JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: DIFFERENTIAL CALCULUS

ACTIVITY BOOK

LEARNER/TEACHER

TERM 2

FIRST PRINCIPLES

FINDING EQUATION OF THE CUBIC FUNCTION AND EQUATION OF A TANGENT

RULES OF DIFFERENTIATION

GRAPHICAL INTERPRETATION

SKETCHING OF THE CUBIC FUNCTION

OPTIMISATION

JENN MAREMATLOU

A Division of Marematlou Group Holdings

CONTENTS PAGE

SECTION 1: First Principles
- Activities 4

SECTION 2: Rules of differentiation
- Activities 5

SECTION 3: Sketching of the cubic functions
- Activities 6 - 7

SECTION 4: Finding equation of the cubic function and equation of a tangent
- Activities 8 – 12

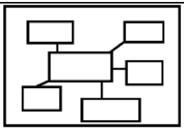
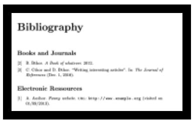
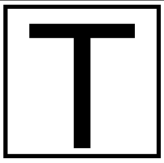
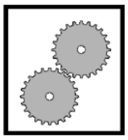
SECTION 5: Graphical Interpretation
- Activities 13 – 16

SECTION 6: Optimisation
- Activities 17 - 21

Annexure A: 2015, 2016 AND 2017 JUNE EXAM PAPERS (DIFFERENTIAL CALCULUS QUESTIONS) 22 - 27

Annexure B: Selected Questions on Differential Calculus 28 - 34

ICON DESCRIPTION

 **MIND MAP**	 **EXAMINATION GUIDELINE**	 **CONTENTS**	 **ACTIVITIES**
 **BIBLIOGRAPHY**	 **TERMINOLOGY**	 **WORKED EXAMPLES**	 **STEPS**

SECTION 1: FIRST PRINCIPLES

Outcomes: At the end of the session learners/ teachers must:

- **Be able to find the gradients of basic functions (linear, quadratic, hyperbola and cubic) from first principles**

Activities

QUESTION 1

1.1 Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$

1.2 Given: $f(x) = -\frac{2}{x}$

1.2.1 Determine $f'(x)$ from first principles.

1.2.2 For which value(s) of x will $f'(x) > 0$? Justify your answer.

1.3 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$.

1.4 Differentiate f by first principles where $f(x) = x^2 - 2x$.

1.5 Determine $f'(x)$ from first principles if $f(x) = -4x^2$.

SECTION 2: RULES OF DIFFERENTIATION

Outcomes: At the end of the session learners/ teachers must:

- **Be able to determine gradients using rules of differentiation**

Activities

QUESTION 1

1.1 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations.

1.2 Use the rules of differentiation to determine the following:

$$D_x\left[(x-2)(x+3)\right]$$

1.3 Evaluate: $\frac{dy}{dx}$ if $y = x^2 - \frac{1}{2x^3}$

1.4 Evaluate $\frac{dy}{dx}$ if $y = \frac{x^6}{2} + 4\sqrt{x}$.

1.5 Evaluate:

1.5.1 $\frac{dy}{dx}$ if $y = \frac{3}{2x} - \frac{x^2}{2}$

1.5.2 $f'(1)$ if $f(x) = (7x+1)^2$

1.6 Evaluate $\frac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$.

1.7 Given: $g(x) = \frac{x^2 + x - 2}{x - 1}$

1.7.1 Calculate $g'(x)$ for $x \neq 1$.

1.7.2 Explain why it is not possible to determine $g'(1)$.

SECTION 3: SKETCHING OF THE CUBIC FUNCTION

Outcomes: At the end of the session learners/ teachers must:

- **Be able to sketch graphs of cubic polynomial functions using differentiation to determine the co-ordinate of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques.**

Activities

QUESTION 1

Given: $f(x) = -x^3 + x^2 + 8x - 12$

1.1 Calculate the x-intercepts of the graph of f.

1.2 Calculate the coordinates of the turning points of the graph of f.

1.3 Sketch the graph of f, showing clearly all the intercepts with the axes and turning points.

1.4 Write down the x-coordinate of the point of inflection of f.

1.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$.

QUESTION 2

Given: $f(x) = -x^3 - x^2 + x + 10$

2. 1 Write down the coordinates of the y-intercept of f.

2. 2 Show that (2 ; 0) is the only x-intercept of f.

2. 3 Calculate the coordinates of the turning points of f.

2. 4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points.

QUESTION 3

Given: $f(x) = -x^3 + 3x^2 - 4.$

3.1 Show that $f(-1) = 0$

3.2 Hence, or otherwise, determine the x- and y-intercepts of the graph of f.

3.3 Determine the coordinates of the turning points of the graph of f.

3.4 Draw a neat sketch graph of f. Clearly show all the intercepts with the axes and the turning points on the graph.

3.5 For which values of x is f increasing?

QUESTION 4

Given: $g(x) = (x-6)(x-3)(x+2)$

4.1 Calculate the y-intercept of g.

4.2 Write down the x-intercepts of g.

4.3 Determine the turning points of g.

4.4 Sketch the graph of g.

4.5 For which values of x is $g(x).g'(x) < 0$?

SECTION 4: FINDING EQUATION OF THE CUBIC FUNCTION AND EQUATION OF A TANGENT

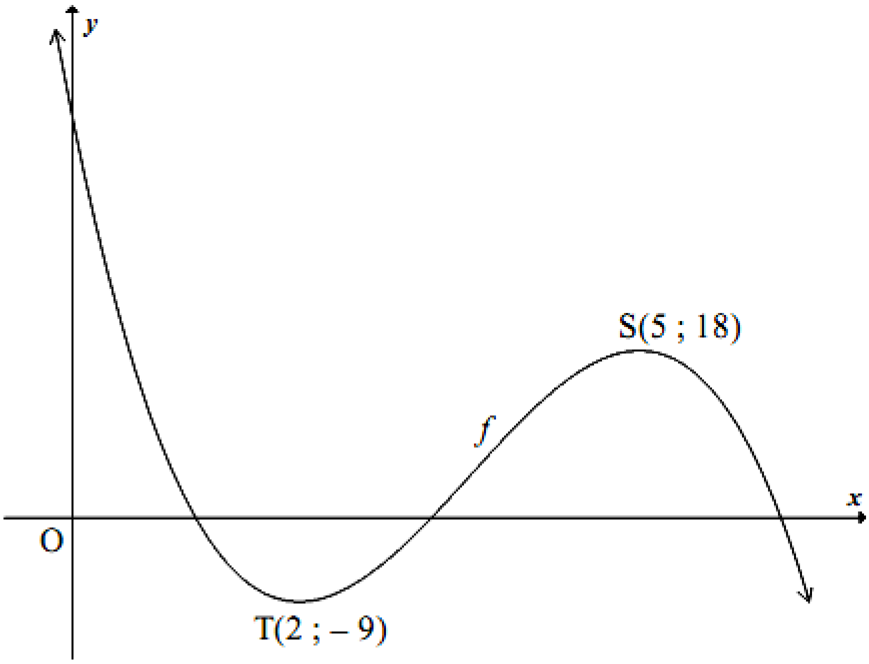
Outcomes: At the end of the session learners/ teachers must:

- **Be able to find parameters of the function, and thus the equation of a function**
- **Be able to find equations of tangents to functions**

