JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

DIFFERENTIAL CALCULUS

MEMORANDUM/ANSWER BOOKLET

TEACHER

TERM 2

FIRST PRINCIPLES

FINDING EQUATION OF THE CUBIC FUNCTION AND EQUATION OF A TANGENT

RULES OF DIFFERENTIATION

GRAPHICAL INTERPRETATION

SKETCHING OF THE CUBIC FUNCTION

OPTIMISATION

JENN MAREMATLOU TRAINING INSTITUTE
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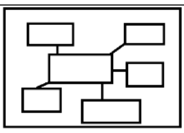
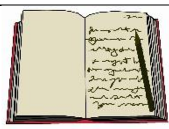
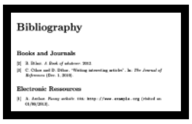
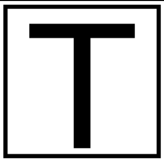
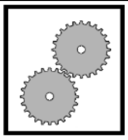
ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

SESTION 1: FIRST PRINCIPLES

QUESTION 1

1.1

$$f(x) = 2x^3$$

$$f(x+h) = 2(x+h)^3$$

✓ substitution

$$= 2\left(x^3 + 3x^2h + 3xh^2 + h^3\right)$$

$$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$$

✓ expansion

$$f(x+h) - f(x) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3$$

$$= 6x^2h + 6xh^2 + 2h^3$$

✓ formula

$$f'(x) = \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$$

$$= \lim_{h \to 0} \frac{h\left(6x^2 + 6xh + 2h^2\right)}{h}$$

✓ $6x^2 + 6xh + 2h^2$

$$= \lim_{h \to 0}\left(6x^2 + 6xh + 2h^2\right)$$

✓ answer

$$f'(x) = 6x^2$$

(5)

OR

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

✓ formula

$$= \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h}$$

✓ substitution

$$= \lim_{h \to 0} \frac{2\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - 2x^3}{h}$$

✓ expansion

$$= \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$$

$$= \lim_{h \to 0} \frac{h\left(6x^2 + 6xh + 2h^2\right)}{h}$$

✓ $6x^2 + 6xh + 2h^2$

$$= \lim_{h \to 0}\left(6x^2 + 6xh + 2h^2\right)$$

✓ answer

$$f'(x) = 6x^2$$

(5)

1.2.1

$$f(x) = -\frac{2}{x}$$ ✓ substitution

$$f(x+h) = -\frac{2}{(x+h)}$$ ✓ simplification

$$f(x+h) - f(x) = -\frac{2}{(x+h)} - \left(-\frac{2}{x}\right)$$ ✓ formula

$$= \frac{-2x + 2(x+h)}{x(x+h)}$$ ✓ common factor

$$= \frac{-2x + 2x + 2h}{x(x+h)}$$ ✓ answer

(5)

$$= \frac{2h}{x(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \left(\frac{2}{x^2 + xh}\right)$$ ✓ formula

✓ substitution

$$= \frac{2}{x^2}$$ ✓ simplification

OR

✓ common factor

✓ answer

(5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left[-\frac{2}{(x+h)}\right] - \left(-\frac{2}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2x + 2(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2x + 2x + 2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \left(\frac{2}{x^2 + xh} \right)$$

$$= \frac{2}{x^2}$$

1.2.2

$$f'(x) = \frac{2}{x^2}$$

$x^2 \geq 0$ for $x \in R$ — ✓ $x^2 \geq 0$ or $\frac{2}{x^2} \geq 0$ for $x \in R$

$f'(x) > 0$ for $x \in R;\ x \neq 0$ — ✓ $f'(x) > 0$ for $x \in R;\ x \neq 0$

(2)

1.3

$f(x) = 9 - x^2$ — ✓ substitution

$f(x+h) = 9 - (x+h)^2$ — ✓ simplification

$= 9 - x^2 - 2xh - h^2$

$f(x+h) - f(x) = -2xh - h^2$ — ✓ formula

$f'(x) = \lim_{h \to 0} \frac{-2xh - h^2}{h}$ — ✓ common factor

$= \lim_{h \to 0} \frac{h(-2x - h)}{h}$ — ✓ answer

$= \lim_{h \to 0} (-2x - h)$

$= -2x$

(5)

OR

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ ✓ formula

$$=\lim_{h\to 0}\frac{9-(x+h)^2-(9-x^2)}{h}$$ ✓ substitution

$$=\lim_{h\to 0}\frac{9-(x^2+2xh+h^2)-9+x^2}{h}$$ ✓ simplification

$$=\lim_{h\to 0}\frac{-2xh-h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(-2x-h)}{h}$$ ✓ common factor

$$=\lim_{h\to 0}(-2x-h)$$

$$=-2x$$ ✓ answer

(5)

1.4

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ ✓ method

$$=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)-x^2+2x}{h}$$ ✓ substitution

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}$$ ✓ simplification

$$=\lim_{h\to 0}\frac{2xh+h^2-2h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h-2)}{h}$$ ✓ factorising

$$=\lim_{h\to 0}(2x-2+h)$$

$$=2x-2$$ ✓ answer

(5)

1.5

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-4(x+h)^2-(-4x^2)}{h}$$
$$=\lim_{h\to 0}\frac{-4(x^2+2xh+h^2)+4x^2}{h}$$
$$=\lim_{h\to 0}\frac{-4x^2-8xh-4h^2+4x^2}{h}$$
$$=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$$
$$=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$$
$$=\lim_{h\to 0}(-8x-4h)$$
$$=-8x$$

✓ formula
✓ substitution
✓ expansion
✓ $-8x-4h$
✓ answer
(5)

Note:
Incorrect notation:
no lim written:
penalty 2 marks
lim written before equals sign:
penalty 1 mark

Note:
A candidate who gives $-8x$ only:
0/5 marks

Note:
A candidate who omits brackets in the line $\lim_{h\to 0}(-8x-4h)$:
NO penalty

OR

$$f(x)=-4x^2$$
$$f(x+h)=-4(x+h)^2$$
$$=-4x^2-8xh-4h^2$$
$$f(x+h)-f(x)=-8xh-4h^2$$
$$f'(x)=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$$
$$=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$$
$$=\lim_{h\to 0}(-8x-4h)$$
$$=-8x$$

✓ substitution
✓ expansion
✓ formula
✓ $-8x-4h$
✓ answer
(5)

SECTION 2: RULES OF DIFFERENTIATION

Question 1

1.1

$$y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

$$y = \frac{1}{2}.x^{\frac{1}{2}} - \frac{1}{6}.x^{-3}$$ ✓ Simplification

$$\frac{dy}{dx} = \frac{1}{4}x^{-\frac{1}{2}} + \frac{3}{6}x^{-4}$$ ✓ $\frac{1}{4}x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{4}x^{-\frac{1}{2}} + \frac{1}{2}x^{-4}$$ ✓ $\frac{1}{2}x^{-4}$ or $\frac{3}{6}x^{-4}$

$$\frac{dy}{dx} = \frac{1}{4\sqrt{x}} + \frac{1}{2x^4}$$

(3)

> Note:
> If removed coefficients, or moved the numbers from the denominator to the numerator:
> **Continued accuracy applies for each correct derivative**
> **Max 2/3**
> If leave out $\frac{dy}{dx}$ penalise 1 mark.

1.2

$$D_x[(x-2)(x+3)]$$

$$= D_x[x^2 + x - 6]$$ ✓ simplification

$$= 2x + 1$$ ✓✓answer

(3)

1.3

$$y = x^2 - \frac{1}{2x^3}$$

$$y = x^2 - \frac{1}{2}x^{-3}$$

$$\frac{dy}{dx} = 2x + \frac{3}{2}x^{-4}$$ ✓ $2x$ ✓ $+\frac{3}{2}x^{-4}$

OR

$$\frac{dy}{dx} = 2x + \frac{3}{2x^4}$$

OR

$$\frac{dy}{dx} = 2x - (-3)\frac{1}{2}x^{-4}$$

(2)
[7]

1.4 $y = \frac{x^6}{2} + 4\sqrt{x}$

$y = \frac{1}{2}x^6 + 4x^{\frac{1}{2}}$ ✓ $+4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 3x^5 + 2x^{-\frac{1}{2}}$ ✓ $3x^5$ ✓ $2x^{-\frac{1}{2}}$

Note:
If $\frac{dy}{dx}$ or y' is left out, penalty 1 mark

If a candidate shows evidence of how to differentiate from an incorrect function which involves breakdown, then max 1 / 3

1.5.1 $y = \frac{3}{2x} - \frac{x^2}{2}$

$= \frac{3}{2}x^{-1} - \frac{1}{2}x^2$ ✓ $\frac{3}{2}x^{-1}$

$\frac{dy}{dx} = -\frac{3}{2}x^{-2} - x$ ✓ $-\frac{3}{2}x^{-2}$ ✓ $-x$

$= -\frac{3}{2x^2} - x$

(3)

1.5.2 $f(x) = (7x+1)^2$

$= 49x^2 + 14x + 1$ ✓multiplication

$f'(x) = 98x + 14$ ✓ $98x$ ✓14

$f'(1) = 98(1) + 14$

$= 112$ ✓ answer

(4)

OR

$f(x) = (7x+1)^2$

$f'(x) = 2(7x+1).(7)$ By the chain rule ✓✓ chain rule

$f'(x) = 98x + 14$

$f'(1) = 98(1) + 14$

$= 112$ ✓✓ answer

(4)

1.6 $\frac{dy}{dx} = -4x^{-5} + 6x^2 - \frac{1}{5}$ ✓ $-4x^{-5}$ ✓ $6x^2$ ✓ $-\frac{1}{5}$

$= \frac{-4}{x^5} + 6x^2 - \frac{1}{5}$

Note: notation error penalise 1 mark

Note: candidates do NOT need to give their answer with positive exponents

(3)

1.7.1 $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

$= \dfrac{(x+2)(x-1)}{x-1}$ ✓ simplification

$= x + 2 \quad (x \neq 1)$ ✓ answer

$g'(x) = 1 \quad (x \neq 1)$ (2)

1.7.2 The function is undefined at $x = 1$. ✓ answer

OR

Division by zero is undefined. (1)

OR

The denominator cannot be zero.

OR

In the definition of the derivative, $g'(1) = \lim\limits_{h \to 0} \dfrac{g(1+h) - g(1)}{h}$, but $g(1)$ does not exist.

SECTION 3: SKETCHING OF THE CUBIC FUNCTION

QUESTION 1

1.1 $0 = -x^3 + x^2 + 8x - 12$

$x^3 - x^2 - 8x + 12 = 0$

$(x-2)(x^2 + x - 6) = 0$

$(x-2)(x-2)(x+3) = 0$

$x = 2$ or $x = -3$

x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$

✓ any one of factors
✓ quadratic factor
✓ linear factors
✓✓ x-answers

(5)

OR

$0 = -x^3 + x^2 + 8x - 12$

$x^3 - x^2 - 8x + 12 = 0$

$(x+3)(x^2 - 4x + 4) = 0$

$(x+3)(x-2)(x-2) = 0$

$x = 2$ or $x = -3$

x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$

1.2 $f'(x) = -3x^2 + 2x + 8$

$0 = 3x^2 - 2x - 8$

$0 = (x-2)(3x+4)$

$x = 2$ or $x = -\frac{4}{3}$

turning points are $(2 ; 0)$ and $\left(-\frac{4}{3} ; -\frac{500}{27}\right)$

OR $(2 ; 0)$ and $(-1,33 ; -18,52)$

✓ $f'(x) = 0$
✓ $-3x^2 + 2x + 8 = 0$ or $3x^2 - 2x - 8 = 0$
✓ factors

✓ x-values

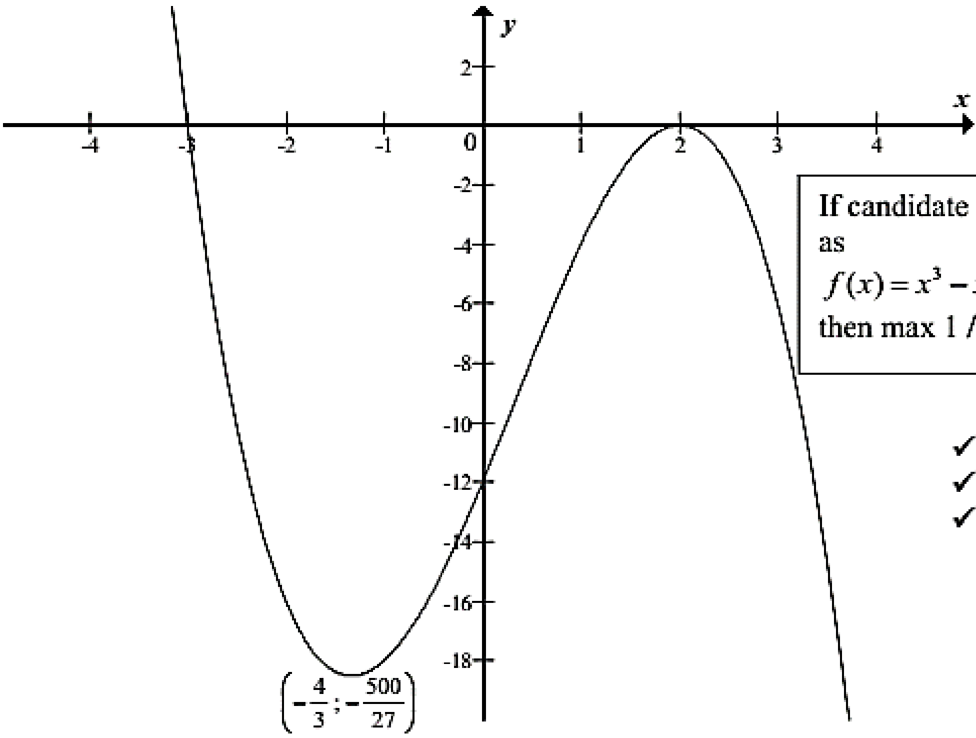
✓ y-values

(5)

