	✓ OD = OE ✓ radii ✓ BF = FE ✓ BD ‖ AO (4)
	OR $B\hat{F}O = E\hat{F}O = 90°$ (BD ‖ AO) OF is common BO = OE (radii) $\Delta BOF \equiv \Delta EOF$ (90°HS) BF = FE ($\equiv \Delta$s)	✓ $B\hat{F}O = E\hat{F}O = 90°$ (BD ‖ AO) ✓ BO = OE ✓ $\Delta BOF \equiv \Delta EOF$ ✓ BF = FE (4)
	OR $\hat{B}_2 = \hat{A} = x$ (proven) $\hat{O}_2$ is common $\Delta AOB \parallel\!\mid \Delta BOF$ (AAA) $A\hat{B}O = B\hat{F}O$ $A\hat{B}O = 90°$ (proven) $A\hat{B}O = B\hat{F}O = 90°$ BF = FE (line from circ cent ⊥ ch bisects ch)	✓ $\Delta AOB \parallel\!\mid \Delta BOF$ ✓ $A\hat{B}O = B\hat{F}O$ ✓ BF = FE ✓ line from circ cent ⊥ ch bisects ch (4)
5.1.4	In ΔCBD and ΔCEB 1. $\hat{E} = \hat{B}_4 = x$ (proven in 8.2.1) 2. $\hat{C}$ is common 3. $\hat{D}_4 = C\hat{B}E = 90° + x$ $\Delta CBD \parallel\!\mid \Delta CEB$ (AAA)	✓ $\hat{E} = \hat{B}_4 = x$ ✓ $\hat{C}$ is common Or ✓ $\hat{D}_4 = C\hat{B}E = 90° + x$ Any two of the above (2)
5.1.5	$\frac{EB}{BD} = \frac{CE}{CB}$ (sim Δs ∴ sides in proportion) EB.CB = CE.BD but EB = 2EF (F is the midpoint of BE) 2EF.CB = CE.BD	✓ $\frac{EB}{BD} = \frac{CE}{CB}$ ✓ EB.CB = CE.BD ✓ EB = 2EF (3) **[21]**

QUESTION 6

6.1	$M\hat{E}C = 90°$ (tan ⊥ rad) $M\hat{D}C = 90°$ (line from cent bisects ch) $M\hat{E}C + M\hat{D}C = 180°$ ∴ MDCE a cyclic quad (opp ∠s of quad supplementary) **OR** $M\hat{E}C = 90°$ (tan ⊥ rad) $M\hat{D}A = 90°$ (line from cent bisects ch) $M\hat{E}C = M\hat{D}A$ ∴ MDCE a cyclic quad (ext ∠ quad = int opp)
6.2	$MD^2 = MB^2 - DB^2$ (Pythagoras; ΔMBD) $MC^2 = MD^2 + DC^2$ (Pythagoras; ΔMDC) $= MB^2 - DB^2 + DC^2$
6.3	$DB = 30$ (given) $MB = 40$ (radii) $MC^2 = (40)^2 + (50)^2 - (30)^2$ $= 3\ 200$ $MC = 40\sqrt{2} = 56{,}57$ $MC^2 = ME^2 + CE^2$ (Pythagoras) $CE^2 = 3\ 200 - 1\ 600$ $CE^2 = 1\ 600$ $CE = 40$ mm

QUESTION 7



7.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) **OR** ($\angle$s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$s; CA || DF)	✓ $\hat{A} = x$ ✓ tan ch th ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ alt $\angle$s; CA || DF (6)
7.2	In ΔBHD and Δ FED 1. $\hat{B}_2 = \hat{F}$ ($\angle$s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$s) ΔBHD ||| ΔFED ($\angle\angle\angle$)	✓ $\hat{B}_2 = \hat{F}$ ✓ $\angle$s in same seg ✓ $\hat{D}_3 = \hat{D}_1$ ✓ = chs subt = $\angle$s ✓ $\angle\angle\angle$ (5)
7.3	$\frac{FE}{BH} = \frac{FD}{BD}$ (||| Δs) But FE = AB (given) $\frac{AB}{BH} = \frac{FD}{BD}$ AB.BD = FD.BH	✓ $\frac{FE}{BH} = \frac{FD}{BD}$ ✓ FE = AB (2) **[13]**