Activities

QUESTION 1

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



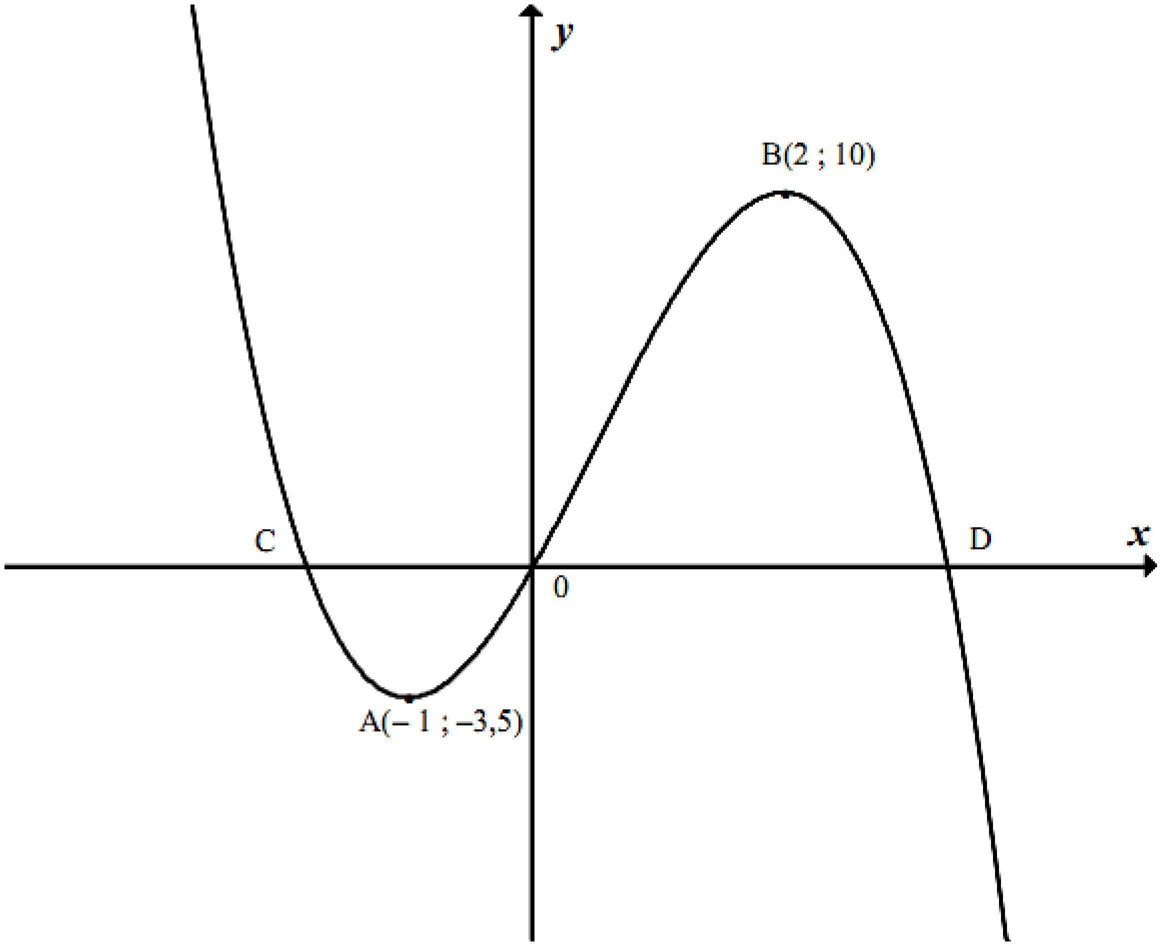
1.1 Show that $a = 21$, $b = -60$ and $c = 43$.

1.2 Determine an equation of the tangent to the graph of f at $x = 1$.

1.3 Determine the x-value at which the graph of f has a point of inflection.

QUESTION 2

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. A(– 1 ; 3,5) and B(2 ; 10) are the turning points of h. The graph passes through the origin and further cuts the x-axis at C and D.



2.1 Show that $a = \dfrac{3}{2}$ and $b = 6$.

2.2 Calculate the average gradient between A and B.

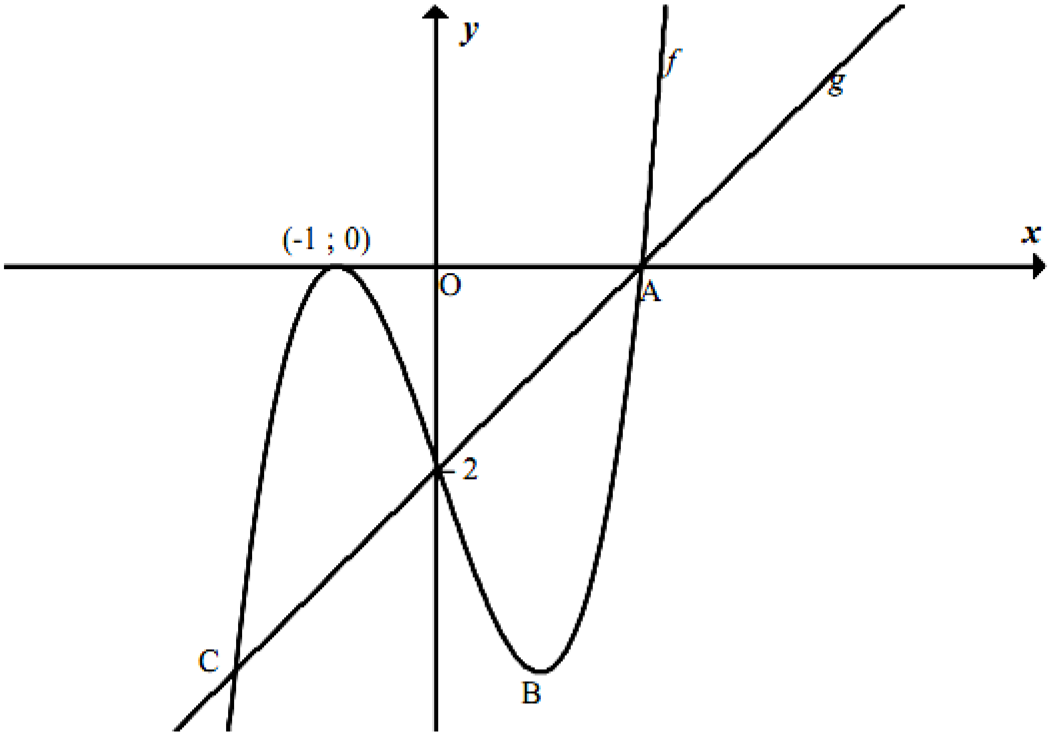
2.3 Determine the equation of the tangent to h at $x = -2$.

2.4 Determine the x-value of the point of inflection of h.

2.5 Use the graph to determine the values of p for which the equation $-x^3 + \dfrac{3}{2}x^2 + 6x + p = 0$ will have ONE real root.

QUESTION 3

The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and $(-1\ ;0)$ are the x-intercepts of f. The graphs of f and g intersect at A and C.