NOTE:
If = 0 is omitted in 11.2: penalty 1 mark

If not in coordinate form but coordinates implied: OK

1.3



If candidate used function as $f(x) = x^3 - x^2 - 8x + 12$ then max 1 / 3

✓ shape
✓ y-intercept
✓ turning pts

(3)

1.4 $f''(x) = 0$ or $f''(x) = 0$

$6x - 2 = 0$ or $-6x + 2 = 0$

$x = \frac{1}{3}$ $x = \frac{1}{3}$

OR

$$x = \frac{2 - \frac{4}{3}}{2}$$

$$x = \frac{1}{3}$$

✓ method
✓ answer

Answer only: Full marks

(2)

Note:
If write down $f''(x) = 6x - 2$ or $f''(x) = -6x + 2$ then 1 / 2

1.5 $(2\,;\,-3)$ and $\left(-\frac{4}{3};-\frac{581}{27}\right)$ ✓✓ each answer (2)

OR

$(2\,;\,-3)$ and $(-1{,}33\,;-21{,}52)$

QUESTION 2

2.1 (0;10) ✓ $(0;10)$ (1)

2.2 $0=-x^3-x^2+x+10$

$0=-(x-2)(x^2+3x+5)$ ✓ $(x-2)$ ✓ (x^2+3x+5)

$x-2=0$ or $x^2+3x+5=0$

$x=2$

$x=\frac{-3\pm\sqrt{3^2-4(1)(5)}}{2(1)}$ ✓ $x=\frac{-3\pm\sqrt{-11}}{2}$

$=\frac{-3\pm\sqrt{-11}}{2}$

which has no solution ✓ no solution (4)

Therefore the only x-intercept of f is $(2;0)$

2.3 $f'(x)=-3x^2-2x+1$ ✓ $f'(x)=-3x^2-2x+1$

$0=-3x^2-2x+1$ ✓ $f'(x)=0$

$0=(3x-1)(x+1)$ ✓ factors

$x=\frac{1}{3}$ or $x=-1$ ✓ x-values

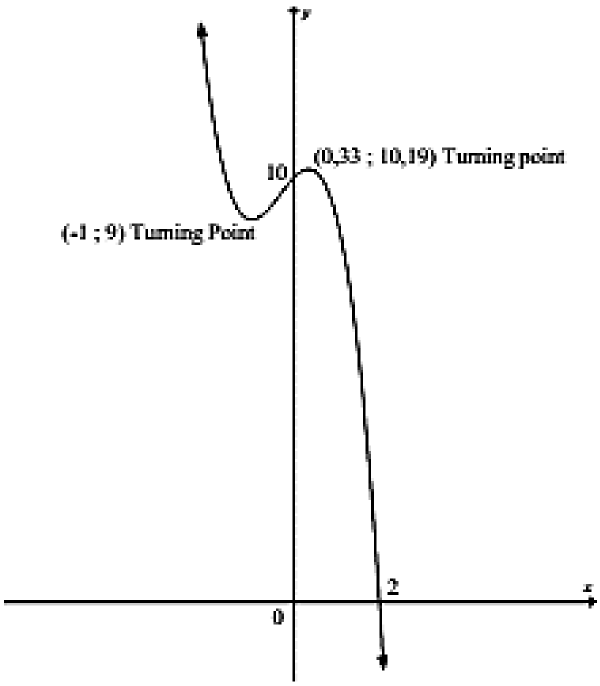
$y=-\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)+10$ or $y=-(-1)^3-(-1)^2+(-1)+10$

$=\frac{275}{27}$ $=9$

$\left(\frac{1}{3};10\frac{5}{27}\right)$ $(-1;9)$ ✓ $\left(\frac{1}{3};10\frac{5}{27}\right)$ ✓ $(-1\,;\,9)$

(6)

2.4



✓ shape
✓ intercepts
✓ turning points

(3)

QUESTION 3

3.1 $f(x) = -x^3 + 3x^2 - 4$

$f(x) = -(-1)^3 + 3(-1)^2 - 4 = 0$

3.2 $y\ intercept, x = 0$

$f(0) = -(0)^3 + 3(0)^2 - 4 = -4$

$(0; -4)$

$f(x) = (x+1)(-x^2 + bx - 4)$

$f(x) = -x^3 + bx^2 - 4x - x^2 + bx - 4$

$b - 1 = 3$

$b = 4$

$f(x) = (x+1)(-x^2 + 4x - 4)$

$x\ intercept, y = 0$

$x = -1\ or\ -x^2 + 4x - 4 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

$x\ intercepts, (-1; 0)\ and\ (2; 0)$

3.3 $at\ the\ turning\ point\ f'(x) = 0$

$$3x(x-2) = 0$$

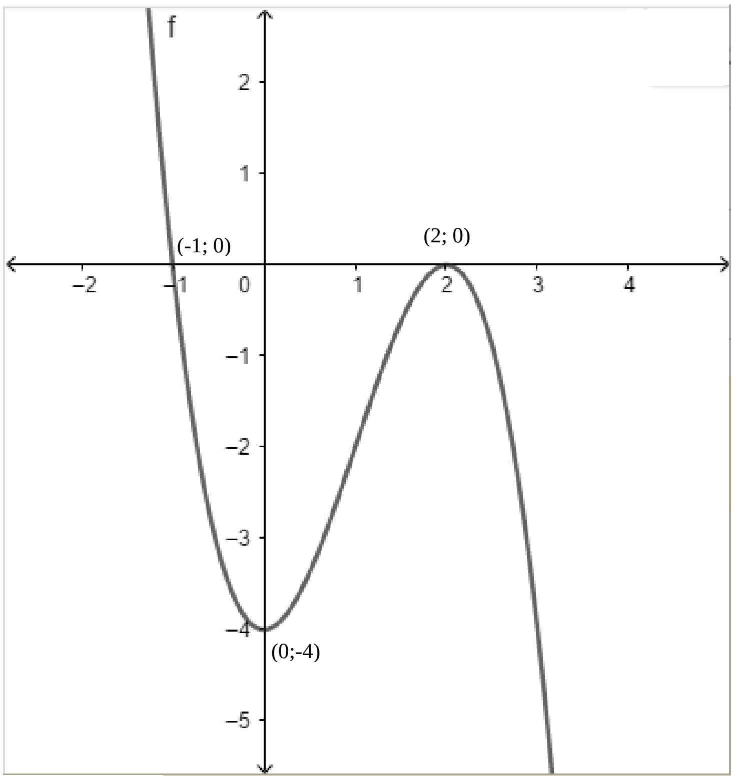
$$x = 0 \ \ or\ x = 2$$

$$f(0) = -(0)^3 + 3(0)^2 - 4 = -4$$

$$f(x) = -(2)^3 + 3(2)^2 - 4 = 0$$

$the\ turning\ points\ are\ (0;-4)\ and\ (2;0)$

3.4



3.5 $0 < x < 2$

QUESTION 4

4.1 $(-6)(-3)(+2) = 36$ ✓ $(-6)(-3)(+2)$

y-intercept is 36 ✓ y-intercept is 36

(1)

OR

$g(x) = (x-6)(x^2 - x - 6)$ ✓ trinomial

$g(x) = x^3 - 7x^2 + 36$ ✓ 36

y-intercept : $(0;36)$

(1)

4.2 $g(x) = 0$ ✓ $g(x) = 0$

$x = 6$ or $x = 3$ or $x = -2$ ✓ all x-intercepts

intercepts are $(6\ ;\ 0)$ and $(3\ ;\ 0)$ and $(-2;\ 0)$

(2)

4.3 $g(x)=(x-6)(x^2-x-6)$

$=x^3-7x^2+36$

$g'(x)=3x^2-14x$

$0=x(3x-14)$

$x=0$ or $x=\dfrac{14}{3}$

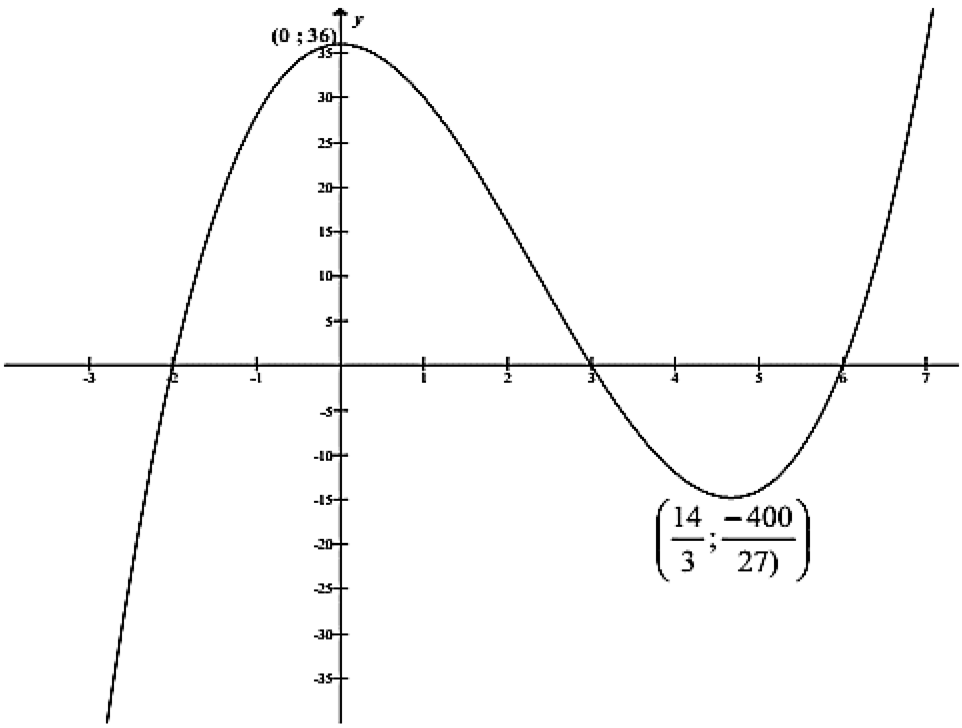
Turning points are $(0\ ;36)$ and $\left(\dfrac{14}{3};-\dfrac{400}{27}\right)$

✓ $x^3-7x+36$
✓ $g'(x)=3x^2-14x$
✓ $g'(x)=0$
✓ answers

✓✓ points

(6)

4.4



✓ x-intercepts
✓✓ turning points
✓ shape

4.5 $g(x).g'(x<0$

$x<-2$ *or* $0<x<3$ *or* $\dfrac{14}{3}<x<6$

1 mark for each inequality

(3)

SECTION 4: FINDING EQUATION OF THE CUBIC FUNCTION AND EQUATION OF A TANGENT

QUESTION 1

1.1 $f(x) = -2x^3 + ax^2 + bx + c$

$f'(x) = -6x^2 + 2ax + b$

$= -6(x-5)(x-2)$

$= -6(x^2 - 7x + 10)$

$= -6x^2 + 42x - 60$

$2a = 42$

$a = 21$

$b = -60$

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

$a = 21$; $b = -60$; $c = 43$

> **Note:**
> A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve:
> award max 2/7 marks

✓ $f'(x) = -6x^2 + 2ax + b$

✓✓ $-6(x-5)(x-2)$

✓ $b = -60$

✓ $2a = 42$

✓ subs (5 ; 18) or (2 ; -9)

✓ $c = 43$

(7)

OR

$f'(x) = -6x^2 + 2ax + b$

$f'(2) = -6(2)^2 + 2a(2) + b$

$0 = -24 + 4a + b$

$b = 24 - 4a$

$f'(5) = -6(5)^2 + 2a(5) + b$

$0 = -150 + 10a + b$

$0 = -150 + 10a + (24 - 4a)$

$0 = -126 + 6a$

$6a = 126$

$a = 21$

$b = -60$

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

$a = 21$; $b = -60$; $c = 43$

> **Note:**
> If derivative equal to zero is not written: penalize once only

✓ $f'(x) = -6x^2 + 2ax + b$

✓ $f'(2) = 0$

✓ $f'(5) = 0$

✓ $6a = 126$

✓ $b = -60$

✓ subs (5 ; 18) or (2 ; -9)

✓ $c = 43$

(7)

1.2 $f'(x)=-6x^2+42x-60$	✓ $f'(x)=-6x^2+42x-60$
$m_{\tan}=-6(1)^2+42(1)-60$	✓ subs $f'(1)$
$=-24$	✓ $m_{\tan}=-24$
$f(1)=-2(1)^3+21(1)^2-60(1)+43$	
$=2$	✓ $f(1)=2$
Point of contact is (1 ; 2)	
$y-2=-24(x-1)$ **OR** $y=-24x+c$	✓ $y-2=-24(x-1)$
$y=-24x+26$ $2=-24(1)+c$	OR $y=-24x+26$
$c=26$	
$y=-24x+26$	(5)
1.3 $f'(x)=-6x^2+42x-60$	
$f''(x)=-12x+42$	✓ $f''(x)=-12x+42$
$0=-12x+42$	
$x=\frac{7}{2}$	✓ $x=\frac{7}{2}$
	(2)
OR	
$x=\frac{2+5}{2}$	✓ $x=\frac{2+5}{2}$
$x=\frac{7}{2}$	✓ $x=\frac{7}{2}$
	(2)
OR	
$x=\frac{-21}{3(-2)}$	✓ $x=\frac{-21}{3(-2)}$
$=\frac{7}{2}$	✓ $x=\frac{7}{2}$
	(2)