QUESTION 8

8 .1	AF = FC FE ǁ CD AE = ED	(hoeklyne van parm) (Eweredigheidstelling; FE ǁ CD) **of** (lyn uit middel-punt van een sy ǁ aan tweede sy halveer die derde sy) **of** (omgekeerde middelpuntstelling)
8.2	$\frac{AC}{CP}=\frac{1}{2}$ $\frac{AD}{DQ}=\frac{1}{2}$ $\frac{AC}{CP}=\frac{AD}{DQ}$ CD ǁ PQ CD ǁ FE $\therefore$ PQ ǁ FE	(egee) (gegee) (omgekeerde eweredigheidstel) **of** (sye eweredig) (gegee)

8.3	In ΔAEF en ΔAPQ 1. $\hat{A}$ is gemeenskaplik 2. $A\hat{E}F = A\hat{Q}P$ (ooreenk $\angle$e; FE ǁ PQ) 3. $A\hat{F}E = A\hat{P}Q$ (ooreenk $\angle$e; FE ǁ PQ) $\therefore \Delta AEF \parallel\!\mid \Delta AQP$ ($\angle\angle\angle$) $\frac{FE}{PQ}=\frac{AF}{AP}$ (ǁǀ Δs) $\frac{FE}{60}=\frac{1}{6}$ FE = 10 cm

QUESTION 9

9.1	Draw a point P on FG such that FP = LM and a point Q on FH such that FQ = LN. In ΔFPQ and ΔLMN 1. $\hat{F} = \hat{L}$ (given) 2. $FP = LM$ (construction) 3. $FQ = LN$ (construction) $\therefore \Delta FPQ \equiv \Delta LMN$ (SAS) $F\hat{P}Q = L\hat{M}N$ $(\equiv \Delta s)$ But $F\hat{G}H = L\hat{M}N$ (given) $F\hat{P}Q = F\hat{G}H$ $PQ \parallel GH$ (corresponding angles =) $\frac{FP}{FG} = \frac{FQ}{FH}$ (PQ || GH ; Prop Th) $\frac{LM}{FG} = \frac{LN}{FH}$
9.2	$\frac{VP}{PR} = \frac{VT}{TK}$ (PT || RK;Prop Th) $\frac{2x-10}{9} = \frac{4}{6}$ $2x-10 = 6$ $2x = 16$ $x = 8$ **OR** $\frac{VP}{VR} = \frac{VT}{VK}$ (PT || RK; Prop Th) $\frac{2x-10}{2x-1} = \frac{4}{10}$ $20x - 100 = 8x - 4$ $12x = 96$ $x = 8$

QUESTION 10

10.1	$\hat{AOB} = 2x$ (∠circ centre = 2 ∠ circumference) $\hat{T} = 180° - 2x$ (opp∠ cyclic quad suppl)
10.2	$\hat{CAT} = x$ (∠ sum Δ) $\hat{K}_1 = x$ (ext∠ cyclic quad) $\hat{CAT} = \hat{K}_1$ BK ∥ AC (corresponding ∠s =)

10.3	In ΔBKT and ΔCAT 1. $\hat{CAT} = \hat{K}_1$ $(= x)$ 2. $\hat{T}$ is common 3. $\hat{ACT} = \hat{B}_4$ (∠ sum Δ) ΔBKT ||| ΔCAT (∠∠∠)
10.4	$\frac{AC}{KB} = \frac{AT}{KT}$ (||| Δs) $\frac{AC}{KB} = \frac{7}{2}$

QUESTION 11

11.1 In Δ ABQ,

$$\frac{BR}{RA} = \frac{BT}{TQ} \quad \text{..... (RT || AQ, proportional intercept theorem)}$$

$$\frac{1}{2} = \frac{k}{TQ}$$

$$\therefore TQ = 2k$$

11.2.1 InΔ CRT,

$$\frac{CP}{PR} = \frac{5k}{2k} \quad \text{.... (RT || AQ, proportional intercept theorem)}$$

