JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: ANALYTICAL GEOMETRY

SOLUTIONS BOOK

LEARNER/TEACHER

TERM 2 WORK

Lines, Triangles and Quadrilaterals

Equation of a Circle

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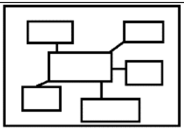
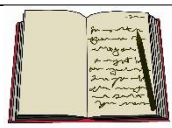
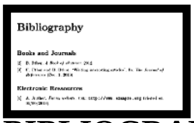
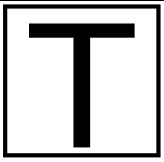
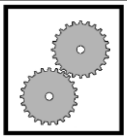
ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

TOPIC 1: Lines, Triangles and Quadrilaterals

Activity 1

$$m_{KL} = \frac{5-(-3)}{-3-2} = -\frac{8}{5}$$

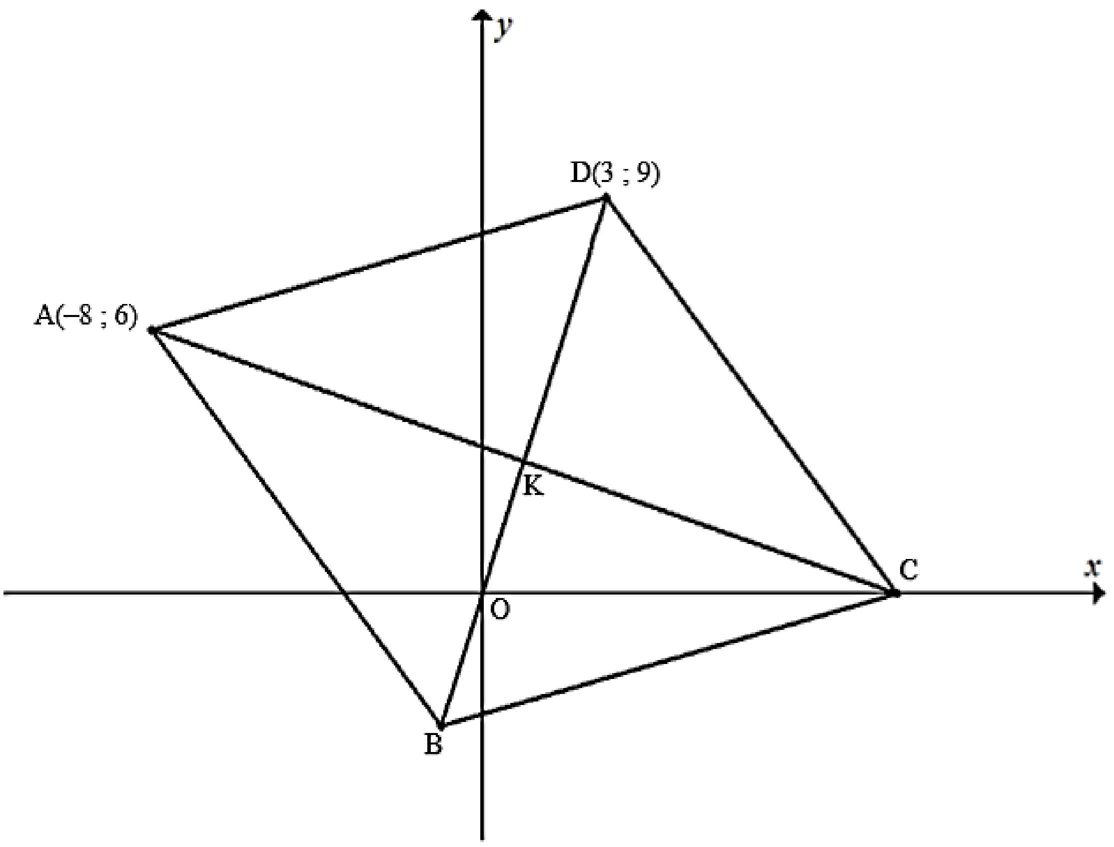
$$m_{LN} = \frac{-3-(-9)}{2-5} = -\frac{6}{3} = -2$$