QUESTION 2

2.1 $h'(x)=-3x^2+2ax+b$	✓ $h'(x)$
$h'(-1)=-3(-1)^2+2a(-1)+b$	✓ substitution of $x=-1$
$0=-3-2a+b$	✓ $h'(x)=0$
$2a-b=-3$ ... (i)	✓ simplification
$h'(2)=-3(2)^2+2a(2)+b$	
$0=-12+4a+b$	✓ substitution
$4a+b=12$ ... (ii)	
(ii) + (i): $6a=9$	✓ solving simultaneously
$a=\frac{3}{2}$	(6)
$\therefore 2\left(\frac{3}{2}\right)-b=-3$	
$b=6$	

	OR $h(-1) = -(-1)^3 + a(-1)^2 + b(-1) = \frac{-7}{2}$ $\therefore a - b = \frac{-9}{2}$ $2a - 2b = -9 \quad \cdots(i)$ $h(2) = -(2)^3 + a(2)^2 + b(2) = 10$ $4a + 2b = 18 \quad \cdots(ii)$ (i) + (ii): $6a = 9$ $a = \frac{3}{2}$ $\left(\frac{3}{2}\right) - b = \frac{-9}{2}$ $b = 6$	✓ substitution of $x = -1$ ✓ $h(-1) = \frac{-7}{2}$ ✓ simplification ✓ substitution of $x = 2$ and $h(2) = 10$ ✓ simplification ✓ solving simultaneously (6)
2.2	Average Gradient $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= \frac{13{,}5}{3}$ $= \frac{9}{2}$	✓ substitution ✓ answer (2)
2.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\ ;\ 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	✓ $h'(x)$ ✓ substitution ✓ gradient ✓ point ✓ answer (5)
2.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ **OR** $x = \frac{-1 + 2}{2}$ $x = \frac{1}{2}$	✓ second derivative ✓ = 0 ✓ answer (3)
2.5	$p > 3{,}5$ or $p < -10$	✓✓ answer (2)

QUESTION 3

3.1	$0 = x - 2$ $x = 2$ A(2 ; 0)	✓ answer (1)
3.2	$f(-1) = 0: -a + c = 2$ $f(2) = 0: \quad 8a - 2c = 2$ $a = 1, \quad c = 3$	✓ $-a + c = 2$ ✓ $8a - 2c = 2$ ✓ $a = 1$ ✓ $c = 3$
	OR	
	$a(x+1)(x+1)(x-2) = 0$ $a(0+1)(0+1)(0-2) = -2$ $-2a = -2$ $a = 1$ $f(x) = (x^2 + 2x + 1)(x - 2)$ $= x^3 - 3x - 2$ $c = -3$	✓ factors ✓ substitution ✓ a ✓ $c = -3$ (4)
3.3	$f'(x) = 0$ $3x^2 - 3 = 0$ $x^2 - 1 = 0$ $(x+1)(x-1) = 0$ B(1 ; − 4)	✓ $f'(x) = 0$ ✓ $x^2 - 1$ ✓ answer (3)
3.4	$x - 2 = x^3 - 3x - 2$ $0 = x^3 - 4x$ $0 = x(x^2 - 4)$ $0 = x(x-2)(x+2)$ $x_C = -2, \ y_C = (-2)^2 - 3(-2) - 2 = -4$ C(− 2 ; − 4) $m_{BC} = \dfrac{-4-(-4)}{1-(-2)}$ $= 0$ BC is parallel to the x-axis.	✓ equating f and g ✓ standard form ✓ factors ✓ $x_C = -2$ ✓ $y_C = -4$ ✓ $m = 0$ ✓ conclusion (7)
	OR Following from C(− 2 ; − 4), B and C have the same y – coordinate, viz. − 4. So BC is parallel to the x-axis. OR	(7)

$(x-2)=(x-2)(x+1)^2$

$\therefore (x+1)^2 = 1$ for $x \neq 2$ (7)

$\therefore x+1=\pm 1$

$\therefore x=0$ *or* $x=-2$

$y = \ \ -4$

3.5 $f''(x)=0$ — ✓ $f''(x)=0$

$6x=0$

$x=0$ — ✓ answer (2)

3.6 $k<-4$ or $k>0$ — ✓✓ answer ✓ or (3)

3.7 $f'(x)<0$

$-1<x<1$ — ✓✓answer (2)

OR

$3(x^2-1)<0$ — ✓✓answer (2)

if $(x+1)(x-1)<0$

$-1<x<1$

QUESTION 4

4.1 $f(x)=a(x+1)^2(x-3)$ — ✓✓ substitution of x-values

$-6=a(0+1)^2(0-3)$ — ✓ subs $(0\,;-6)$

$-6=-3a$

$a=2$ — ✓ $a=2$

$f(x)=2(x^2+2x+1)(x-3)$ — ✓ simplification

$=2x^3-2x^2-10x-6$ (5)

4.2 $f'(x)=6x^2-4x-10$ — ✓ $f'(x)=6x^2-4x-6$

$6x^2-4x-10=0$ — ✓ $f'(x)=0$

$3x^2-2x-5=0$ — ✓ factors

$(3x-5)(x+1)=0$

$x=\frac{5}{3}$ *or* $x=-1$ — ✓ x-value ✓ y-value

$\mathrm{B}\left(\frac{5}{3}\,;-\frac{512}{27}\right)$ **OR** $\mathrm{B}(1{,}67\,;-18{,}96)$ (5)

4.3 $h(x)=2x^3-2x^2-10x-6-(6x-6)$ ✓ $h(x)=2x^3-2x^2-16x$

$=2x^3-2x^2-16x$ ✓ $h'(x)=6x^2-4x-16$

$h'(x)=6x^2-4x-16$ ✓ $h'(x)=0$

$0=3x^2-2x-8$ ✓ factors

$0=(3x+4)(x-2)$

$x=-\frac{4}{3}$ or $x=2$ ✓ correct x-value

$\therefore x=-\frac{4}{3}$ (5)

QUESTION 5

5.1 $f(x)=-(x-1)(x-2)(x-4)$ ✓ $-(x-1)(x-2)(x-4)$

$f(x)=-(x^2-3x+2)(x-4)$ ✓ $a=7$

$f(x)=-x^3+7x^2-14x+8$ ✓ $b=-14$

✓ $c=8$

(4)

5.2 $f(x)=-x^3+7x^2-14x+8$ ✓ $f'(x)=0$

$f'(x)=0$ ✓ $-3x^2+14x-14=0$

$-3x^2+14x-14=0$

$3x^2-14x+14=0$ ✓ subs into formula

$x=\frac{14\pm\sqrt{14^2-4(3)(14)}}{2(3)}$

$=\frac{14\pm\sqrt{28}}{6}$ ✓ x – value

$=\frac{7\pm\sqrt{7}}{3}$ ✓ x – value

(5)

$x=1{,}45$ or $x=3{,}22$

5.3 $x<1{,}45$ or $x>3{,}22$ ✓ critical values

✓✓ notation

(3)

QUESTION 6

6.1 $y=5(1)-8$ ✓ subs 1

$=-3$ (1

Point of contact is $(1\,;-3)$

6.2 $-3 = 2(1)^3 + p(1)^2 + q(1) - 7$ ✓ subs (1 ; – 3)

$2 = p + q$

$g'(x) = 6x^2 + 2px + q$ ✓ $g'(x) = 6x^2 + 2px + q$

$g'(1) = 5$

$5 = 6(1)^2 + 2p(1) + q$ ✓ subs $x = 1$ and $y = 5$

$-1 = 2p + q$ ✓ simplification

$p = -3$ ✓ p-value

$q = 5$ ✓ q-value

(6)

QUESTION 7

7.1 $f'(-1) = -7$ ✓ $f'(x) = 2ax + b$

$f'(x) = 2ax + b$ ✓ substitution of $x = -1$

$-7 = -2a + b$ ✓ $-7 = -2a + b$

$f(-1) = -7(-1) + 3$ ✓ $f(-1) = 10$

$= 10$

$\therefore\ a - b + 5 = 10$

$a - b = 5$............[1]

$-2a + b = -7$.........[2]

$-a = -2$.........[1]+[2]

$a = 2$ ✓ $a = 2$

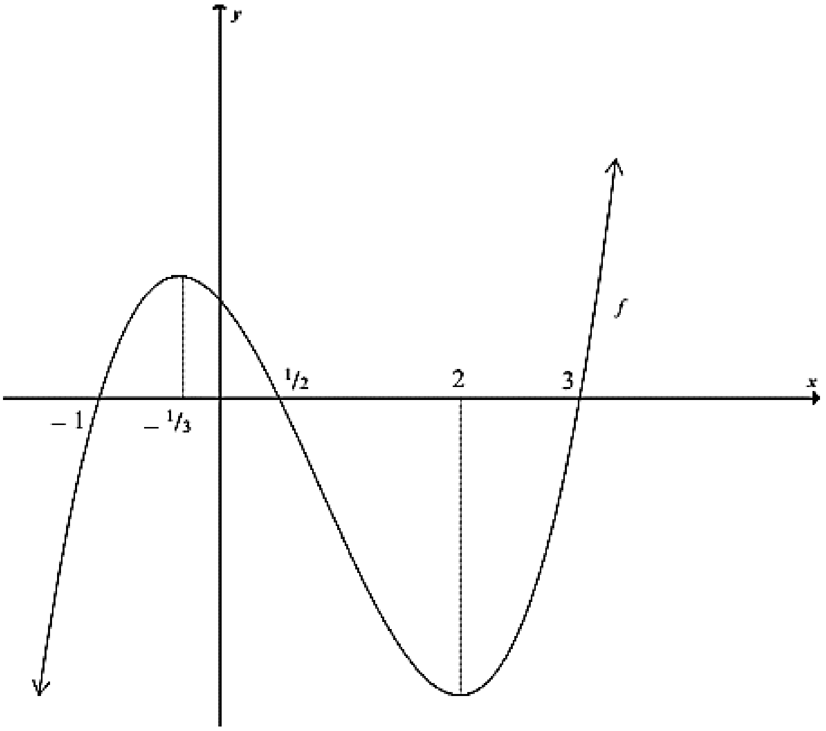
$b = -3$ ✓ $b = -3$

(6)

SECTION 5: GRAPHICAL INTERPRETATION

QUESTION 1

1.1



✓ x-intercepts
✓ turning point
✓✓ shape

[4]

QUESTION 2

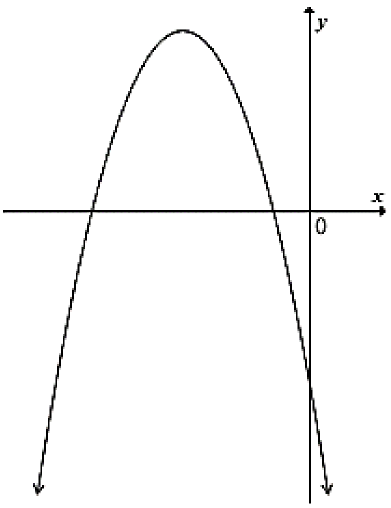
2.1 $f'(x) = 3ax^2 + 2bx + c$ ✓

$f'(x) = 3ax^2 +$

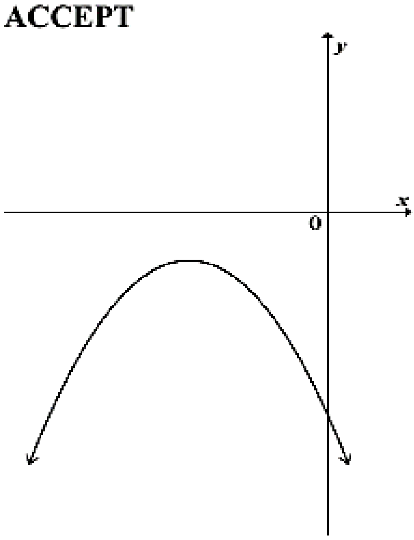
$a < 0$ shape (max TP)

$c < 0$ y - intercept is negative

$b < 0$ axis of symmetry on LHS of y - axis



ACCEPT



✓ shape (max TP)

✓ axis of symmetry on LHS if y-axis

✓ y – intercept is below x-axis

(4)

QUESTION 3

3.1.1 $f(x) = -x^3 - x^2 + 16x + 16$

$f'(x) = -3x^2 - 2x + 16$

$0 = -3x^2 - 2x + 16$

$3x^2 + 2x - 16 = 0$

$(3x+8)(x-2) = 0$

$x = -\frac{8}{3}$ or $x = 2$

✓ $f'(x) = -3x^2 - 2x + 16$
✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$
✓ factors
✓ x values

Note: if neither $f'(x) = 0$ nor $0 = -3x^2 - 2x + 16$ explicitly stated, award maximum 3/4 marks

(4)

OR

$f(x) = -x^3 - x^2 + 16x + 16$

$f'(x) = -3x^2 - 2x + 16$

$0 = -3x^2 - 2x + 16$

$0 = 3x^2 + 2x - 16$

$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-16)}}{2(3)}$

$x = -\frac{8}{3}$ or $x = 2$

✓ $f'(x) = -3x^2 - 2x + 16$
✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$
✓ subs into formula
✓ x values

