JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: ANALYTICAL GEOMETRY

ACTIVITY BOOK

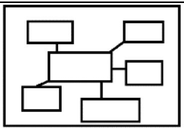
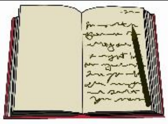
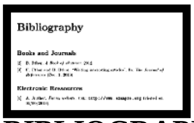
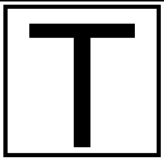
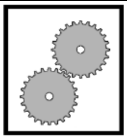
LEARNER/TEACHER

TERM 2 WORK

Straight line geometry, Equation of a circle and Mixed questions

CONTENTS **PAGE**

ICON DESCRIPTION

 **MIND MAP**	 **EXAMINATION GUIDELINE**	 **CONTENTS**	 **ACTIVITIES**
 **BIBLIOGRAPHY**	 **TERMINOLOGY**	 **WORKED EXAMPLES**	 **STEPS**

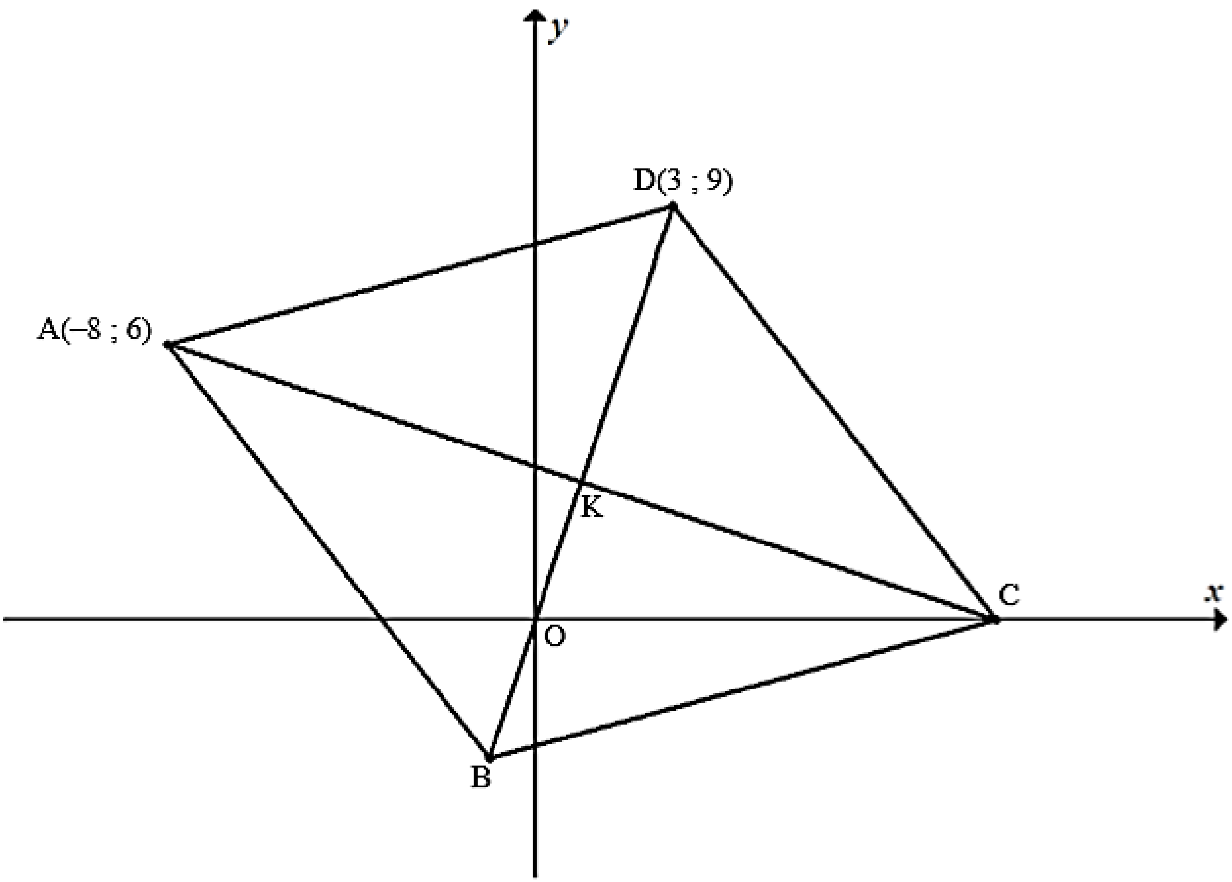
TOPIC 1: LINES, TRIANGLES AND QUADRILATERALS

Activity 1

Determine whether K(-3 ; 5), L(2 ; -3) and N(5 ; -9) are collinear.

Activity 2

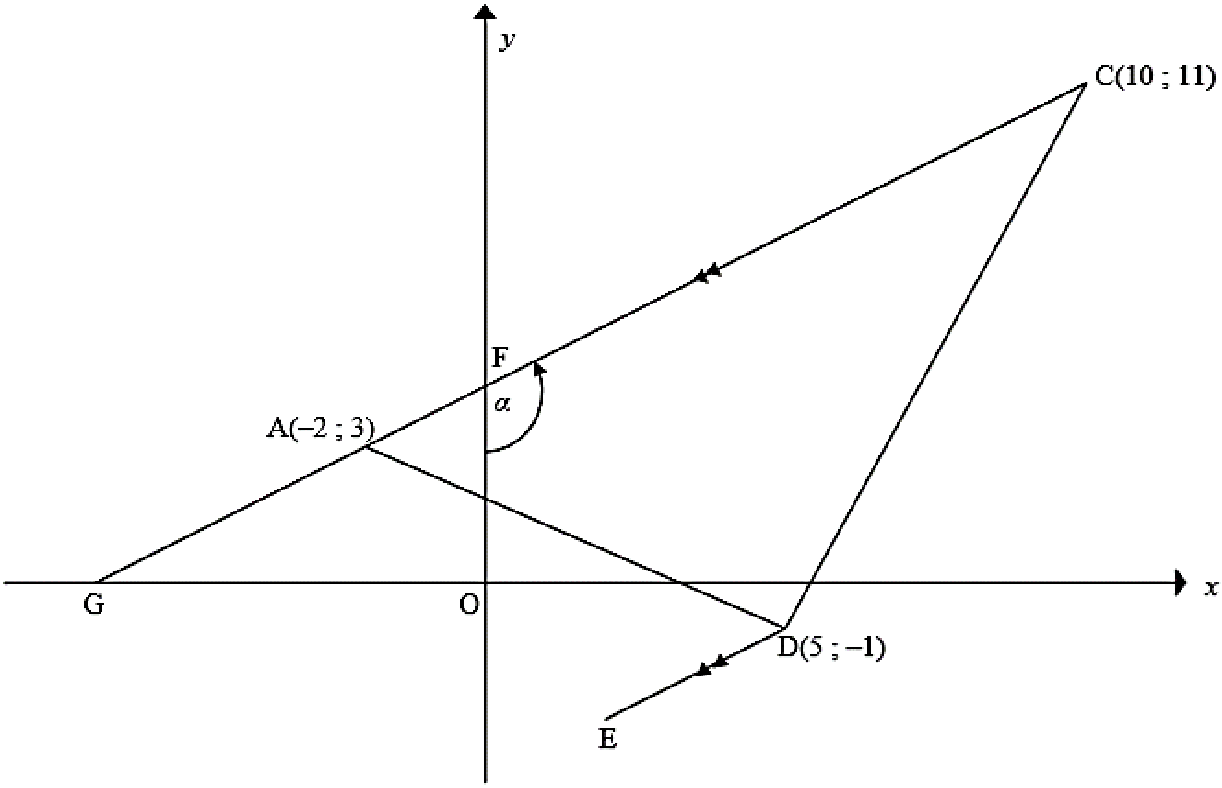
In the diagram A(–8 ; 6), B, C and D(3 ; 9) are the vertices of a rhombus. The equation of BD is $3x - y = 0$. The diagonals of the rhombus intersect at point K.



2.1. Calculate the perimeter of ABCD. Leave your answer in simplest surd form.
2.2. Determine the equation of diagonal AC in the form $y = mx + c$.
2.3. Calculate the coordinates of K if the equation of AC is $x + 3y = 10$.
2.4. Calculate the coordinates of B.
2.5. Determine, showing ALL your calculations, whether rhombus ABCD is a square or not.

Activity 3

In the diagram, $A(-2\ ;\ 3)$, $C(10\ ;\ 11)$ and $D(5\ ;\ -1)$ are the vertices of ΔACD. CA intersects the y-axis in F and CA produced cuts the x-axis in G. The straight line DE is drawn parallel to CA. $C\hat{F}O = \alpha$.



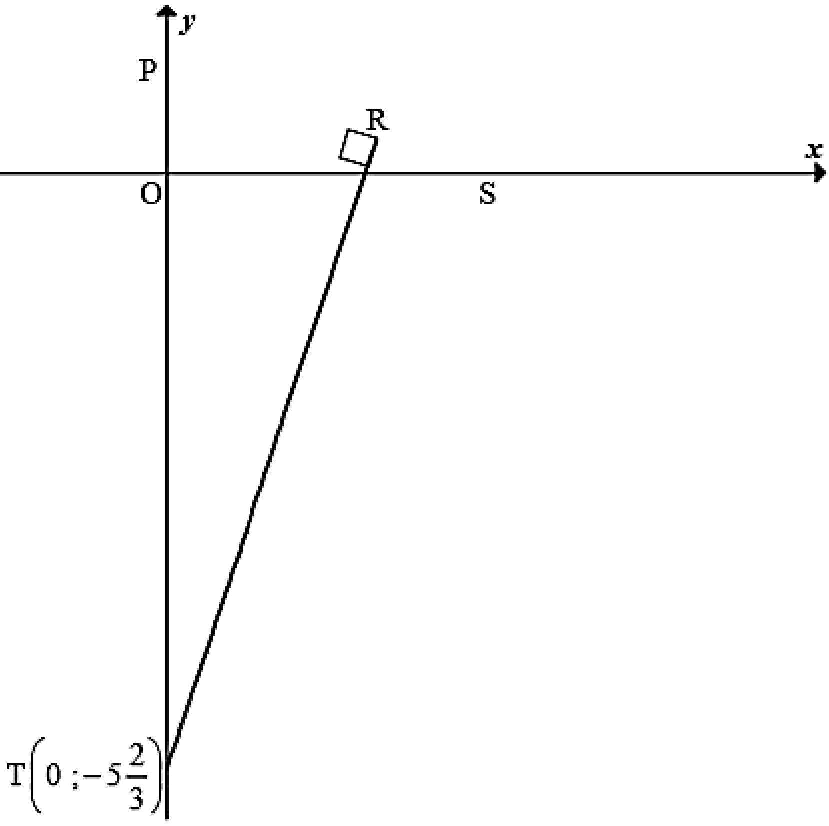
3.1. Calculate the gradient of the line AC.
3.2. Determine the equation of line DE in the form $y = mx + c$.
3.3. Calculate the size of α.
3.4. B is a point in the first quadrant such that ABDE, in the order, forms a rectangle.
Calculate, giving reasons, the:

3.4.1. Coordinates of M, the midpoint of BE.
3.4.2. Length of diagonal BE.

Activity 4

In the diagram, the straight line SP is drawn having S and P as its x- and y-intercepts respectively. The equation of SP is $x + ay - a = 0,\ a > 0$. It is also given that $OS = 3OP$.

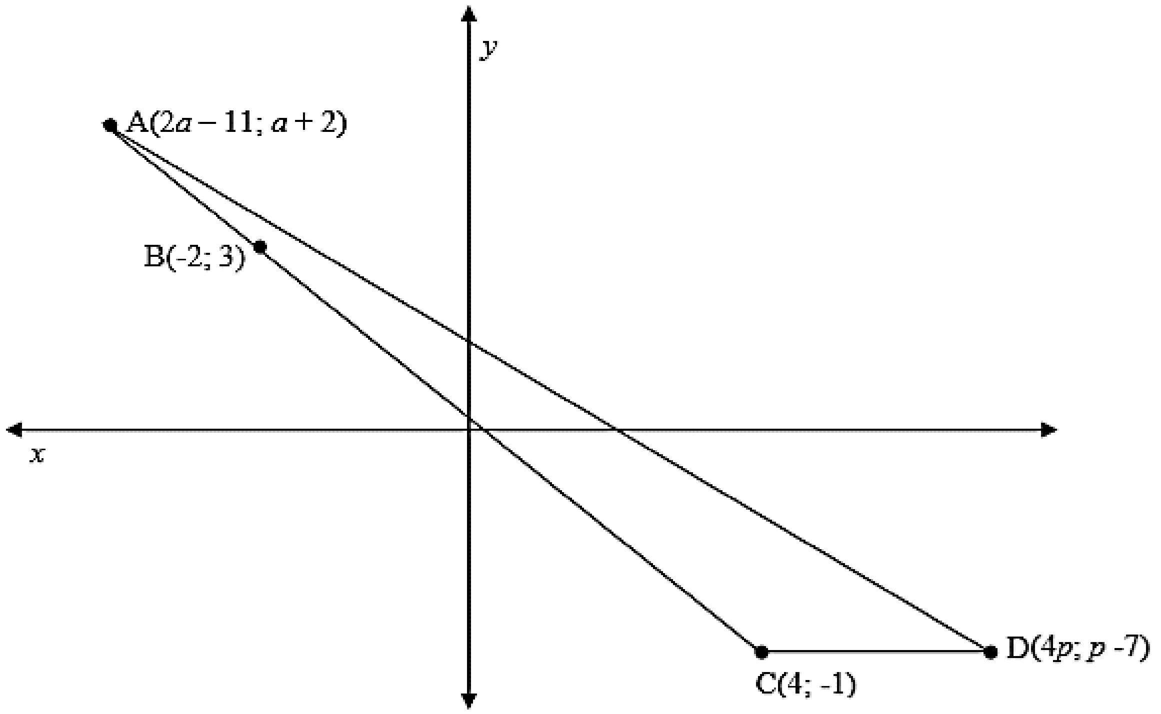
The straight line RT is drawn with R on SP and $RT \perp PS$. RT cuts the y-axis in $T\left(0\,;-5\frac{2}{3}\right)$.



4.1. Calculate the coordinates of P.
4.2. Calculate the value of a.
4.3. Determine the equation of RT in the form $y = mx + c$ if it is given that $a = 3$.
4.4. Calculate the area of ΔPRT if it is given that $R\left(2;\frac{1}{3}\right)$.
4.5. Calculate, giving reasons, the radius of a circle passing through the points P, R and T.

Activity 5

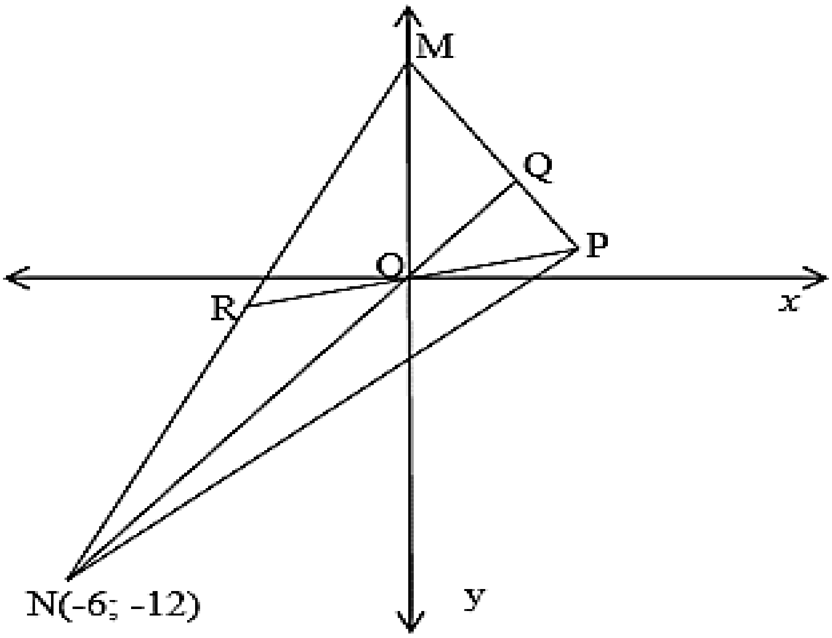
The points A($2a - 11$; $a + 2$), C(4; -1) and D($4p$; $p - 7$) are the vertices of ΔACD with B(-2; 3) on AC.



5.1. If points A, B and C are collinear, find the value a.
5.2. Determine the equation of the line AC.
5.3. Hence, determine the coordinates of midpoint M of AB.
5.4. Determine the value of p if CD is parallel to the $x - axis$.

Activity 6

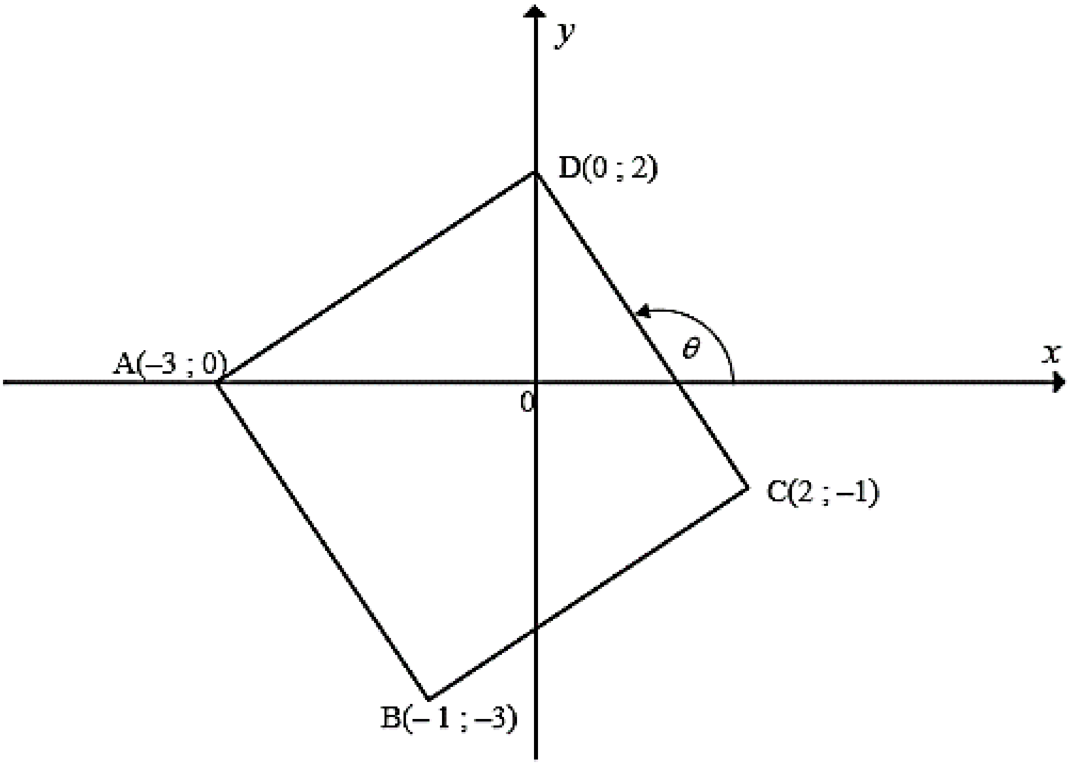
In the diagram, M, N and P are vertices of ΔMNP, with N(-6; -12).
M is a point on the y-axis. The equation of the line MN is $3x - y + 6 = 0$.
MR = NR and NQ $\perp$ MP. PR and NQ intersect at the origin O.