$$m_{KN} = \frac{5-(-9)}{-3-5} = -\frac{14}{8} = -\frac{7}{4}$$

$$\therefore m_{KL} \neq m_{LN} \neq m_{KN}$$

Points K, L and N are not collinear

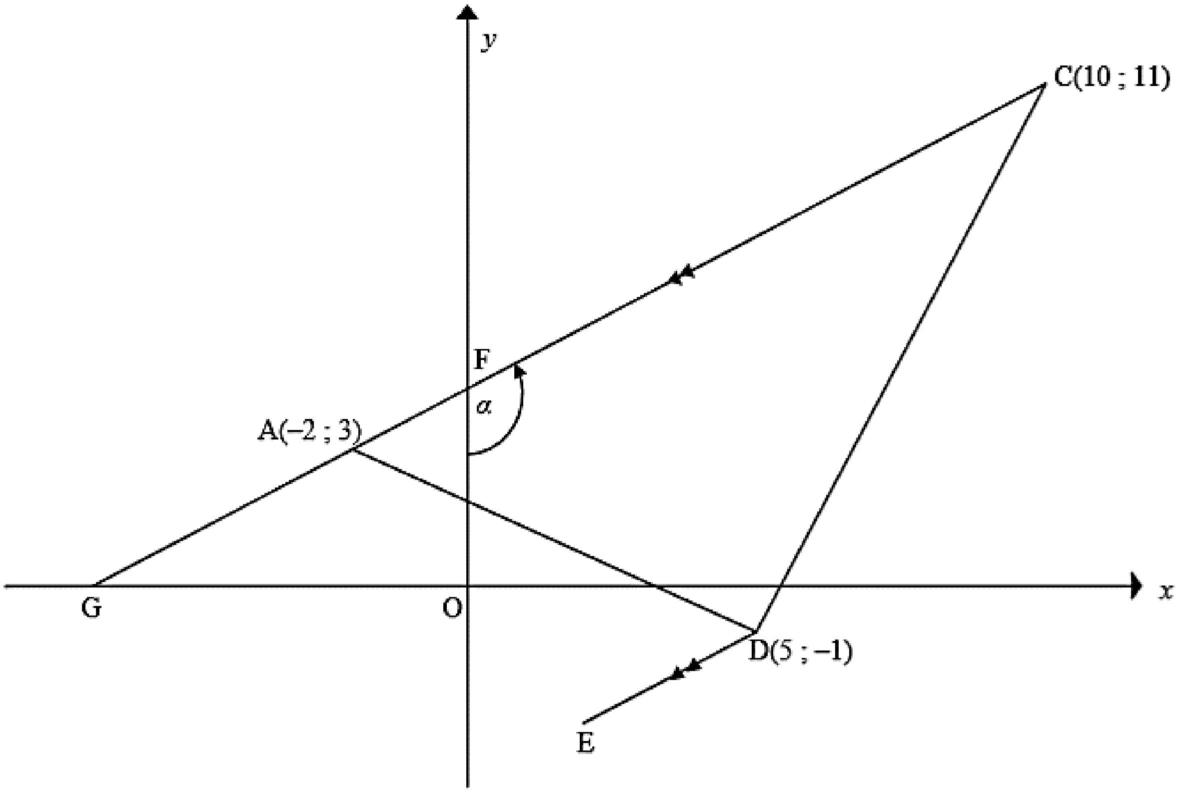
Activity 2



2.1.	$d_{AD}=\sqrt{(3-(-8))^2+(9-6)^2}$ $d_{AD}=\sqrt{130}$ $\text{Perimeter}=4\times sides$ $\text{Perimeter}=4\times\sqrt{130}$ $\text{Perimeter}=4\sqrt{130}$
2.2.	BD: $y=3x$ and $m_{BD}=3$ $m_{AC}\times m_{BD}=-1$ (AC $\perp$ BD) $\therefore m_{AC}=-\frac{1}{3}$ $y-6=-\frac{1}{3}(x-(-8))$ $y=-\frac{1}{3}x-\frac{8}{3}+6$ $y=-\frac{1}{3}x+\frac{10}{3}$ OR $x+3y=10$
2.3.	$y_1=y_2$ $3x=-\frac{1}{3}x+\frac{10}{3}$ $9x+x=10$ $\therefore x_K=1$ $y=3(1)$ $y_K=3$ $\therefore$ K(1;3)
2.4.	$\frac{x_B+3}{2}=1$ and $\frac{y_B+9}{2}=3$ $\therefore x_B=-1$ $\therefore y_B=-3$ B(−1;−3) **OR** By Inspection: B(−1;−3)
2.5.	$m_{AD}=\frac{9-6}{3-(-8)}=\frac{3}{11}$ $m_{AB}=\frac{6-(-3)}{-8-(-1)}=-\frac{9}{7}$ $\therefore m_{AD}\times m_{AB}\neq-1$

	The rhombus is not a square.

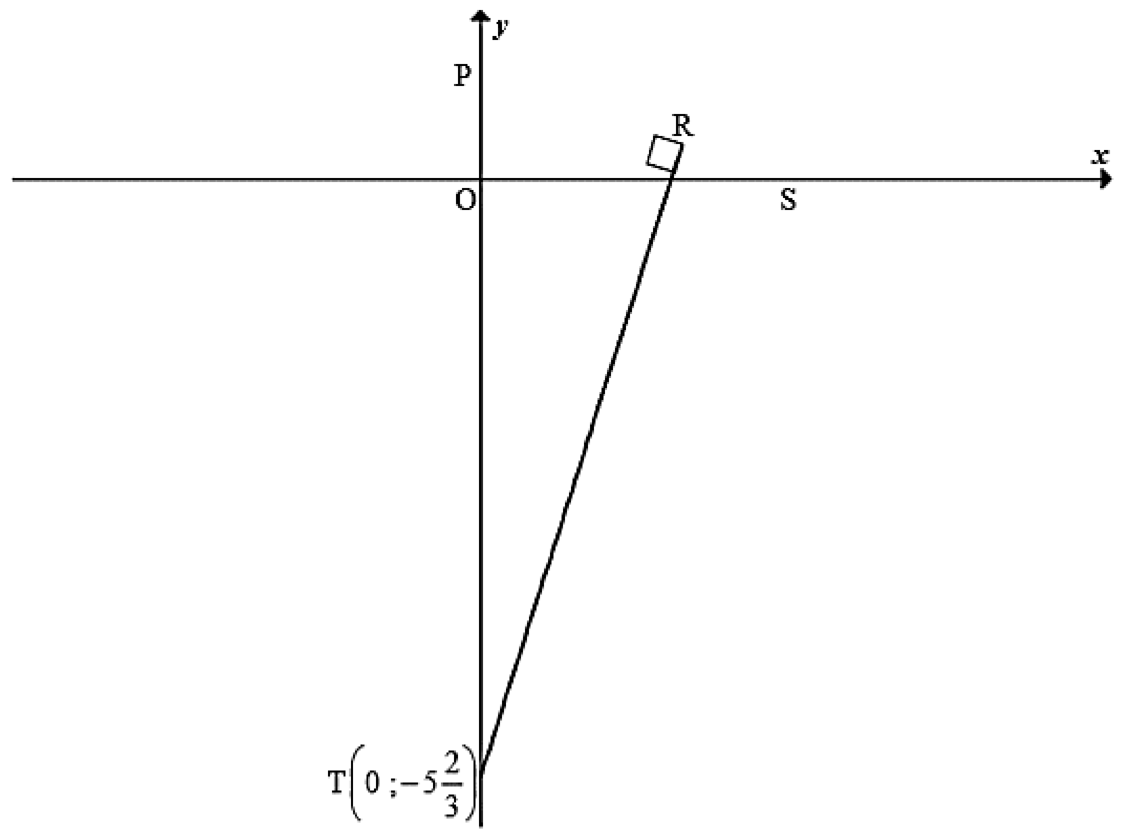
Activity 3	



3.1.	$m_{AC} = \frac{11-3}{10-(-2)} = \frac{8}{12} = \frac{2}{3}$
3.2.	$m_{AC} = m_{DE} = \frac{2}{3}$ (AC//DE) $y-(-1) = \frac{2}{3}(x-5)$ $y = \frac{2}{3}x - \frac{10}{3} - 1$ $y = \frac{2}{3}x - \frac{13}{3}$

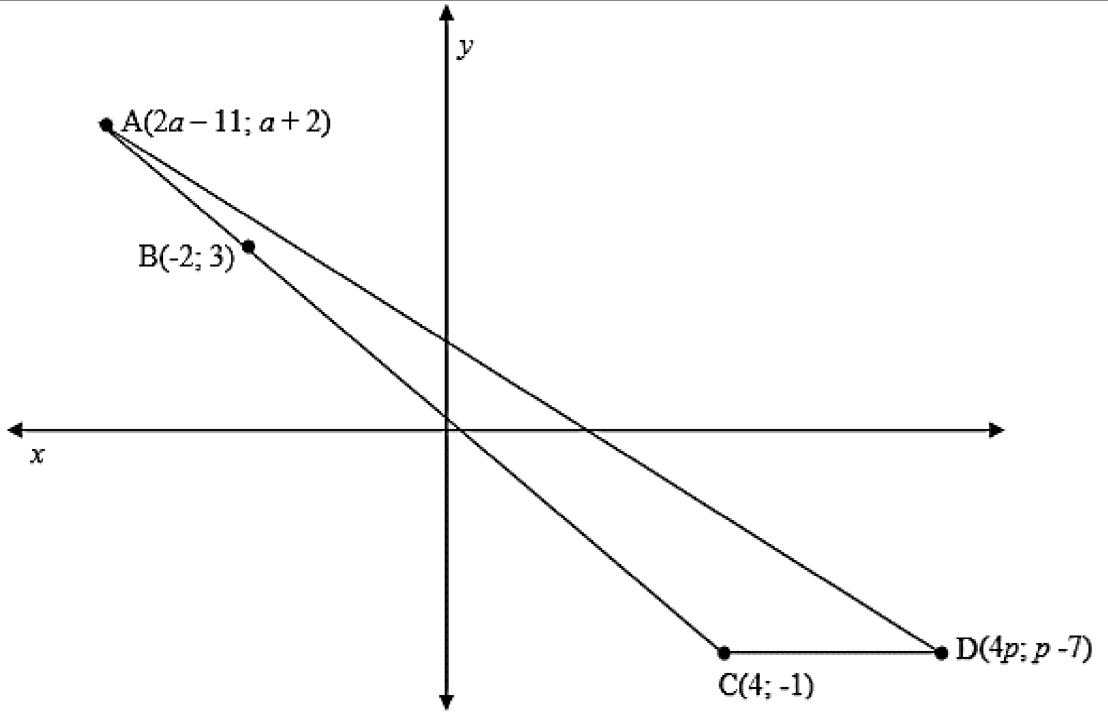
3.3.	$\tan A\hat{G}O = \frac{2}{3}$ $A\hat{G}O = \tan^{-1}\left(\frac{2}{3}\right)$ $A\hat{G}O = 33.69°$ $\alpha = 90° + 33.69°$ (ext $\angle$ of a Δ) $\alpha = 123.69°$
3.4.1.	$mid_{BE} = mid_{AD}$ $mid_{AD}\left(\frac{-2+5}{2};\frac{3-1}{2}\right)$ $M\left(\frac{3}{2};1\right)$
3.4.2.	$d_{BE} = d_{AD}$ $d_{AD} = \sqrt{(5-(-2))^2+(-1-3)^2}$ $d_{AD} = \sqrt{65}$