3.1 Determine the coordinates of A.

3.2 Show by calculation that $a = 1$ and $c = 3$.

3.3 Determine the coordinates of B, a turning point of f.

3.4 Show that the line BC is parallel to the x-axis.

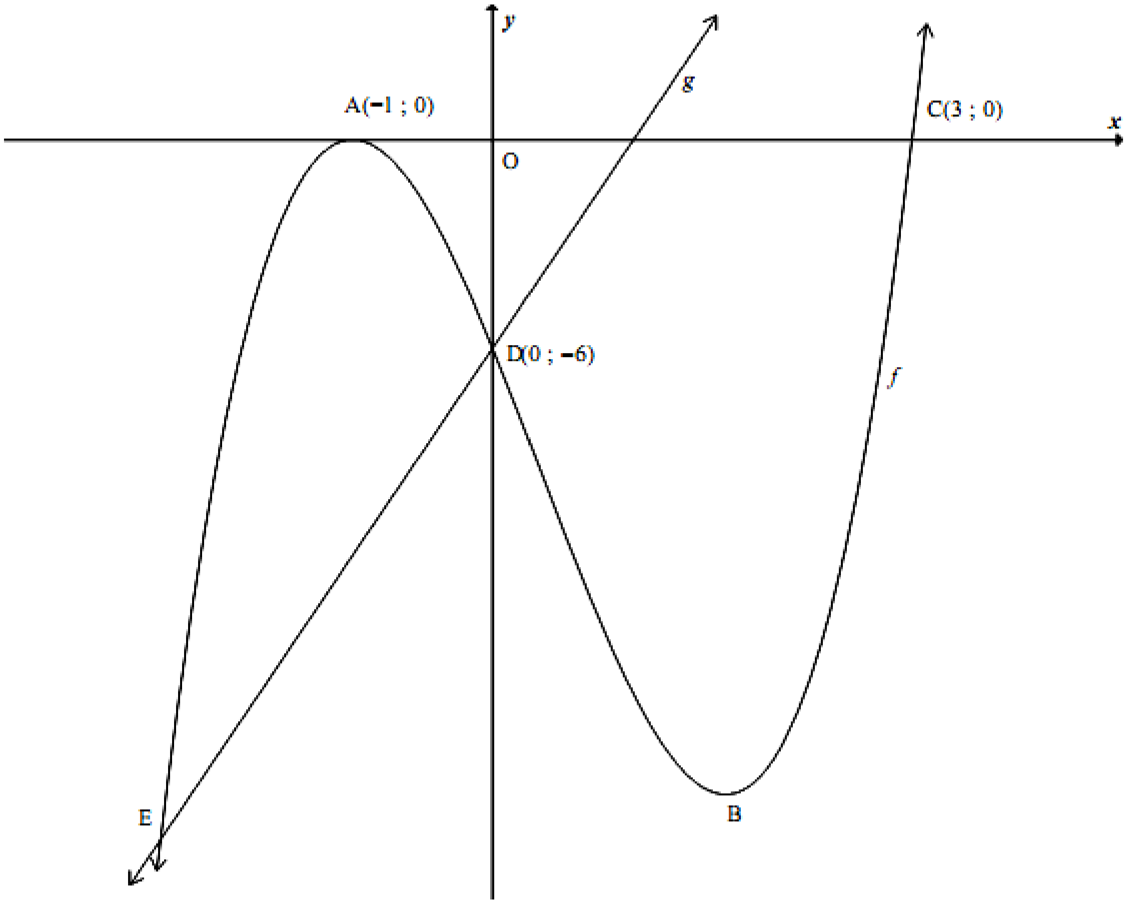
3.5 Find the x-coordinate of the point of inflection of f.

3.6 Write down the values of k for which $f(x) = k$ will have only ONE root.

3.7 Write down the values of x for which $f^{/}(x) < 0$.

QUESTION 4

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A$(-1 ; 0)$ and C$(3 ; 0)$ are the x-intercepts of f.
The graph of f has turning points at A and B.
D$(0 ; -6)$ is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



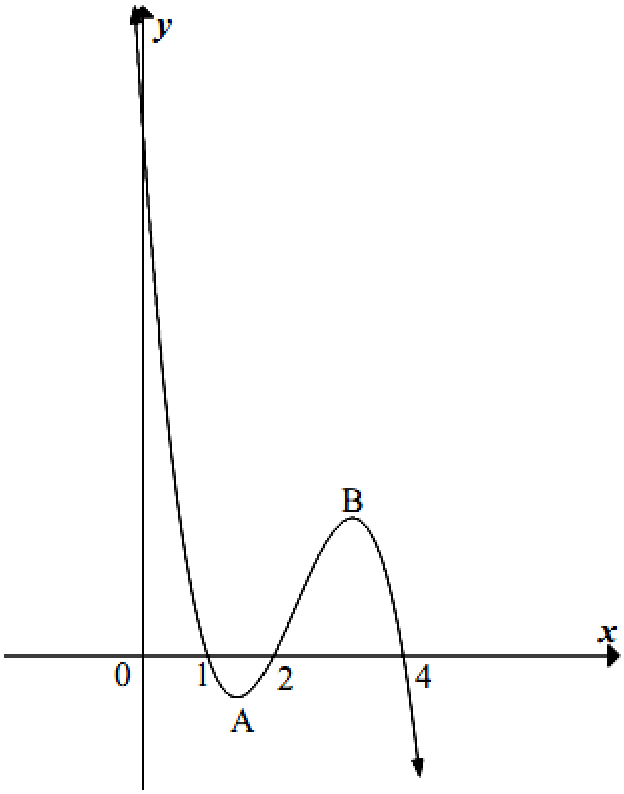
4.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$.

4.2 Calculate the coordinates of the turning point B.

4.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$.

QUESTION 5

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.



5.1 Calculate the values of a, b and c.

5.2 Calculate the x-coordinates of A and B, the turning points of f.

5.3 For which values of x will $f'(x) < 0$?

QUESTION 6

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

6.1 Show that $(1\ ;-3)$ is the point of contact of the tangent to the graph.

6.2 Hence or otherwise, calculate the values of p and q.

QUESTION 7

7.1 Calculate the values of a and b if $f(x) = ax^2 + bx + 5$ has a tangent at $x = -1$ which is defined by the equation $y = -7x + 3$

SECTION 5: GRAPHICAL INTERPRETATION

Outcomes: At the end of the session: the learner must

- **Be able to read information from graphs**
- **Be able to sketch cubic functions from given properties**

Activities

QUESTION 1

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3}; 2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts.

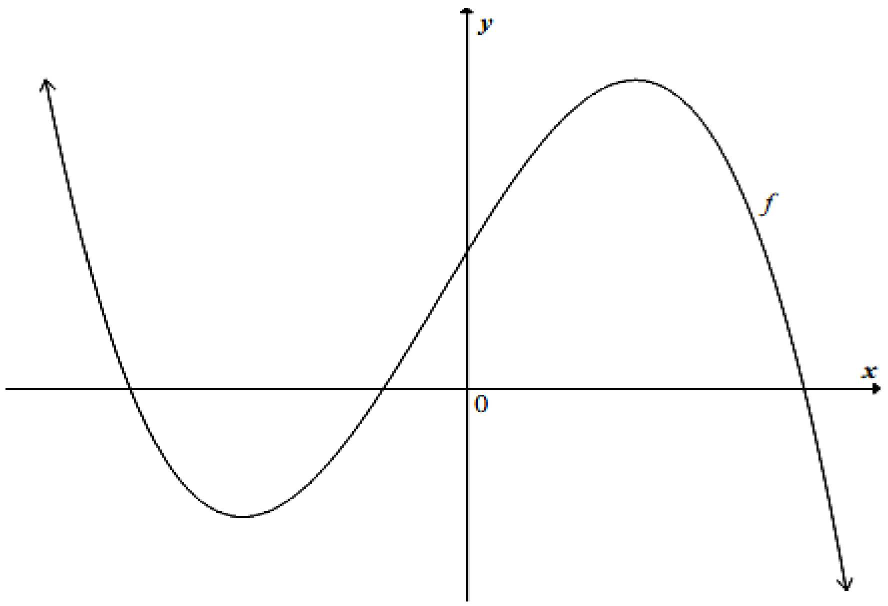
QUESTION 2

2.1 Given: $f(x) = ax^3 + bx^2 + cx + d$

Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers.

QUESTION 3

3.1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



3.1.1 Calculate the x-coordinates of the turning points of f.

3.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum.