(4)

3.1.2 $f''(x) = 0$

$-6x - 2 = 0$

$x = -\frac{1}{3}$

✓ $f''(x) = -6x - 2$
✓ $-6x - 2 = 0$
✓ answer

(3)

OR

$x = \frac{-\frac{8}{3} + 2}{2}$

$x = -\frac{1}{3}$

✓ $x = \frac{-\frac{8}{3} + 2}{2}$
✓✓ answer

(3)

OR

$f'(x) = -3x^2 - 2x + 16$

$x = \frac{-(-2)}{2(-3)}$

$= -\frac{1}{3}$

✓✓ $x = \frac{-(-2)}{2(-3)}$
✓ answer

(3)

OR

✓✓ $x = \frac{-(-1)}{3(-1)}$
✓ answer

(3)

$$f(x) = -x^3 - x^2 + 16x + 16$$

$$x = \frac{-(-1)}{3(-1)}$$

$$= -\frac{1}{3}$$

3.2.1 $g(x) = -2x^2 - 9x + 5$

$g(-1) = -2(-1)^2 - 9(-1) + 5$

$= 12$

$g'(x) = -4x - 9$

$m_{\tan} = -4(-1) - 9$

$= -5$

$y = -5x + c$

$12 = -5(-1) + c$

$c = 7$

$y = -5x + 7$

✓ $g(-1) = 12$
✓ $g'(x) = -4x - 9$

✓ $m_{\tan} = -5$

✓ answer

(4)

OR

$g(x) = -2x^2 - 9x + 5$

$g(-1) = -2(-1)^2 - 9(-1) + 5$

$= 12$

$g'(x) = -4x - 9$

$m_{\tan} = -4(-1) - 9$

$= -5$

$y - 12 = -5(x + 1)$

$y = -5x + 7$

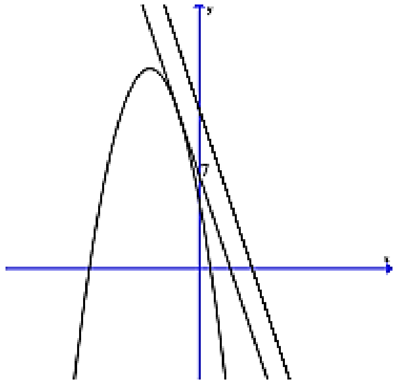
✓ $g(-1) = 12$
✓ $g'(x) = -4x - 9$

✓ $m_{\tan} = -5$

✓ answer

(4)

3.2.2



✓ sketch

✓ 7
✓correct inequality
(3)

$q > 7$

✓ method

✓ 7
✓correct inequality
(3)

OR

$y = -5x + q$ and $y = -2x^2 - 9x + 5$

$-5x + q = -2x^2 - 9x + 5$

$q = -2(x+1)^2 + 7$

$\therefore q > 7$

✓ method

OR

$y = -5x + q$ and $y = -2x^2 - 9x + 5$

$-5x + q = -2x^2 - 9x + 5$

$2x^2 + 4x + q - 5 = 0$

$x = \dfrac{-4 \pm \sqrt{16 - 4(2)(q-5)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{56 - 8q}}{4}$

$56 - 8q < 0$

$q > 7$

✓ 7
✓correct inequality
(3)

✓ method

OR

Since $g(-1) = 12$ and at $x = -1$, tangent equation is $y = -5x + 7$

$y = -5x + q$ not intersecting g $\Rightarrow$

$12 < -5(-1) + q$

$12 - 5 < q$

$7 < q$

✓ 7
✓correct inequality
(3)

3.2 $h'(x) = 12x^2 + 5$

For all values of x: $x^2 \geq 0$

$12x^2 \geq 0$

$12x^2 + 5 \geq 5$

$12x^2 + 5 > 0$

For all values of x: $h'(x) > 0$

All tangents drawn to h will have a positive gradient.
It will never be possible to draw a tangent with a negative gradient to the graph of h.

✓ $h'(x) = 12x^2 + 5$

✓ clearly argues that $h'(x) > 0$

✓ conclusion

(3)

OR

$h'(x) = 12x^2 + 5$

Suppose $h'(x) < 0$ and try to solve for x :

$12x^2 + 5 < 0$

$$x^2 < -\frac{5}{12}$$

but x^2 is always positive

$\therefore$ no solution for x

$\therefore h'(x) \geq 0$ for all $x \in R$

i.e. there are no tangents with negative slopes

✓ $h'(x) = 12x^2 + 5$

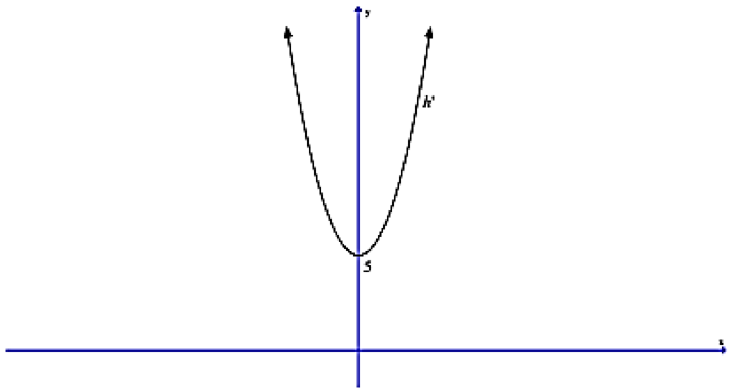
✓ clearly argues that $h'(x) < 0$ is impossible

✓ conclusion

(3)

OR

$h'(x) = 12x^2 + 5$



Since clearly $h'(x) > 0$ for all $x \in R$,
it will never be possible to draw a tangent with a negative gradient to the graph of h.

✓ $h'(x) = 12x^2 + 5$

✓ argues $h'(x) > 0$ by drawing a sketch

✓ conclusion

(3)

QUESTION 4

4.1 x-value of turning point:

$$x = \frac{-4+1}{2}$$

$$= -\frac{3}{2}$$

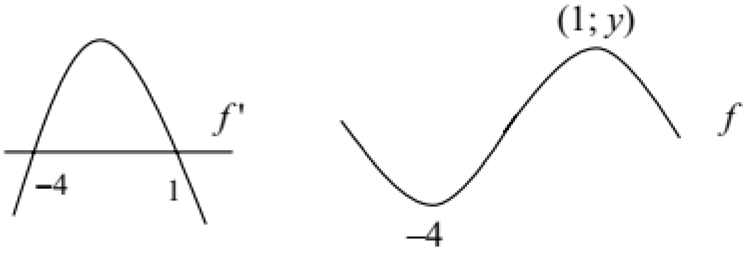
$$\therefore x > -\frac{3}{2} \quad \text{OR} \quad \therefore x \in \left(-\frac{3}{2}; \infty\right)$$

✓ $x > -\frac{3}{2}$ OR $\left(-\frac{3}{2}; \infty\right)$ (1)

4.2 f has a local minimum at $x = -4$ because:

✓ $x = -4$

✓✓ graph (3)



OR

$f'(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$.

$f'(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$

i.e. 

$\therefore f$ has a local minimum at $x = -4$

✓ $x = -4$

✓ $f'(x) < 0$ for $x < -4$

✓ $f'(x) > 0$ for $-4 < x < 1$ (3)

OR

Gradient of f changes from negative to positive at $x = -4$

✓ $x = -4$

✓gradient negative for $x < -4$

✓ gradient positive for $-4 < x < 1$ (3)

OR

$f'(-4) = 0$

$f''(-4) > 0$ so graph is concave up at $x = -4$, so f has a local minimum at $x = -4$.

✓ $f'(-4) = 0$

✓ $f''(-4) > 0$

✓ $x = -4$ (3)

QUESTION 5

5.1 $x = 1$ and/or $x = 2$ ✓✓ answer (2)

5.2 When $x < 1$, $f'(x) > 0$ and so f is increasing ✓ $f'(x) > 0$

When $1 < x < 2$, $f'(x) < 0$ and so f is decreasing ✓ $f'(x) < 0$

When $x > 2$, $f'(x) > 0$ and so f is increasing

At $x = 1$: local maximum ✓answer

At $x = 2$: local minimum ✓ answer

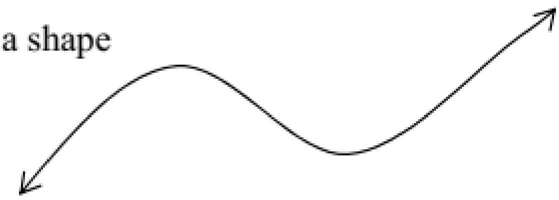
(4)

OR

$f'(x) = ax^2 + bx + c$ is minimum-valued ✓ $f'(x)$ minimum-valued

$\therefore\ a > 0$ ✓ $a > 0$

$\therefore f$ has a shape



At $x = 1$: local maximum ✓ answer

At $x = 2$: local minimum ✓ answer

(4)

OR ✓✓ number line

$f'(x)$	+	0	–	0	+
x		1		2	

At $x = 1$: local maximum ✓ answer

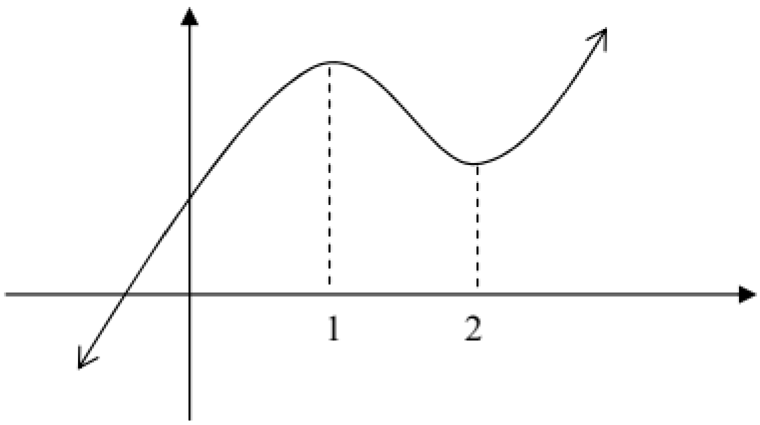
At $x = 2$: local minimum ✓ answer

(4)

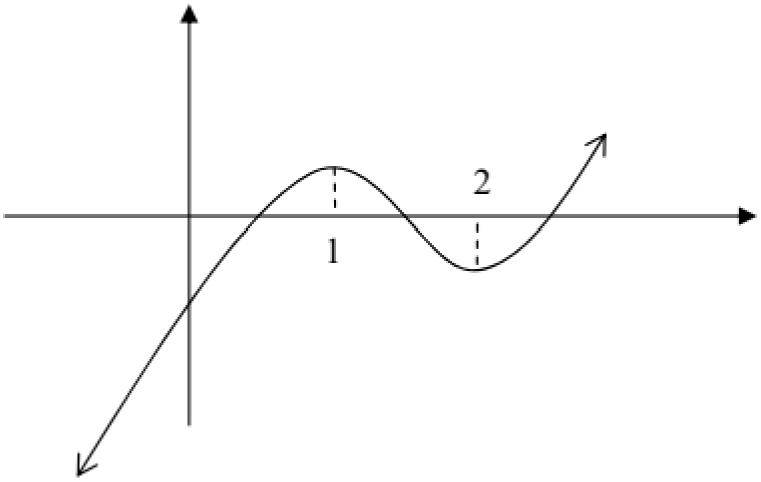
5.3 $x = \dfrac{1+2}{2} = 1{,}5$ ✓ answer

(1)

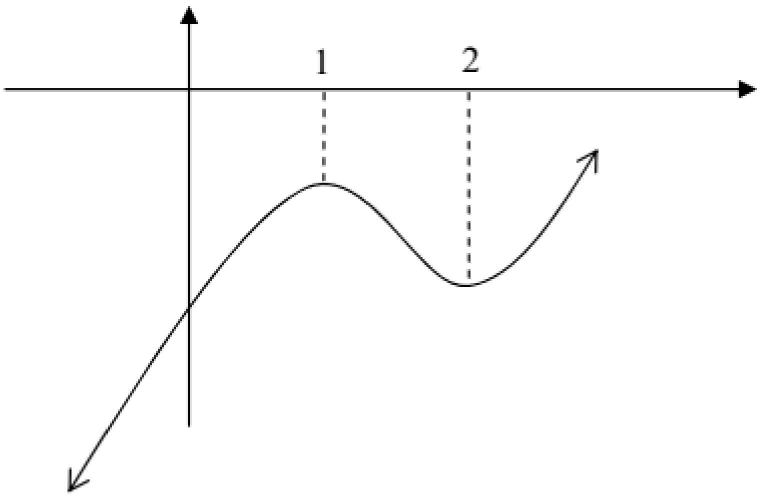
5.4



OR



OR



✓ shape
✓ x-values of turning points correct

(2)

QUESTION 6

6.1 The y-intercept of g is E$(0\,;-4)$

OR

$x = 0$ and $y = -4$

✓ answer

(1)

6.2 $y = a(x+2)(x-6)$

$-4 = a(0+2)(0-6)$

$-4 = -12a$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x+2)(x-6)$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

OR

✓ setting up of equation
✓ subs $(0\,;-4)$

✓ $a = \frac{1}{3}$

✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

$$g'(0) = -4 = c$$
$$g'(x) = ax^2 + bx - 4$$
$$g'(-2) = 0$$
$$4a - 2b - 4 = 0$$
$$b = 2a - 2$$
$$g''(2) = 0$$
$$2a(2) + b = 0$$
$$b = -4a$$
$$2a - 2 = -4a$$
$$a = \frac{1}{3}$$
$$b = -\frac{4}{3}$$
$$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$$