Activity 4



4.1.	$P(0;y_P)$ $x+ay-a=0$ $0+ay-a=0$ $\therefore y=1$ and $P(0;1)$
4.2.	Given: OS=3OP $\therefore$ OS=3 and $S(3;0)$ $x+ay-a=0$ $3+a(0)-a=0$ $\therefore a=3$ $x+3y-3=0$
4.3.	$x+3y-3=0$ $y=-\frac{1}{3}x+1$ $m_{PS}\times m_{RT}=-1$ (PS $\perp$ RT) $\therefore m_{RT}=3$ $y-\left(-\frac{17}{3}\right)=3(x-0)$ $y=3x-\frac{17}{3}$
4.4.	$d_{PR}=\sqrt{(2-0)^2+\left(\frac{1}{3}-1\right)^2}=\frac{2\sqrt{10}}{3}$ $d_{RT}=\sqrt{(2-0)^2+\left(\frac{1}{3}-\left(-\frac{17}{3}\right)\right)^2}=2\sqrt{10}$ Area ΔPRT $=\frac{1}{2}$(RT)(PR) $=\frac{1}{2}(2\sqrt{10})\left(\frac{2\sqrt{10}}{3}\right)=\frac{20}{3}$
4.5.	$r=\frac{d}{2}=\frac{PT}{2}$ ($\angle$ at semi-circle) $d_{PT}=y_P-y_T=1-\left(-\frac{17}{3}\right)=\frac{20}{3}$ $r=\frac{20}{3}\div 2=\frac{10}{3}$

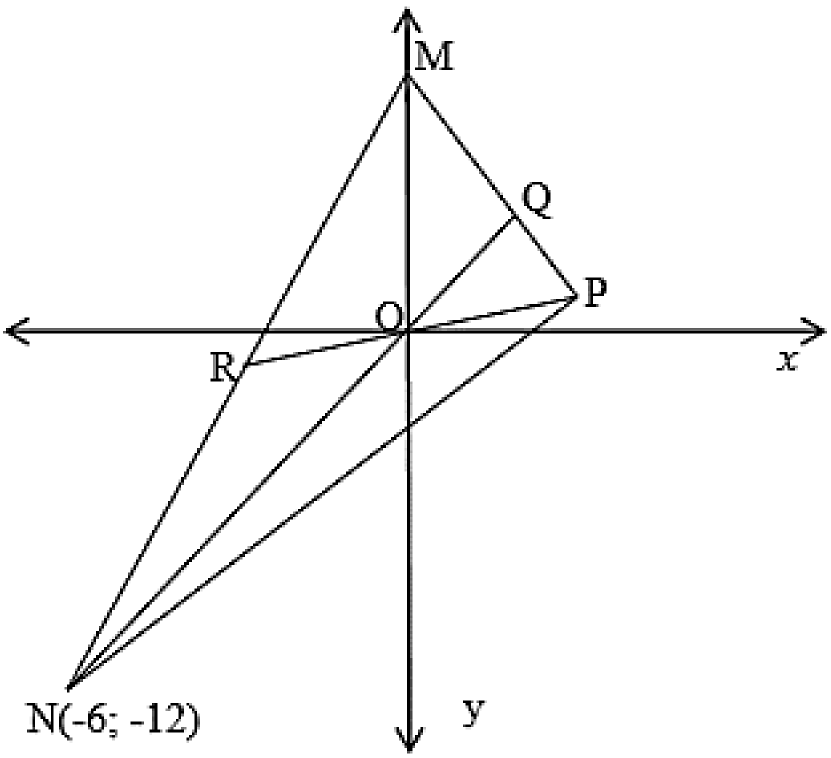
Activity 5	



5.1.	$m_{AB} = m_{BC}$ $\frac{(a+2)-3}{(2a-11)-(-2)} = \frac{3-(-1)}{-2-4}$ $\frac{a-1}{2a-9} = \frac{4}{-6}$ $-a+6 = 8a-36$ $14a = 42$ $\therefore a = 3$ $\therefore \text{A}(-5;5)$
5.2.	$m_{AC} = -\frac{2}{3}$ $y-(-1) = -\frac{2}{3}(x-4)$ $y = -\frac{2}{3}x + \frac{8}{3} - 1$ $y = -\frac{2}{3}x + \frac{5}{3}$

5.3.	$\mathrm{M}\left(\frac{-5-2}{2};\frac{5+3}{2}\right)$ $\mathrm{M}\left(-\frac{7}{2};4\right)$
5.4.	$\mathrm{CD}//x-axis$ $p-7=-1$ $\therefore p=6$ $\therefore \mathrm{D}(24;-1)$

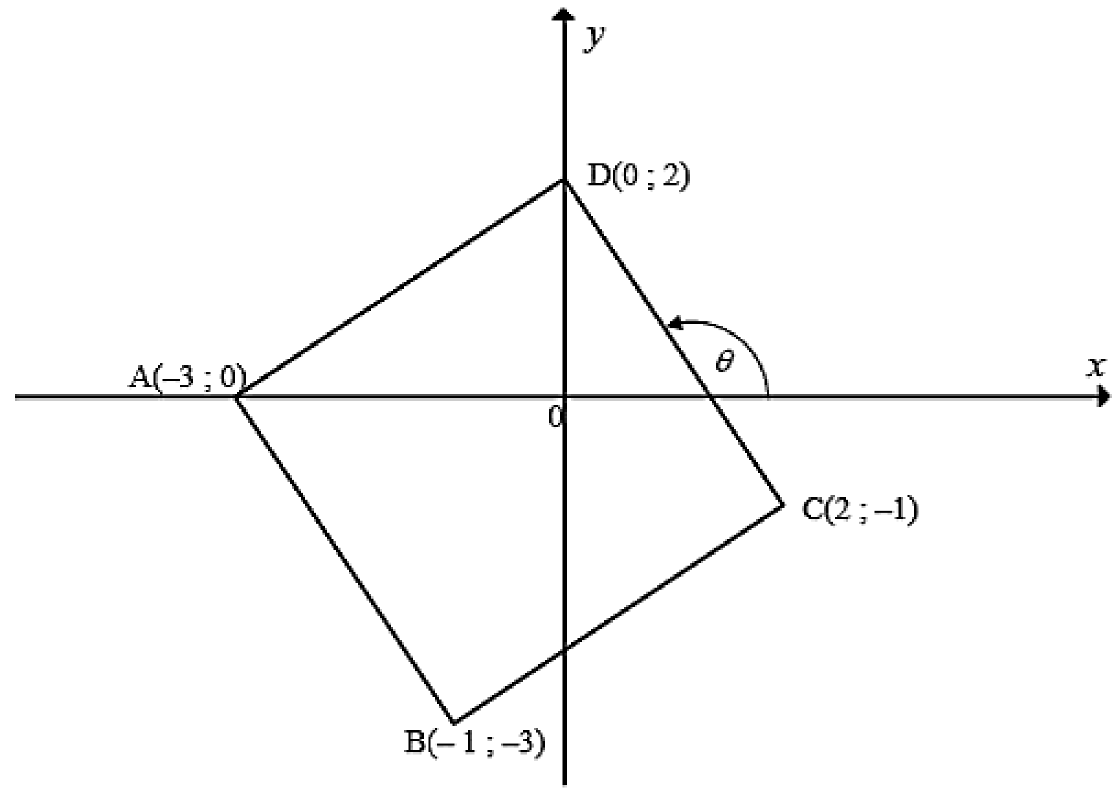
Activity 6



6.1.	$m_{NQ}=m_{NO}=\frac{0-(-12)}{0-(-6)}=2$
6.2.	$m_{MP}\times m_{NQ}=-1$ (NQ $\perp$ MP) $\therefore m_{MP}=-\frac{1}{2}$