$$\therefore \frac{CP}{PR} = \frac{5}{2}$$

11.2.2

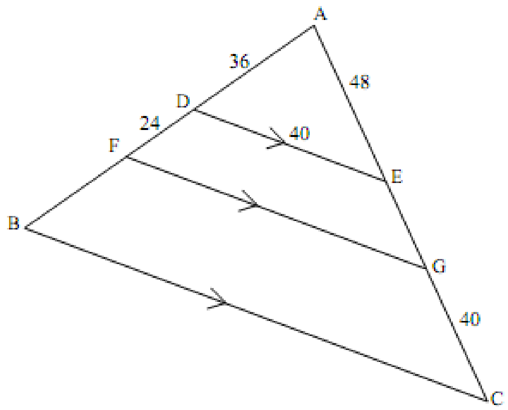
$$\frac{\text{Area } \Delta \text{ RCT}}{\text{Area } \Delta \text{ ABC}} = \frac{\text{Area } \Delta \text{ RCT}}{\text{Area } \Delta \text{ BRC}} \times \frac{\text{Area } \Delta \text{ BRC}}{\text{Area } \Delta \text{ ABC}}$$

$$= \frac{7}{8} \times \frac{1}{3}$$

$$= \frac{7}{24}$$

.. (the ratio of the areas of triangles having equal altitude ….)

QUESTION 12



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12.1	$\frac{EG}{48}=\frac{24}{36}$ (DE ‖ FG) $EG=\frac{48\times 24}{36}$ $EG = 32$ cm	✓ S/R ✓ answer (2)
12.2	$\frac{BC}{DE}=\frac{AC}{AE}$ $BC=\frac{120\times 40}{48}$ $= 100$ cm	✓ statement ✓✓ substitution ✓ answer (4)
	OR $\frac{AB}{AD}=\frac{AC}{AE}$ $AB=\frac{120\times 36}{48}$ $AB = 90$ $\Delta ABC \parallel\!\mid \Delta ADE$ (∠∠∠) $\frac{BC}{DE}=\frac{AB}{AD}$ (sides in proportion) $BC=\frac{90\times 40}{36}$ $BC = 100$ cm	✓ S ✓ S ✓ 90 ✓ answer (4)
	OR $\Delta ABC \parallel\!\mid \Delta ADE$ (∠∠∠) $\frac{BC}{DE}=\frac{AC}{AE}$ (sides in proportion) $BC=\frac{120\times 40}{36}$ $BC = 100$ cm	✓ S ✓ S ✓ substitution ✓ answer (4) **[6]**

QUESTION 13

13.1 In Δ BPE and Δ BDA

$\hat{B}_1$ is common

$\hat{P}_2 = \hat{D} = 90°$ ......(given perpendicular, $\angle$ in a semi - circle)

$B\hat{A}D = \hat{E}_3$ ....(remaining angles)

$\therefore \Delta BPE /// \Delta BDA$ ..... (equiangular)

13.2 $\Delta BPE /// \Delta BDA$ .... (from 9.1)

$\therefore \frac{BP}{BD} = \frac{PE}{DA}$ ...... (sides in proportion)

13.3 $AB = \frac{BD.BE}{BP}$

$AB^2 = \frac{BD^2.BE^2}{BP^2}$

In Δ PBE; $BE^2 = BP^2 + PE^2$ ....(Theorem of Pythagoras)

$AB^2 = \frac{BD^2.(BP^2 + PE^2)}{BP^2}$

$AB^2 = \frac{BD^2.BP^2}{BP^2} + \frac{BD^2.PE^2}{BP^2}$

$AB^2 = BD^2 + \frac{BD^2.PE^2}{BP^2}$

QUESTIO 14

14.1	$\frac{WS}{SP}=\frac{3}{2}$ $\frac{WS}{SP}=\frac{WT}{RT}=\frac{3}{2}$ $WT=\frac{3\times 6}{2}$ $WT = 9$ cm	(ST || PR; Prop th)
14.2	$\frac{WS}{SP}=\frac{WR}{RQ}=\frac{3}{2}$ $\frac{9+6}{RQ}=\frac{3}{2}$ $RQ = 10$ cm $WQ = 10 + 9 + 6$ $= 25$ cm	(SR || PQ; Prop th)

QUESTION 15

15.1	Is equal to the angle subtended by the chord in the alternate segment
15.2.1	$\hat{A}_2 = x$ (tangent chord theorem) $\hat{A}_5 = x$ (vertically opp. angles) $\hat{P}_2 = x$ (tangent chord theorem)
15.2.2	$PT = TA$ (tangents drawn from same point) $\hat{P}_1 = \hat{A}_3$ (angles opp equal sides) ; $PT = TA$ $\hat{A}_3 = \hat{A}_6$ (vertical opp angles) $\hat{A}_6 = \hat{R}_2$ (tangent chord theorom) $\therefore \hat{P}_1 = \hat{R}_2$ $\therefore$ APTR is a cyclic quadrilateral (converse : ext angle of cycl.quad.)