6.1. Calculate the gradient of NQ.
6.2. Calculate the gradient of MP.
6.3. Calculate the angle of inclination of MP.
6.4. Hence, determine the equation of the line MP.
6.5. Hence, determine the coordinates of P.
6.6. Determine the coordinates of R.

Activity 7

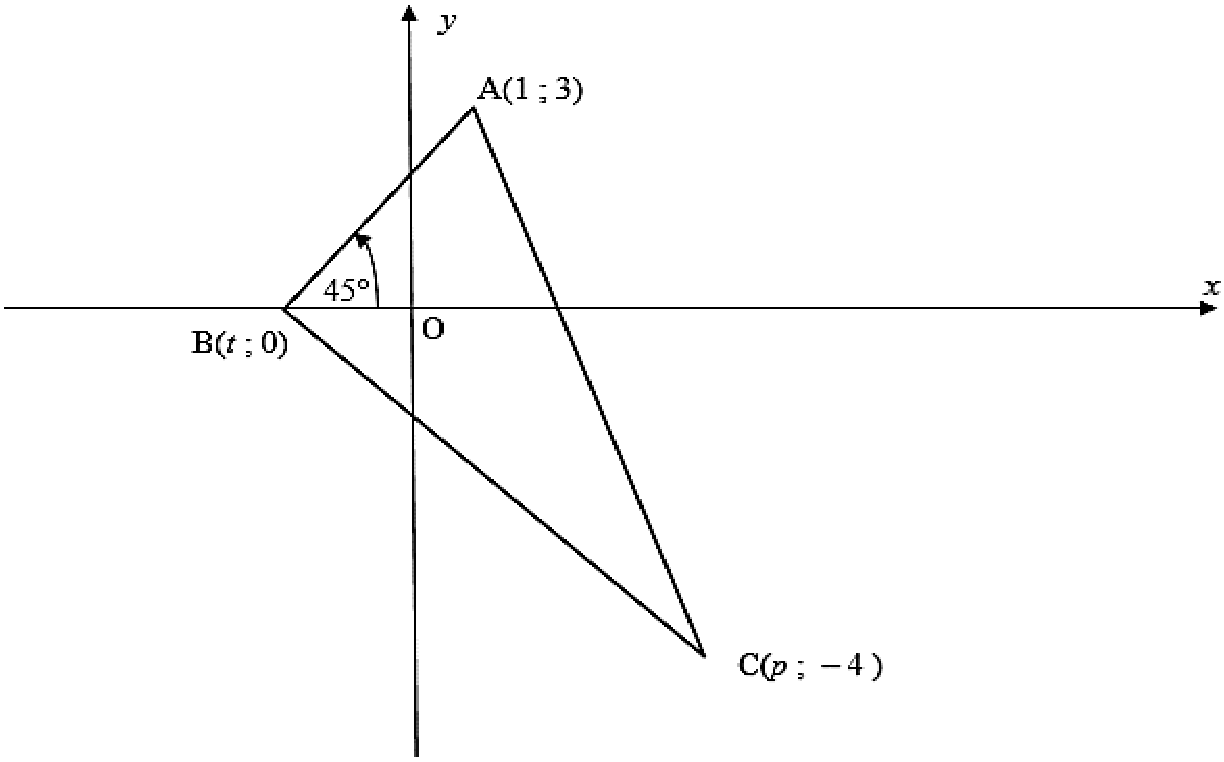
ABCD is a quadrilateral with vertices A(– 3 ; 0), B(– 1 ; – 3), C(2 ; –1) and D(0 ; 2).



7.1. Determine the coordinates of M, the midpoint of AC.
7.2. Show that AC and BD bisect each other.
7.3. Prove that $A\hat{D}C = 90°$.
7.4. Show that ABCD is a square.
7.5. Determine the size of θ, the angle of inclination of DC, correct to one decimal place.
7.6. Does C lie inside or outside the circle with centre (0;0) and radius 2? Justify your answer.

Activity 8	

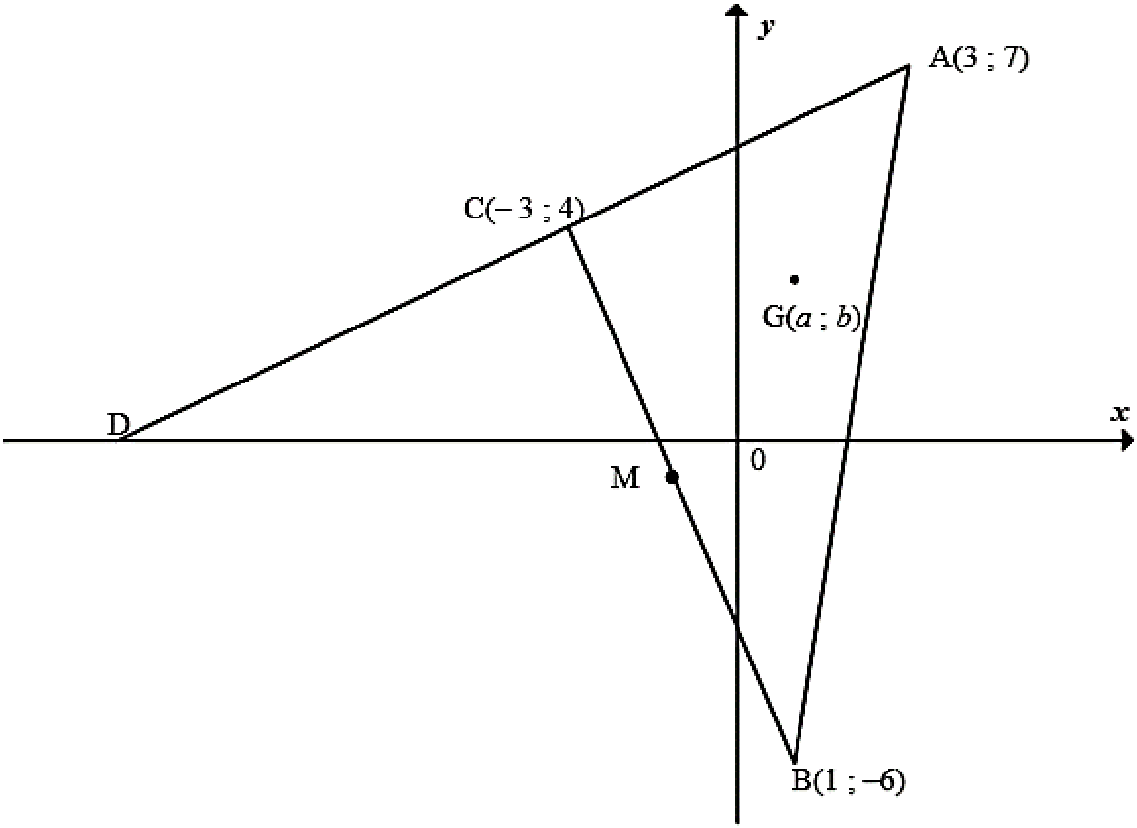
ABC is a triangle with vertices A(1 ; 3), B(t ; 0) and C(p ; −4), with $p > 0$, in a Cartesian plane. AB makes an angle of 45° with the positive x-axis. $AC = \sqrt{50}$.



8.1. Determine the gradient of AB.
8.2. Calculate the value of t .
8.3. Calculate p , the $x-coordinate$ of point C.
8.4. Hence, determine the midpoint of BC.
8.5. Determine the equation of the line parallel to AB, passing through point C.

Activity 9	

In the diagram below, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D.



9.1. Calculate the gradient of:

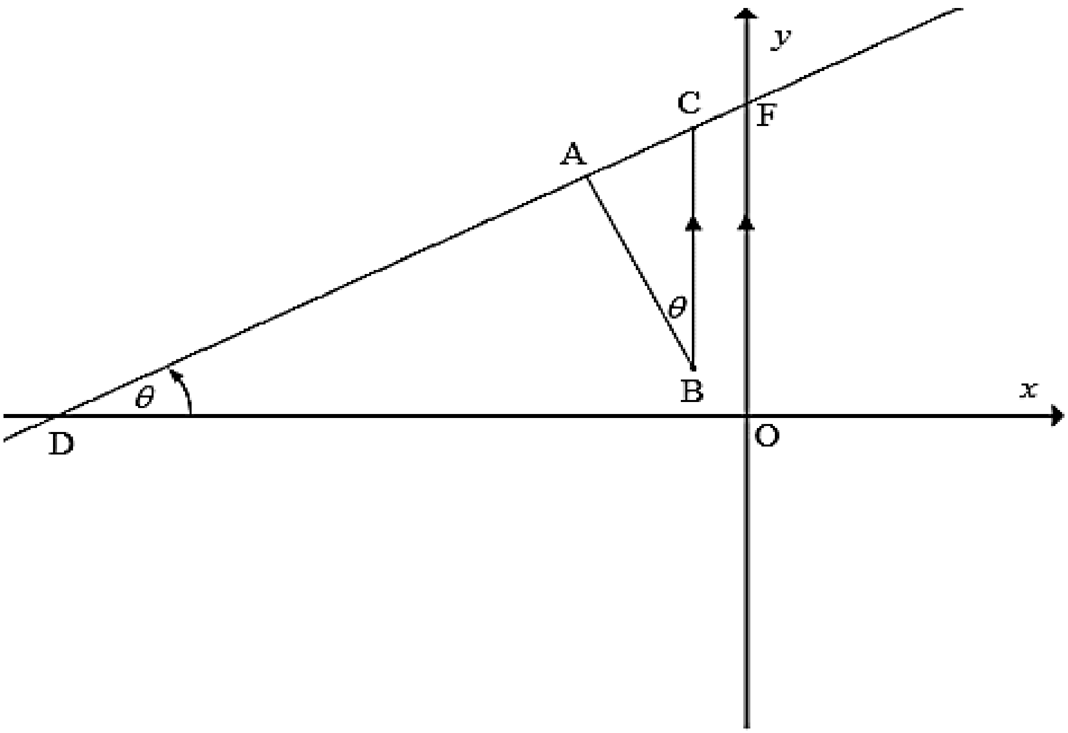
9.1.1. AD.
9.1.2. BC.

9.2. Calculate the size of $D\hat{C}B$.
9.3. Write down an equation of the straight line AD.
9.4. Determine the coordinates of M, the midpoint of BC.
9.5. If $G(a;b)$ is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$.
9.6. Hence, calculate TWO possible values of b if $GC = \sqrt{17}$.

Activity 10	

The circle with centre B(–1 ; 1) and radius $\sqrt{20}$ is shown. BC is parallel to the y-axis and CB = 5. The tangent to the circle at A passes through C.

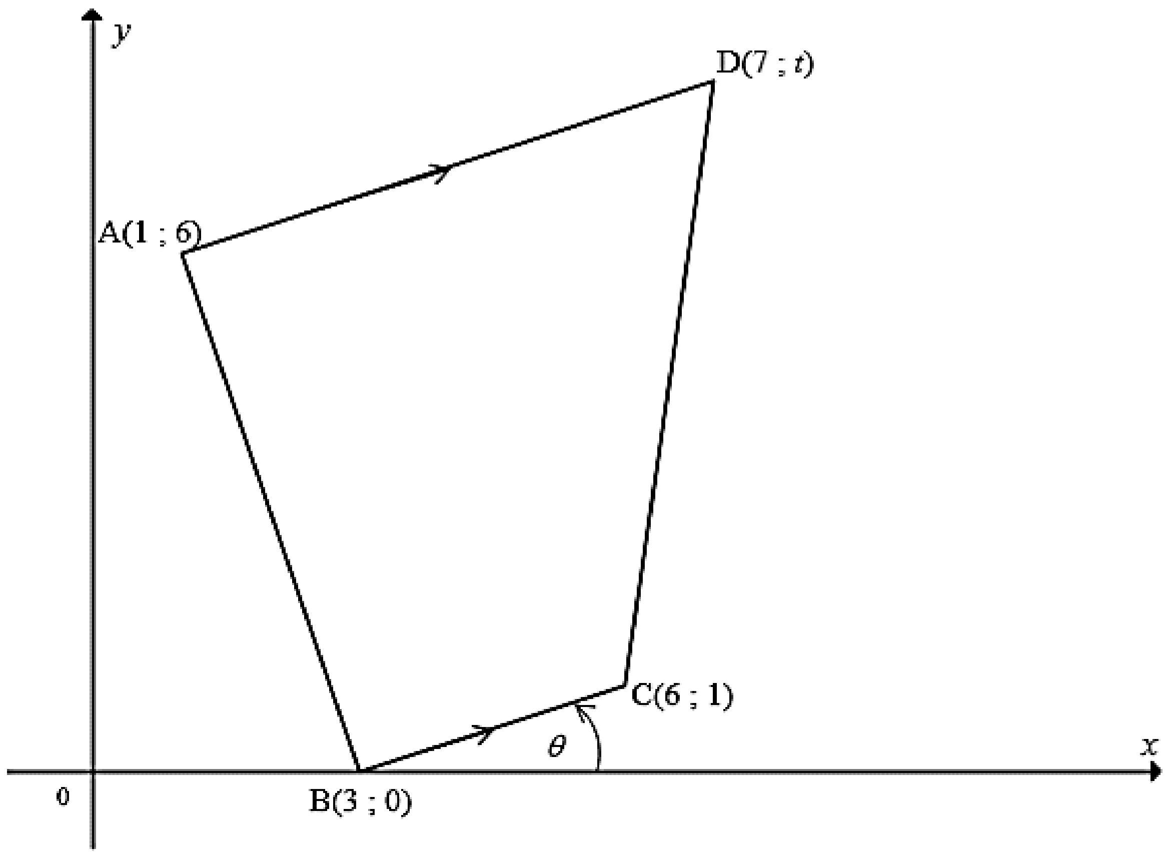
$A\hat{B}C = A\hat{D}O = \theta$



10.1. Determine the coordinates of C.
10.2. Calculate the length of CA.
10.3 Write down the value of $\tan\theta$.
10.4. Show that the gradient of AB is -2.
10.5. Determine the coordinates of A.
10.6. Calculate the ratio of ΔABC to the area of ΔODF. Simplify your answer.

Activity 11	

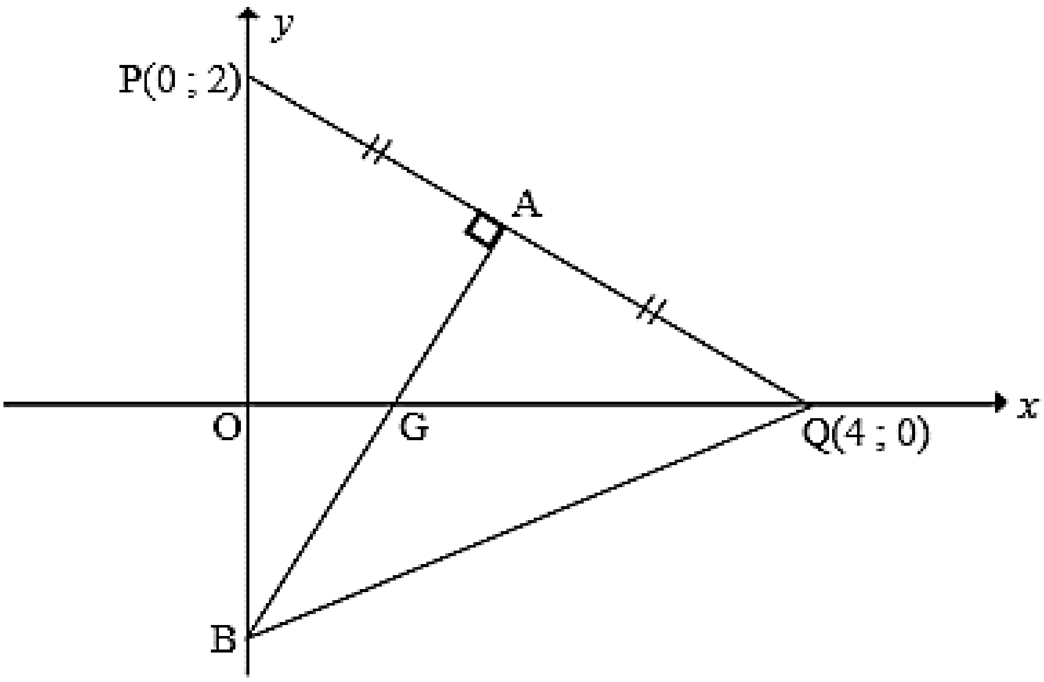
ABCD is a quadrilateral with vertices A(1 ; 6), B(3 ; 0), C(6 ; 1) and D(7 ; t) in a Cartesian plane. AD || BC.



11.1. Calculate the gradient of BC.
11.2. Determine the equation of AD in the form $y = \ldots$.
11.3. Show that $t = 8$.
11.4. Calculate the lengths of AD, BC and AB.
11.5. Show that AB is perpendicular to BC.
11.6. Calculate the area of the quadrilateral ABCD (Simplify your answer).
11.7. Determine θ, the angle of inclination of BC.

Activity 12	

The diagram below shows the points P(0 ; 2) and Q(4 ; 0). Point A is the midpoint of PQ. The line AB is perpendicular to PQ and intersects the x-axis at G and the y-axis at B.



12.1. Show that the gradient of PQ is $-\frac{1}{2}$.

12.2. Determine the coordinates of A.

12.3. Determine the equation of the line AB.

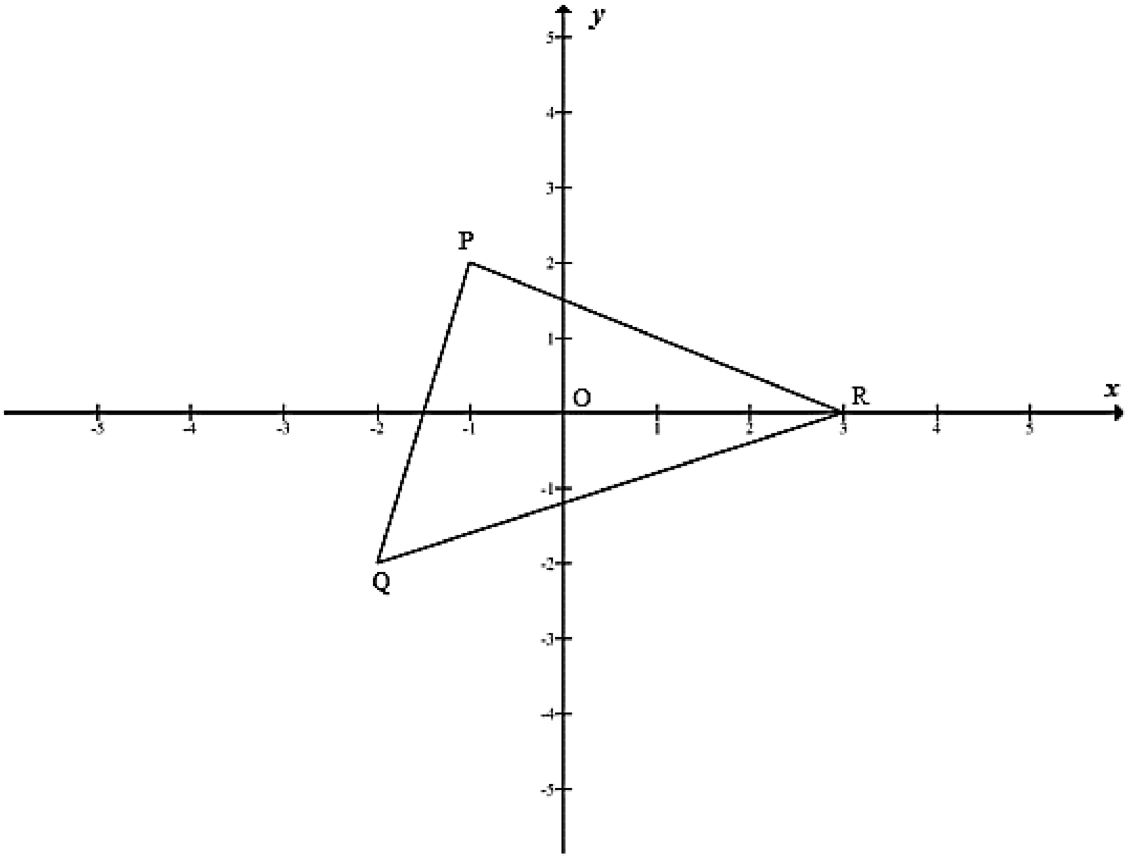
12.4. Calculate the length of BQ.