3.2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

3.2.1 Determine the equation of the tangent to the graph of g at $x = -1$.

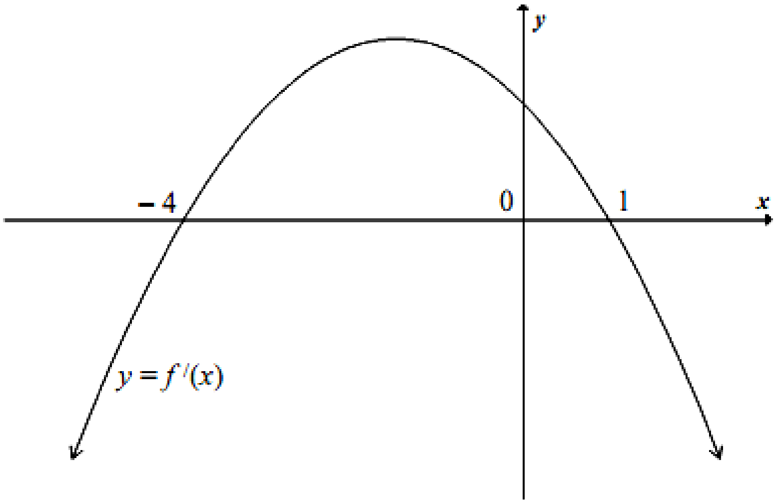
3.2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola?

3.3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations.

QUESTION 4

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



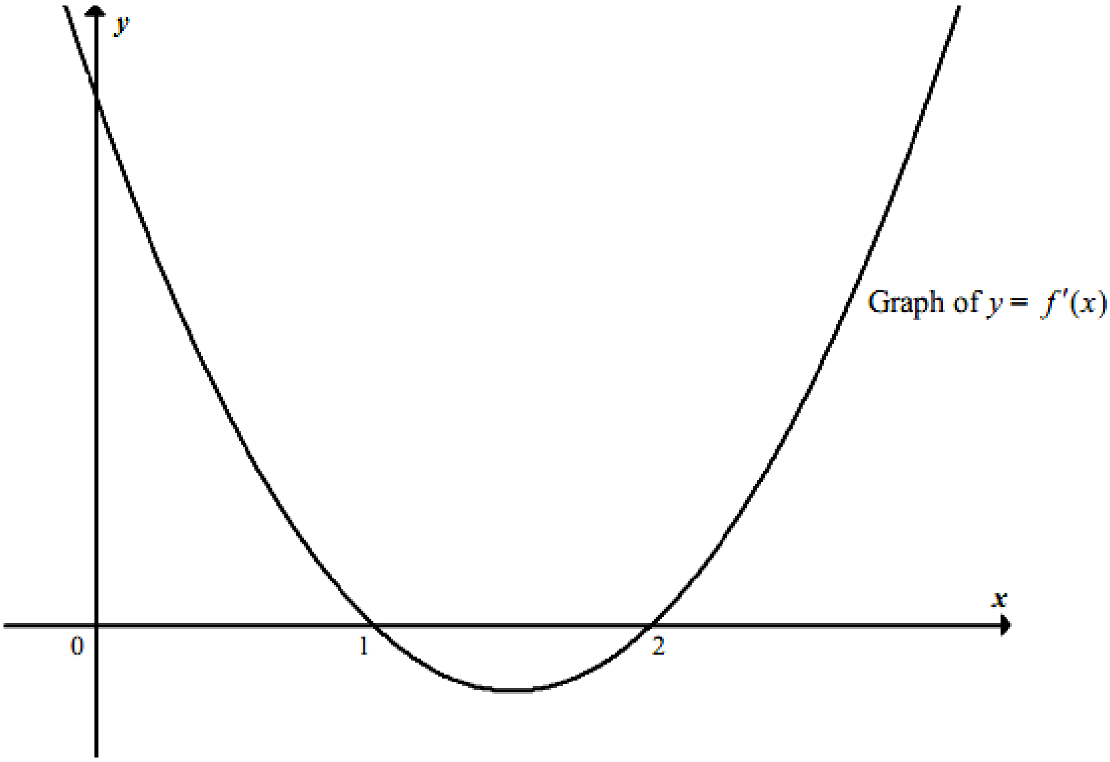
Use the graph to answer the following questions:

4.1 For which values of x is the graph of $y = f'(x)$ decreasing?

4.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer.

QUESTION 5

In the sketch below, the graph $y = ax^2 + bx + c$ represents the derivative, f', of f where f is a cubic function.



5.1 Write down the x-coordinates of the stationary points of f.

5.2 State whether each stationary point in QUESTION 10.1 is a local minimum or a local maximum. Substantiate your answer.

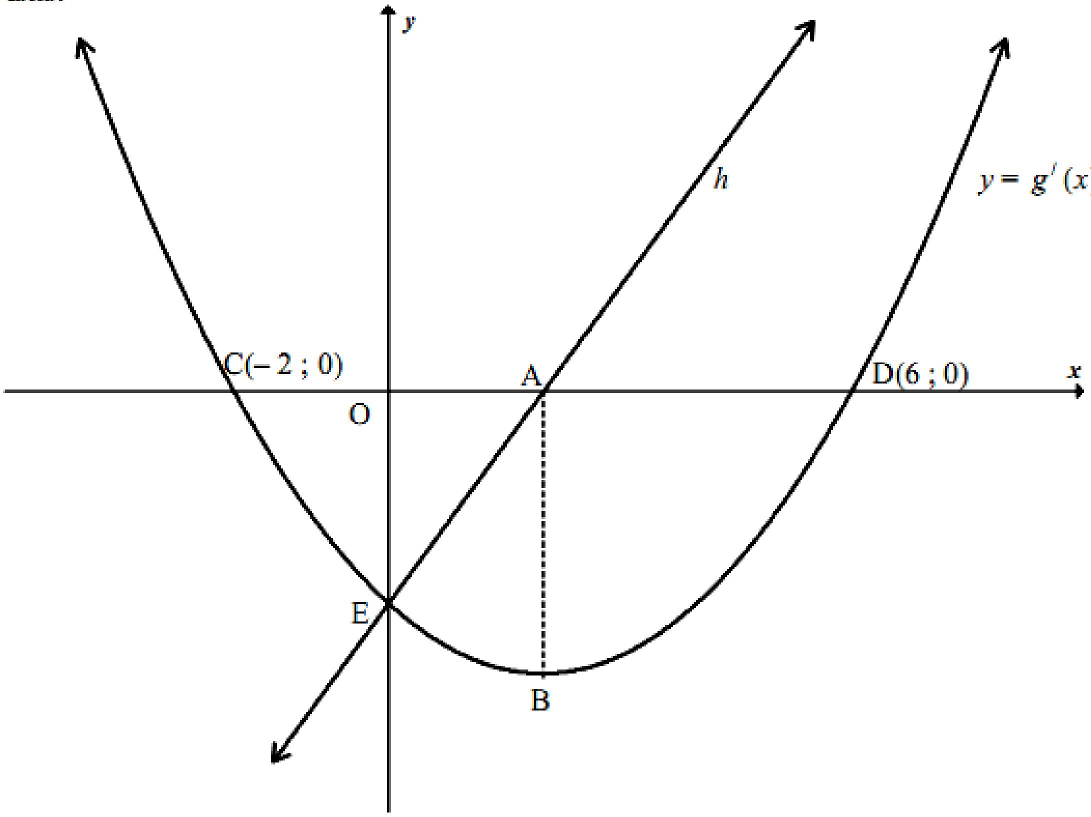
5.3 Determine the x-coordinate of the point of inflection of f.

5.4 Hence, or otherwise, draw a sketch graph of f.

QUESTION 6

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

The graphs of h and g' have a common y-intercept at E.
C$(-2\,;0)$ and D$(6\,;0)$ are the x-intercepts of the graph of g'.
A is the x-intercept of h and B is the turning point of g'.
AB || y-axis.



6.1 Write down the coordinates of E.

6.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$.

6.3 Write down the x-coordinates of the turning points of g

6.4 Write down the x-coordinate of the point of inflection of the graph of g

6.5 Explain why g has a local maximum at $x = -2$.

SECTION 6: OPTIMISATION

Outcomes: At the end of the session:

- **Be able to solve practical problems concerning Optimisation and rate of change, including calculus of motion**

Activities

QUESTION 1

1.1 The depth h of petrol in a large tank, t days after the tank was refilled, is given by $h(t) = 12 - \frac{t}{4} - \frac{t^3}{6}$ metres for $0 \leq t \leq 4$.