6.3.	Inclination of MP: θ $\theta = \tan^{-1}\left(-\frac{1}{2}\right) + 180°$ $\theta = 153.43°$
6.4.	$\mathrm{M}(0; y_M)$ $3x - y + 6 = 0$ $3(0) - y + 6 = 0$ $\therefore y_M = 6$ $\therefore \mathrm{M}(0;6)$ $y - 6 = -\frac{1}{2}(x - 0)$ $y = -\frac{1}{2}x + 6$
6.5.	RP: $y = x$ $y_1 = y_2$ $x = -\frac{1}{2}x + 6$ $\frac{3}{2}x = 6$ $\therefore x_P = 4$ and $y_P = 4$ $\therefore \mathrm{P}(4;4)$
6.6.	$\mathrm{R}\left(\frac{0-6}{2}; \frac{6-12}{2}\right)$ $\mathrm{R}(-3;-3)$

Activity 7



7.1.	M $\left(\frac{2-3}{2};\frac{-1+0}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$
7.2.	Midpoint BD $=\left(\frac{-1+0}{2};\frac{-3+2}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$ $\therefore$ Midpoint of AC and BD are the same point therefore AC and BD bisect each other **OR**

$$AM = \sqrt{\left(-3+\frac{1}{2}\right)^2 + \left(0+\frac{1}{2}\right)^2}$$

$$AM = \sqrt{6,5}$$

$$CM = \sqrt{\left(2+\frac{1}{2}\right)^2 + \left(-1+\frac{1}{2}\right)^2}$$

$$CM = \sqrt{6,5}$$

$$BM = \sqrt{\left(-1+\frac{1}{2}\right)^2 + \left(-3+\frac{1}{2}\right)^2}$$

$$BM = \sqrt{6,5}$$

$$DM = \sqrt{\left(0+\frac{1}{2}\right)^2 + \left(2+\frac{1}{2}\right)^2}$$

$$DM = \sqrt{6,5}$$

AC and BD bisect each other

7.3.

$$m_{AD} = \frac{2-0}{0+3}$$

$$m_{AD} = \frac{2}{3}$$

$$m_{CD} = \frac{-1-2}{2-0}$$

$$m_{CD} = -\frac{3}{2}$$

$$m_{AD} \times m_{CD}$$

$$= \frac{2}{3} \times -\frac{3}{2}$$

$$= -1$$

$$\therefore AD \perp CD$$

$$\therefore \ A\hat{D}C = 90^\circ$$

OR

$$\tan\theta = m_{CD}$$

$$\tan\theta = -\frac{3}{2}$$

$$\theta = 123,69^\circ$$

$$\tan D\hat{A}C = \frac{2}{3}$$

$$D\hat{A}C = 33,69^\circ$$

$$A\hat{D}C = 123,69^\circ - 33,69^\circ$$

$$A\hat{D}C = 90^\circ$$

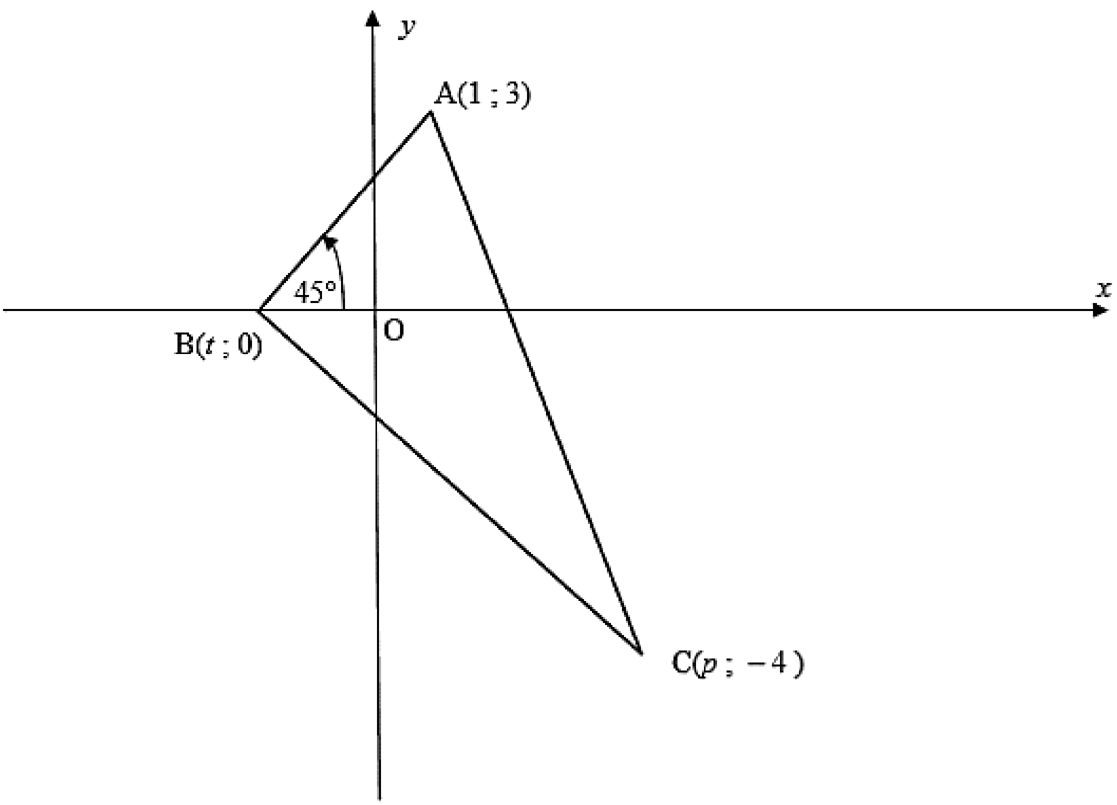
OR

	$AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $AC^2 = (0-(-1))^2 + (-3-2)^2$ $AC^2 = 26$ $AD^2 + DC^2$ $= 13 + 13$ $= 26$ $= AC^2$ $\therefore$ AD $\perp$ DC $\therefore\ A\hat{D}C = 90°$
7.4.	$BD = \sqrt{(2+3)^2 + (0+1)^2}$ $= \sqrt{26}$ $AC = \sqrt{(-3-2)^2 + (0+1)^2}$ $= \sqrt{26}$ diagonals are equal diagonals bisect each other (Proved in 1.2) (i.e. ABCD is a rectangle) $m_{AC}.m_{BD}$ $= \frac{1}{-5} \times \frac{5}{1}$ $= -1$ AC $\perp$ BD **OR** $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ The figure is a rectangle and one pair of adjacent sides are equal in length $\therefore$ it is a square. **OR**