12.5. Show that ΔBPQ is isosceles.

12.6. If PBQR is a rhombus, determine the coordinates of R.

Activity 13

In the diagram below ΔPQR with vertices $P(-1\ ;\ 2)$, $Q(-2\ ;\ -2)$ and $R(3\ ;\ 0)$ is given.

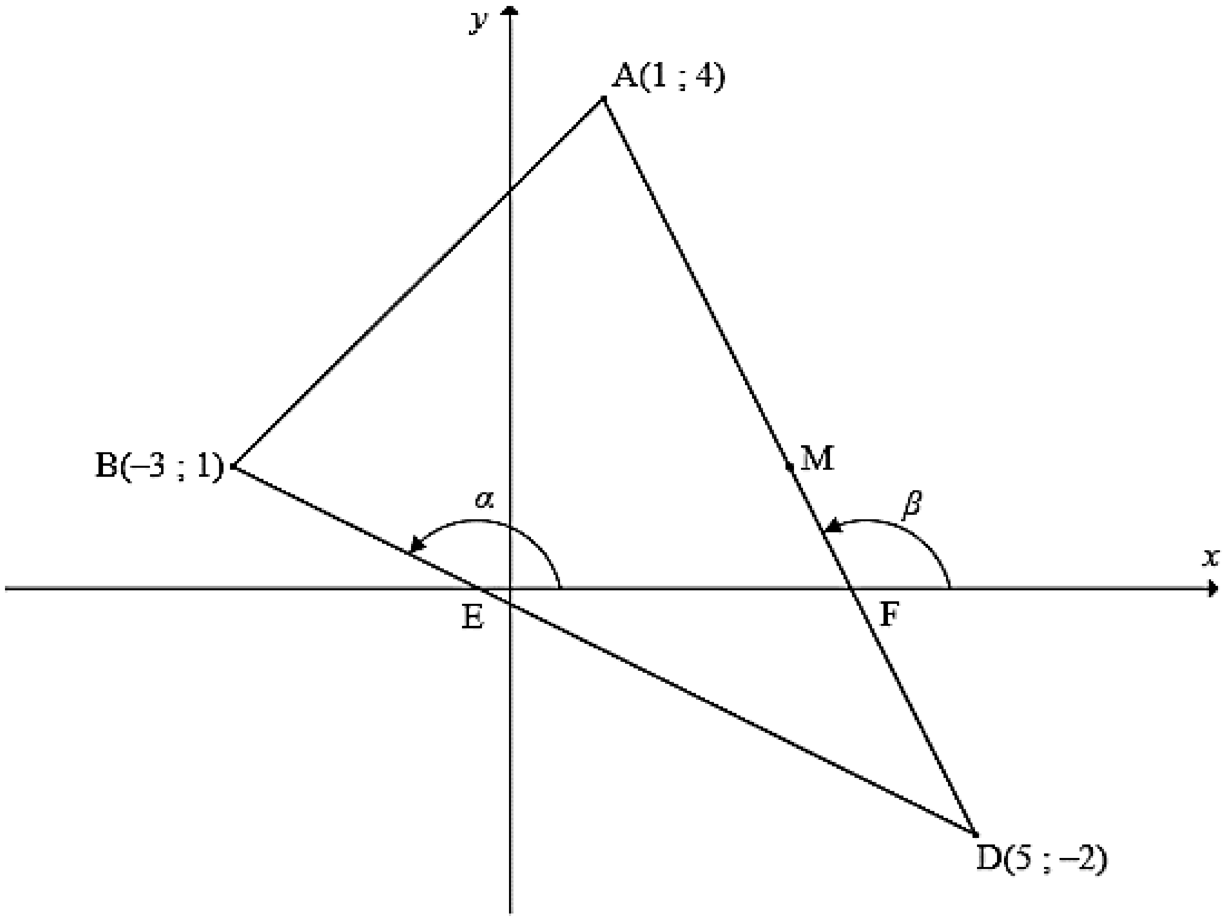


13.1. Calculate the angle that PQ makes with the positive $x-axis$.
13.2. Determine the coordinates of M, the midpoint of PR.
13.3. Determine the perimeter of ΔPQR to the nearest whole number.
13.4. Determine an equation of the line parallel to PQ that passes through M.

Activity 14

In the figure below, A(1 ; 4), B(–3 ; 1) and D(5 ; –2) are the coordinates of the vertices of ΔABD.

- BD and AD intersect the x-axis at E and F respectively.
- The angle of inclination of BD with the x-axis at E is α.
- The angle of inclination of AD with the x-axis at F is β.

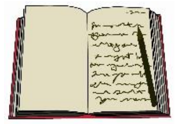


14.1. Calculate the gradient of AD.
14.2. Determine the length of the line segment AD. Leave your answer in in surd form if necessary.
14.3. Determine the coordinates of M, the midpoint of AD.
14.4. C is a point such that line BC is parallel to AD.
Determine the equation of line BC in the form $ax+by+c=0$.
14.5. 14.5.1. Calculate the size of β.
14.5.2. Calculate ALL the angles of ΔDEF.

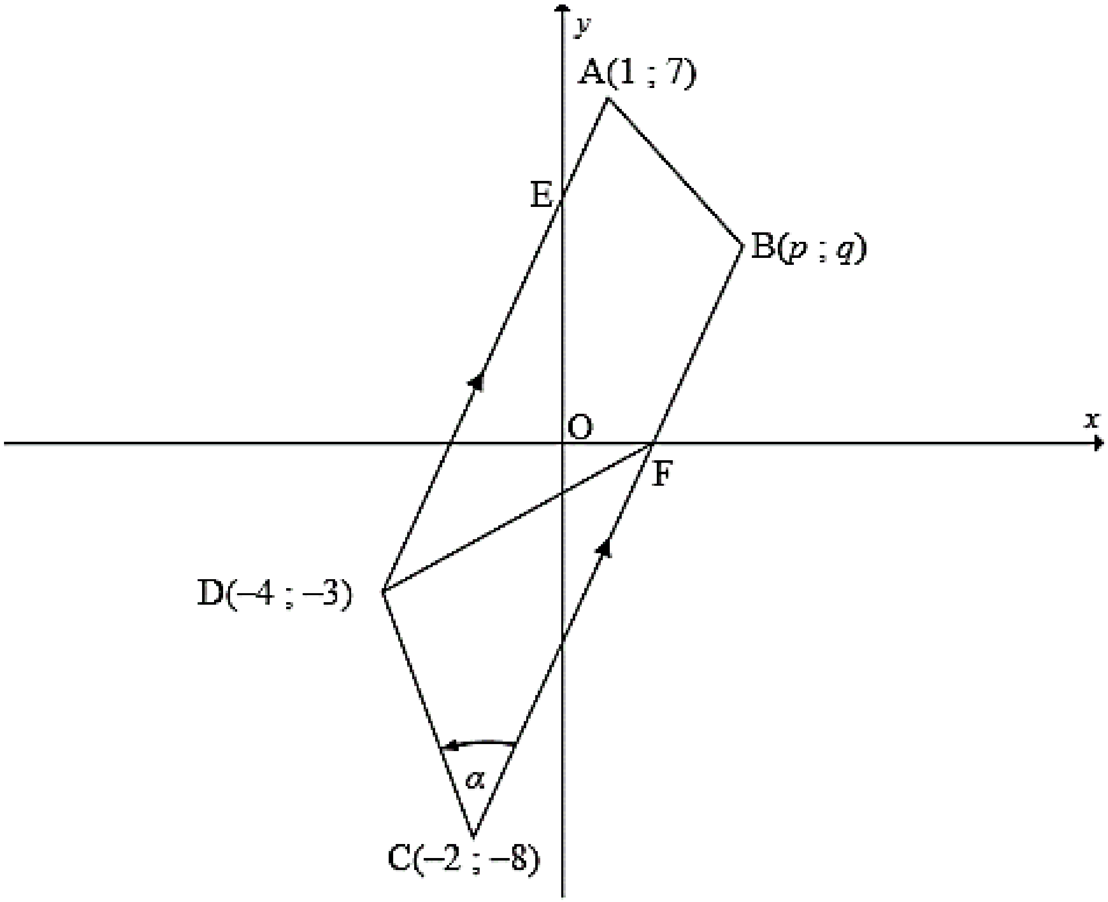
Gr 12: Equation of a circle

14.6. Determine the equation of a circle with centre M, which passes through the point A and D.
Give your answer in the form: $(x-a)^2+(y-b)^2=r^2$.

14.7. Does the point B lie inside, outside or on the circle in QUESTION 14.6?
Show ALL calculations to justify your answer.

Activity 15	

In the diagram below, trapezium ABCD with AD // BC is drawn. The coordinates of the vertices are A(1 ; 7); B(p ; q); C(–2 ; –8) and D(–4 ; –3). BC intersects the x-axis at F. $D\hat{C}B = \alpha$.



15.1. Calculate the gradient of AD.

15.2. Determine the equation of BC in the form $y = mx + c$.

15.3. Determine the coordinates of point F.

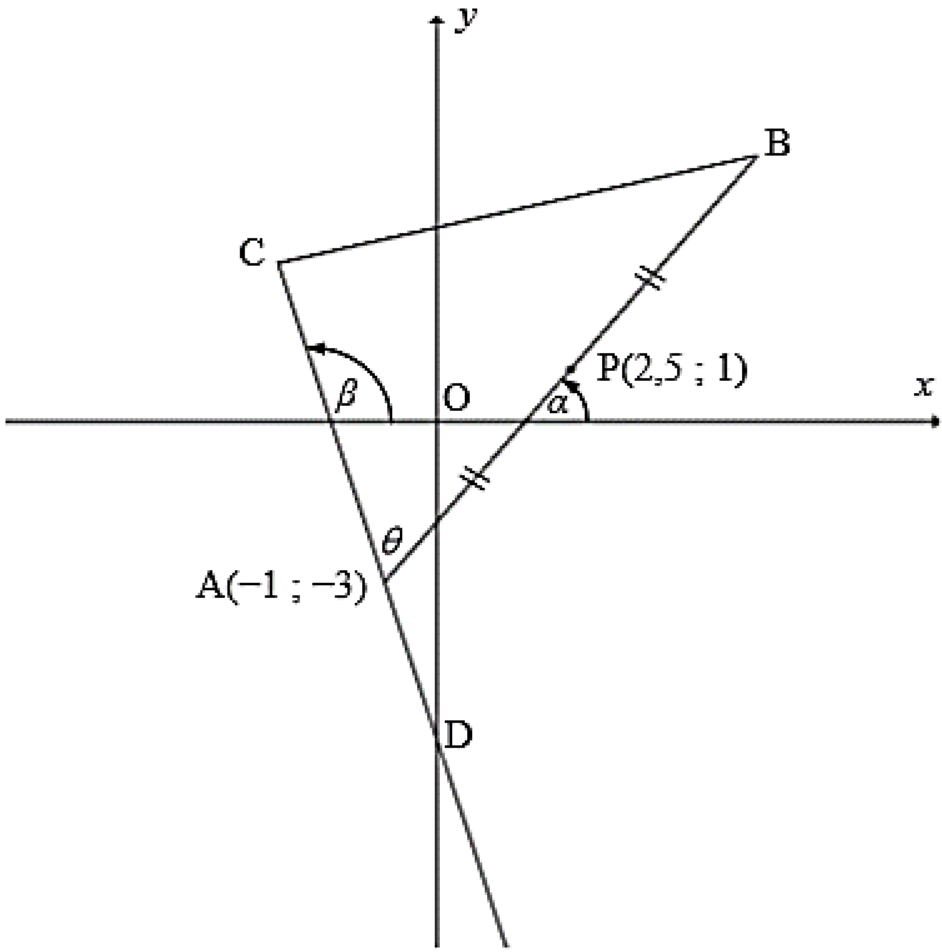
15.4. $AB^{/}CD$ is a parallelogram with $B^{/}$ on BC.
Determine the coordinates of $B^{/}$, using a transformation $(x; y) \rightarrow (x + a; y + b)$ that sends A to $B^{/}$.

15.5. Show that $\alpha = 48{,}37°$.

15.6. Calculate the area of ΔDCF.

Activity 16

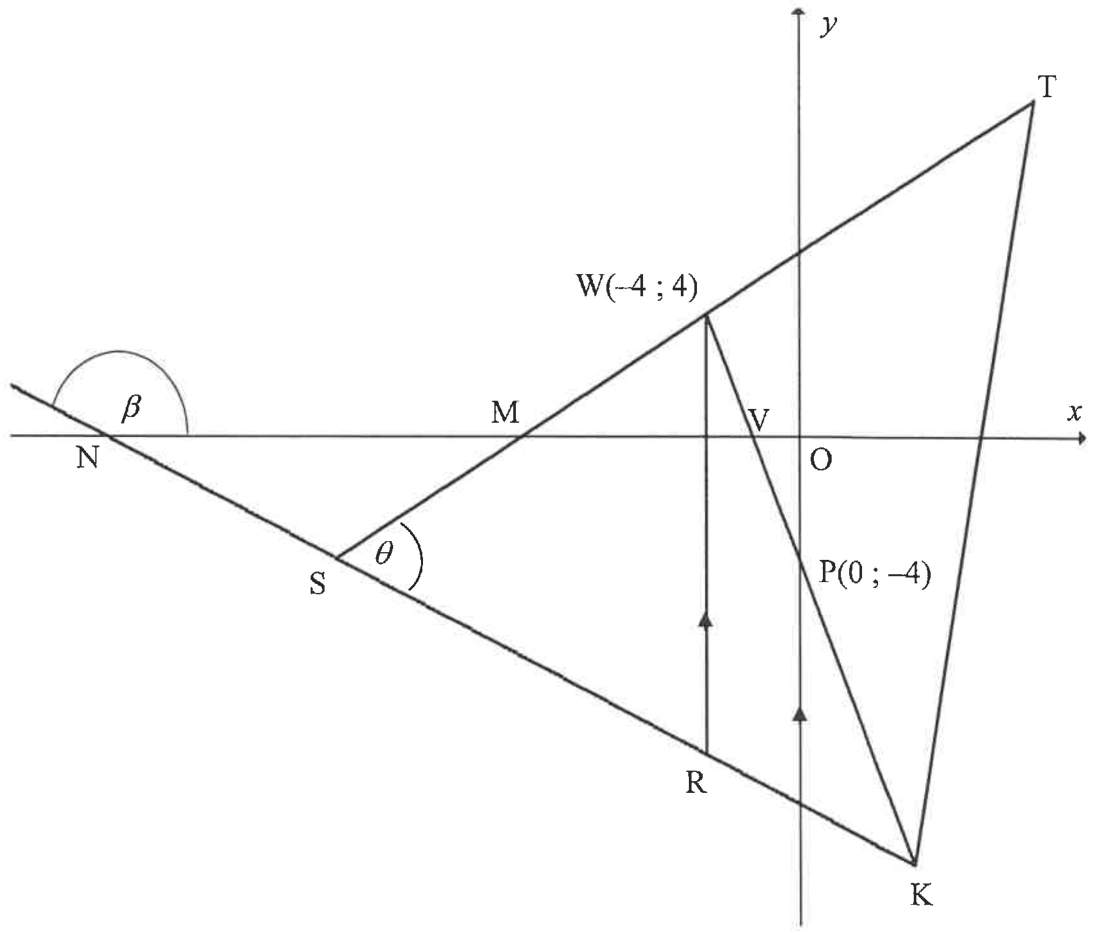
In the diagram below, A(−1 ; −3), B and C are the vertices of a triangle.
P(2,5 ; 1) is the midpoint of AB. CA extended cuts the y-axis at D.
The equation of CD is $y = -3x + k$. $C\hat{A}B = \theta$.
α and β are the angles that AB and AC respectively make with the x-axis.



16.1. Determine the value of k.
16.2. Determine the coordinates of B.
16.3. Determine the gradient of AB.
16.4. Calculate the size of θ.
16.5. Calculate the length of AD. Leave your answer in surd form.
16.6. If $AC=2AD$ and $AB=\sqrt{113}$, calculate the length of CB.

QUESTION 3

ΔTSK is drawn. The equation of ST is $y = \frac{1}{2}x + 6$ and ST cuts the x-axis at M. W(–4 ; 4) lies on ST and R lies on SK such that WR is parallel to the y-axis. WK cuts the x-axis at V and the y-axis at P(0 ; –4). KS produced cuts the x-axis at N. $T\hat{S}K = \theta$.