✓ substitution $x = -2$ and $g'(x) = 0$

✓ $g''(2) = 0$

✓ $a = \frac{1}{3}$

✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

OR

$$c = -4$$
$$4a - 2b - 4 = 0$$
$$36a + 6b - 4 = 0$$
$$48a - 16 = 0$$
$$a = \frac{1}{3}$$
$$b = -\frac{4}{3}$$
$$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$$

✓ setting up of equation
✓ simultaneous equation

✓ $a = \frac{1}{3}$

✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

OR

$$y = a(x+2)(x-6)$$
$$= a(x^2 - 4x - 12)$$
$$= ax^2 - 4ax - 12a$$
$$-12a = -4$$
$$a = \frac{1}{3}$$
$$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$$

OR

$\frac{dy}{dx} = 2ax + b$

$0 = 2a(2) + b$

$b = -4a$

✓ $b = -4a$

EITHER

subs $(6 ; 0)$

$0 = 36a + 6b - 4$

$4 = 36a + 6b$

$2 = 18a + 3b$

$2 = 18a + 3(-4a)$

$2 = 6a$

$a = \frac{1}{3}$

$b = -\frac{4}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

OR

$0 = 4a - 2b - 4$

$0 = 4a - 2(-4a) - 4$

$12a = 4$

$a = \frac{1}{3}$

$b = -\frac{4}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

✓ simultaneous equation

✓ $a = \frac{1}{3}$

✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

✓ setting up of equation

✓ $ax^2 - 4ax - 12a$

✓ $a = \frac{1}{3}$

✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

6.3 At turning point $g'(x) = 0$

$x = -2$ and $x = 6$

Answer only:
Full marks

If only 1 value given, max 1 / 2

✓ $g'(x) = 0$

✓ $x = 6$ and $x = -2$

(2)

6.4 $x = \frac{-2+6}{2}$

$x = 2$

✓ $x = \frac{-2+6}{2}$

✓ answer

(2)

OR

x-value of point of inflection of g is at A.

$g''(x) = 0$

$\frac{2x}{3} - \frac{4}{3} = 0$

$2x - 4 = 0$

$2x = 4$

$x = 2$

✓ $2x - 4 = 0$

✓ answer

(2)

OR

$$x = -\frac{b}{2a}$$

$$x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$$

$$x = 2$$

OR

$$g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$$

$$x = 2$$

✓ $x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$

✓ answer

(2)

✓ $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$

✓ answer

(2)

6.5 $g'(x) > 0$ for $x < -2$, so g is increasing for $x < -2$.

$g'(x) < 0$ for $x > -2$, so g is decreasing for $x > -2$.

$\therefore$ g has a local maximum at $x = -2$ because the graph is increasing followed by decreasing

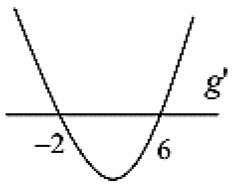
✓ $g'(x) > 0$

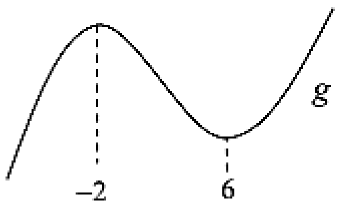
✓ g is incr for $x < -2$

✓ g is decr for $x > -2$

(3)

OR





$\therefore$ g has a local maximum at $x = -2$

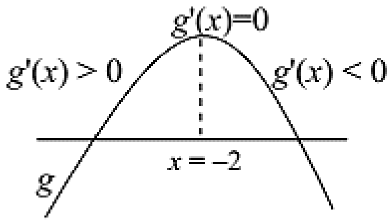
✓ $g'(x) > 0$ for $x < -2$

✓ $g'(x) < 0$ for $x > -2$

✓ max at $x = -2$

(3)

OR



✓ $g'(-2) = 0$

✓ $g''(-2) < 0$

✓ max at $x = -2$

(3)

SECTION 6: OPTIMISATION

QUESTION 1

1.1.1 Depth after 3 days $= 12 - \frac{1}{4}(3) - \frac{1}{6}(3)^3 = \frac{27}{4} = 6{,}75$ m

✓ answer (1)

1.1.2 Rate of decrease in depth $= h'(t) = -\frac{1}{4} - \frac{1}{2}t^2$

$$= -\frac{1}{4} - \frac{1}{2}(2)^2$$

Rate of decrease in depth after 2 days

$$= -\frac{9}{4}$$

$$= -2{,}25 \text{ metres/day}$$

Rate of decrease in depth $= 2{,}25$ metres per day

✓ $h'(t)$
✓derivative
✓ substitution of $t = 2$
✓answer (2,25)
✓ units (metres per day)
(5)

1.2

$$g(x) = ax^2 + \frac{b}{x}$$

$$g(x) = ax^2 + bx^{-1}$$

$$g'(x) = 2ax - bx^{-2}$$

$$0 = 2a(4) - \frac{b}{(4)^2}$$

$$8a = \frac{b}{16}$$

$$b = 128a$$

$$96 = a(4)^2 + \frac{b}{4}$$

$$96 = 16a + \frac{1}{4}(128a)$$

$$96 = 48a$$

$$a = 2$$

$$b = 256$$

OR

Note:
In the equation $g'(x) = 0$; $= 0$ must be shown in the equation.

✓ $g'(x) = 2ax - bx^{-2}$
✓ $0 = g'(x)$
✓ $2a(4) - \frac{b}{(4)^2}$
✓ subs (4 ; 96)
✓ $a = 2$
✓ $b = 256$
(6)

$$g'(x)=2ax-\frac{b}{x^2}$$

$$g'(4)=8a-\frac{b}{16}=0$$

$$g(4)=16a+\frac{b}{4}=96$$

$$32a-\frac{b}{4}=0$$

$$48a=96$$

$$a=2$$

$$b=256$$

✓ $g'(x)=2ax-\frac{b}{x^2}$
✓ $g'(4)=8a-\frac{b}{16}$
✓ $g'(x)=0$
✓ $g(4)=16a+\frac{b}{4}=96$
✓ $a=2$
✓ $b=256$
(6)

QUESTION 2

2.1 $s(0)=5(0)^3-65(0)^2+200(0)+100$

$=100$ metres

NOTE:
If subs $t=8$, then answer = 100: 0 / 2

✓ $t=0$
✓ answer
(2)

Answer only: full marks

2.2 $s(t)=5t^3-65t^2+200t+100$

$s'(t)=15t^2-130t+200$

$s'(4)=15(4)^2-130(4)+200$

$=-80$ metres per minute

✓ $s'(t)=15t^2-130t+200$
✓ substitution $t=4$
✓ answer (-80)
(3)

NOTE:
If used average rate of change between $t=0$ and $t=4$: 0 / 3
If subs $t=4$ into $s(t)$: 0 / 3

2.3 The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change.

OR

The **vertical** velocity of the car at $t=4$ is 80 metres per minute.

✓ speed 80 metres per minute
✓ downwards
(2)

NOTE:
Mark this CA even if answer to QUESTION 12.2 is completely inaccurate.

2.4 $s'(t)=15t^2-130t+200$

$s''(t)=30t-130$

$130=30t$

$t=4,3\dot{3}$ minutes

OR

$$t=\frac{-(-130)}{2(15)}$$

$t=4,3\dot{3}$ minutes

✓ $s''(t)=30t-130$
✓ $s''(t)=0$
✓ answer
(3)

QUESTION 3

3.1 $V(0) = 100 - 4(0)$
$= 100$ litres

✓ answer (1)

3.2 Rate in – rate out
$= 5 - k \quad l/\min$

$V'(t) = -4 \ l/\min$

✓ $5 - k$
✓ -4
✓ units stated once (3)

3.3 $5 - k = -4$
$k = 9 \quad l/\min$

✓ $5 - k = -4$
✓ $k = 9$ (2)

OR

Volume at any time t = initial volume + incoming total – outgoing total

$100 + 5t - kt = 100 - 4t$

$5t - kt = -4t$

$9t - kt = 0$

$t(9 - k) = 0$

At 1 minute from start, $t = 1$, $9 - k = 0$,
so $k = 9$

✓ $100 + 5t - kt = 100 - 4t$
✓ $k = 9$ (2)

OR

Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$.

✓✓ $k = 9$ (2)

QUESTION 4

4.1 $s(t) = 2t^2 - 18t + 45$
$s'(t) = 4t - 18$
$s'(0) = 4(0) - 18$
$= -18 \ m/s$

Note: answer only award 0/3 marks

✓ $s'(t)$
✓ subs $t = 0$ into $s'(t)$ formula
✓ answer (3)

4.2 $s''(t) = 4 \text{ m/s}^2$

✓ answer (1)

4.3 $4t - 18 = 0$
$4t = 18$
$t = \frac{9}{2}$ seconds or 4,5 seconds

✓ $s'(t) = 0$
✓ answer (2)

OR

$s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$

$t = \frac{9}{2}$ seconds or 4,5 seconds

✓ $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$
✓ answer (2)

OR

$s(t)=2t^2-18t+45$

$t=-\frac{-18}{2(2)}$ ✓ $t=-\frac{-18}{2(2)}$

$t=\frac{9}{2}$ seconds or 4,5 seconds ✓ answer

(2)

QUESTION 5

5.1 $AB^2=(t^2-0)^2+(t-3)^2$ ✓substitution

$AB^2=t^4+t^2-6t+9$ ✓ simplification

(2)

5.2 $\frac{d}{dt}AB^2=4t^3+2t-6$ ✓ $\frac{d}{da}AB^2$

$0=4t^3+2t-6$ ✓ $\frac{d}{da}AB^2=0$

$0=2t^3+t-3$ ✓ simplification

$0=(t-1)(2t^2+2t+3)$ ✓ factorisation

$t=1$ ✓ answer

There is no solution for $2t^2+2t+3=0$

(5)

QUESTION 6

6.1 $40-x$ ✓answer

(1)

6.2 $P(x)=(40-x)(144+4x)$ ✓concept of multiplication

$=4(40-x)(36+x)$ ✓ $(144+4x)$

$=5\,760+16x-4x^2$ ✓ answer

(3)

6.3 $P'(x)=16-8x$ ✓ $P'(x)=16-8x$

$P'(x)=0$ ✓ $P'(x)=0$

$16-8x=0$

$8x=16$

$x=2$ ✓ $x=2$

Cost = 144 + 4 (2)

= R 152 ✓answer

(4)

OR

Max at $x=\frac{40-36}{2}=2$ ✓ $x=40$ & 36 are solutions to $P(x)=0$

✓✓ $x=\frac{40-36}{2}=2$

Cost = 144 + 4 (2)

= R 152 ✓ answer

(4)

✓✓ explanation

OR

	Number of watches	Cost	Income
Year 0:	40	144	5 760
Year 1:	39	148	5 772
Year 2:	38	152	5 776
Year 3:	37	156	5 772

✓ $x = 2$
✓ R 152

(4)

Max Income at $x = 2$
Max cost = R 152

QUESTION 7

7.1 Length of box = $3x$

Volume $= l \times b \times h$

$$9 = 3x \cdot x \cdot h$$

$$9 = 3x^2 h$$

$$h = \frac{3}{x^2}$$

✓ length of box = $3x$
✓ $9 = 3x \cdot x \cdot h$
✓ $h = \frac{3}{x^2}$

(3)

7.2 $C = (2(3xh) + 2xh) \times 50 + (2 \times 3x^2) \times 100$

$$= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$$

$$= \frac{1200}{x} + 600x^2$$

✓ $(2(3xh) + 2xh) \times 50$
✓ $(2 \times 3x^2) \times 100$
✓ substitution of $h = \frac{3}{x^2}$ (3)

OR

$C = (h \times 8x) \times 50 + (2 \times 3x^2) \times 100$

$$= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$$

$$= \frac{1200}{x} + 600x^2$$

✓ $(h \times 8x) \times 50$
✓ $(2 \times 3x^2) \times 100$
✓ substitution of $h = \frac{3}{x^2}$ (3)

7.3 $C = 1200x^{-1} + 600x^2$

$$\frac{dC}{dx} = -1200x^{-2} + 1200x$$

$$0 = -1200x^{-2} + 1200x$$

$$1200x^3 = 1200$$

$$x^3 = 1$$

$$x = 1$$

Therefore the width of the box is 1 metre.

✓ $\frac{dC}{dx} = -1200x^{-2} + 1200x$
✓ $\frac{dC}{dx} = 0$
✓ $x^3 = 1$
✓ $x = 1$

(4)

QUESTION 8

8.1 $m = -\frac{b}{a}$ ✓ $m = -\frac{b}{a}$

$y - b = \frac{-b}{a}(x - 0)$

$y = \frac{-b}{a}x + b$

OR

$y = mx + b$

$0 = ma + b$

$m = \frac{-b}{a}$

$y = -\frac{b}{a}x + b$

OR $\frac{x}{a} + \frac{y}{b} = 1$

✓ answer

(2)

8.2 $\text{A} = xy$ ✓ area formula

$A = x\left(\frac{-bx}{a} + b\right)$ ✓ substitution

$= -\frac{b}{a}x^2 + bx$

$\frac{dA}{dx} = -\frac{2b}{a}x + b$ ✓ $\frac{dA}{dx} = -\frac{2b}{a}x + b$

$0 = -\frac{2b}{a}x + b$ ✓ $\frac{dA}{dx} = 0$

$-ba = -2bx$

$x = \frac{a}{2}$ ✓ x-value

$y = -\frac{b}{a}\left(\frac{a}{2}\right) + b$

$= \frac{b}{2}$ ✓ y-value

$\text{P}\left(\frac{a}{2}; \frac{b}{2}\right)$ which is the midpoint of MN

(6)

OR

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{y}{b} = 1 - \frac{x}{a}$

To maximise xy, we maximise

$\frac{xy}{ab} = \frac{x}{a}\left(\frac{y}{b}\right) = \frac{x}{a}\left(1 - \frac{x}{a}\right)$

This is a maximum when $\frac{x}{a} = \frac{1}{2}$ i.e. $x = \frac{a}{2}$

By the midpoint theorem, P is then the midpoint of MN.