	$AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $AB^2 = (-3-(-1))^2 + (0-(-3))^2$ $AB^2 = 13$ $BC^2 = (2-(-1))^2 + (-1-(-3))^2$ $BC^2 = 13$ All four sides equal and one internal angle equal to 90° **OR** The diagonals bisect one another $A\hat{D}C = 90°$ $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $\therefore$ adjacent sides equal in length $\therefore$ ABCD is a square
7.5.	$\tan\theta = \frac{2+1}{0-2}$ $\tan\theta = -\frac{3}{2}$ $\theta = -56{,}30993247\ldots + 180°$ $\theta = 123{,}7°$ **OR** $\tan D\hat{A}O = \frac{2}{3}$ $D\hat{A}O = 33{,}7°$ $A\hat{D}C = 90°$ $\theta = 90° + 33{,}7°$ $\theta = 123{,}7°$
7.6.	$OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC = 2{,}236067977$ $OC > 2$ C lies outside the circle **OR**

	$OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $OC^2 > 4$ C lies outside the circle **OR** $x^2 + y^2 = 4$ $(2)^2 + (-1)^2 = 5 > 4$ C lies outside the circle

Activity 8	

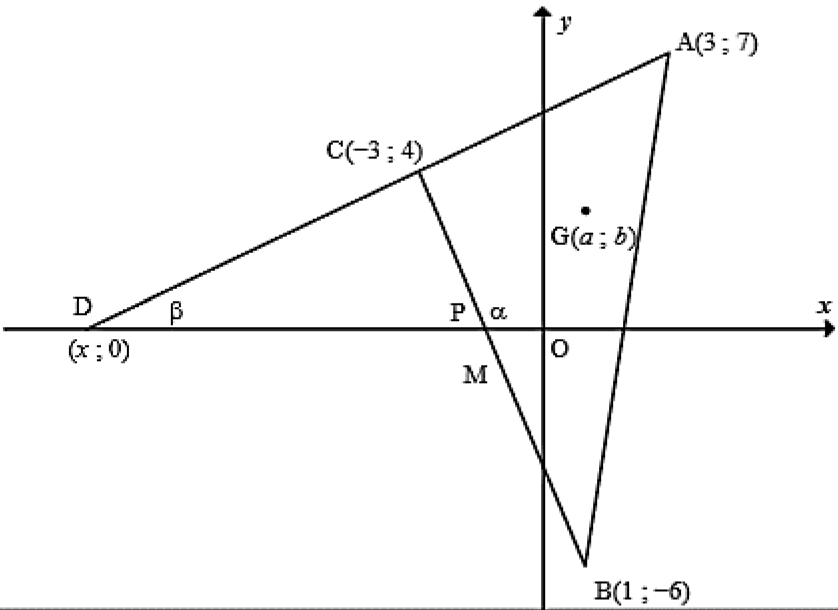


8.1.	$\tan 45° = m_{AB}$ $= 1$ **OR** $m_{AB} = \frac{3-0}{1-t} = \frac{3}{1-t}$

8.2.	$\frac{3-0}{1-t}=\tan 45°=1$ $1-t=3$ $t=-2$ **OR** $y=mx+c$ $3=(1)(1)+c$ $c=2$ $y=x+2$ $(t;0)$ in $y=mx+2$ $0=t+2$ $t=-2$
8.3.	$\sqrt{(1-p)^2+(3+4)^2}=\sqrt{50}$ $(1-p)^2+(3+4)^2=50$ $1-2p+p^2+49=50$ $p^2-2p=0$ $p(p-2)=0$ $p\neq 0$ or $p=2$ **OR** $(1-p)^2+(3+4)^2=50$ $(1-p)^2=50-49$ $(1-p)^2=1$ $1-p=1$ or $1-p=-1$ $p\neq 0$ or $p=2$ **OR** $Let\ p=2$ $AC=\sqrt{(1-2)^2+(3+4)^2}$ $=\sqrt{1+49}$ $=\sqrt{50}$ which is true $\therefore p=2$

8.4.	midpoint of $BC = \left(\frac{-2+2}{2};\frac{0-4}{2}\right)$ midpoint of $BC = (0\,;-2)$
8.5.	Gradient of line $= m_{AB} = 1$ Equation of line is: $y+4=1(x-2)$ $y = x-6$ **OR** $y = mx + c$ $y = x - p - 4$

Activity 9	



9.1.1.	$m_{AD} = m_{AC}$ $= \frac{7-4}{3-(-3)}$ $= \frac{3}{6}$ $= \frac{1}{2}$ **OR** $m_{AD} = m_{AC}$ $= \frac{4-7}{-3-(3)}$ $= \frac{-3}{-6}$ $= \frac{1}{2}$

9.1.2.	$m_{BC} = \frac{-6-4}{1-(-3)}$ $= \frac{-10}{4}$ $= \frac{-5}{2}$ **OR** $m_{BC} = \frac{4-(-6)}{-3-(1)}$ $= \frac{10}{-4}$ $= \frac{-5}{2}$
9.2.	$m_{AD} = \frac{1}{2} = \tan C\hat{D}O$ $C\hat{D}O = 26{,}56505....^\circ$ $m_{BC} = \frac{-5}{2} = \tan\alpha$ $\alpha = 111{,}814\,095^\circ$ $D\hat{C}B = 111{,}8014095....^\circ - 26{,}56505....^\circ$ $= 85{,}236359^\circ$ $= 85{,}24^\circ$ $\approx 85{,}2^\circ$ **OR** $\tan C\hat{D}O = \frac{1}{2}$ $C\hat{D}O = 26{,}56505....^\circ$ $\tan(180^\circ - \alpha) = \frac{5}{2}$ $180^\circ - \alpha = 68{,}19859051...^\circ$ $D\hat{C}B = 180^\circ - (26{,}56505....^\circ + 68{,}19859051...^\circ)$ $= 85{,}236359^\circ$ $= 85{,}24^\circ$ **OR** $D\hat{C}B = \alpha - C\hat{D}O$ $\tan D\hat{C}B = \frac{m_{CB} - m_{CD}}{1 + m_{CB}.m_{CD}}$ $= \frac{-\frac{5}{2} - \frac{1}{2}}{1 + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right)}$ $= 12$ $D\hat{C}B = 85{,}24^\circ$ **OR**

	$AC = \sqrt{45} \quad BC = \sqrt{116} \quad AB = \sqrt{173}$ $\cos A\hat{C}B = \dfrac{AC^2 + BC^2 - AB^2}{2AC.BC}$ $= \dfrac{45 + 116 - 173}{2(\sqrt{45})(\sqrt{116})}$ $= -0{,}083045...$ $A\hat{C}B = 94{,}76...°$ $D\hat{C}B = 180° - 94{,}76...°$ $= 85{,}24°$ **OR** $D(-11 ; 0)$ $DC = \sqrt{80} \quad BC = \sqrt{116} \quad DB = \sqrt{180}$ $\cos D\hat{C}B = \dfrac{DC^2 + BC^2 - DB^2}{2DC.BC}$ $= \dfrac{80 + 116 - 180}{2(\sqrt{80})(\sqrt{116})}$ $= 0{,}08304547985...$ $D\hat{C}B = 85{,}24°$ **OR**