3.1 Calculate the gradient of WP. (2)

3.2 Show that WP $\perp$ ST. (2)

3.3 If the equation of SK is given as $5y + 2x + 60 = 0$, calculate the coordinates of S. (4)

3.4 Calculate the length of WR. (4)

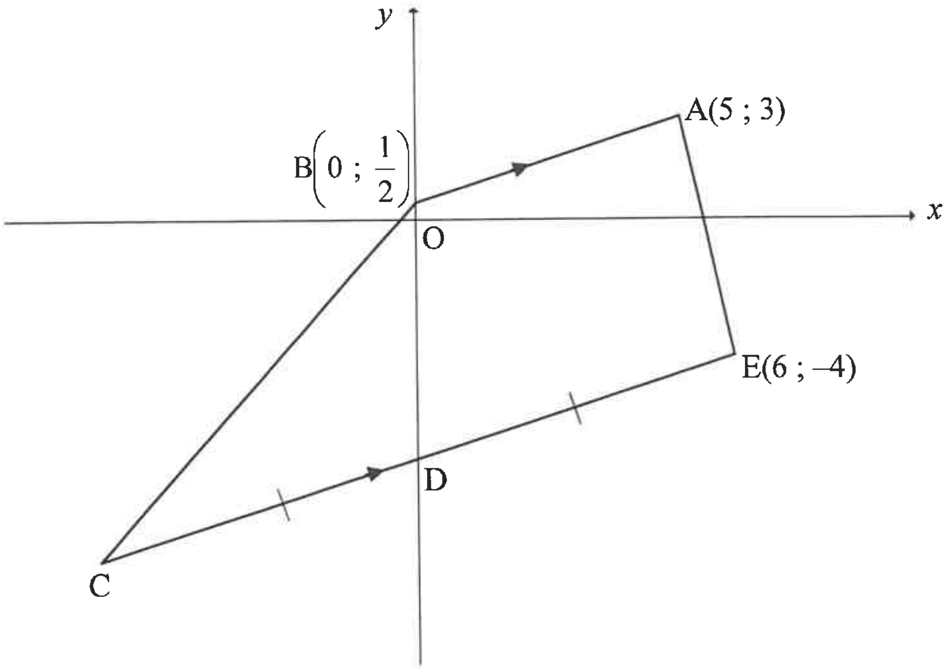
3.5 Calculate the size of θ. (5)

3.6 Let L be a point in the third quadrant such that SWRL, in that order, forms a parallelogram. Calculate the area of SWRL. (4)

[21]

QUESTION 3

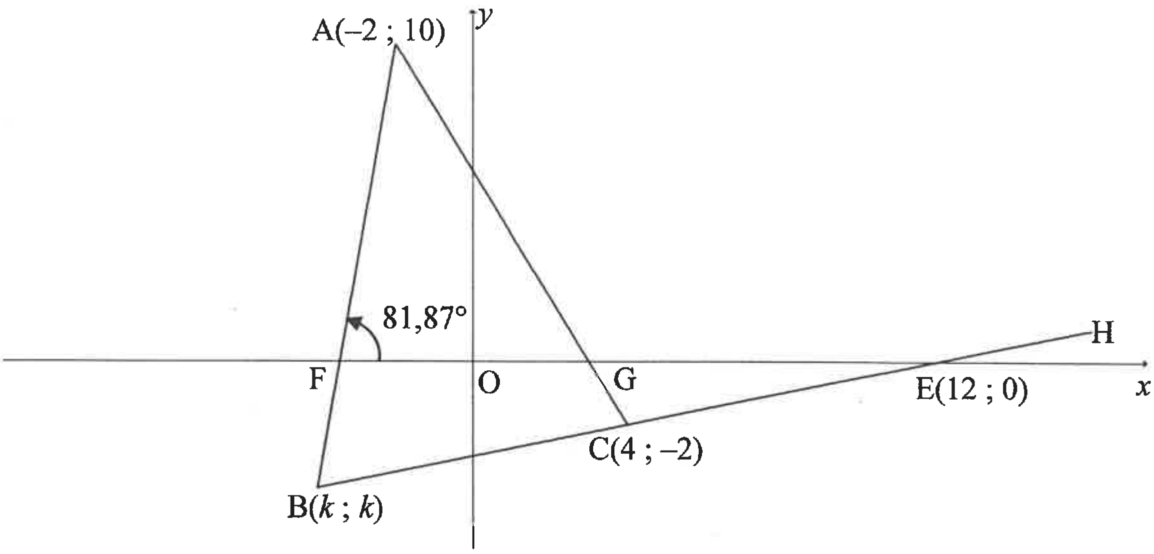
In the diagram, A(5 ; 3), $B\left(0\,;\,\frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA || CE. D is the y-intercept of CE and CD = DE.



3.1	Calculate the gradient of AB.	(2)
3.2	Determine the equation of CE in the form $y = mx + c$.	(3)
3.3	Calculate the:	
	3.3.1 Coordinates of C	(3)
	3.3.2 Area of quadrilateral ABCD	(4)
3.4	If point K is the reflection of E in the y-axis:	
	3.4.1 Write down the coordinates of K	(2)
	3.4.2 Calculate the:	
	(a) Perimeter of ΔKEC	(4)
	(b) Size of $K\hat{C}E$	(3) **[21]**

QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

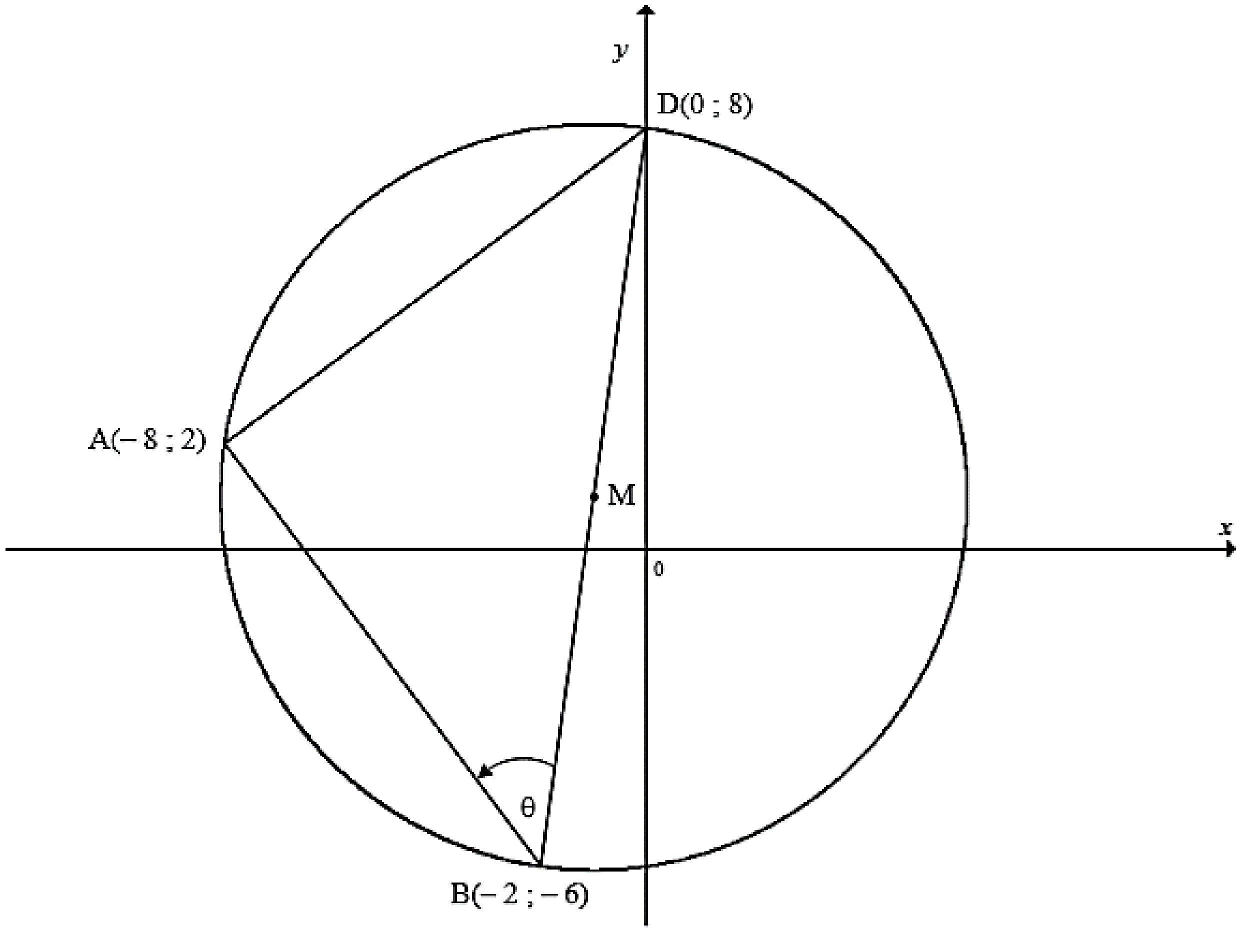
(b) Tangent to the circle at point B(k ; k) (3)

[24]

TOPIC 2: EQUATION OF A CIRCLE

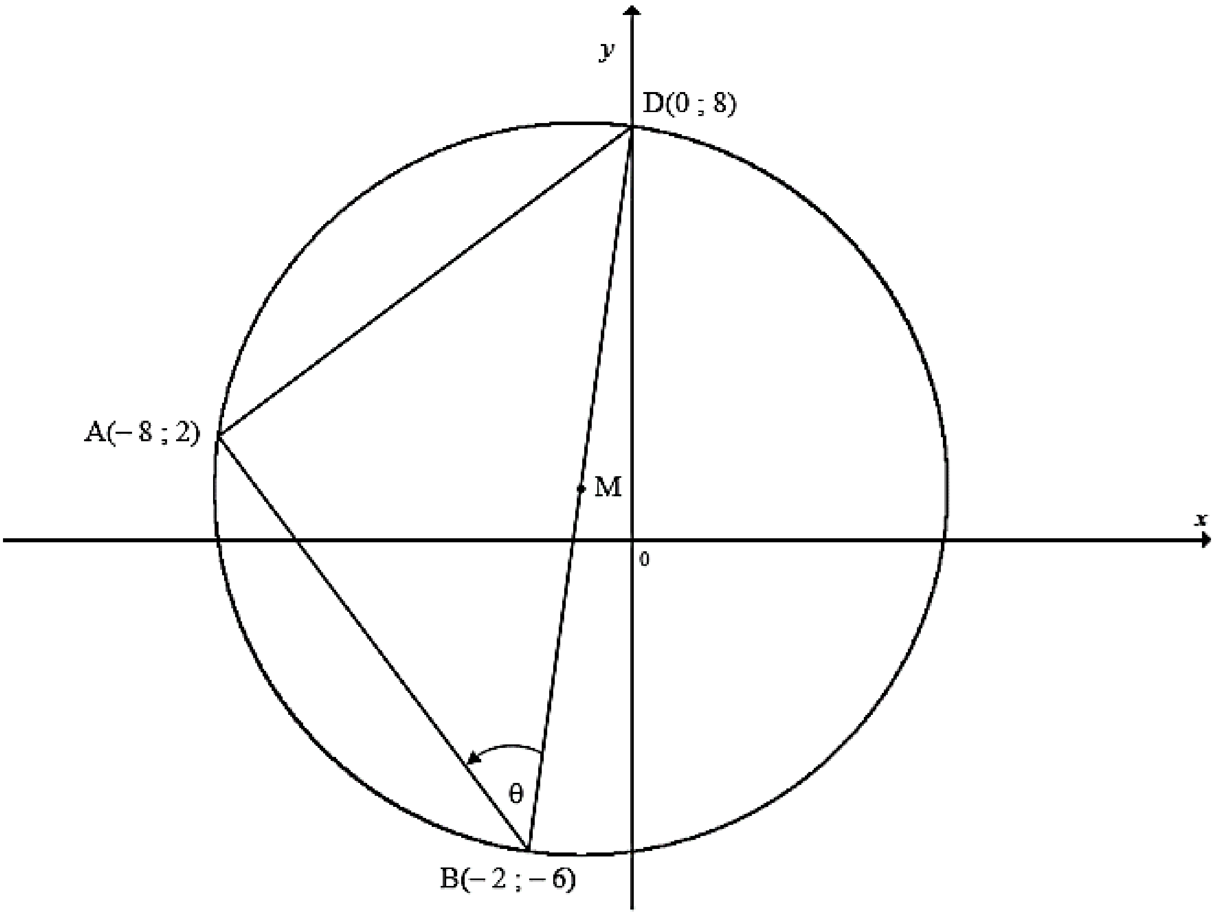
Activity 1

A(−8 ; 2), B(−2 ; −6) and D(0 ; 8) are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M, as shown in the figure below.

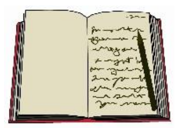


1.1. Calculate the coordinates of M.
1.2. Show that $(-8;2)$ lie on the line $y = 7x + 58$.
1.3. What is the relationship between the line $y = 7x + 58$ and the circle centred at M? Motivate your answer.
1.4. Calculate the lengths of AD and AB.
1.5. Prove that $D\hat{A}B = 90°$.
1.6. Write down the size of angle θ.
1.7. A circle centred at a point Z inside ΔABD, is drawn to touch sides AB, BD and DA at N, M and T respectively. Given that BMZN is a kite, calculate the radius of this circle. A diagram sheet is provided on DIAGRAM SHEET 1.

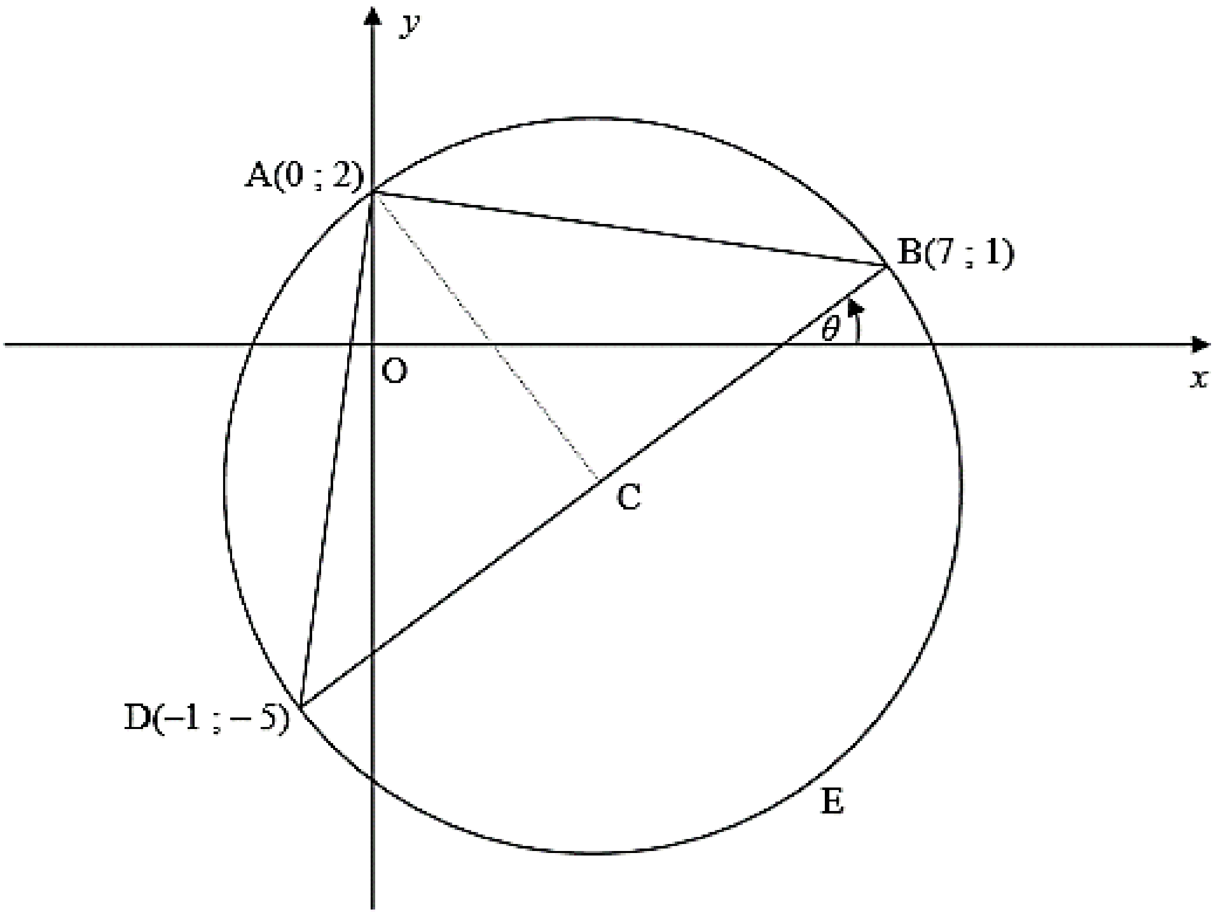
DIAGRAM SHEET 1



Activity 2

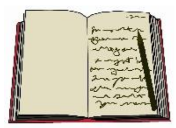


The circle that passes through the points A(0 ; 2), B(7 ; 1) and D(−1 ; −5) is given below.

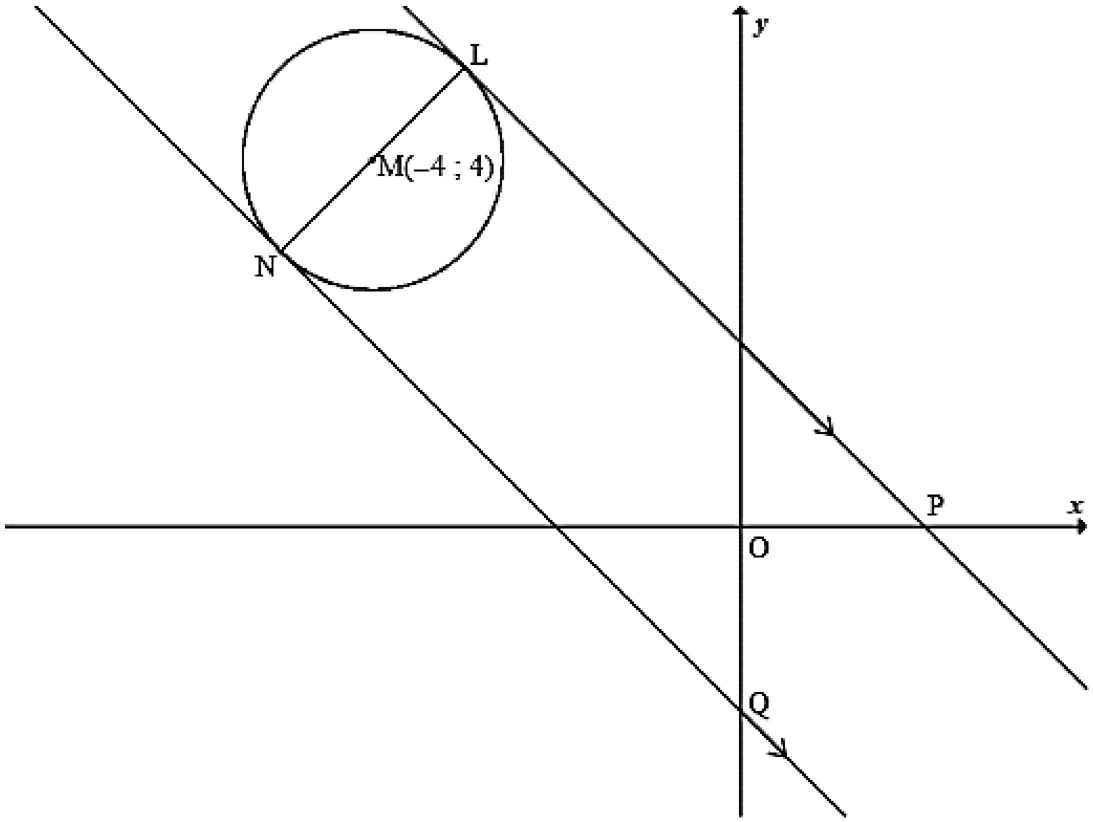


2.1. Calculate C, the coordinates of the midpoint of BD.
2.2. Show that $CA=CB$.
2.3. Hence, give the equation of the circle.
2.4. Calculate the angle θ that BD makes with the positive $x-axis$.
2.5. If AC is extended to meet the circle at E, calculate the coordinates of E.
2.6. Explain why ABED is a rectangle.
2.7. Determine the equation of the tangent to the circle at B in the form $y=...$.