QUESTION 9

9.1 $\pi r^2 h = 6$ ✓ $h = \frac{6}{\pi r^2}$ (1)

$h = \frac{6}{\pi r^2}$

9.2 $S = 10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ ✓✓ $10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$

$= 10\left[2\pi rh + 6\pi r^2\right]$

$= 20\pi rh + 60\pi r^2$ ✓ $20\pi rh + 60\pi r^2$

$= 20\pi r\left(\frac{6}{\pi r^2}\right) + 60\pi r^2$ ✓ substitution/*substitusie*

$= 60\pi r^2 + \frac{120}{r}$ (4)

OR

Area of/*van* 10 spheres/*sfere* $= 10 \times 4 \times \pi \times r^2 = 40\pi r^2$ ✓area of 10 spheres/ *area van 10 sfere*

Area of/*van* 10 cylinders/*silinders* $= 10(2\pi r^2 + 2\pi rh)$ ✓ area of 10 cylinders/ *area van 10 silinders*

$= 10(2\pi r^2 + 2\pi r\frac{6}{\pi r^2})$ ✓ substitution/*substitusie*

$= 20\pi r^2 + \frac{120}{r}$

Total area/*Totale area* $= 40\pi r^2 + 20\pi r^2 + \frac{120}{r}$ ✓simplification/*vereenvoudiging*

$= 60\pi r^2 + \frac{120}{r}$ (4)

9.3 $S' = 120\pi r - 120r^{-2} = 0$ ✓ $120\pi r - 120r^{-2}$ ✓ $= 0$

$120\pi r - \frac{120}{r^2} = 0$

$120\pi r^3 - 120 = 0$

$r^3 = \frac{120}{120\pi}$ ✓ $r^3 = \frac{120}{120\pi}$

$\therefore r = \frac{1}{\pi^{\frac{1}{3}}} = 0,68$ cm ✓answer/*antwoord*

QUESTION 10

10.1

$$V = \pi r^2 h + 2 \times \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$V = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$\frac{\pi}{6} = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$\pi r^2 h = \frac{\pi}{6} - \frac{4}{3}\pi r^3$$

$$h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$$

$$h = \frac{1}{6r^2} - \frac{4r}{3}$$

✓ volume equation

✓ substitution of $\frac{\pi}{6}$

✓ $h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{\pi r^2}$

(3)

10.2

$$S = 2 \times 2\pi r^2 + 2\pi r h$$

$$S = 4\pi r^2 + 2\pi r h$$

$$S = 4\pi r^2 + 2\pi r\left(\frac{1}{6r^2} - \frac{4r}{3}\right)$$

$$S = 4\pi r^2 + \frac{\pi}{3r} - \frac{8\pi r^2}{3}$$

$$= \frac{4}{3}\pi r^2 + \frac{\pi}{3r}$$

✓ surface area equation

✓ substitution of h

✓ simplification

(3)

10.3

$$S = \frac{4}{3}\pi r^2 + \frac{\pi}{3} r^{-1}$$

$$\frac{dS}{dr} = \frac{8\pi r}{3} - \frac{\pi}{3r^2} = 0$$

$$8r = \frac{1}{r^2}$$

$$8r^3 = 1$$

$$r = \frac{1}{2}$$

Then $S = \frac{4}{3}\pi\left(\frac{1}{2}\right)^2 + \frac{\pi}{3}(2)$

$S = \pi$ square metres

$= 3{,}14$ square metres

✓ $\frac{\pi}{3} r^{-1}$

✓ $\frac{dS}{dr} = \frac{\pi}{3}\left(8r - \frac{1}{r^2}\right)$

or

$\frac{dS}{dr} = \frac{\pi}{3}\left(8r - r^{-2}\right)$

✓ $\frac{dS}{dr} = 0$

✓ $8r = \frac{1}{r^2}$

✓ $r = \frac{1}{2}$

✓ $S = \pi$

(6)

QUESTION 11

11.1 $r = 2x$ ✓ $r = 2x$

Area rectangle $= 8x$ ✓ area rectangle $= 8x$

Radius small circle $= \frac{2}{3}r$ ✓ radius small circle $= \frac{2}{3}r$

$A(x) =$ area rectangle $-$ area circle ✓ formula

$$A(x) = 8x - \left[\pi r^2 + \pi\left(\frac{2}{3}r\right)^2\right]$$

$$A(x) = 8x - \pi(2x)^2 - \pi\left(\frac{2}{3}(2x)\right)^2$$

$$A(x) = 8x - 4\pi x^2 - \frac{16}{9}\pi x^2$$ ✓ $\frac{16}{9}\pi x^2$

$$A(x) = 8x - \frac{52\pi}{9}x^2$$

(5)

11.2 $A'(x) = 8 - \frac{104}{9}\pi x$ ✓ $A'(x) = 8 - \frac{104}{9}\pi x$

$0 = 8 - \frac{104}{9}\pi x$ ✓ $A'(x) = 0$

$x = \frac{72}{104\pi}$ ✓ answer

$x = \frac{9}{13\pi}$

$x = 0{,}2203683827...$

$x = 0{,}22$ m

(3)

11.3 Area of circles

$= \frac{52\pi}{9}x^2$ ✓ substitution

$= \frac{52\pi}{9}(0{,}22)^2$ ✓ answer

$= 0{,}88 \text{ m}^2$

OR

Area of circles

$= \frac{52\pi}{9}x^2$

$= \frac{52\pi}{9}\left(\frac{9}{13\pi}\right)^2$

$= \frac{36}{13\pi} \text{ m}^2$

(2)

ANNEXURE A: 2015, 2016 AND 2017 JUNE , AND SELECTED QUESTIONS

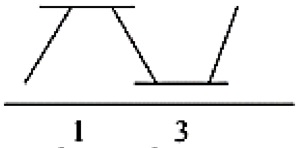
2015 June Exam Paper 1 (Differential Calculus)

QUESTION/*VRAAG 8*

8.1	$f(x+h)=\dfrac{4}{x+h}$	
	$f(x+h)-f(x)=\dfrac{4}{x+h}-\dfrac{4}{x}$	✓ $\dfrac{4}{x+h}-\dfrac{4}{x}$
	$=\dfrac{4x\text{-}4(x+h)}{x(x+h)}$	✓ $\dfrac{4x-4(x+h)}{x(x+h)}$
	$=\dfrac{4x-4x-4h}{x(x+h)}$	
	$=\dfrac{-4h}{x(x+h)}$	✓ $\dfrac{-4}{x(x+h)}$
	$\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{-4h}{x(x+h)}}{h}$	
	$=\dfrac{-4h}{xh(x+h)}$	
	$\dfrac{-4}{x(x+h)}$	
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$	✓formula
	$=\lim\limits_{h\to 0}\dfrac{-4}{x(x+h)}$	✓answer
	$=\dfrac{-4}{x^2}$	(5)
	OR/*OF*	

	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\frac{4}{x+h}-\frac{4}{x}}{h}$ $=\lim_{h\to 0}\frac{\frac{4x-4(x+h)}{x(x+h)}}{h}$ $=\lim_{h\to 0}\frac{4x-4x-4h}{hx(x+h)}$ $=\lim_{h\to 0}\frac{-4h}{xh(x+h)}$ $=\lim_{h\to 0}\frac{-4}{x(x+h)}$ $=\frac{-4}{x^2}$	✓formula ✓subst. into formula ✓ $\frac{4x-4(x+h)}{x(x+h)}$ ✓ $\frac{-4}{x(x+h)}$ ✓answer (5)
8.2.1	$y=5x^2+5x+2$ $\frac{dy}{dx}=10x+5$	✓ $10x$ ✓ 5 (2)
8.2.2	$D_x\left[\sqrt[3]{x^2}-\frac{1}{2}x\right]$ $=D_x\left[x^{\frac{2}{3}}-\frac{1}{2}x\right]$ $=\frac{2}{3}x^{\frac{-1}{3}}-\frac{1}{2}$	✓ $x^{\frac{2}{3}}$ ✓ $\frac{2}{3}x^{\frac{-1}{3}}$ ✓ $-\frac{1}{2}$ (3)
8.3	$p(x)=x^3+2x$ $p'(x)=3x^2+2$ $3x^2\geq 0$ or / *of* $x^2\geq 0$ for all/*vir alle* $x\in\mathbf{R}$ $\therefore 3x^2+2\geq 2>0$ for all/*vir alle* $x\in\mathbf{R}$ i.e. $p'(x)>0$ for all/*vir alle* $x\in\mathbf{R}$ i.e. all tangents to p have gradient greater than (or equal to) 2. Thus there is no tangent to p that has negative gradient. *Alle raaklyne aan p sal dus 'n gradiënt groter (of gelyk aan) 2 hê. Daar sal dus geen raaklyn aan p wees met 'n negatiewe gradiënt nie.*	✓ $p'(x)=3x^2+2$ ✓states & justifies $p'(x)>0$ ✓linking derivative to gradient of tangent/*verband tussen gradiënt en afgeleide* (3) **[13]**

QUESTION/*VRAAG* 9

9.1	$x=1$ or $x=3$	✓ $x=3$ ✓ $x=1$ (2)
9.2	$1<x<3$	✓✓answer (2)
9.3	For a point x close to 3/*Vir 'n punt naby aan 3*: If $x<3$, $f'(x)<0 \Rightarrow f$ decreasing/*dalend* If $x>3$, $f'(x)>0 \Rightarrow f$ increasing/*stygend*  Therefore: f has a local minimum at/*f het lokale minimum by* $x=3$	✓f dec for $x<3$ *f dalend vir* $x<3$ f incr for $x>3$ *f stygend vir* $x>3$ ✓ $x=3$ local min (2)
	OR/*OF*	
	At $x=3$, the gradient function changes from negative to positive therefore the function will have a local minimum point at $x=3$/ *By* $x=3$ *verander die gradiëntfunksie van negatief na positief dus sal die funksie 'n lokale minimum punt hê by* $x=3$.	✓ at $x=3$ gradient changes from neg to pos ✓ $x=3$ local min (2)
	OR/*OF*	
	$f'(3)=0$ and $f''(3)>0$ therefore the function will have a local minimum point at $x=3$ / $f''(3)>0$ *dus sal die funksie 'n lokale minimum punt hê by* $x=3$.	✓ $f''(3)>0$ ✓ $x=3$ local min (2)
9.4	$f''(x)=0$ at the turning point of/*by die draaipunt van* $f'(x)$ Using symmetry/*Deur simmetrie* $x=\frac{1+3}{2}$ $=2$	✓answer (1)
9.5	Concave up if/*Konkaaf op as* $f''(x)>0$ $x>2$	✓ $f''(x)>0$ ✓answer (2) **[9]**

QUESTION/*VRAAG 10*

	Given: $M(t)=t^3-9t^2+3000 \quad ; \quad 0\le t\le 30$	
10.1	$M(0)=0^3-9(0)^2+3000$ $=3000g$ or $3kg$	✓answer (1)
10.2	$t^3-9t^2+3000=3000$ $t^3-9t^2=0$ $t^2(t-9)=0$ $t=0$ or $t=9$ Baby's mass will return to the birth mass on the 9th day/ *Baba se massa keer terug na massa by geboorte op die 9de dag.*	✓ $M(t)=3000$ ✓ $t^3-9t=0$ ✓factors ✓ $t=9$ (4)
10.3	$M'(t)=0$ $3t^2-18t=0$ $3t(t-6)=0$ $t=0$ or $t=6$ Baby's mass will be a minimum on the 6th day/ *Baba se massa sal 'n minimum wees op die 6de dag.*	✓ $M'(t)=0$ ✓ $3t^2-18t$ ✓factors ✓ $t=6$ (4)
10.4	$M'(t)=3t^2-18t$ $M''(t)=6t-18$ $0=6t-18$ $t=3$ **OR / *OF*** Using symmetry/*Deur simmitrie*: $t=\frac{0+6}{2}$ $=3$	✓ $6t-18$ ✓answer (2) ✓ $\frac{0+6}{2}$ ✓answer (2) **[11]**