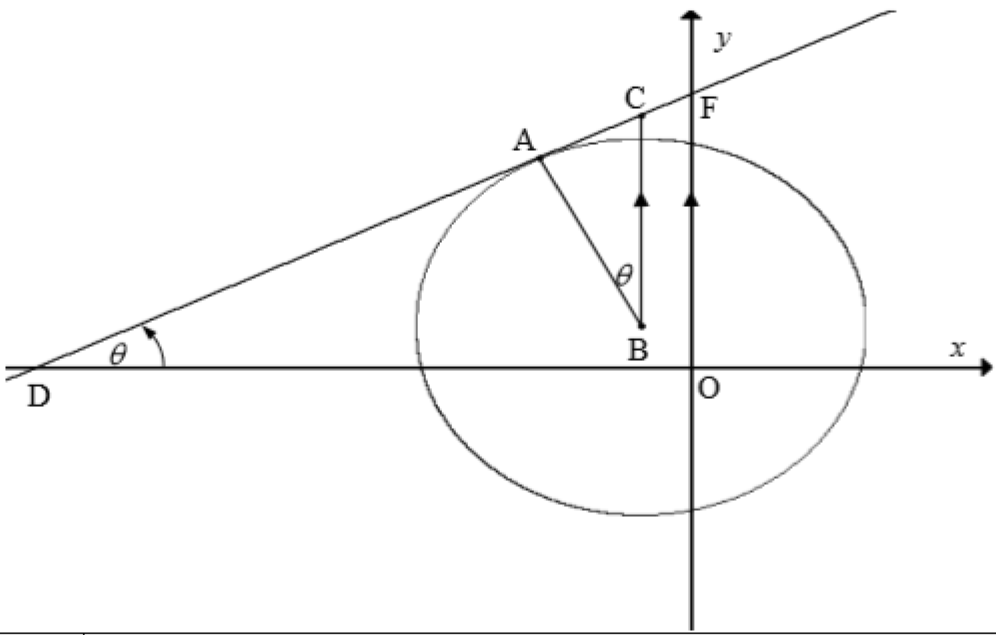
	Equation AC: $2y = x + 11$ $D(-11\ ;\ 0)$ $C(-3\ ;\ 4)$ $DC^2 = (x_C - x_D)^2 + (y_C - y_D)^2$ $= (-3+11)^2 + (4-0)^2$ $= 80$ Equation BC: $2y = -5x - 7$ $P\left(-\frac{7}{5};0\right)$ $PC^2 = (-3+\frac{7}{5})^2 + (4-0)^2$ $= \frac{464}{25}$ $DP^2 = (-\frac{7}{5}+11)^2$ $= \frac{2304}{25}$ In ΔDCP: $DP^2 = DC^2 + CP^2 - 2DC.CP.\cos D\hat{C}P$ $\frac{2304}{25} = \frac{2000}{25} + \frac{464}{25} - 2\left(\frac{\sqrt{2000}}{5}\right)\left(\frac{\sqrt{464}}{5}\right).\cos D\hat{C}P$ $D\hat{C}P = 85{,}23635...$ $D\hat{C}P = 85{,}24°$

9.3.	$y-7=\frac{1}{2}(x-3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y-4=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y=\frac{1}{2}x+c$ $(7)=\frac{1}{2}(3)+c$ $c=\frac{11}{2}$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$
9.4.	$\mathrm{M}(x;y)=\left(\frac{-3+1}{2};\frac{4-6}{2}\right)$ $\mathrm{M}(x;y)=(-1;-1)$
9.5.	$m_{AM}=\frac{7-(-1)}{3-(-1)}=2$ $y=2x+c$ $-1=2(-1)+c$ $\therefore c=1$ $y=2x+1$ G(a ; b) lies on the line $\therefore b=2a+1$ **OR**

	$\frac{7-b}{3-a}=\frac{b+1}{a+1}$ $(7-b)(a+1)=(b+1)(3-a)$ $7a+7-ab-b=3b-ab+3-a$ $8a-4b=-4$ $2a-b=-1$ $b=2a+1$ **OR** Using the point $(-1\,;-1)$ $\frac{b+1}{a+1}=\frac{8}{4}$ $\frac{b+1}{a+1}=2$ $b+1=2a+2$ $b=2a+1$ **OR** Using the point $(3\,;7)$ $\frac{7-b}{3-a}=\frac{8}{4}$ $\frac{7-b}{3-a}=2$ $7-b=6-2a$ $b=2a+1$
9.6.	$\text{GC}=\sqrt{17}$ $\text{GC}^2=17$ $(a+3)^2+(b-4)^2=17$ $(a+3)^2+(2a+1-4)^2=17$ $a^2+6a+9+4a^2-12a+9-17=0$ $5a^2-6a+1=0$ $(5a-1)(a-1)=0$ $a=\frac{1}{5}$ or $a=1$ $\therefore b=\frac{7}{5}$ or $b=3$ **OR**

	$a=\frac{b-1}{2}$ $17=(a+3)^2+(b-4)^2$ $17=\left(\left(\frac{b-1}{2}\right)+3\right)^2+(b-4)^2$ $17=\left(\frac{b+5}{2}\right)^2+(b-4)^2$ $17=\frac{b^2+10b+25+4b^2-32b+64}{4}$ $68=5b^2-22b+89$ $0=5b^2-22b+21$ $0=(5b-7)(b-3)$ $\therefore b=\frac{7}{5}$ or $b=3$

Activity 10	



10.1.	$x_C=x_B=-1$ $y_C=y_B+5=6$ $\therefore C(-1;6)$

10.2.	BA ⊥ CA (tangent ⊥ radius) $\therefore CA^2 = BC^2 - AB^2$ (Pythagoras) $= (5)^2 - (\sqrt{20})^2 = 5$ $\therefore CA = \sqrt{5}$ or 2,24 units
10.3.	$\tan\theta = \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$
10.4. 10.5.	$m_{DC} \times m_{AB} = -1$ $m_{DC} = \tan\theta = \frac{1}{2}$ $m_{DC} = \frac{1}{2}$ $m_{AB} = -2$ Eq. of DC: $y - 6 = \frac{1}{2}(x+1)$ $y = \frac{1}{2}x + \frac{13}{2}$ Eq. of AB: $y - 1 = -2(x+1)$ $y = -2x - 1$ $-2x - 1 = \frac{1}{2}x + \frac{13}{2}$ $-\frac{5}{2}x = \frac{15}{2}$ $x = -3$ $y = -2(-3) - 1$ $y = 5$ $\therefore A(-3\,;5)$ Answer only: (–3 ; 5): 1 mark **OR**