Activity 3



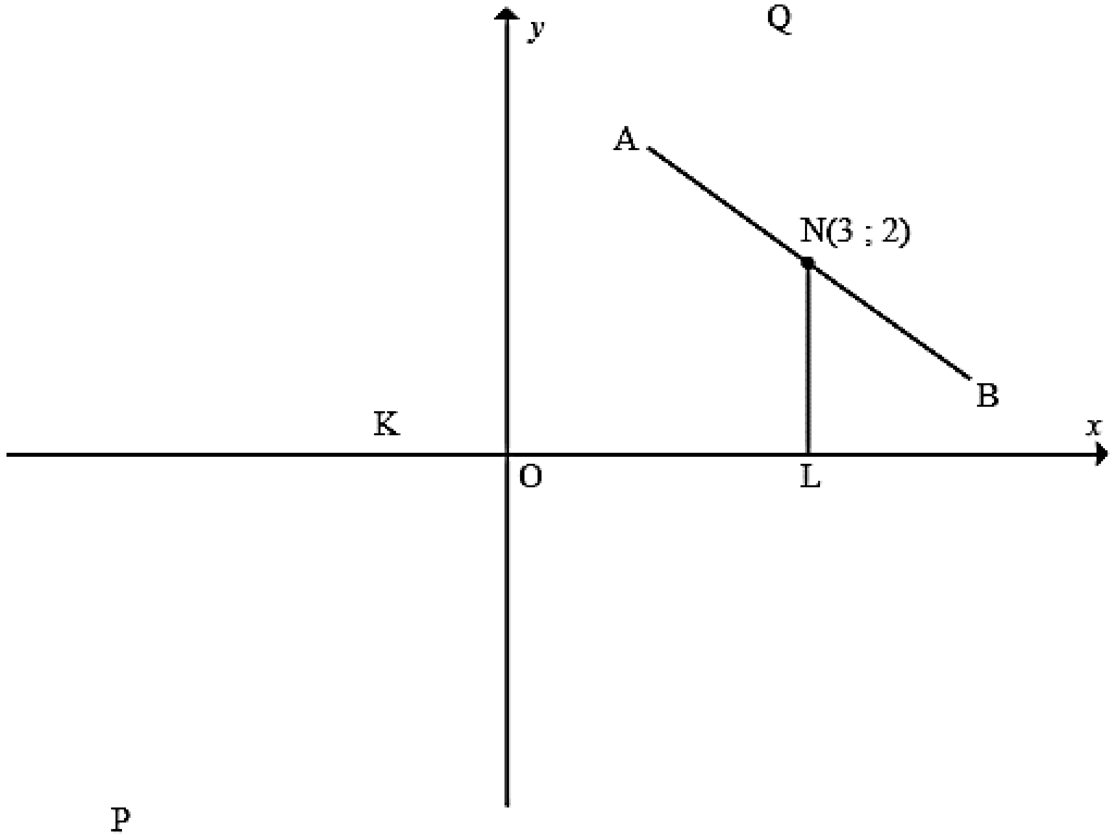
The line LP, with equation $y + x - 2 = 0$, is a tangent at L to the circle with centre M(– 4 ;4). LN is a diameter of the circle. Also LP || NQ, where P lies on the x-axis, and Q lies on the y-axis.



3.1. Determine the equation of the diameter LN.
3.2. Calculate the coordinates of L.
3.3. Determine the equation of the circle.
3.4. Write down the coordinates of N.
3.5. Write down the equation of NQ.
3.6. If the length of the diameter is doubled and the circle is translated horizontally 6 units to the right, write down the equation of the new circle.

Activity 4

A circle centred at N(3 ; 2) touches the x-axis at point L. The line PQ, defined by the equation $y=\frac{4}{3}x+\frac{4}{3}$, is a tangent to the same circle at point A.

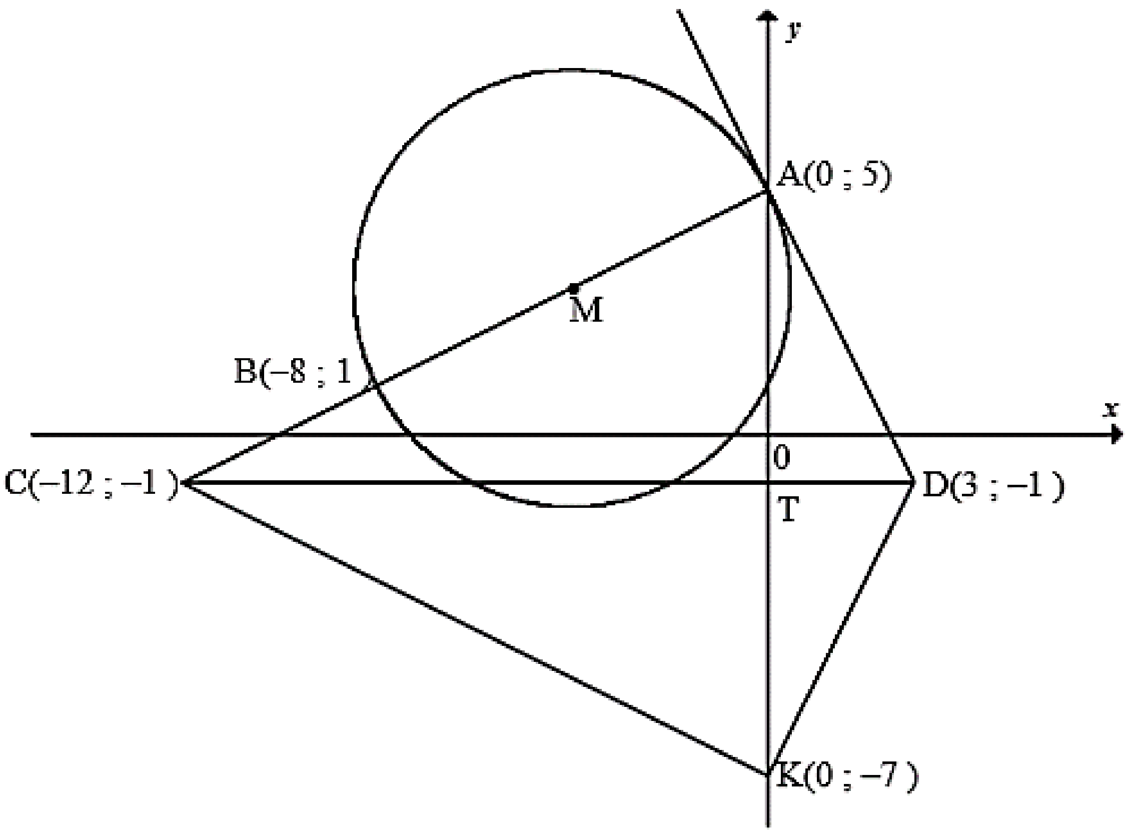


Join the line PQ and answer the questions that follow:

4.1. Why is NL perpendicular to OL?
4.2. Determine the coordinates of L.
4.3. Determine the equation of the circle with centre N in the form $(x-a)^2+(y-b)^2=r^2$.
4.4. Calculate the length of KL.
4.5. Determine the equation of the diameter AB in the form $y=mx+c$.
4.6. Show that the coordinates of A are $\left(\frac{7}{5};\frac{16}{5}\right)$.
4.7. Calculate the length of KA.
4.8. Why is KLNA a kite?
4.9. Show that $A\hat{B}K=45°$.
4.10. If the given circle is reflected about the $x-axis$, give the coordinates of the centre of the new circle.

Activity 5

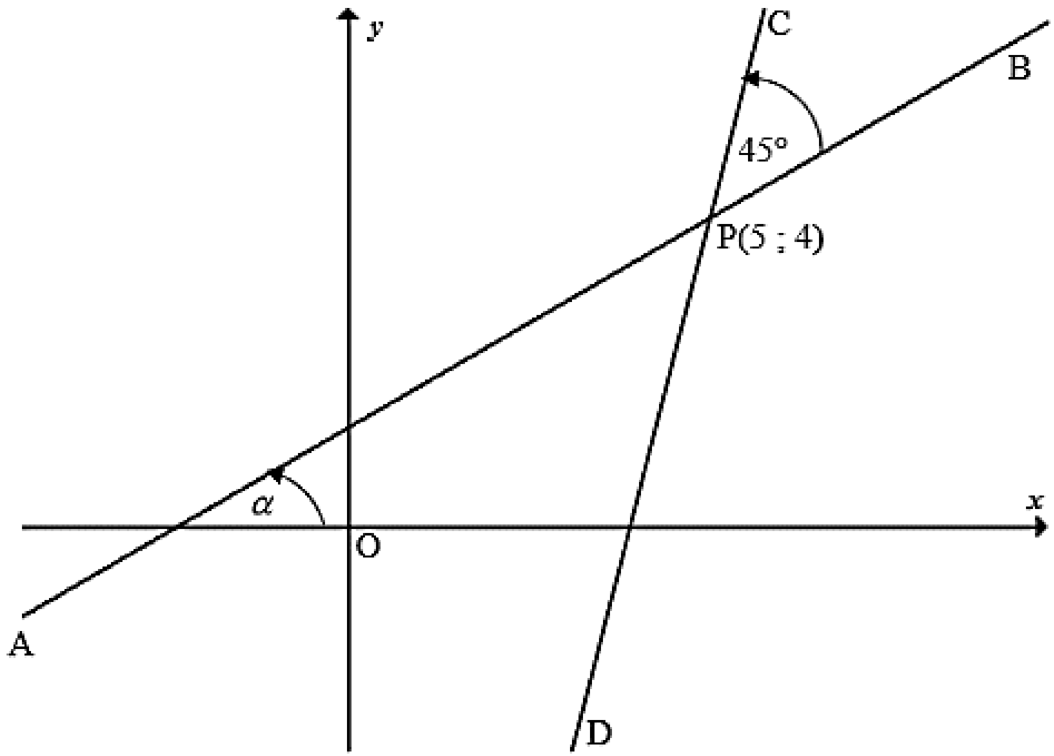
A(0 ; 5) and B(–8 ; 1) are two points on the circumference of the circle centre M, in a Cartesian plane. M lies on AB. DA is a tangent to the circle at A. The coordinates of D are (3 ; –1) and the coordinates of C are (–12 ; –1). Points C and D are joined. K is the point (0 ; –7). CTD is a straight line.



5.1. Show that the coordinates of M, the midpoint of AB, are $(-4;3)$.
5.2. Determine the equation of the tangent AD.
5.3. Determine the length of AM.
5.4. Determine the equation of circle centre M in the form $ax^2 + by^2 + cx + dy + e = 0$.
5.5. Quadrilateral ACKD is one of the following:
Parallelogram; kite; rhombus; rectangle
Which one is it? Justify your answer.

Activity 6

The straight line AB has the equation $5y-3x-5=0$. Another straight line CD is drawn to intersect AB at P(5 ; 4) such that the acute angle between AB and CD is 45°.



6.1. Determine the gradient of the line CD

6.2. Hence, or otherwise, determine the equation of the line CD.

Activity 7

7.1. Determine the centre and radius of the circle with the equation $x^2+y^2+8x+4y-38=0$.

7.2. A second circle has the equation $(x-4)^2+(y-6)^2=26$. Calculate the distance between the centres of the two circles.

7.3. Hence, show that the circles described in QUESTION 7.1 and QUESTION 7.2 intersect each other.

7.4. Show that the two circles intersect along the line $y=-x+4$.

Activity 8

8.1. The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

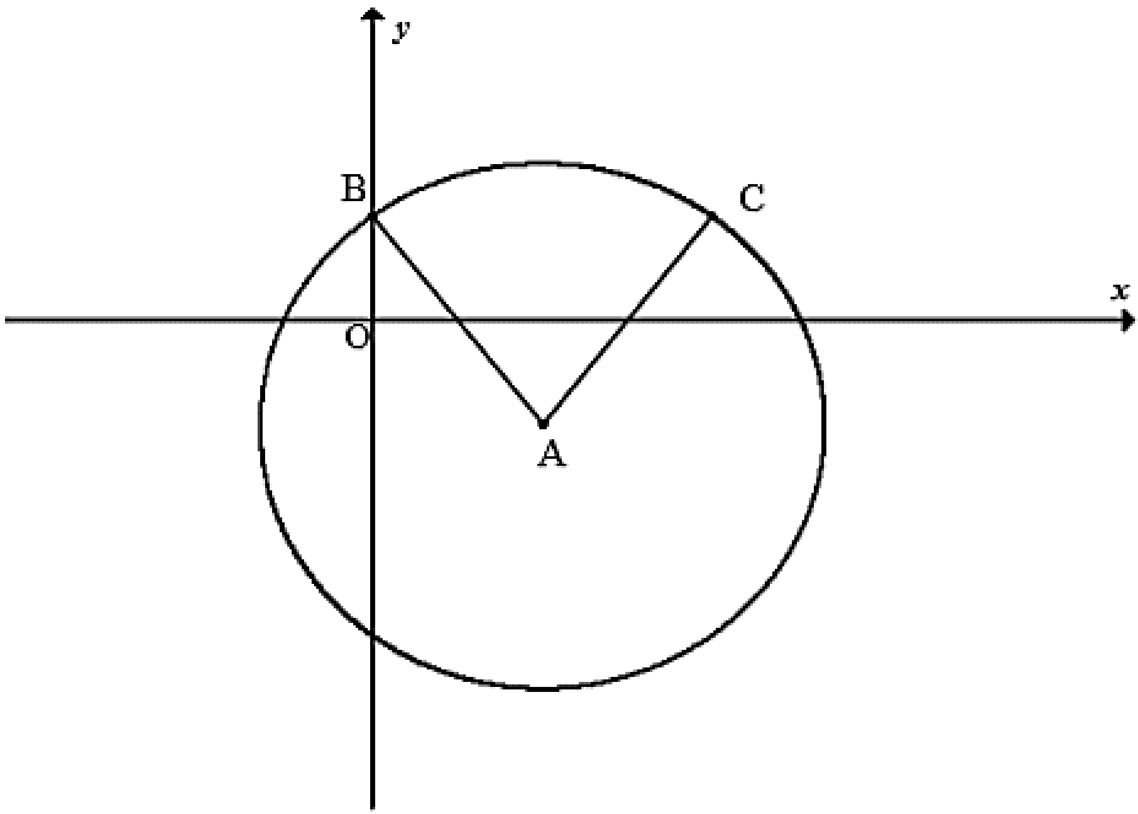
8.1.1. Prove that the point $(2;-9)$ is on the circumference of the circle.

8.1.2. Determine an equation of the tangent to the circle at the point $(2;-9)$.

8.2. Calculate the length of the tangent AB drawn from the point $A(6;4)$ to the circle with equation $(x-3)^2 + (y+1)^2 = 10$.

Activity 9

The circle, with centre A and equation $(x-3)^2 + (y+2)^2 = 25$ is given in the following diagram. B is a y-intercept of the circle.



9.1. Determine the coordinates of B.

9.2. Write down the coordinates of C, if C is the reflection of B in the line $x = 3$.

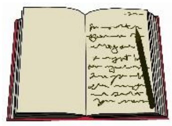
9.3. The circle is enlarged through the origin by a factor of $\frac{3}{2}$.

Write down the equation of the new circle, centre $A^{/}$, in the form $(x-a)^2+(y-b)^2=r^2$.

9.4. In addition to the circle with centre A and equation $(x-3)^2+(y+2)^2=25$, you are given the circle $(x-12)^2+(y-10)^2=100$ with centre B.

9.4.1 Calculate the distance between the centres A and B.

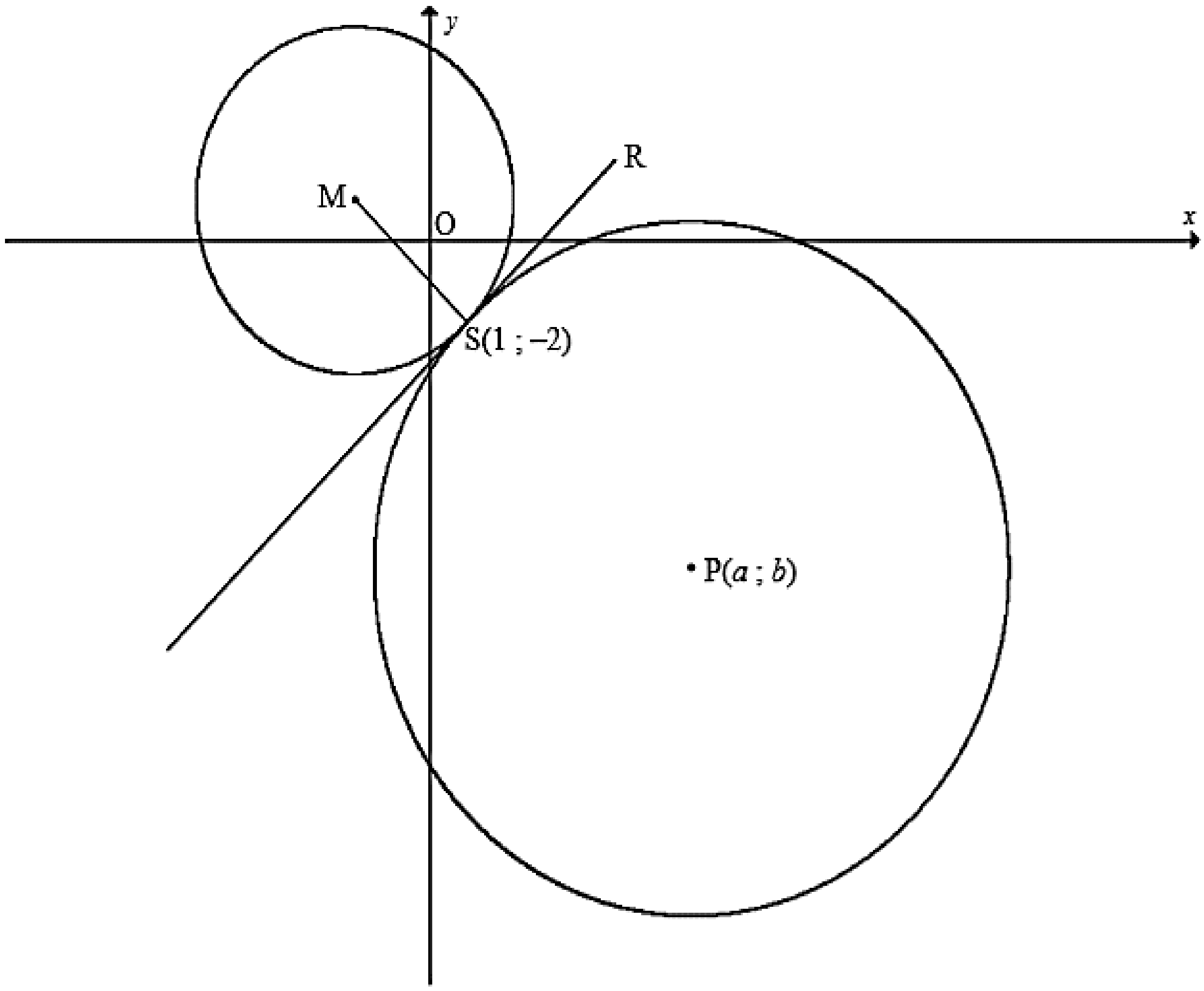
9.4.2. In how many points do these two circles intersect? Justify your answer.

Activity 10	

In the figure below, a circle with centre M is drawn. The equation of the circle is $(x+2)^2+(y-1)^2=r^2$.