1.1.1 What is the depth after 3 days?

1.1.2 What is the rate of decrease in the depth after 2 days? (Give your answer in the correct units.)

1.2 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b.

QUESTION 2

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

2.1 How high is the car above sea level when it starts its journey on the mountainous pass?

2.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass.

2.3 Interpret your answer to QUESTION 12.2.

2.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum?

QUESTION 3

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

3.1 What is the initial volume of the water in the tank?

3.2 Write down TWO different expressions for the rate of change of the volume of water in the tank.

3.3 Determine the value of k (that is, the rate at which water flows out of the tank).

QUESTION 4

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

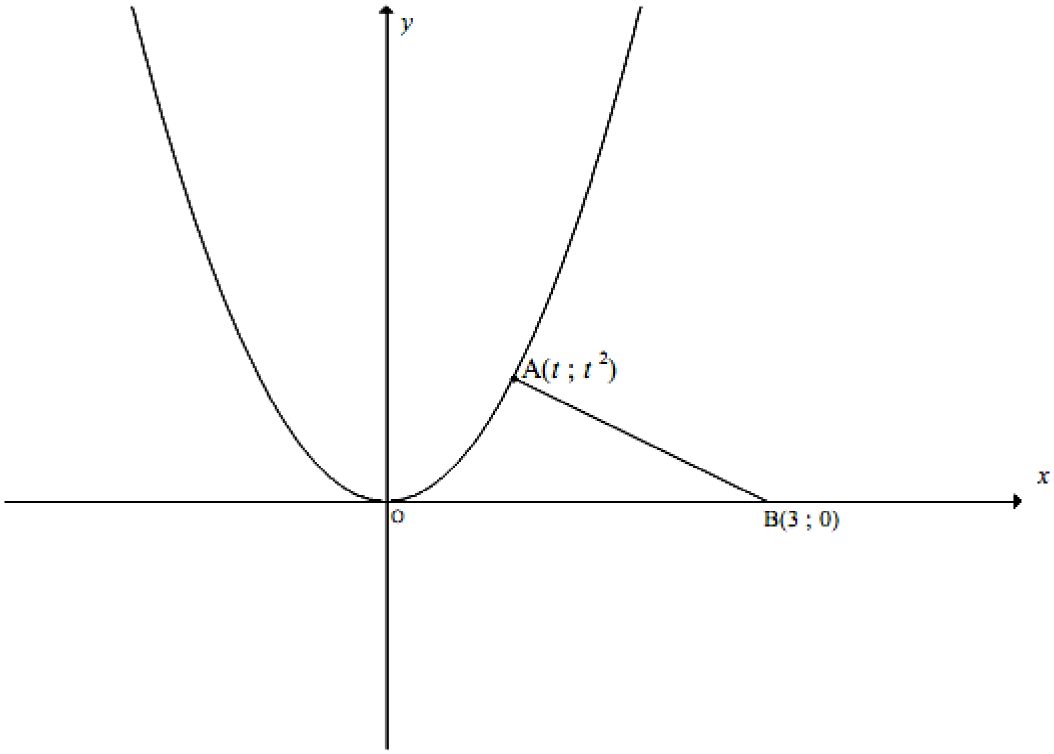
4.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.)

4.2 Determine the rate at which the velocity of the particle is changing at t seconds.

4.3 After how many seconds will the particle be closest to the fixed point?

QUESTION 5

Sketched is the graph of $y = x^2$. A(t ; t^2) and B(3 ; 0) are shown.



5.1 A(t ; t^2) is a point on the curve $y = x^2$ and the point B(3 ; 0) lies on the x-axis. Show that $AB^2 = t^4 + t^2 - 6t + 9$.

5.2 Hence, determine the value of t which minimises the distance AB.

QUESTION 6

A small business currently sells 40 watches per year. Each of the watches is sold at R144. For each yearly price increase of R4 per watch, there is a drop in sales of one watch per year.

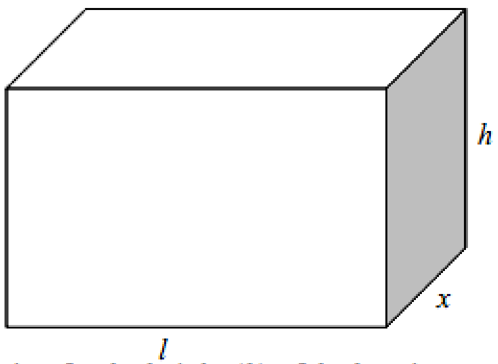
6.1 How many watches are sold x years from now?

6.2 Determine the annual income from the sale of watches in terms of x.

6.3 In what year and at what price should the watches be sold in order for the business to obtain a maximum income from the sale of watches?

QUESTION 7

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3. Let the width of the box be x metres.



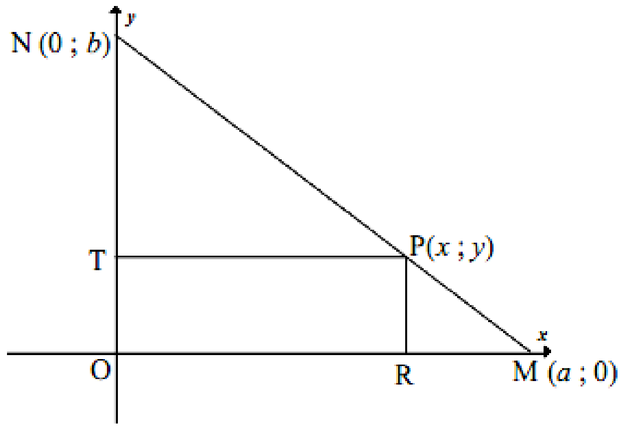
7.1 Determine an expression for the height (h) of the box in terms of x.

7.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$.

7.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum.

QUESTION 8

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.

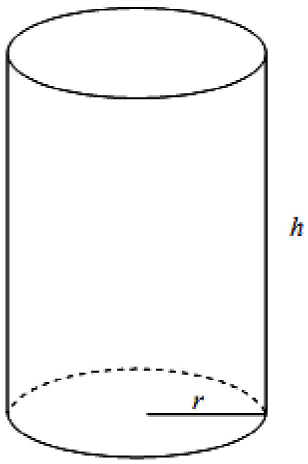


8.1 Determine an equation of MN in terms of a and b.

8.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN.

QUESTION 9

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



9.1 Show that the height of the glass, h, can be expressed as $h = \frac{200}{\pi r^2}$.

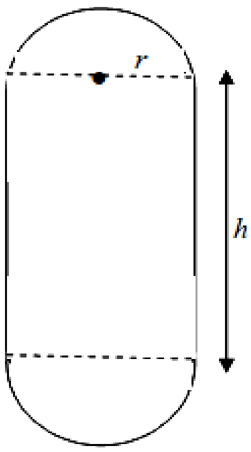
9.2 Show that the total surface area of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$.

9.3 Hence determine the value of r for which the total surface area of the glass is a minimum.

QUESTION 10

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

10.1 Show that $h = \frac{\quad}{6r^2} - \frac{r}{3}$.

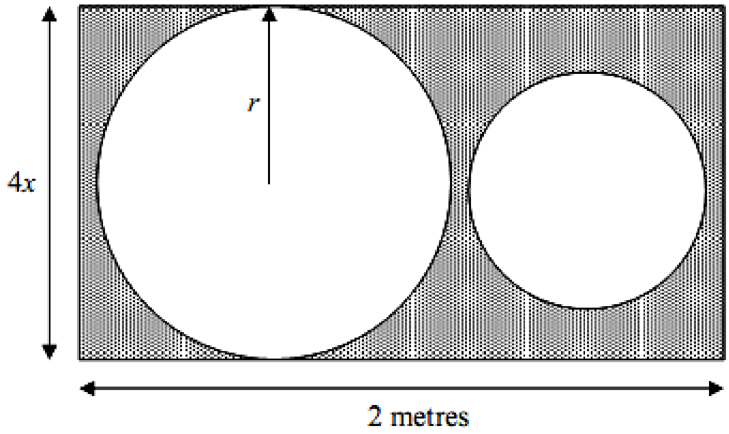
10.2 Hence, show that the outer surface area of the satellite can be given as $S = \frac{4\pi r^2}{3} + \frac{\pi}{3r}$.