2016 June Exam Paper 1 (Differential Calculus)

QUESTION/*VRAAG* 7

7.1	$f(x+h)=3(x+h)^2-5=3(x^2+2xh+h^2)-5$ $=3x^2+6xh+3h^2-5$ $f(x+h)-f(x)=3x^2+6xh+3h^2-5-3x^2+5$ $=6xh+3h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{6xh+3h^2}{h}$ $=\lim_{h\to 0}\frac{h(6x+3h)}{h}$ $=\lim_{h\to 0}(6x+3h)$ $=6x$ **OR/OF** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{3(x+h)^2-5-(3x^2-5)}{h}$ $=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-5-3x^2+5}{h}$ $=\lim_{h\to 0}\frac{6xh+3h^2}{h}$ $=\lim_{h\to 0}\frac{h(6x+3h)}{h}$ $=\lim_{h\to 0}(6x+3h)$ $=6x$	✓ $3x^2+6xh+3h^2-5$ ✓ $6xh+3h^2$ ✓ $\frac{f(x+h)-f(x)}{h}$ ✓ common factor/ $(6x+3h)$ ✓ answer (5) ✓ $\frac{f(x+h)-f(x)}{h}$ ✓ $3x^2+6xh+3h^2-5$ ✓ $6xh+3h^2$ ✓ common factor/ $(6x+3h)$ ✓ answer (5)
7.2.1	$y=2x^5+\frac{4}{x^3}$ $y=2x^5+4x^{-3}$ $\frac{dy}{dx}=10x^4-12x^{-4}$	✓ $2x^5+4x^{-3}$ ✓ $10x^4$ ✓ $-12x^{-4}$ (3)

7.2.2	$y=\left(\sqrt{x}-x^{2}\right)^{2}$ $y=\left(x^{\frac{1}{2}}-x^{2}\right)^{2}$ $=x-2x^{\frac{5}{2}}+x^{4}$ $\frac{dy}{dx}=1-5x^{\frac{3}{2}}+4x^{3}$	✓ $x-2x^{\frac{5}{2}}+x^{4}$ ✓ 1 ✓ $-5x^{\frac{3}{2}}$ ✓ $4x^{3}$ (4) **[12]**

QUESTION/*VRAAG* 8

8.1	$y=12$	✓answer (1)
8.2	$12=(0-2)^2(0-k)$ $k=-3$ $(x-2)^2(x+3)=0$ $x=-3$ **OR/OF** $y=0$ $(x-2)^2(x-k)=0$ $(x^2-4x+4)(x-k)=0$ $x^3-kx^2-4x^2+4kx+4x-4k=0$ But $-4k$ is the y-intercept *Maar* $-4k$ *is die y-afsnit* $-4k=12$ $k=-3$ $x=-3$	✓ substituting (0;12) ✓ $k=-3$ ✓ $x=-3$ (3) ✓$-4k$ ✓ $-4k=12$ or $k=-3$ ✓ $x=-3$ (3)
8.3	$f(x)=x^3+3x^2-4x^2-12x+4x+12$ $f(x)=x^3-x^2-8x+12$ $f'(x)=3x^2-2x-8$ $3x^2-2x-8=0$ $(3x+4)(x-2)=0$ $x=-\frac{4}{3}$ or $x=2$ $y=\frac{500}{27}$ or $18{,}52$ or $18\frac{14}{27}$ $C\left(-\frac{4}{3};18{,}52\right)$	✓ $f(x)=x^3-x^2-8x+12$ ✓derivative ✓derivative equal to 0 ✓factors or formula ✓ $x=-\frac{4}{3}$ ✓ $y=\frac{500}{27}$ or $18{,}52$ or $18\frac{14}{27}$ (6)

8.4	$f''(x)=6x-2$ $6x-2<0$ $x<\frac{1}{3}$ f is concave down when $x<\frac{1}{3}$ *f is konkaaf na onder vir $x<\frac{1}{3}$*	✓ $6x-2$ ✓✓ $x<\frac{1}{3}$ (3)
	OR/OF	
	$f''(x)=6x-2$ $6x-2=0$ $x=\frac{1}{3}$ f is concave down when $x<\frac{1}{3}$ *f is konkaaf na onder vir $x<\frac{1}{3}$*	✓ $6x-2$ ✓✓ $x<\frac{1}{3}$ (3)
	OR/OF	
	$x=\frac{x_c+x_d}{2}$ $=\frac{-\frac{4}{3}+2}{2}$ $=\frac{1}{3}$ or/of $x=-\frac{b}{3a}$ $=-\frac{-1}{3(1)}$ $=\frac{1}{3}$ f is concave down when $x<\frac{1}{3}$ *f is konkaaf na onder vir $x<\frac{1}{3}$*	✓ $\frac{-\frac{4}{3}+2}{2}$ or $-\frac{-1}{3(1)}$ ✓✓ $x<\frac{1}{3}$ (3) **[13]**

QUESTION/*VRAAG* 9

9.1	$V=\pi r^2 h$ $\pi r^2 h=340$ $h=\dfrac{340}{\pi r^2}$	✓formula ✓equating to 340 ✓ $h=\dfrac{340}{\pi r^2}$ (3)
9.2	$\text{A}=2\pi r^2+2\pi rh$ $=2\pi r^2+2\pi r\left(\dfrac{340}{\pi r^2}\right)$ $=2\pi r^2+\dfrac{680}{r}$ $\text{A}'(r)=4\pi r-\dfrac{680}{r^2}$ $\text{A}'(r)=0$ for minimum surface area/ *vir minimum buite-oppervlakte* $4\pi r-\dfrac{680}{r^2}=0$ $r^3=\dfrac{680}{4\pi}=\dfrac{170}{\pi}$ $=54{,}11268$ $r=3{,}78\text{ cm}$	✓ $2\pi r^2+2\pi rh$ ✓substituting h ✓ $4\pi r-\dfrac{680}{r^2}$ ✓ $\text{A}'(r)=0$ ✓ $r^3=\dfrac{680}{4\pi}$ ✓answer (6) **[9]**

2017 June Exam Paper 1 (Differential Calculus)

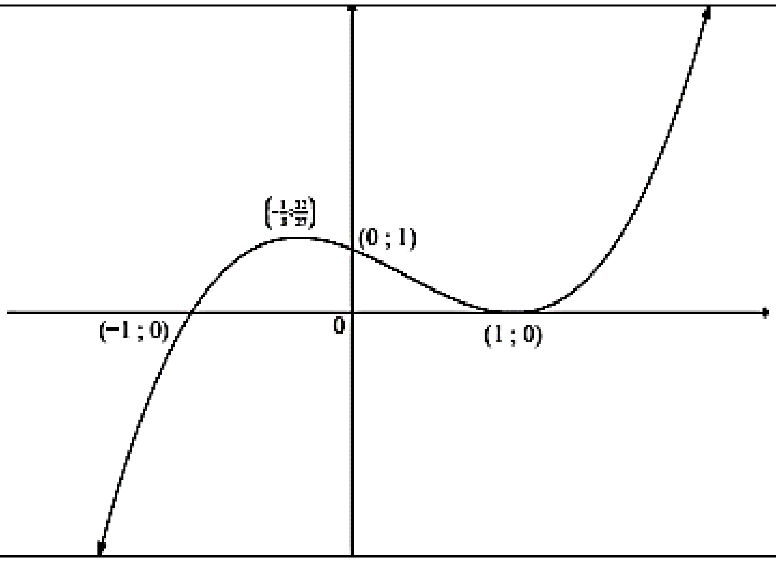
QUESTION/*VRAAG* 8

8.1	$f(x+h)=3-2(x+h)^2$ $=3-2x^2-4xh-2h^2$ $f(x+h)-f(x)=3-2x^2-4xh-2h^2-3+2x^2$ $=-4xh-2h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=\lim_{h\to 0}(-4x-2h)$ $=-4x$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{3-2(x+h)^2-(3-2x^2)}{h}$ $=\lim_{h\to 0}\frac{3-2x^2-4xh-2h^2-3+2x^2}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=\lim_{h\to 0}(-4x-2h)$ $=-4x$	✓ $3-2x^2-4xh-2h^2$ ✓ $-4xh-2h^2$ ✓ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓ $\lim_{h\to 0}(-4x-2h)$ ✓ $-4x$ (5) ✓ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓ $3-2x^2-4xh-2h^2$ ✓ $-4xh-2h^2$ ✓ $\lim_{h\to 0}(-4x-2h)$ ✓ $-4x$ (5)
8.2	$y=\frac{12x^2+2x+1}{6x}$ $=2x+\frac{1}{3}+\frac{1}{6x}$ $=2x+\frac{1}{3}+\frac{1}{6}x^{-1}$ $\frac{dy}{dx}=2-\frac{1}{6}x^{-2}$ $=2-\frac{1}{6x^2}$	✓ $\frac{12x^2}{6x}+\frac{2x}{6x}+\frac{1}{6x}$ ✓ $\frac{1}{6}x^{-1}$ ✓ 2 ✓ $-\frac{1}{6}x^{-2}$

8.3	$y = x^3 + bx^2 + cx - 4$ $y' = 3x^2 + 2bx + c$ $y'' = 6x + 2b$ At point of inflection: $y'' = 6x + 2b = 0$ Substitute $x = 2$: $6(2) + 2b = 0$ $2b = -12$ $b = -6$ $y = x^3 - 6x^2 + cx - 4$ Substitute $(2; 4)$: $4 = 2^3 - 6(2)^2 + c(2) - 4$ $2c = 24$ $c = 12$ $y = x^3 - 6x^2 + 12x - 4$	✓ $y' = 3x^2 + 2bx + c$ ✓ $y'' = 6x + 2b$ ✓ $y'' = 0$ ✓ sub $x = 2$ into $y'' = 0$ ✓ value of b ✓ substitute $(2;4)$ ✓ value of c (7) **[16]**

QUESTION/*VRAAG* 9

9.1	(0 ; 1)	✓ answer (1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x-1) - (x-1)$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR** $f(x) = x^3 - x^2 - x + 1$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR**	✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5) ✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)

	$f(x)=x^3-x^2-x+1$ $f(x)=(x+1)(x^2-2x+1)$ $f(x)=(x+1)(x-1)(x-1)$ $f(x)=0$ $(x-1)(x-1)(x+1)=0$ x-intercepts: $(-1;0)$; $(1;0)$	✓ $(x+1)$ ✓ (x^2-2x+1) ✓ $(x-1)(x-1)(x+1)$ ✓$(-1;0)$ ✓$(1;0)$ (5)
9.3	$f(x)=x^3-x^2-x+1$ $f'(x)=3x^2-2x-1$ $f'(x)=0$ $(3x+1)(x-1)=0$ $x=-\frac{1}{3}$ or $x=1$ $y=\frac{32}{27}$ $y=0$ $\left(-\frac{1}{3};\frac{32}{27}\right)$ $(1;0)$	✓ $f'(x)=3x^2-2x-1$ ✓ $f'(x)=0$ ✓factorisation ✓x value ✓x value ✓$y=\frac{32}{27}$ (6)
9.4	 	✓y– and x–intercepts ✓shape ✓turning points (3)
9.5	$f'(x)<0$ $-\frac{1}{3}<x<1$ **OR/*OF*** $(-\frac{1}{3};1)$	✓ $x>-\frac{1}{3}$ ✓ $x<1$ (2) ✓ $(-\frac{1}{3}$ ✓ $1)$ (2) **[17]**

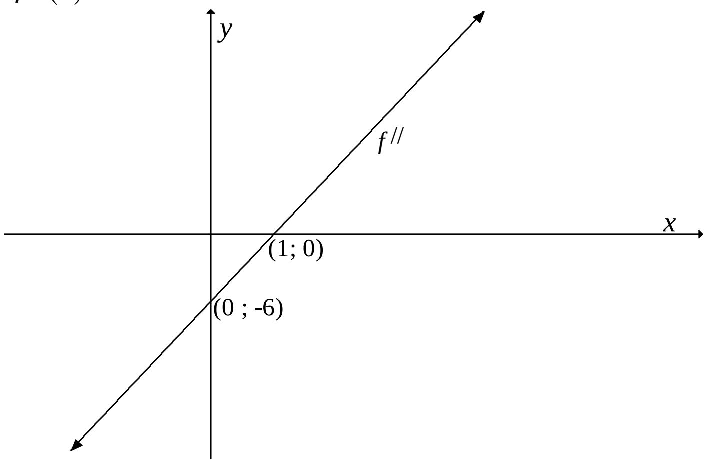
QUESTION/*VRAAG* 10

10.1	$60 = 2b + 2r + \frac{1}{2}(2\pi r)$ $2b = 60 - 2r - \pi r$ $b = 30 - r - \frac{1}{2}\pi r$	✓ $60 = 2b + 2r + \frac{1}{2}(2\pi r)$ ✓ $b = 30 - r - \frac{1}{2}\pi r$ (2)
10.2	Area = area of rectangle + area of semicircle $A(r) = \text{length} \times \text{breadth} + \frac{1}{2}(\text{area of circle})$ $= (2r)\left(30 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $= 60r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $= 60r - 2r^2 - \frac{1}{2}\pi r^2$ $= 60r - \left(2 + \frac{1}{2}\pi\right)r^2$ For a maximum, $A'(r) = 0$ $60 - 2\left(2 + \frac{1}{2}\pi\right)r = 0$ $60 - (4 + \pi)r = 0$ $r = \frac{60}{4 + \pi}$ $= 8{,}40 \text{ m}$	✓ $(2r)\left(30 - r - \frac{1}{2}\pi r\right)$ ✓ $\frac{1}{2}(\pi r^2)$ ✓ $60r - 2r^2 - \frac{1}{2}\pi r^2$ ✓ $A'(r) = 0$ ✓ $60 - 2\left(2 + \frac{1}{2}\pi\right)r$ ✓answer (6) **[8]**