S(1 ; −2) is a point on the circle.

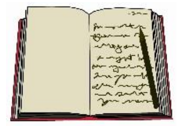
SR is a tangent to the circle.



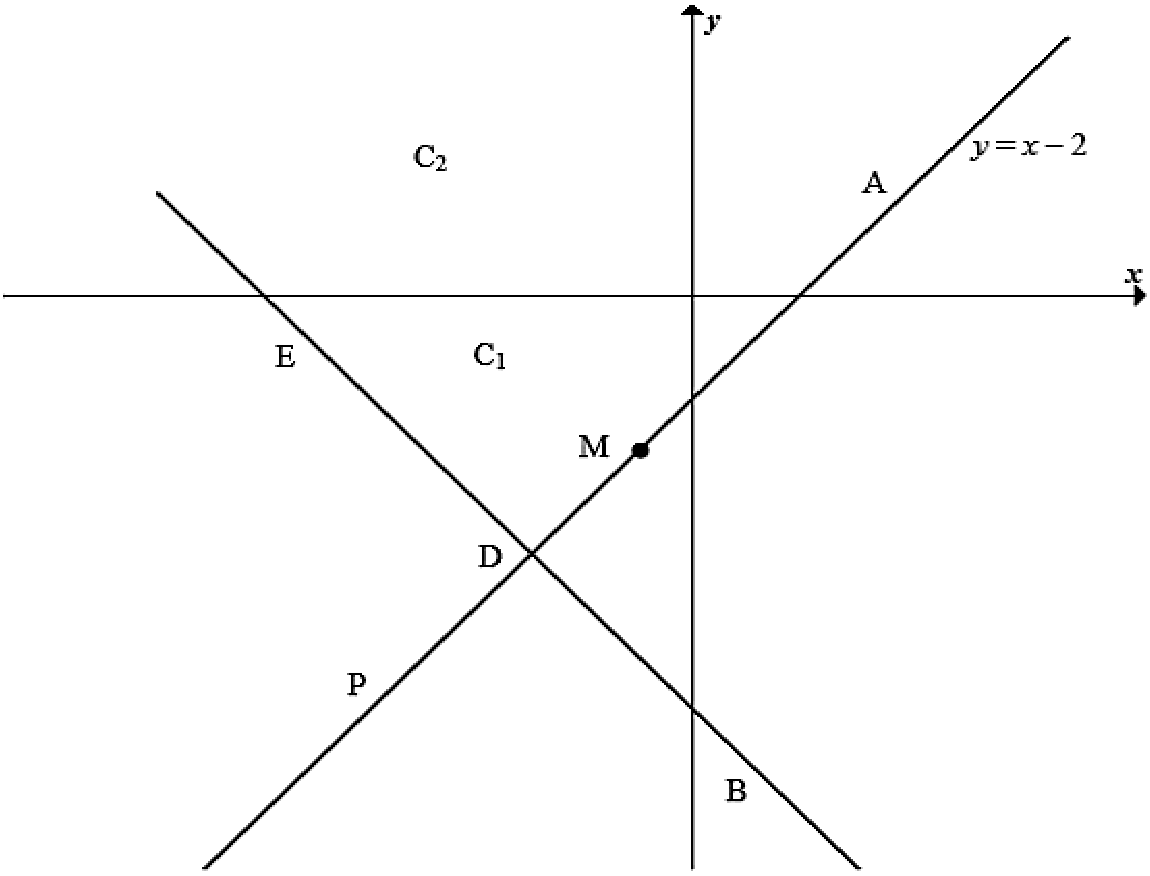
10.1. Write down the coordinates of M and the radius of the circle centre M.

10.2. Determine the equation of the tangent RS in the form $y=mx+c$.

10.3. The circles having centres P and M touch externally at point S. SR is a tangent to both these circles. If $MS:MP = 1:3$, determine the coordinates $(a;b)$ of point P.

Activity 11	

Circles C_1 and C_2 in the figure below have the same centre M. P is a point on C_2. PM intersects C_1 at D. The tangent DB to C_1 intersects C_2 at B. The equation of circle C_1 is given by $x^2 + 2x + y^2 + 6y + 2 = 0$ and the equation of line PM is $y = x - 2$.



11.1. Determine the following:
11.1.1. The coordinates of centre M.
11.1.2. The radius of circle C_1.

11.2. Determine the coordinates of D, the point where line PM and circle C_1 intersect.

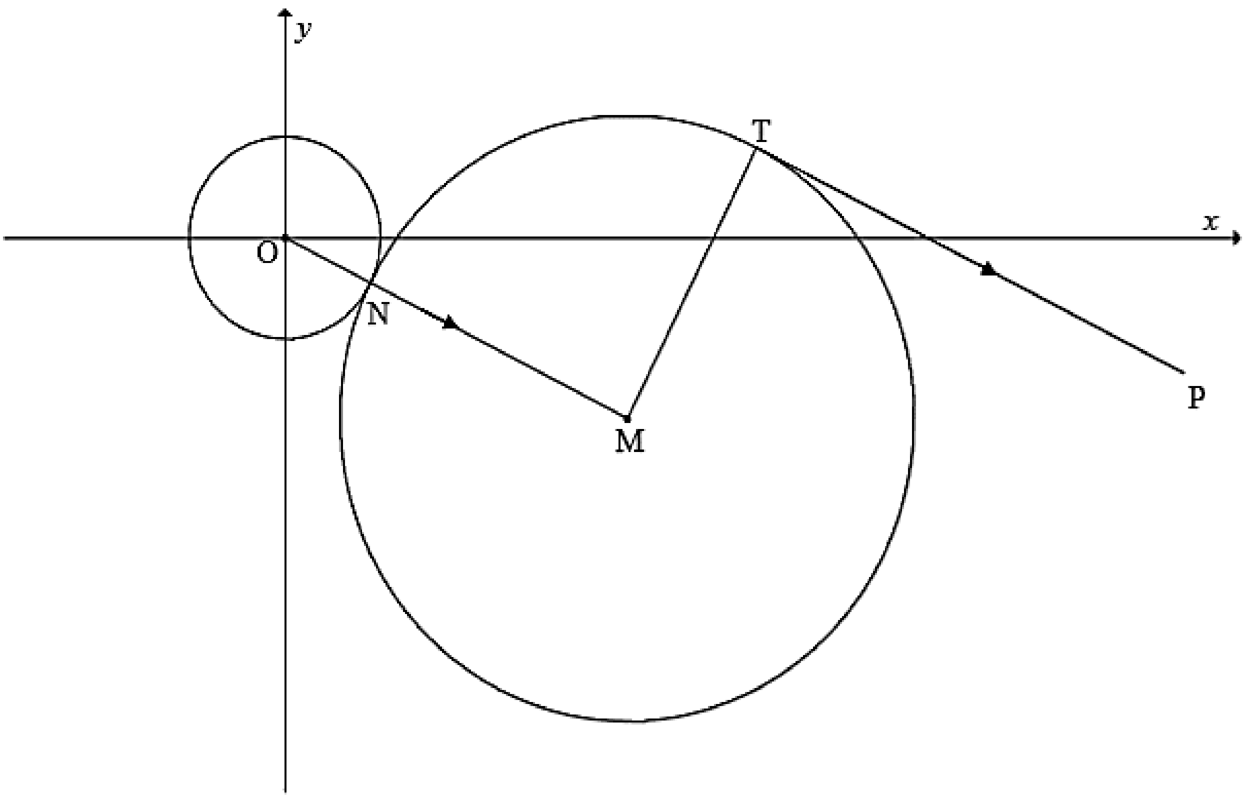
11.3. If it is given that $DB = 4\sqrt{2}$, determine MB, the radius of circle C_2.

11.4. Write down the equation of C_2 in the form $(x-a)^2 + (y-b)^2 = r^2$.

11.5. Is the point $F(2\sqrt{5};0)$ inside circle C_2? Support your answer with calculations.

Activity 12

In the diagram below, the equation of the circle with centre M is $(x-8)^2+(y+4)^2=45$. PT is a tangent to this circle at T and PT is parallel to OM. Another circle, having centre O, touches the circle having centre M at N.

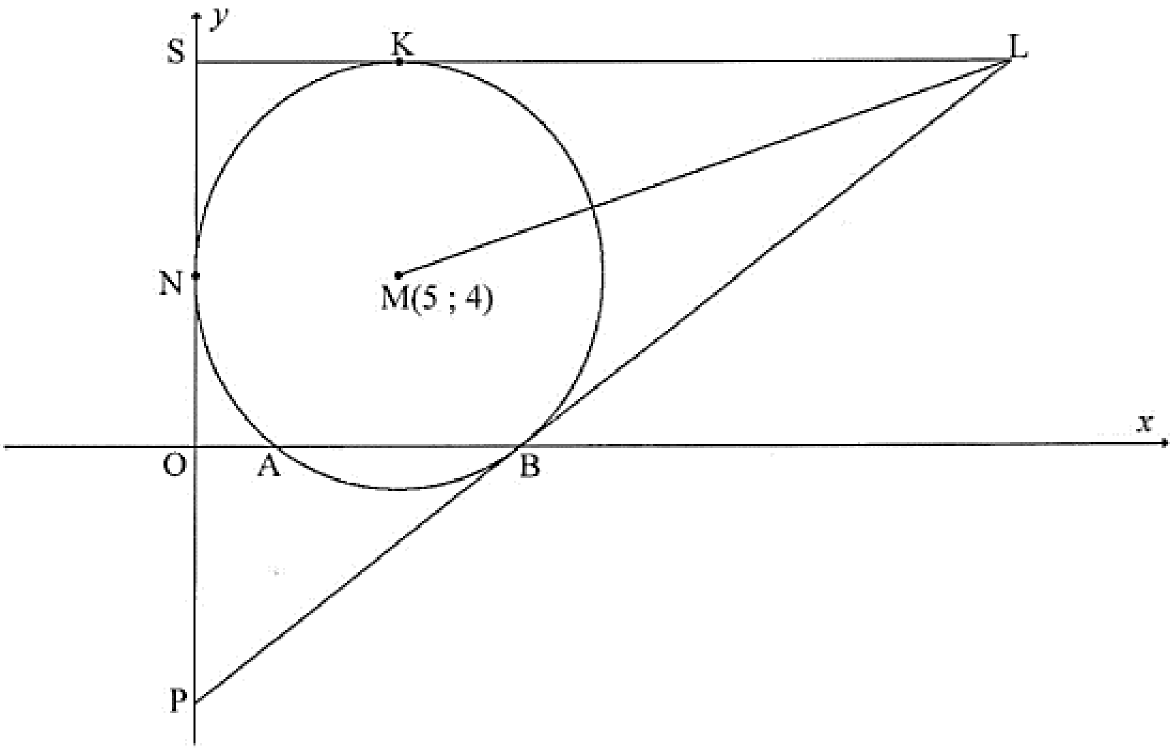


12.1. Write down the coordinates of M.
12.2. Calculate the length of OM. Leave your answers in simplest surd form.
12.3. Calculate the length of ON. Leave your answers in simplest surd form.
12.4. Calculate the size of $O\hat{M}T$.
12.5. Determine the equation of MT in the form $y=mx+c$.
12.6. Calculate the coordinates of T.

MIXED QUESTIONS

Activity 1

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



1.1. Write down the length of the radius of the circle having centre M.

1.2. Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$.

1.3. Calculate the coordinates of A.

1.4. If the coordinates of B are $(8;0)$, calculate:

1.4.1. The gradient of MB.

1.4.2. The equation of the tangent PB in the form $y=mx+c$.

1.5. Write down the equation of tangent SKL.

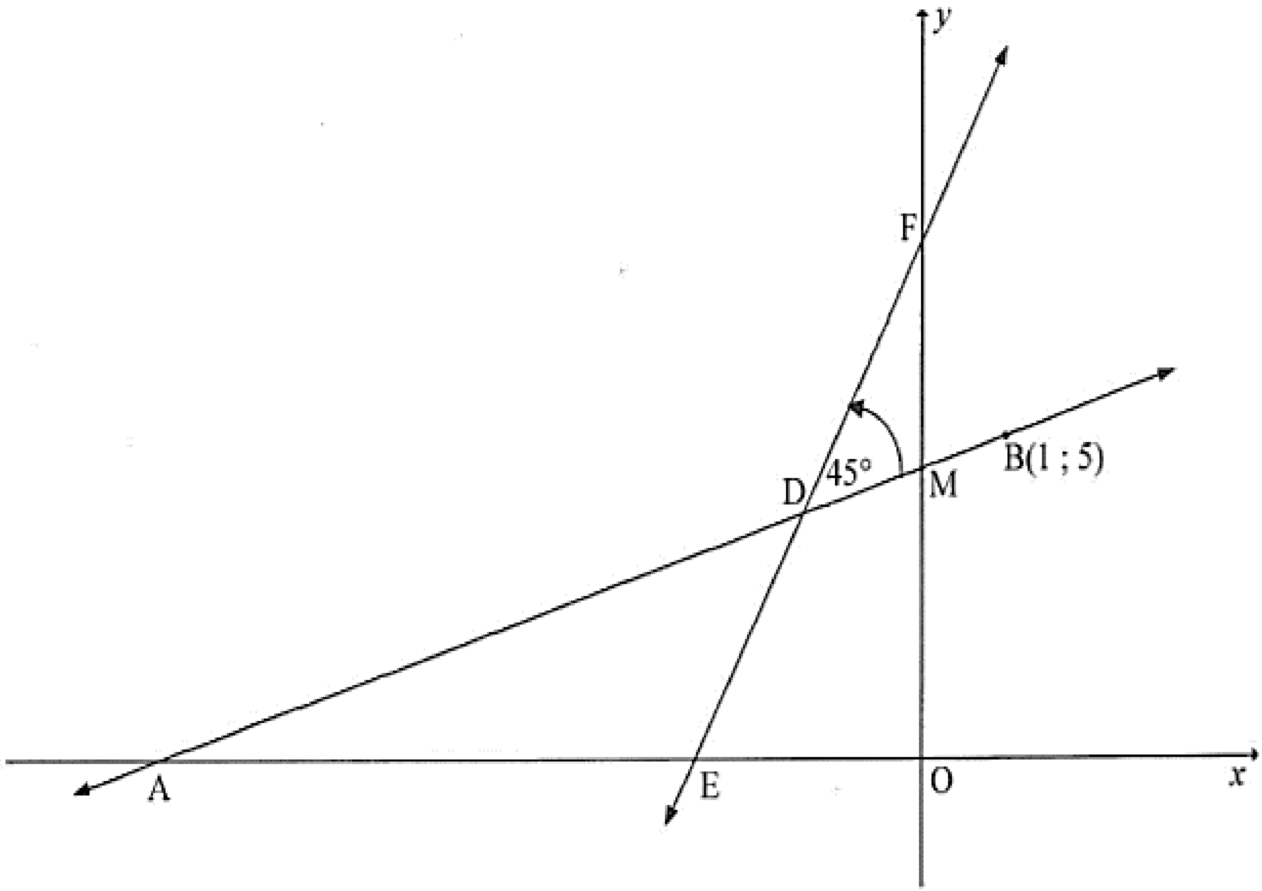
1.6. Show that L is the point $(20;9)$.

1.7. Calculate the length of ML in surd form.

1.8. Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$.

Activity 2

In the diagram below, E and F respectively are the x- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has x- and y-intercepts A and M respectively.



2.1. Determine the coordinates of E.

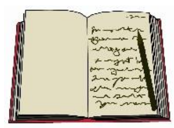
2.2. Calculate the size of $D\hat{A}E$.

2.3. Determine the equation of AB in the form $y = mx + c$.

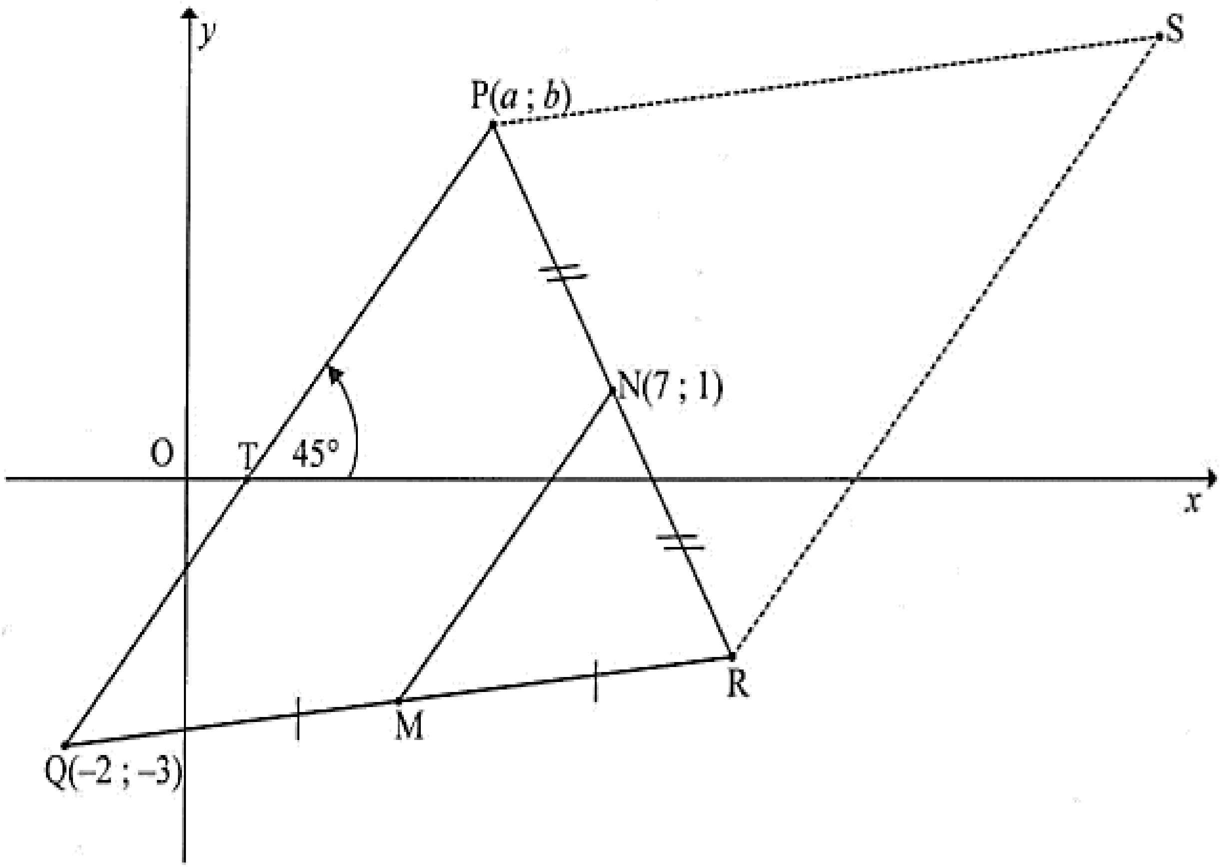
2.4. If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D.

2.5. Calculate the area of quadrilateral DMOE.

Activity 3



In the diagram below, the line joining Q(–2 ; –3) and P(a ; b), a and $b > 0$, makes an angle of 45° with the positive x-axis. QP = $7\sqrt{2}$ units. N(7 ; 1) is the midpoint of PR and M is the midpoint of QR.