10.3 Calculate the minimum outer surface area of the satellite.

QUESTION 11

Devan wants to cut two circles out of a rectangular piece of cardboard of 2 metres long and $4x$ metres wide. The radius of the larger circle is half the width of the cardboard and the smaller circle has a radius that is $\frac{2}{3}$ the radius of the bigger circle.

$A = lb$	$A = \pi r^2$	$P = 2(l + b)$	$C = 2\pi r$



11.1 Show that the area of the shaded region is $A(x) = 8x - \frac{52\pi}{9}x^2$.

11.2 Determine the value of x, such that the area of the shaded region is a maximum.

11.3 Calculate the total area of the circles, if the area of the shaded region is to be a maximum.

ANNEXURE A: 2015, 2016 AND 2017 EXAM PAPERS (DIFFERENTIAL CALCULUS)

2015 June Exam Paper 1 (Differential Calculus)

QUESTION 8

8.1 If $f(x)=\frac{4}{x}$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y=5x^2+5x+2$ (2)

8.2.2 $D_x\left[\sqrt[3]{x^2}-\frac{1}{2}x\right]$ (3)

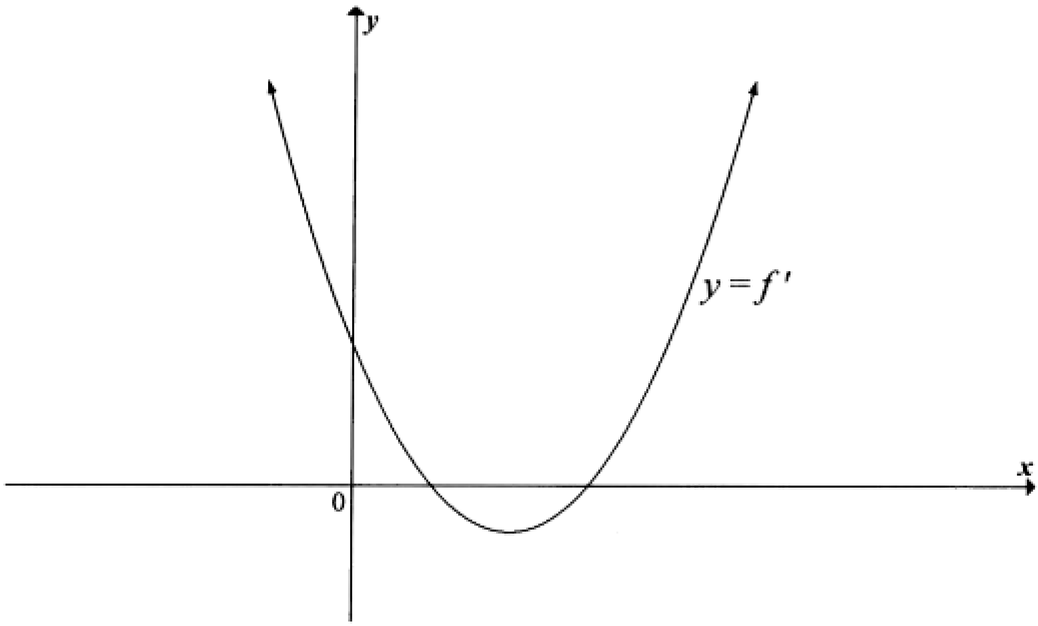
8.3 Given: $p(x)=x^3+2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of p to have a negative gradient. (3)

[13]

QUESTION 9

The graph of $y=ax^2+bx+c$ below represents the derivative of f.
It is given that $f'(1)=0$, $f'(3)=0$ and $f'(0)=6$.



9.1 Write down the x-coordinates of the stationary points of f. (2)

9.2 For which value(s) of x is f strictly decreasing? (2)

9.3 Explain at which value of x the stationary point of f will be a local minimum. (2)

9.4 Determine the x-coordinate of the point of inflection of f. (1)

9.5 For which value(s) of x is f concave up? (2)

[9]

QUESTION 10

The mass of a baby in the first 30 days of life is given by

$M(t) = t^3 - 9t^2 + 3\,000 \quad ; \quad 0 \le t \le 30$.

t is the time in days and M is the mass of the baby in grams.

10.1 Write down the mass of the baby at birth. (1)

10.2 A baby's mass usually decreases in the first few days after birth.
On which day will the baby's mass return to its birth mass? (4)

10.3 On which day will this baby have a minimum mass? (4)

10.4 On which day will the baby's mass be decreasing the fastest? (2)

[11]

2016 June Exam Paper 1 (Differential Calculus)

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 5$ (5)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 2x^5 + \frac{4}{x^3}$ (3)

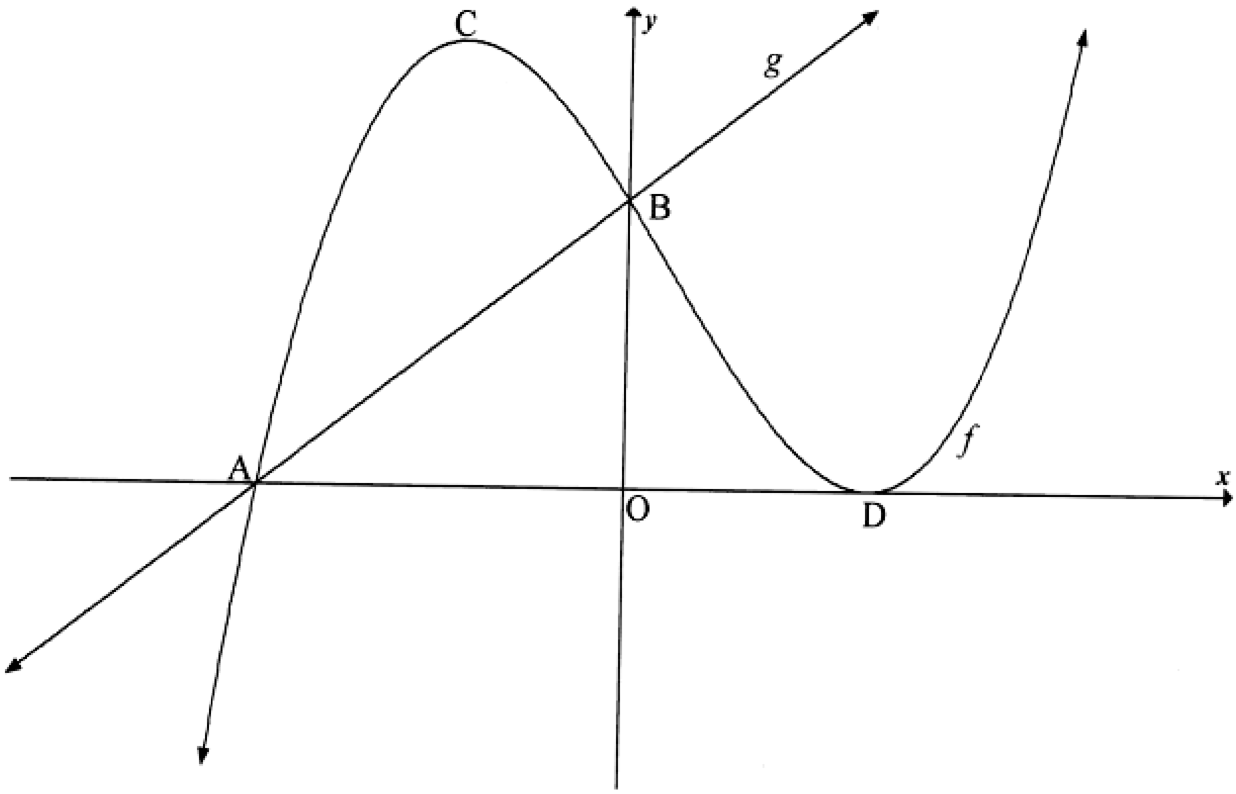
7.2.2 $y = \left(\sqrt{x} - x^2\right)^2$ (4)

[12]

QUESTION 8

Sketched below are the graphs of $f(x) = (x-2)^2(x-k)$ and $g(x) = mx + 12$

- A and D are the x-intercepts of f.
- B is the common y-intercept of f and g.
- C and D are turning points of f.
- The straight line g passes through A.