QUESTION/*VRAAG* 8

8.1	$f(x+h) = 4x^2$ $f(x+h) - f(x) = 4(x+h)^2 - 4x^2$ $= 4(x^2 + 2xh + h^2) - 4x^2$ $= 4x^2 + 8xh + 4h^2 - 4x^2$ $= 8xh + 4h^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \left[\frac{8xh + 4h^2}{h} \right]$ $= \lim_{h \to 0} \left[\frac{h(8x + 4h)}{h} \right]$ $= 8x$ **OR/OF** $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \left[\frac{4(x+h)^2 - 4x^2}{h} \right]$ $= \lim_{h \to 0} \left[\frac{4x^2 8xh + 4h^2 - 4x^2}{h} \right]$ $= \lim_{h \to 0} \left[\frac{8xh + 4h^2}{h} \right]$ $= \lim_{h \to 0} \left[\frac{h(8x + 4h)}{h} \right]$ $= 8x$	✓ $4(x+h)^2$ ✓ $8xh + 4h^2$ ✓ $\frac{f(x+h) - f(x)}{h}$ ✓ $\frac{h(8x+4h)}{h}$ ✓ $8x$ **OR/OF** ✓ $\frac{f(x+h) - f(x)}{h}$ ✓ $4(x+h)^2$ ✓ $8xh + 4h^2$ ✓ $\frac{h(8x+4h)}{h}$ ✓ $8x$ (5)
8.2.1	$D_x \left[\frac{x^2 - 2x - 3}{x - 1} \right]$ $= D_x \left[\frac{(x-3)(x+1)}{x+1} \right]$ $= D_x (x - 3)$ $= 1$	✓ $\frac{(x-3)(x+1)}{x+1}$ ✓ $(x-3)$ ✓ 1 (3)
8.2.2	$f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$ $f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$	✓ $x^{\frac{1}{2}}$ ✓ $\frac{1}{2} x^{-\frac{1}{2}}$ ✓ $-\frac{1}{4} x^{-\frac{3}{2}}$ (3) **[11]**

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+2)(x-1)(x-4)$ $=(x^2+x-2)(x-4)$ $=x^3+x^2-2x-4x^2-4x+8$ $=x^3-3x^2-6x+8$ $b=-3$; $c=-6$; $d=8$	✓✓ $f(x)=(x+2)(x-1)(x-4)$ ✓ expansion ✓ x^3-3x^2-6x+8 (4)
9.2	$f(x)=x^3-3x^2-6x+8$ $f^{/}(x)=0$ $3x^2-6x-6=0$ $x^2-2x-2=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\dfrac{2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$ $=\dfrac{2\pm\sqrt{12}}{2}$ $x=-0{,}73$	✓ $f^{/}(x)=0$ ✓ $3x^2-6x-6$ ✓ substitution into correct formula ✓ $x=-0{,}73$ (4)
9.3	$f(x)=x^3-3x^2-6x+8$ $f(-1)=(-1)^3-3(-1)^2-6(-1)+8$ or $f(-1)=(1)(-2)(-5)$ $=10$ $=10$ $f^{/}(-1)=3(-1)^2-6(-1)-6$ $=3$ $y-10=3(x+1)$ $y=3x+13$	✓ $f(-1)=10$ ✓ $f^{/}(-1)=3$ ✓ substitution ✓ $y=3x+13$ (4)
9.4	$f^{//}(x)=6x-6$  	✓ $f^{//}(x)=6x-6$ ✓x- intercept ✓y- intercept (3)

9.5	f concave upwards $f''(x) > 0$ $6x - 6 > 0$ $x > 1$ NOTE: Answer only 2 / 2	✓ $f''(x) > 0$ ✓ $x > 1$ (2) **[17]**

QUESTION/*VRAAG* 10

	. $f(x) = -3x^3 + x$ $-9x^2 + 1 = 0$ $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ Maximum of f will be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$ $= \frac{2}{9}$ Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) + q = \frac{8}{9}$ $\frac{2}{9} + q = \frac{8}{9}$ $q = \frac{6}{9}$ $= \frac{2}{3}$ For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.	✓ $-9x^2 + 1 = 0$ ✓ $x = \frac{1}{3}$ *or* $x = -\frac{1}{3}$ ✓ Maximum at $x = \frac{1}{3}$ ✓ $f\left(\frac{1}{3}\right) = \frac{2}{9}$ ✓ $\frac{2}{9} + q = \frac{8}{9}$ ✓ $q = \frac{2}{3}$ **[6]**

QUESTION/*VRAAG* 7

Penalty of – 1 for notation only in 7.1

7.1	$f(x)=2x^2-1$ $f(x+h)=2(x+h)^2-1$ $=2(x^2+2xh+h^2)^2-1$ $=2x^2+4xh+2h^2-1$ $f(x+h)-f(x)=2x^2+4xh+2h^2-1-(2x^2-1)$ $=2x^2+4xh+2h^2-1-2x^2+1$ $=4xh+2h^2$ $f^{/}(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to0}\frac{4xh+2h^2}{h}$ $=\lim\limits_{h\to0}\frac{h(4x+2h)}{h}$ $=\lim\limits_{h\to0}(4x+2h)$ $=4x$	✓ $2x^2+4xh+2h^2-1$ ✓ $4xh+2h^2$ ✓ substitution ✓ simplification ✓ answer (5)
7.2.1	$\frac{d}{dx}\left(\sqrt[5]{x^2}+x^3\right)$ $=\frac{d}{dx}\left(x^{\frac{2}{5}}+x^3\right)$ $\frac{dy}{dx}=\frac{2}{5}x^{-\frac{3}{5}}+3x^2$	✓ $x^{\frac{2}{5}}$ ✓ $\frac{2}{5}x^{-\frac{3}{5}}$ ✓ $3x^2$ (3)
7.2.2	$f(x)=\frac{4x^2-9}{4x+6}$ $=\frac{(2x-3)(2x+3)}{2(2x+3)}$ $=\frac{2x-3}{2}$ $=x-\frac{3}{2}$ $f^{/}(x)=1$	✓ $(2x-3)(2x+3)$ ✓ $2(2x+3)$ ✓ simplification to two separate terms ✓ answer (4)
		[12]

QUESTION/*VRAAG* 8

8.1	$-1<x<2$	✓✓ answer (2)
8.2	$x=\frac{-1+2}{2}$ $x=\frac{1}{2}$ **Answer Only: Full Marks**	✓ method ✓ answer (2)
8.3	From the graph $x>\frac{1}{2}$ **Answer Only: Full Marks**	✓✓ answer (2)
8.4	$g(x)=ax^3+bx^2+cx$ $g'(x)=3ax^2+2bx+c=-6x^2+6x+12$ $3a=-6.$ $2b=6$ $c=12$ $a=-2$ $b=3$ $g(x)=-2x^3+3x^2+12x$	✓ $g'(x)=3ax^2+2bx+c$ ✓ $a=-2$ ✓ $b=3$ ✓ $g(x)=-2x^3+3x^2+12x$ (4)
8.5	$g'\left(\frac{1}{2}\right)=-6\left(\frac{1}{2}\right)^2+6\left(\frac{1}{2}\right)+12$ $m=\frac{27}{2}$ or 13,5 $y=-2\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+12\left(\frac{1}{2}\right)$ $y=\frac{13}{2}$ or 6,5 $y-y_1=m(x-x_1)$ $y-6,5=13,5(x-0,5)$ $y=13,5x-0,25$	✓max gradient at $x=\frac{1}{2}$ ✓answer ✓ y value ✓substitution ✓answer (5)
		[15]

QUESTION/VRAAG 9

9.1	$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim\limits_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
	OR/OF	**OR/OF**
	$f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\frac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓3 (3)

9.2.2	$D_x\left[\frac{-\sqrt[3]{x}}{2}+\left(\frac{1}{3x}\right)^2\right]$ $D_x\left[\frac{-x^{\frac{1}{3}}}{2}+\frac{x^{-2}}{9}\right]$ $D_x\left[-\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{9}x^{-2}\right]$ $=-\frac{1}{6}x^{-\frac{2}{3}}-\frac{2x^{-3}}{9}$ $=-\frac{1}{6x^{\frac{2}{3}}}-\frac{2}{9x^3}$	✓ $\frac{-x^{\frac{1}{3}}}{2}$ ✓ $\frac{x^{-2}}{9}$ ✓ $-\frac{1}{6}x^{-\frac{2}{3}}$ ✓ $-\frac{2x^{-3}}{9}$ (4)
		[12]

QUESTION/VRAAG 10

10.1	$h(x)=ax^3+bx^2$ $h^/(x)=3ax^2+2bx$ $h^/(4)=0$ $48a+8b=0$ $6a+b=0$ ...(1) $h(4)=32$ $64a+16b=32$ $4a+b=2$ ...(2) (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h^/(x)$ ✓ $h^/(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A(6 ; 0)	✓ $-x^3+6x^2=0$ ✓ factors ✓ A(6 ; 0) (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0\ ;\ 4)$ or $x\in[0\ ;\ 4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2\ ;\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7\ ;32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

QUESTION/*VRAAG* 9

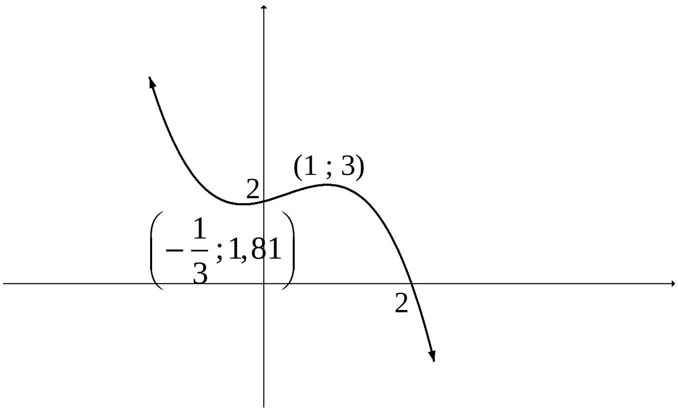
9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f'(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $N\left(\frac{5}{3}\ ;-\frac{256}{27}\right)=(1{,}67\ ;-9{,}48)$	✓ $f'(x)=3x^2-2x-5$ ✓ = 0 ✓ factors ✓ x-value ($x>0$) ✓ y-value (A) (5)
9.3.1	$x<3\ ;\ x\neq-1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty\ ;-1)$ or $(-1\ ;3)$	✓ $x<3$ ✓ $x\neq-1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty\ ;-1)$ ✓ $(-1\ ;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq-1$ or $x\geq\frac{5}{3}$ **OR/*OF*** $(-\infty\ ;-1)$ or $\left(\frac{5}{3};\infty\right)$ OR/OF $(-\infty\ ;-1]$ or $\left[\frac{5}{3};\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty\ ;-1)$ ✓ $\left(\frac{5}{3};\infty\right)$ (2)
9.3.3	$f''(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$ ANSWER ONLY: FULL MARKS **OR/OF** $\frac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$	✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

[65]

9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x+1)(x-3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

QUESTION 8/*VRAAG* 8

8.1	$f'(x)=mx^2+nx+k$ $f'(x)=m\left(x+\frac{1}{3}\right)(x-1)$ $1=m\left(0+\frac{1}{3}\right)(0-1)$ $1=-\frac{1}{3}m$ $\therefore m=-3$ $f'(x)=-3\left(x+\frac{1}{3}\right)(x-1)$ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ $f'(x)=-3x^2+2x+1$ $\therefore n=2$ $\therefore k=1$ **OR/OF** $k=1$ $0=m+n+1$ and $\frac{1}{9}m-\frac{1}{3}n+1=0$ $m+n=-1$ (1) $m-3n=-9$ (2) (1) – (2) $4n=8$ $\therefore n=2$ $m+2=-1$ $\therefore m=-3$	✓ substitution of $\left(-\frac{1}{3}\,;0\right)$ and (1 ; 0) ✓ substitution of (0 ; 1) ✓ $m=-3$ ✓ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ ✓ $n=2$ ✓ $k=1$ (6) **OR/OF** ✓ $k=1$ ✓ $m+n=-1$ ✓ $m-3n=-9$ ✓ $4n=8$ ✓ $n=2$ ✓ $m=-3$ (6)
8.2.1	$f(x)=-x^3+x^2+x+2$ $f\left(-\frac{1}{3}\right)=\frac{49}{27}=1{,}81$ T.P$\left(-\frac{1}{3}\,;\frac{49}{27}\right)$ $f(1)=3$ T.P$(1\,;3)$	✓ x-coordinates of the TP ✓ T.P$\left(-\frac{1}{3}\,;\frac{49}{27}\right)$ ✓ T.P$(1\,;3)$ (3)

8.2.2	$f(x) = -x^3 + x^2 + x + 2$ $-x^3 + x^2 + x + 2 = 0$ $(x-2)(-x^2 - x - 1) = 0$ $x = 2$ or no solution 	✓ $x = 2$ ✓one x-intercept ✓ two turning points ✓y-intercept ✓ shape: neg cubic (5)
8.3.1	$a = \dfrac{-\frac{1}{3}+1}{2}$ $= \dfrac{1}{3}$ **OR/OF** $f^{/}(x) = -3x^2 + 2x + 1$ $f^{//}(x) = -6x + 2$ $f^{//}(a) = -6a + 2 = 0$ $-6a = -2$ $a = \dfrac{1}{3}$	✓ answer (1) **OR/OF** ✓ answer (1)
8.3.2	$b < \dfrac{4}{3}$ units	✓ $\dfrac{4}{3}$ ✓ $b < \dfrac{4}{3}$ (2)
		[17]

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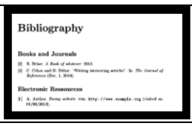
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