QUESTION 16

16.1	$OC = OB$ Hence $AE = BE$ **OR** $C\hat{A}B = 90°$ $O\hat{E}B = C\hat{A}B = 90°$ $\therefore AE = BE$	(radii) (midpoint theorem) (diameter subtends right angle) (corresponding angles AC//OE) (line drawn from centre, perpend. to chord or midpoint theorem)
16.2	In ΔAED and ΔCEB $A\hat{E}D = C\hat{E}B$ $\hat{D} = \hat{B}$ $\hat{A}_3 = \hat{C}_1$ $\therefore \Delta AED \,///\, \Delta CEB$	 (vertically opp angles) (angles in same segment) (angles in same segment) (equi - angular)
16.3	$\frac{AE}{DE} = \frac{CE}{BE}$ $AE.BE = DE.CE$ but $AE = BE$ $\therefore AE^2 = DE.CE$	(deduction) (proven)
16.4	$AE.BE = DE.CE$ But $AE.BE = EF.CE$ $\therefore DE.CE = EF.CE$ $DE = EF$ $\therefore$ E is the midpoint of DF	

QUESTION 17

17.1	In ΔBDA and ΔCDB $B\hat{D}A = C\hat{D}B = 90°$ $\hat{B}_1 = \hat{C}$ (both $= x$) $\hat{A} = \hat{B}_2$ (remaining angles) $\Delta BDA \ /// \ \Delta CDB$ (equiangular)
17.2	$AD : DC = 3 : 2$ $\therefore CD = \frac{2}{3} \times 15 = 10$ But $\frac{BD}{AD} = \frac{CD}{BD}$ $\therefore BD^2 = AD.CD$ $BD^2 = 15.10$ $= 150$ $BD = \sqrt{150}$

17.3	$AB^2 = (\sqrt{150})^2 + (15)^2$ (Theorem of Pythagoras) $= 150 + 225$ $= 375$ $AB = \sqrt{375}$ $\hat{E}_1 = A\hat{B}C = 90°$ $\therefore BC \parallel DE$ $\frac{AE}{AB} = \frac{AD}{AC}$ (proportion theorem) $\frac{AE}{\sqrt{375}} = \frac{15}{25}$ $AE = \frac{15 \times \sqrt{375}}{25} = \sqrt{135} = 3\sqrt{15}$

QUESTION 18

18.1.1	$J\hat{H}F + \hat{F} = 180°$ .... Co-interior angles; JH // EF $\therefore J\hat{H}F = 90°$ ...... $\hat{F} = 90°$ (given)	A✓ S/R A✓ S	(2)
18.1.2	$\hat{R}_2 = \hat{F} = 90°$ ...... ext. ∠ of cyclic quad	AA✓✓ S/R	(2)
18.1.3	In ΔHKG and ΔJHG $\widehat{G_1} = \widehat{G_1}$...... common $\hat{R}_2 = J\hat{H}G = 90°$ ...... proved $\therefore K\hat{H}G = H\hat{J}G$ ...... remaining ∠ $\therefore \Delta HKG /// \Delta JHG$ ...... (∠∠∠)	A✓ S/R A✓ S/R A✓ R	(3)
18.2	$JG^2 = HJ^2 + HG^2$ ...... Pythagoras $= 10^2 + 5^2$ $= 125cm^2$ $JG = \sqrt{125cm^2}$ $= 5\sqrt{5}$cm $\frac{KG}{HG} = \frac{HG}{JG}$ $KG = \frac{HG^2}{JG}$ $= \frac{5^2}{5\sqrt{5}}$ $= \frac{5}{\sqrt{5}}$ $= \frac{5\sqrt{5}}{5}$ $= \sqrt{5}$ cm $= 2{,}24$ cm	A✓ $5\sqrt{5}$ A✓ $\Delta HKG /// \Delta JHG$ CA✓ substitution CA✓ answer in any form	(4) [11]

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