8.1 Write down the y-coordinate of B. (1)

8.2 Calculate the x-coordinate of A. (3)

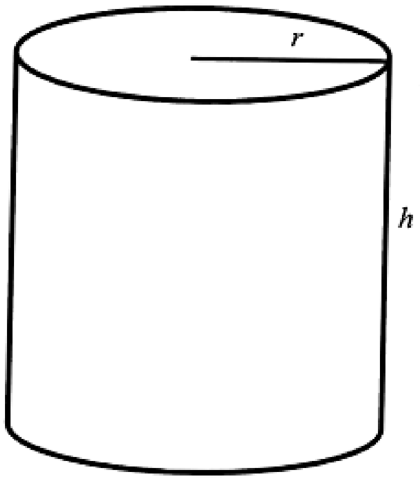
8.3 If $k = -3$, calculate the coordinates of C. (6)

8.4 For which values of x will f be concave down? (3)

[13]

QUESTION 9

A 340 mℓ can with height h cm and radius r cm is shown below.



1 mℓ = 1 cm³

9.1 Determine the height of the can in terms of the radius r. (3)

9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

[9]

2017 June Exam Paper 1 (Differential Calculus)

QUESTION 8

8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)

8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c. (7)

[16]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

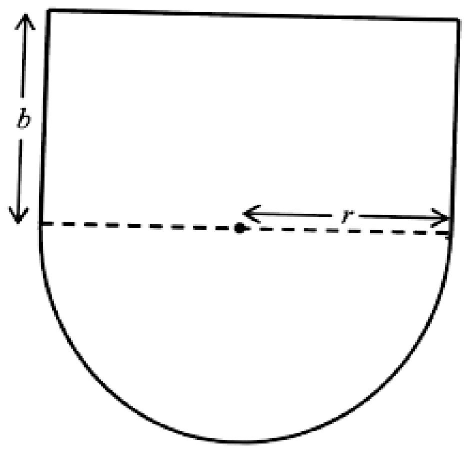
9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

9.5 Write down the values of x for which $f'(x) < 0$. (2)

[17]

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b. The perimeter of the stage is 60 m.

10.1 Determine an expression for b in terms of r. (2)

10.2 For which value of r will the area of the stage be a maximum? (6)

[8]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $\frac{d}{dx}\left(\sqrt[5]{x^2} + x^3\right)$ (3)

7.2.2 $f'(x)$ if $f(x) = \frac{4x^2 - 9}{4x + 6}$; $x \neq -\frac{3}{2}$ (4)

[12]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

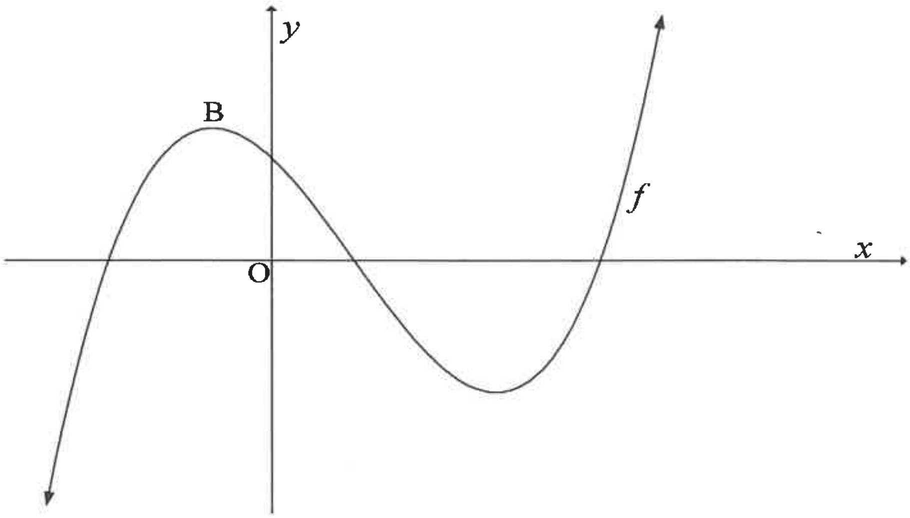
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

[29]

QUESTION 10

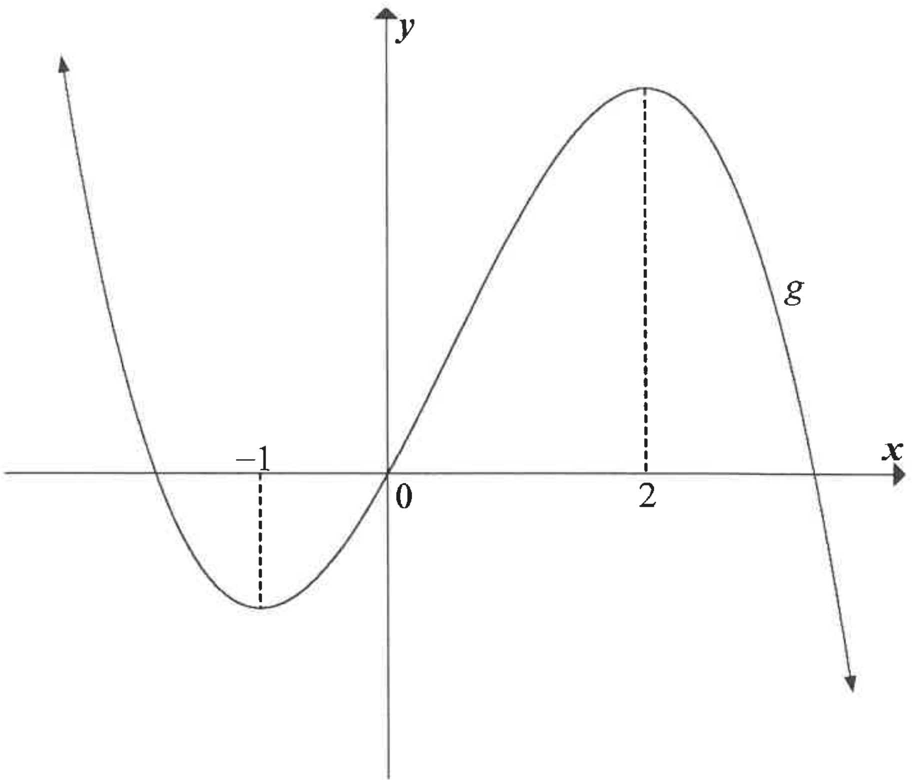
Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$ **[6]**

[30]

QUESTION 8

The graph of $g(x) = ax^3 + bx^2 + cx$, a cubic function having a y-intercept of 0, is drawn below. The x-coordinates of the turning points of g are -1 and 2.