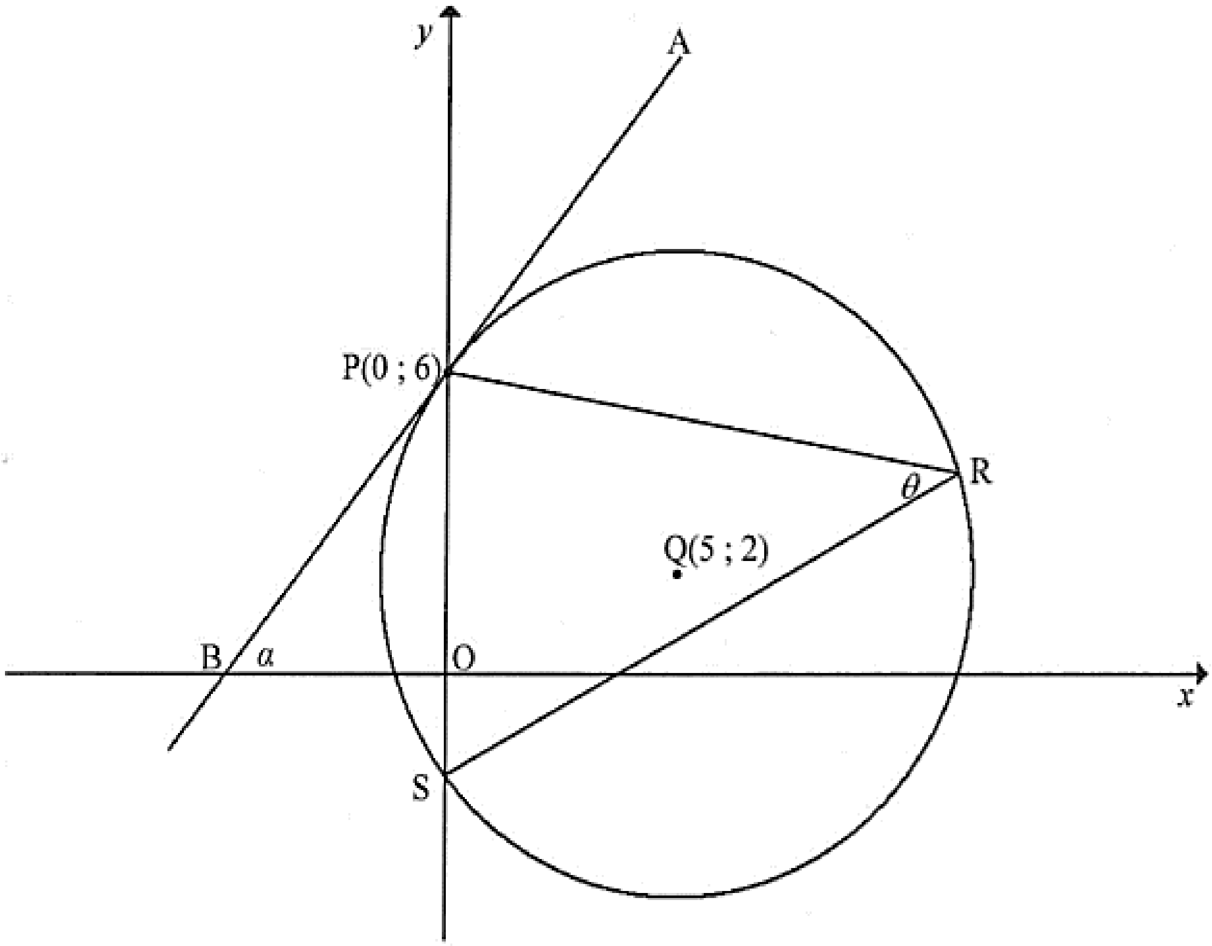


Determine:

3.1. The gradient of PQ.
3.2. The equation of MN in the form $y = mx + c$ and give reasons.
3.3. The length of MN.
3.4. The length of RS.
3.5. The coordinates of S such that PQRS, in this order, is a parallelogram.
3.6. The coordinates of P.

Activity 4

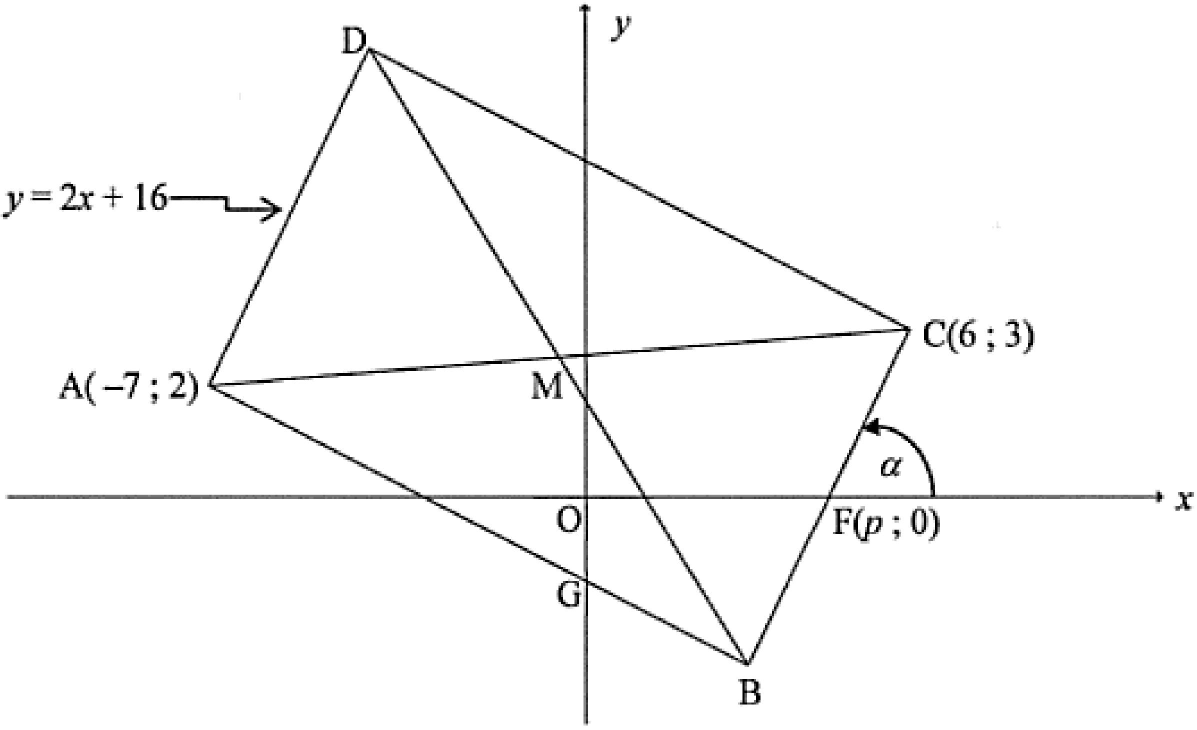
In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1. Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.
4.2. Calculate the coordinates of S.
4.3. Determine the equation of the tangent APB in the form $y = mx + c$.
4.4. Calculate the size of α.
4.5. Calculate, with reasons, the size of θ.
4.6. Calculate the area of ΔPQS.

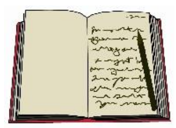
Activity 5

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.

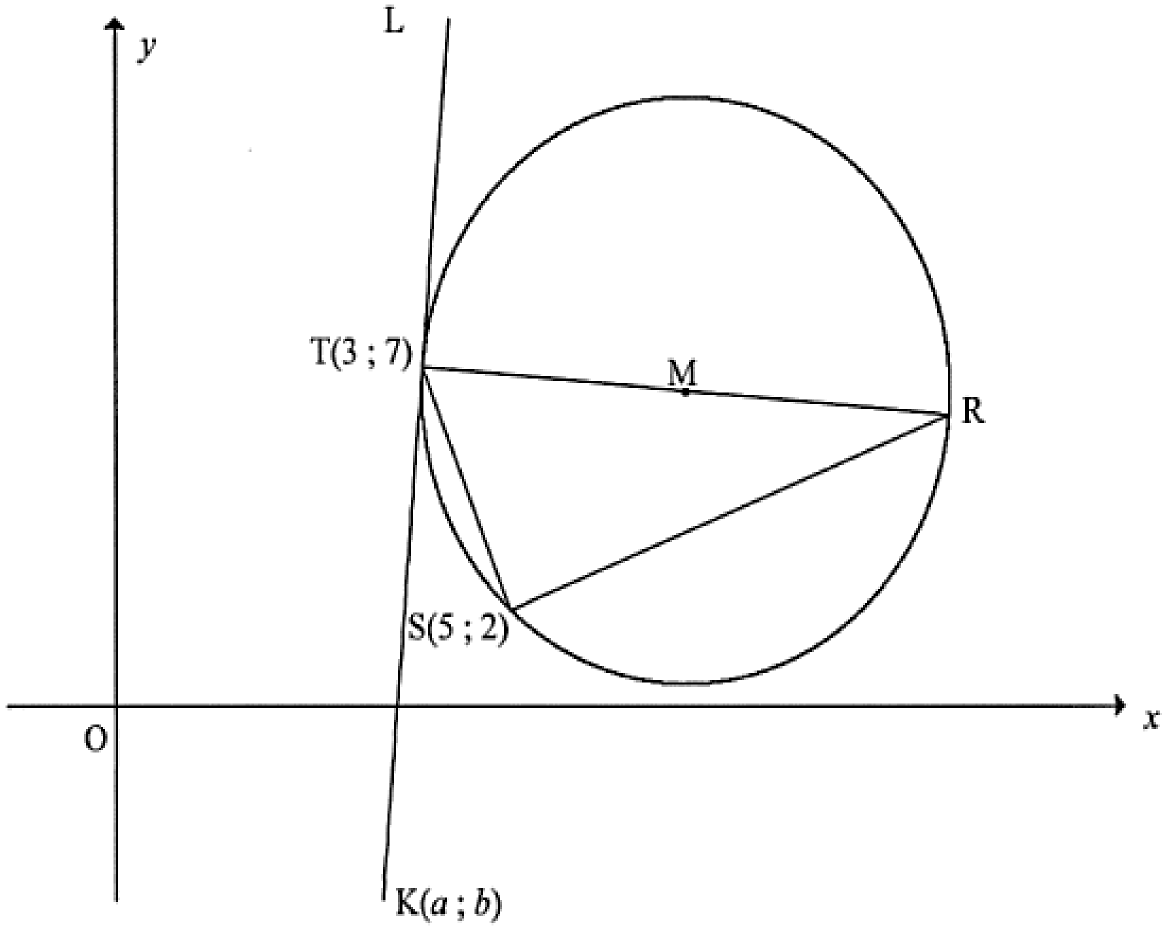


5.1. Calculate the coordinates of M.
5.2. Write down the gradient of BC in terms of p .
5.3. Hence, calculate the value of p .
5.4. Calculate the length of DB.
5.5. Calculate the size of α .
5.6. Calculate the size of $O\hat{G}B$.
5.7. Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2+(y-b)^2=r^2$.
5.8. If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer.

Activity 6



In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.

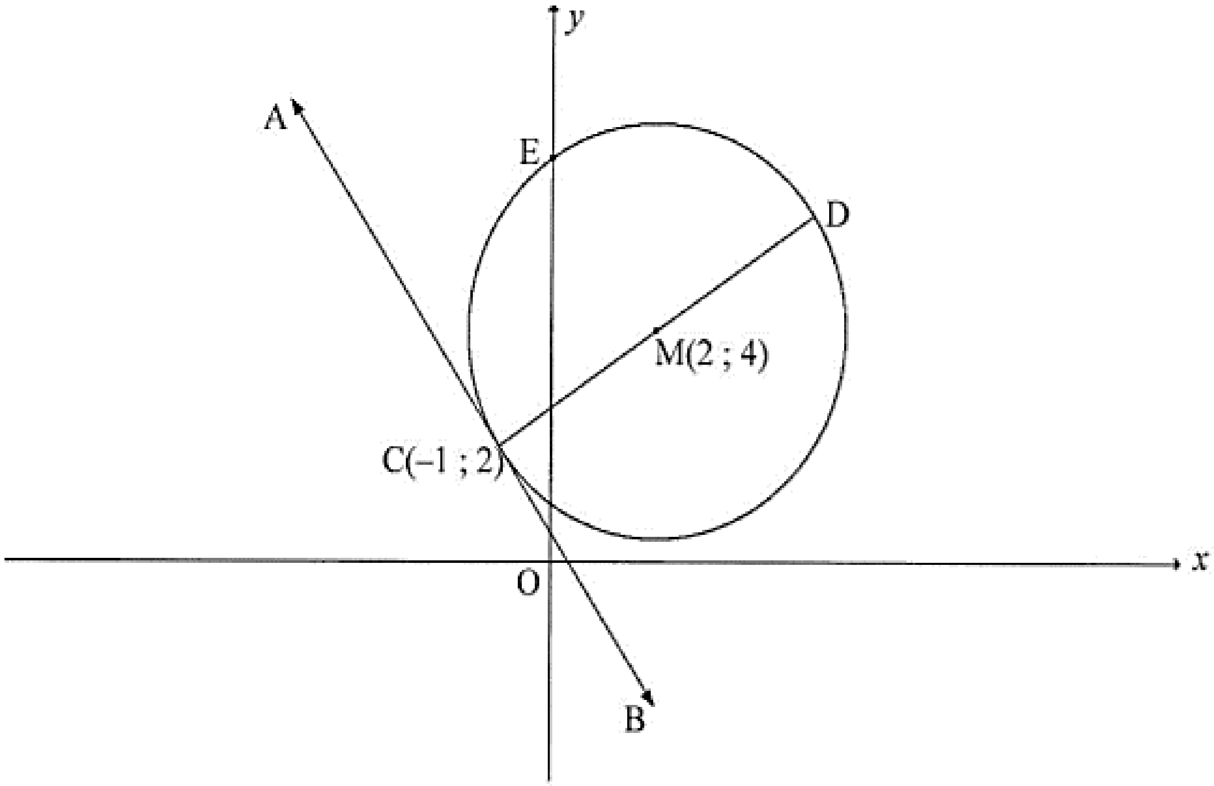


6.1. Give a reason why $T\hat{S}R = 90°$.
6.2. Calculate the gradient of TS.
6.3. Determine the equation of the line SR in the form $y = mx + c$.
6.4. The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

- 6.4.1. Calculate the length of TR in surd form.
- 6.4.2. Calculate the coordinates of R.
- 6.4.3. Calculate $\sin R$.
- 6.4.4. Show that $b = 12a - 29$.
- 6.4.5. If TK=TR, calculate the coordinates of K.

Activity 7

In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



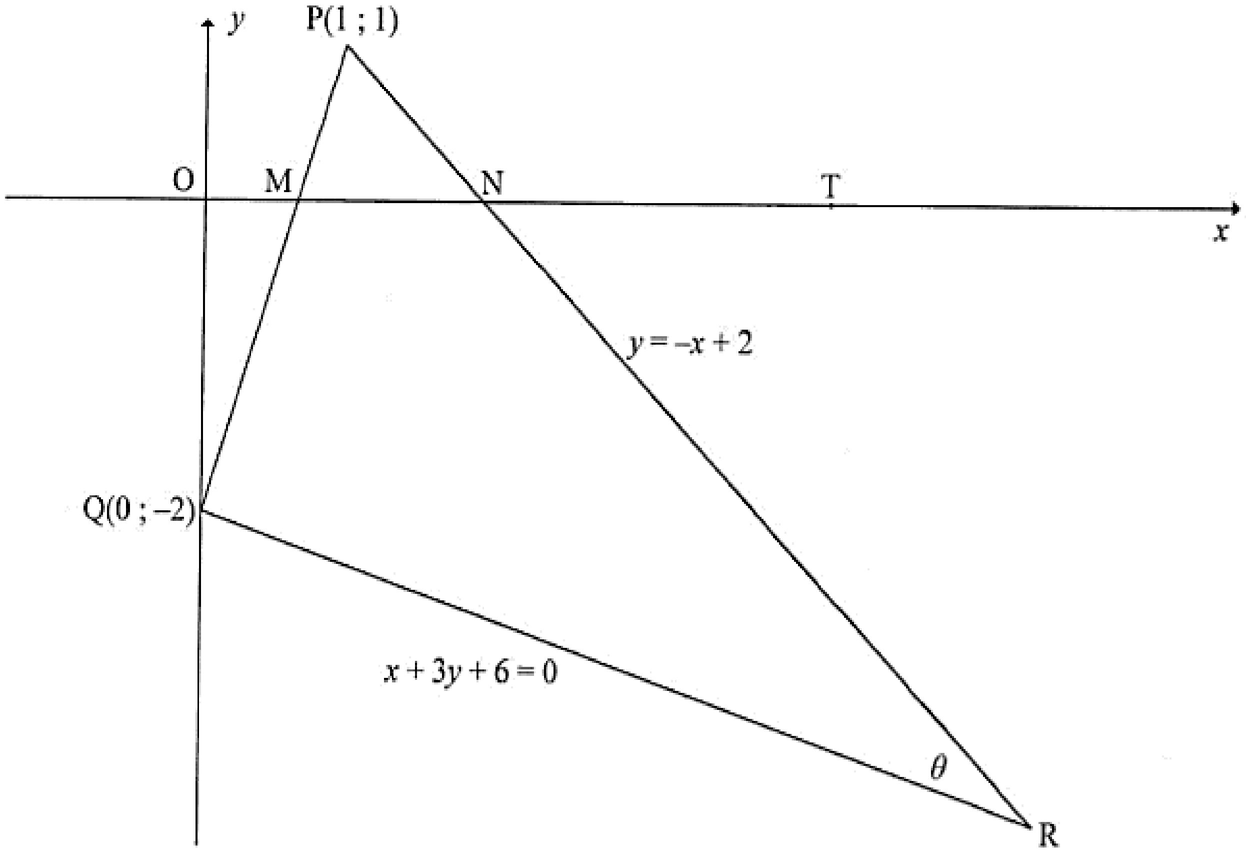
7.1. Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$.
7.2. Write down the coordinates of D.
7.3. Determine the equation of AB in the form $y=mx+c$.
7.4. Calculate the coordinates of E.
7.5. Show that EM is parallel to AB.

Activity 8

Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations.