Eq. of DC: $y-6=\frac{1}{2}(x+1)$

$$y=\frac{1}{2}x+\frac{13}{2}$$

Eq. of circle: $(x+1)^2+(y-1)^2=20$

<u>At A:</u>

$(x+1)^2+(\frac{1}{2}x+\frac{13}{2}-1)^2=20$

$(x+1)^2+(\frac{1}{2}x+\frac{11}{2})^2=20$

$1\frac{1}{4}x^2+\frac{15}{2}x+11\frac{1}{4}=0$

$\therefore x^2+6x+9=0$

$(x+3)^2=0$

$\therefore\ x=-3$

and $y=\frac{1}{2}(-3)+\frac{13}{2}=5$

$\therefore$ A$(-3\ ;5)$

OR

Draw AE $\perp$ BC

$\cos\theta=\frac{2\sqrt{5}}{5}=\frac{AE}{\sqrt{5}}=\frac{BE}{2\sqrt{5}}$

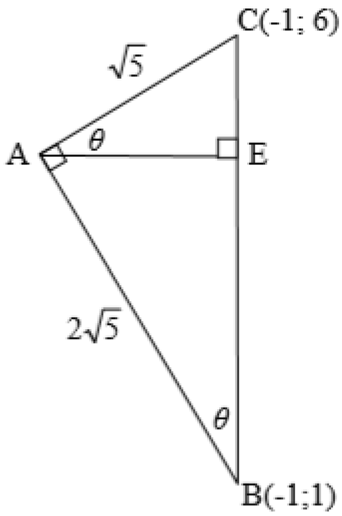
$\therefore AE=\frac{2\times5}{5}=2$

$BE=\frac{4\times5}{5}=4$

$x_A=-1-AE=-1-2=-3$

$\therefore y_A=1+BE=4+1=5$

$\therefore$ A$(-3\ ;5)$



OR

$(x+1)^2+(y-1)^2=20 \qquad (1)$

$y=-2x-1 \qquad (2)$

$(x+1)^2+(-2x-2)^2=20$

$x^2+2x+1+4x^2+8x+4-20=0$

$5x^2+10x-15=0$

$x^2+10x-15=0$

$(x+3)(x-1)=0$

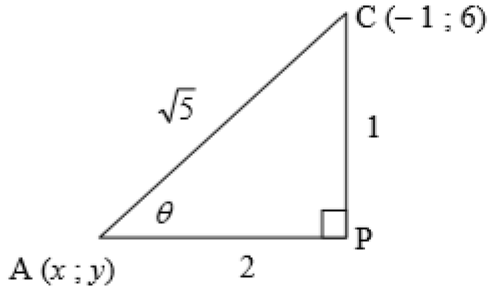
$x=-3$ or $x\neq 1$

subst (1) in (2)

$\therefore y=5$

OR

Equation AC : $y=\frac{1}{2}x+6\frac{1}{2}$



$\tan\theta=\frac{1}{2}$

$\theta=26{,}57°$

$\text{AP}=\sqrt{5}\cos 26{,}57°$

$AP=2$

$\text{CP}=\sqrt{5}\sin 26{,}57°$

$CP=1$

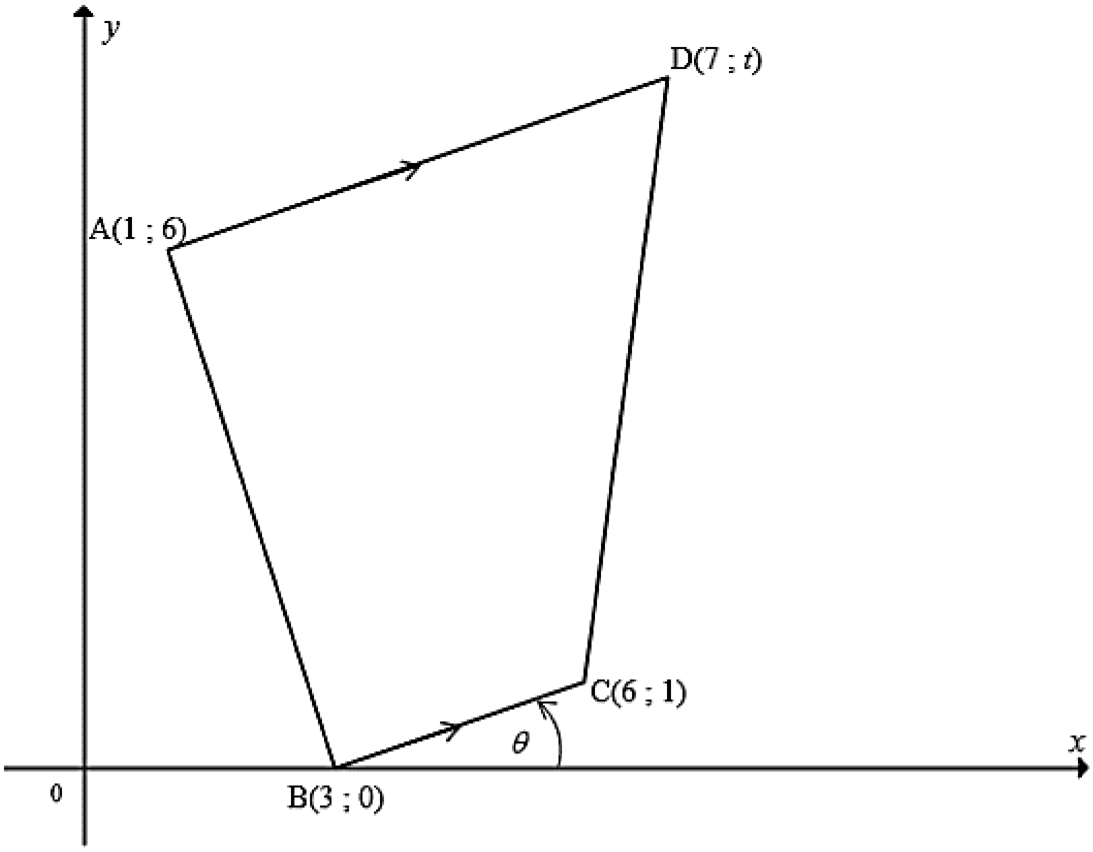
$\therefore x=-1-2=-3$

$y=6-1=5$

$\therefore \text{A}(-3\,;5)$

10.6.	Area $\Delta ABC = \frac{1}{2}(\sqrt{5})(\sqrt{20}) = 5$ Eqn. of DC is $y = \frac{1}{2}x + \frac{13}{2}$ Therefore $OF = \frac{13}{2}$ and $OD = 13$. Area $\Delta ODF = \frac{1}{2}\left(\frac{13}{2}\right)(13) = \frac{169}{4}$ Area ΔABC: Area $\Delta ODF = 5 : \frac{169}{4} = 20 : 169$ **OR** $DF^2 = 13^2 + (\frac{13}{2})^2 = \frac{845}{4}$ $DF = \frac{13.\sqrt{5}}{2}$ $\frac{\Delta ABC}{\Delta ODF} = \frac{\frac{1}{2}(5)(\sqrt{20})\sin\theta}{\frac{1}{2}(13)(\frac{13.\sqrt{5}}{2})\sin\theta}$ $= \frac{20}{169}$ **OR** ΔODF is an enlargement of ΔABC $\therefore$ area ΔABC : area $\Delta ODF = AB^2 : OD^2 = 20 : OD^2$ Equation of DC is $y = \frac{1}{2}x + \frac{13}{2}$ $x_D = -13$ $OD = 13$ $\therefore$ area ΔABC : area $\Delta ODF = AB^2 : OD^2 = 20 : 169$

Activity 11

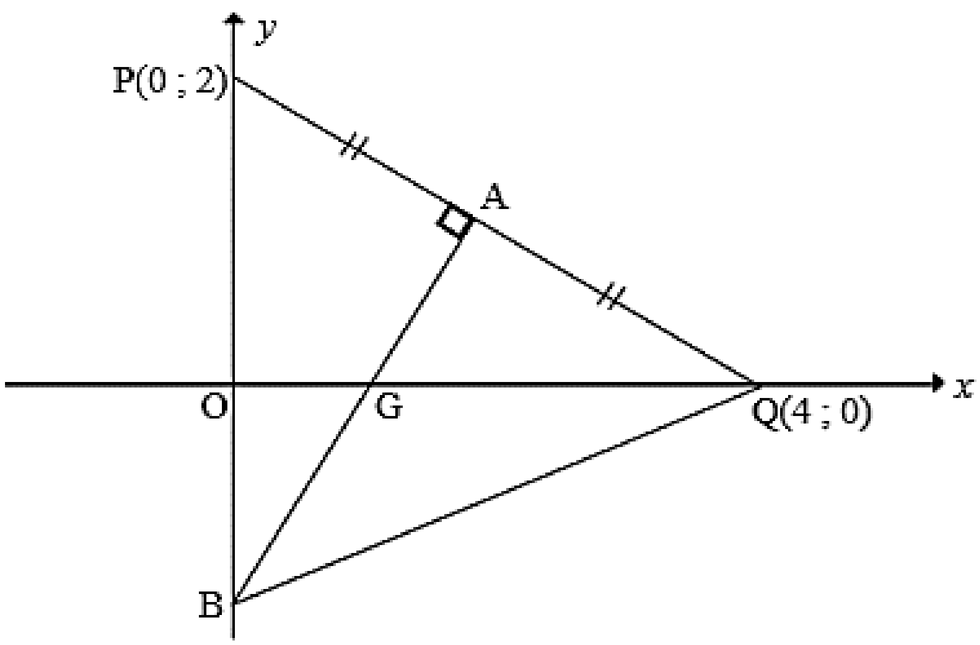


11.1.	$m_{BC} = \frac{1-0}{6-3}$ $m_{BC} = \frac{1}{3}$
11.2.	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3}$ ------------------------------ AB//BC $\therefore$ Equation of AD is: $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$