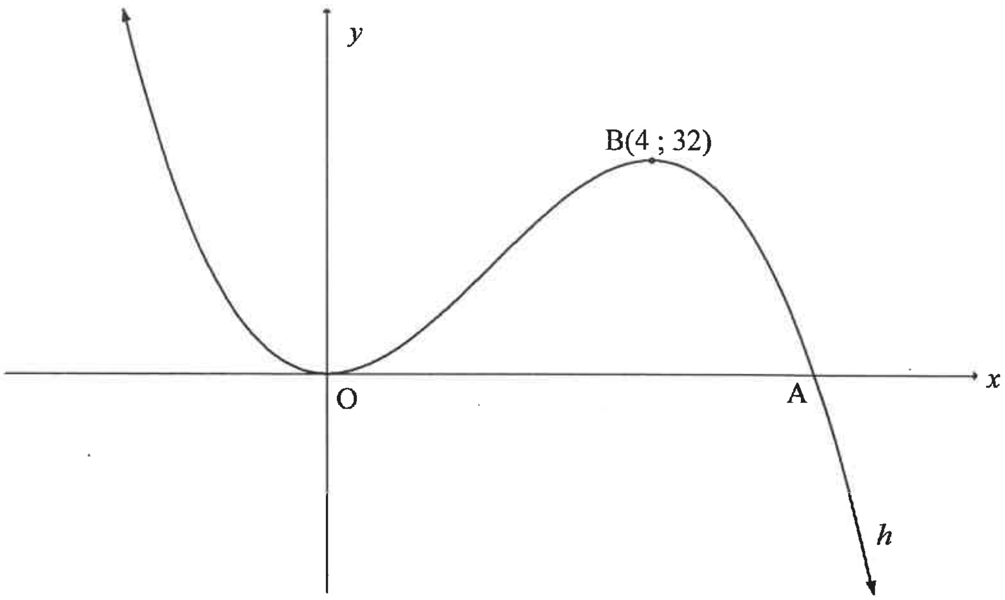
8.1	For which values of x will g increase?	(2)
8.2	Write down the x-coordinate of the point of inflection of g.	(2)
8.3	For which values of x will g be concave down?	(2)
8.4	If $g'(x) = -6x^2 + 6x + 12$, determine the equation of g.	(4)
8.5	Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y = mx + c$.	(5)

[15]

[31]

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.



10.1	Show that $a = -1$ and $b = 6$.		(5)
10.2	Calculate the coordinates of A.		(3)
10.3	Write down the values of x for which h is:		
	10.3.1	Increasing	(2)
	10.3.2	Concave down	(2)
10.4	For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots?		(3)

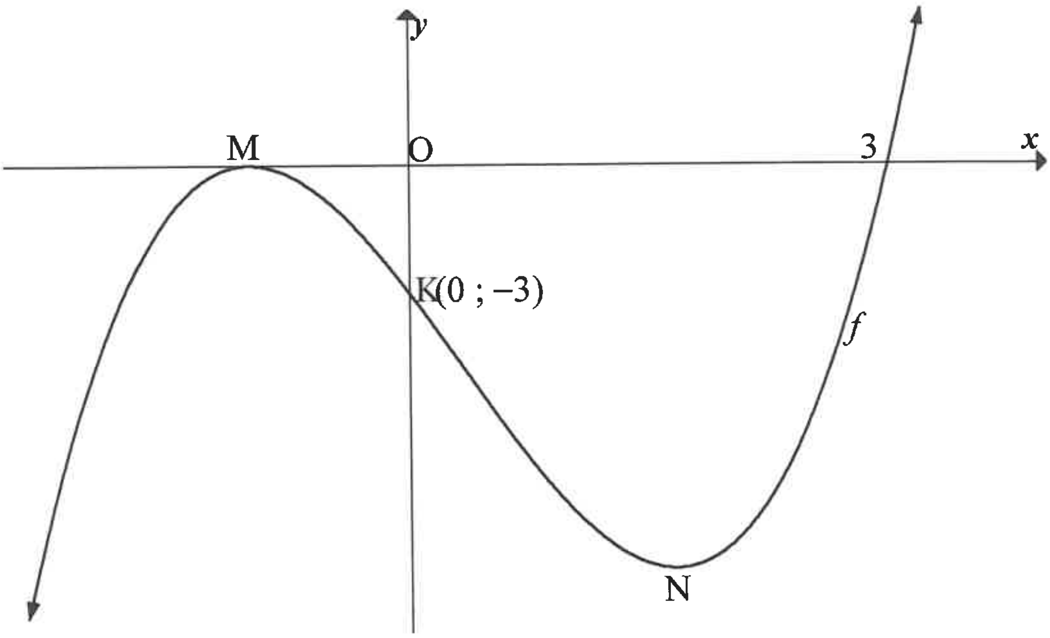
[15]

[32]

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at $(3 ; 0)$ and M, where M lies on the negative x-axis.

K$(0 ; -3)$ is the y-intercept of f.
M and N are the turning points of f.



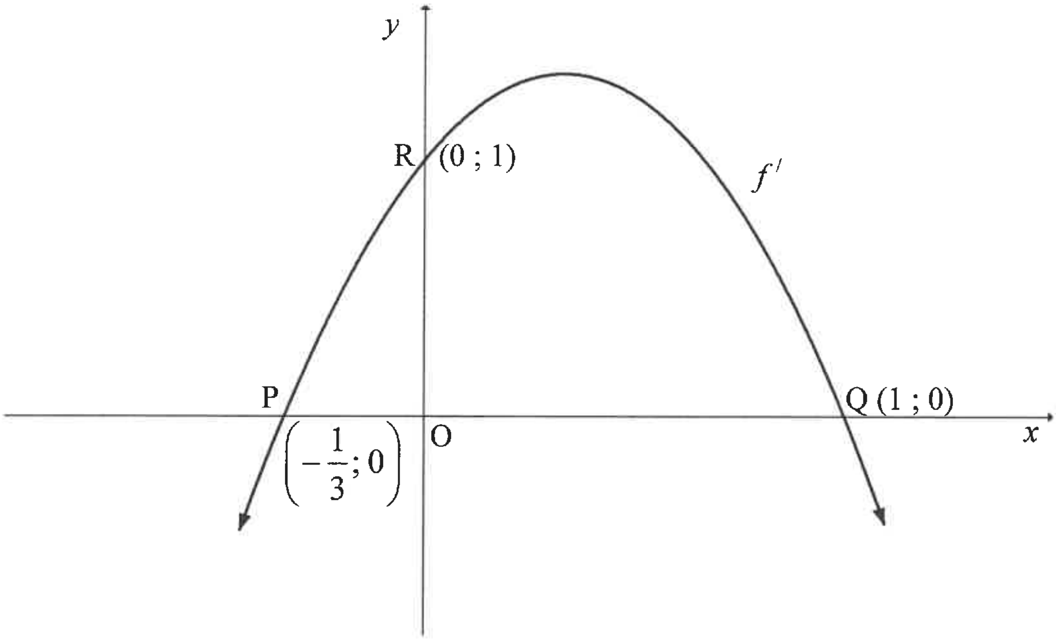
9.1	Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$.	(5)
9.2	Calculate the coordinates of N.	(5)
9.3	For which values of x will:	
	9.3.1 $f(x) < 0$	(2)
	9.3.2 f be increasing	(2)
	9.3.3 f be concave up	(3)
9.4	Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$.	(6) **[23]**

[33]

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}\,;\,0\right)$, $Q(1\,;\,0)$ and $R(0\,;\,1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at $D(a\,;\,b)$.

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

[17]

[34]

ANNEXURE C: INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ *area* $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

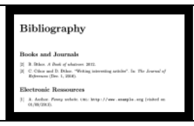
$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

[35]

Bibliography



1. **GRADE 12 MATHEMATICS EXAMINATION GUIDELINES**

2. **MATHEMATICS CAPS DOCUMENT**

3. **GRADE 12 NOVEMBER 2008 – 2022 MATHEMATICS EXAM PAPERS**

4. **GRADE 12 MAR, Jun 2009 – 2022 MATHEMATICS EXAM PAPERS**

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