Activity 9

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



9.1. Determine the gradient of QP.

9.2. Prove that $P\hat{Q}R = 90°$.

9.3. Determine the coordinates of R.

9.4. Calculate the length of PR. Leave your answer in surd form.

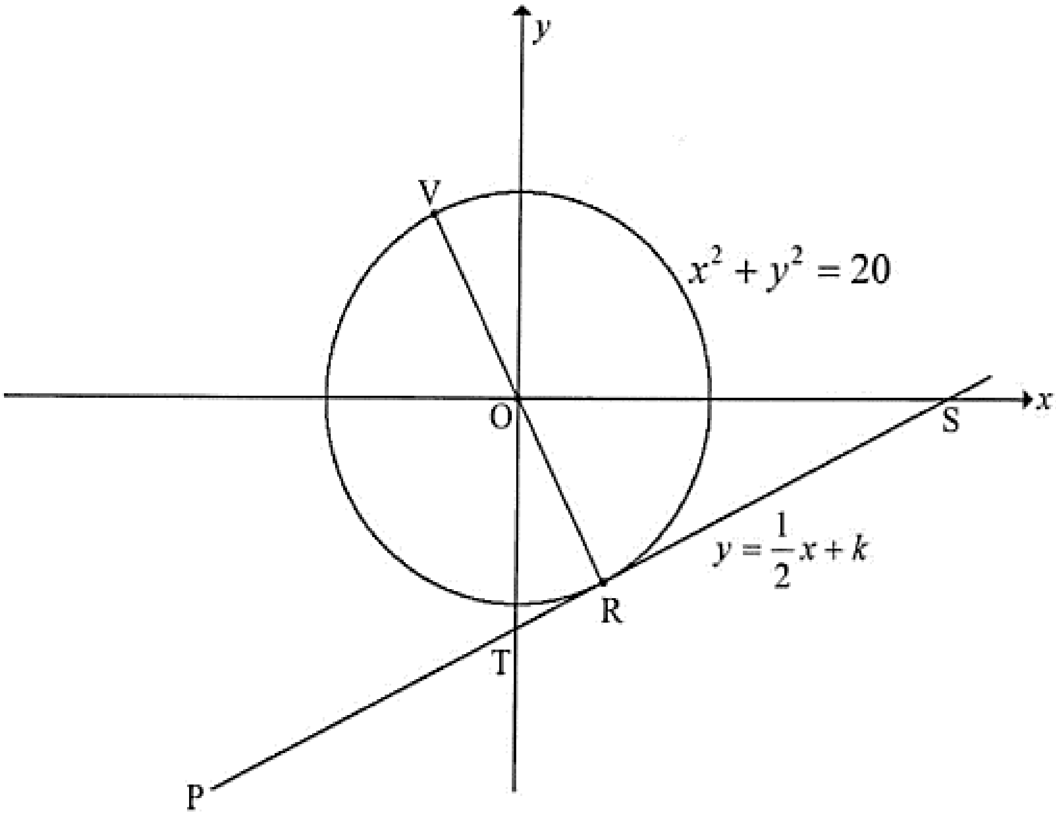
9.5. Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$.

9.6. Determine the equation of a tangent to the circle passing through P, Q and R at P in the form $y = mx + c$.

9.7. Calculate the size of θ.

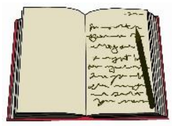
Activity 10

In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.

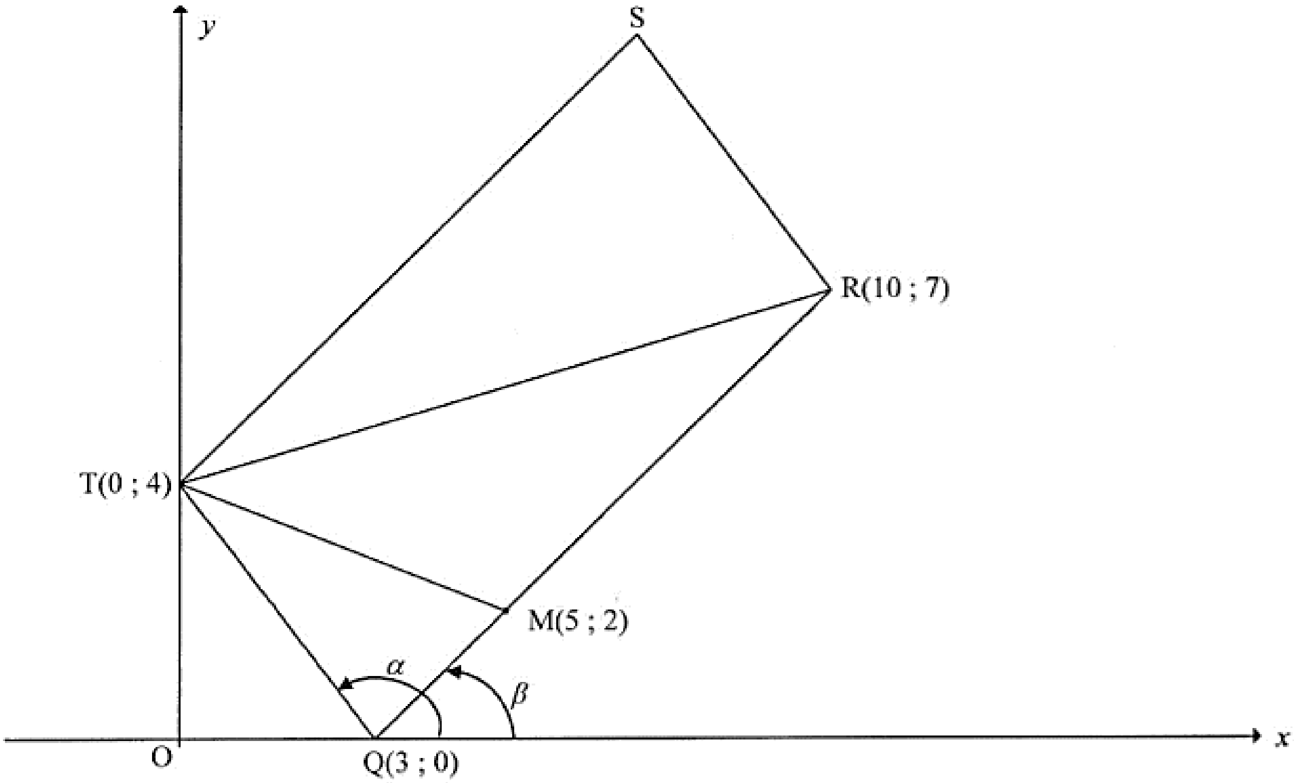


10.1. Determine, giving reasons, the equation of OR in the form $y = mx + c$.
10.2. Determine the coordinates of R.
10.3. Determine the area of ΔOTS, given that $R(2;-4)$.
10.4. Calculate the length of VT.

Activity 11



In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.

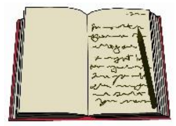


11.1. Calculate the gradient of TQ.
11.2. Calculate the length of RQ. Leave your answer in surd form.
11.3. $F(k;-8)$ is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k.
11.4. Calculate the coordinates of S.
11.5. Calculate the size of $T\hat{S}R$.
11.6. Calculate, in the simplest form, the ratio of:

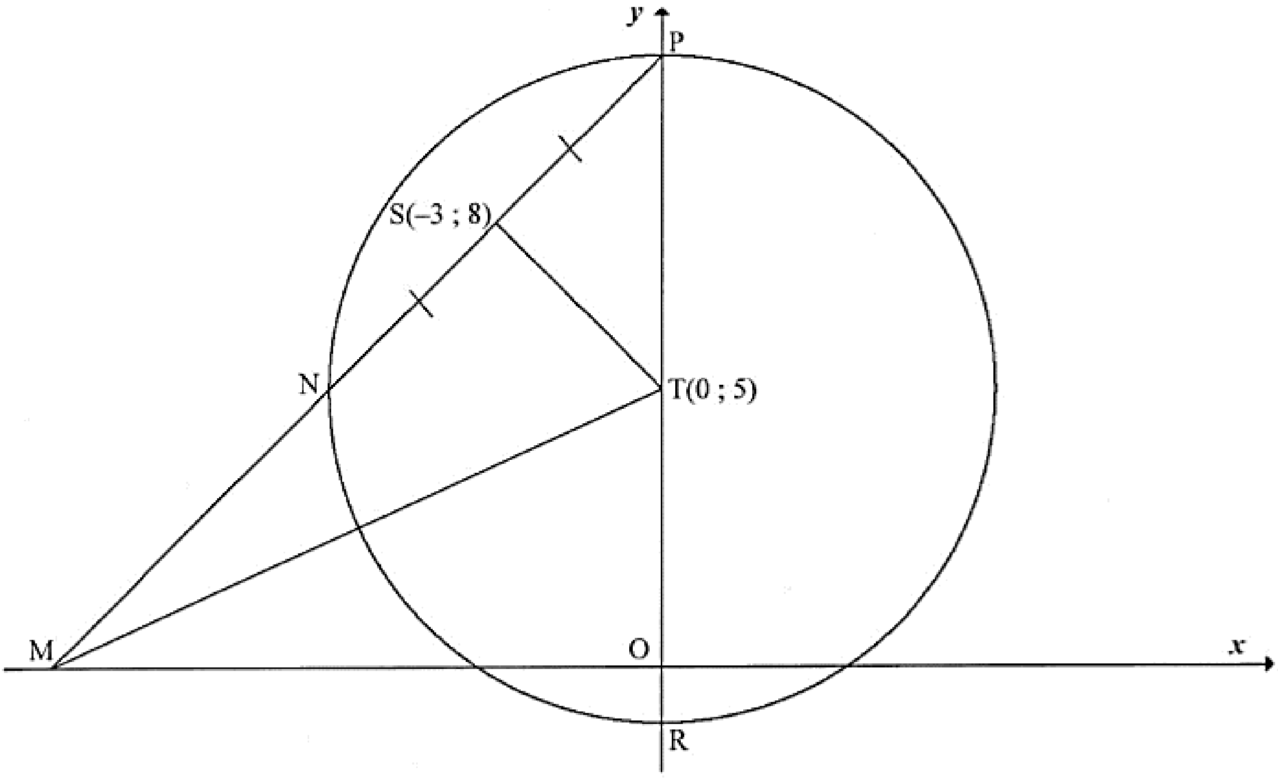
11.6.1. $\dfrac{\text{MQ}}{\text{RQ}}$.

11.6.2. $\dfrac{\text{Area of } \Delta\text{TQM}}{\text{Area of parallelogram RQTS}}$.

Activity 12



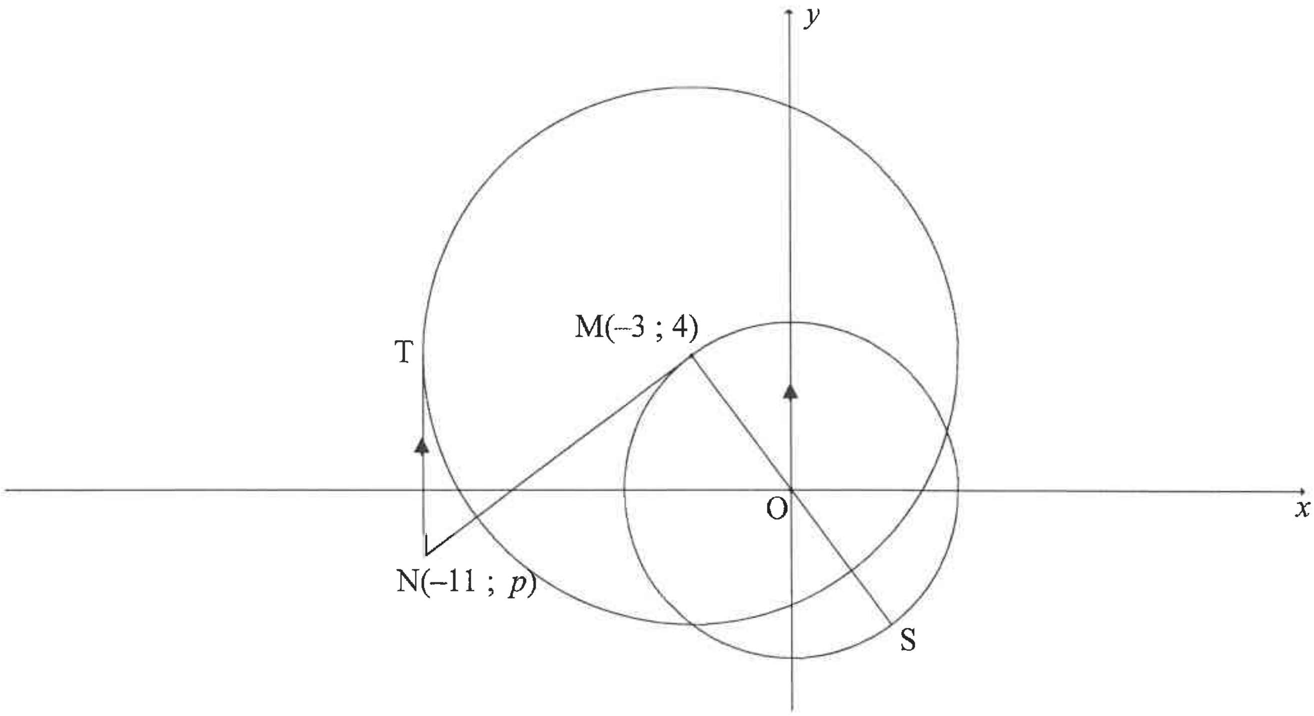
In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



12.1. Give a reason why $TS \perp NP$.
12.2. Determine the equation of the line passing through N and P in the form $y = mx + c$.
12.3. Determine the equations of the tangents to the circle that are parallel to the $x - axis$.
12.4. Determine the length of MT.
12.5. Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle STM in the form $(x-a)^2 + (y-b)^2 = r^2$.

QUESTION 4

M(–3 ; 4) is the centre of the large circle and a point on the small circle having centre O(0; 0). From N(–11 ; p), a tangent is drawn to touch the large circle at T with NT is parallel to the y-axis. NM is a tangent to the smaller circle at M with MOS a diameter.



4.1	Determine the equation of the small circle.	(2)
4.2	Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$	(3)
4.3	Determine the equation of NM in the form $y=mx+c$	(4)
4.4	Calculate the length of SN.	(5)
4.5	If another circle with centre B(–2 ; 5) and radius k touches the circle centred at M, determine the value(s) of k, correct to ONE decimal place.	(5)

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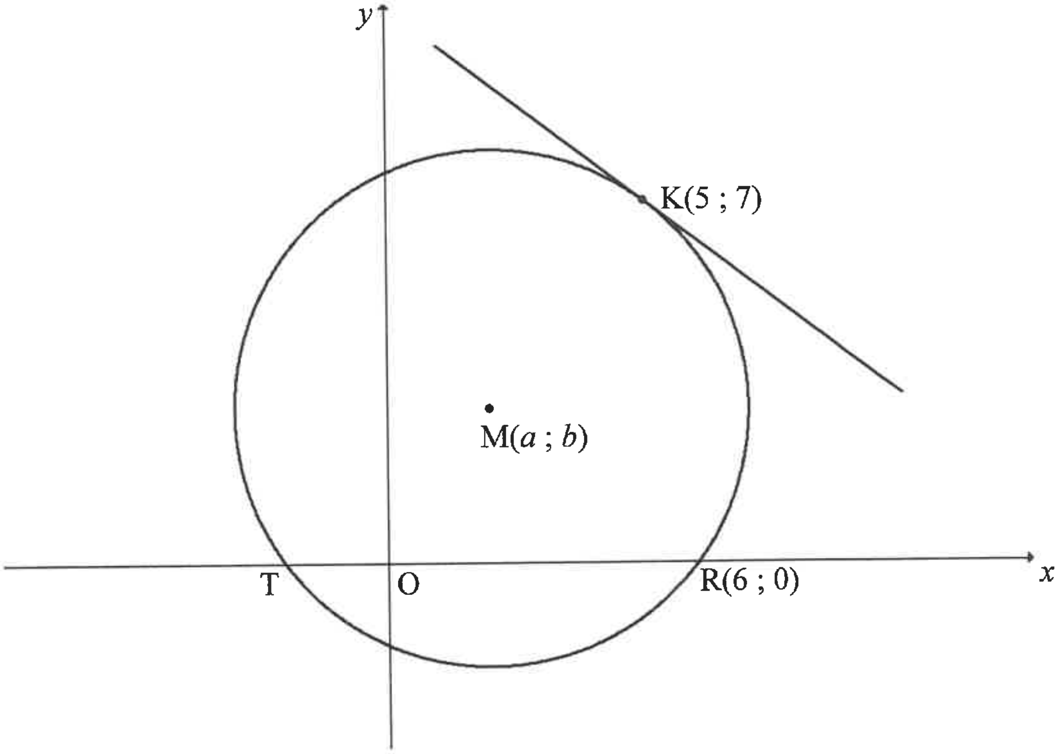
QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

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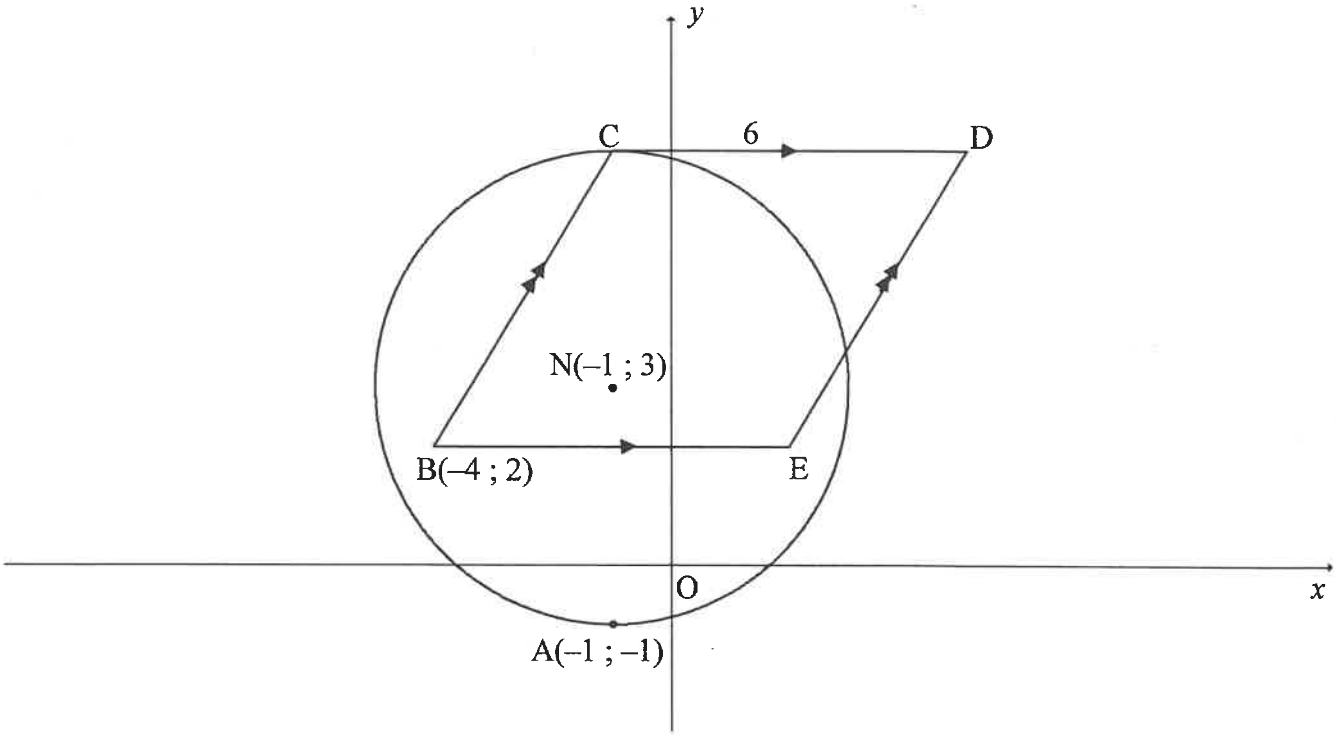
QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

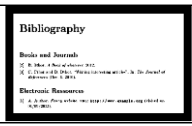
Calculate the:

4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)
[15]

[48]

Bibliography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