	OR $y-6=\frac{1}{3}(x-1)$ $y-6=\frac{1}{3}x-\frac{1}{3}$ $y=\frac{1}{3}x+\frac{17}{3}$
11.3.	$y=\frac{1}{3}x+\frac{17}{3}$ $t=\frac{1}{3}(7)+\frac{17}{3}$ $t=8$ **OR** $\frac{t-6}{7-1}=\frac{1}{3}$ $t-6=2$ $\therefore t=8$
11.4.	$\text{AD}=\sqrt{(8-6)^2+(-1-3)^2}$ $\text{AD}=\sqrt{40}$ $\text{AD}=2\sqrt{10}$ $\text{BC}=\sqrt{(6-3)^2+(1-0)^2}$ $\text{BC}=\sqrt{10}$ $\text{AB}=\sqrt{(6-0)^2+(1-3)^2}$ $\text{AB}=\sqrt{40}$ $\text{AB}=2\sqrt{10}$
11.5.	$m_{AB}=\frac{6-0}{1-3}$ $m_{AB}=-3$ $m_{BC}=\frac{1-0}{6-3}=\frac{1}{3}$ $m_{AB}.m_{BC}=\frac{1}{3}\times-3$ $=-1$ $\therefore AB\perp BC$

11.6.	Area of Quad ABCD = area of ΔADC + area of ABC $=\frac{1}{2}\left(2\sqrt{10}\right)\left(2\sqrt{10}\right)+\frac{1}{2}\left(\sqrt{10}\right)\left(2\sqrt{10}\right)$ $= 20 + 10$ $=30$ square units **OR** Area of ABCD $=\frac{1}{2}(\text{sum of parallel sides})\times h$ $=\frac{1}{2}(2\sqrt{10}+\sqrt{10})2\sqrt{10}$ $=\sqrt{10}(3\sqrt{10})$ $= 30$ square units
11.7.	From 11.1. $m_{BC} = \frac{1}{3}$ $\tan\theta = \frac{1}{3}$ $\therefore \theta = 18{,}43°$

Activity 12	



12.1.	$m_{PQ}=\frac{2-0}{0-4}=-\frac{1}{2}$
12.2.	A: $\left(\frac{0+4}{2};\frac{2+0}{2}\right)$ A $(2;1)$
12.3.	$m_{AB}.m_{PQ}=-1$ $m_{AB}.(-1/2)=-1,\ \therefore m_{AB}=2$ Equation of AB is $y=2x+c$ $\therefore\ 1=2(2)+c$ $c=-3$ Equation of AB is $y=2x-3$. **OR** $m_{AB}.m_{PQ}=-1$ $m_{AB}.(-1/2)=-1,\ \therefore m_{AB}=2$ $y-1=2(x-2)$ $y-1=2x-4$ $y=2x-3$
12.4.	B is the point $(0;-3)$ $\text{BQ}=\sqrt{(0-4)^2+(-3-0)^2}$ $=5$
12.5.	$\text{BP}=\sqrt{(0-0)^2+(-3-2)^2}$ $=5$ $\text{BP}=\text{BQ}$ $\therefore\Delta\text{BPQ}$ is isosceles. **OR** $\text{BP}=2+3$ $=5$ $\text{BP}=\text{BQ}$ $\therefore\Delta\text{BPQ}$ is isosceles

12.6. If PBQR is a rhombus then A is the midpoint of BR.
Let the coordinates of R be $(x\,;y)$

$$\frac{x+0}{2}=2 \qquad \text{and} \qquad \frac{y-3}{2}=1$$

$$x=4 \qquad\qquad y=5$$

$\therefore R(4\,;5)$

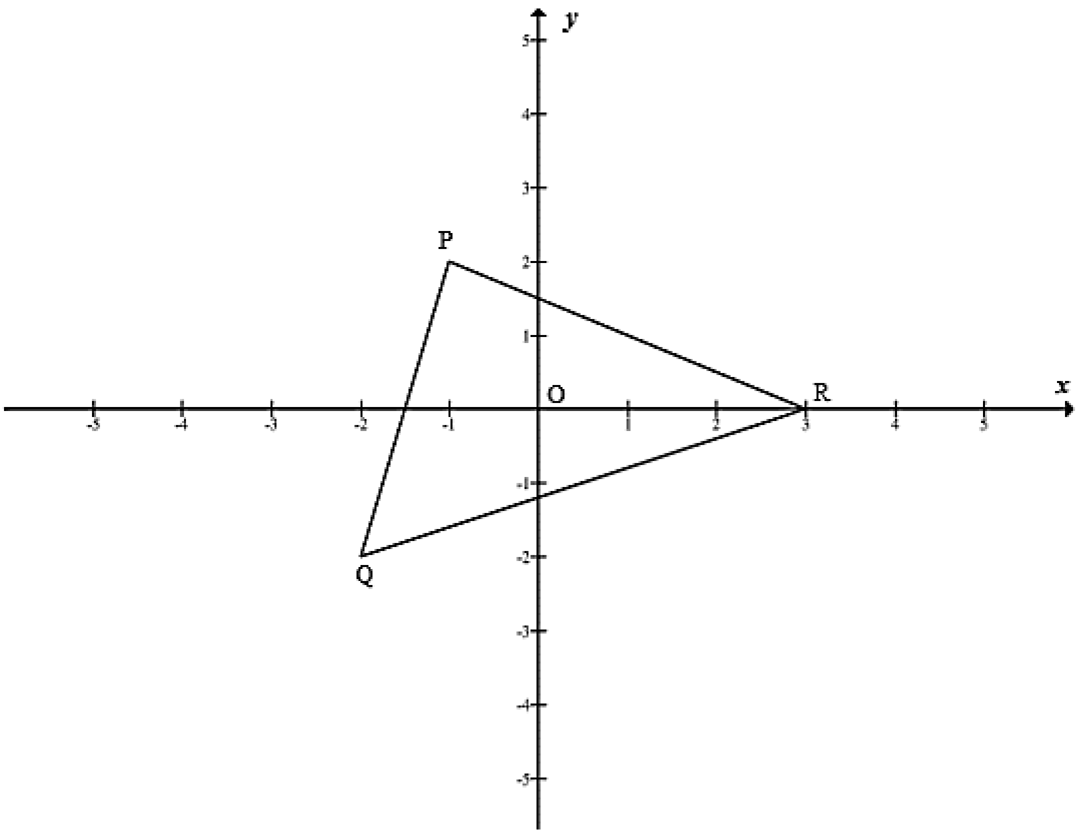
OR

$RQ \parallel PB$ so $x_R = 4$

$RQ = PB = 5$, so $y_R = 5$

$\therefore R(4\,;5)$

Activity 13



13.1. Let β be the angle of inclination of PQ.

$\tan\beta = m_{PQ}$

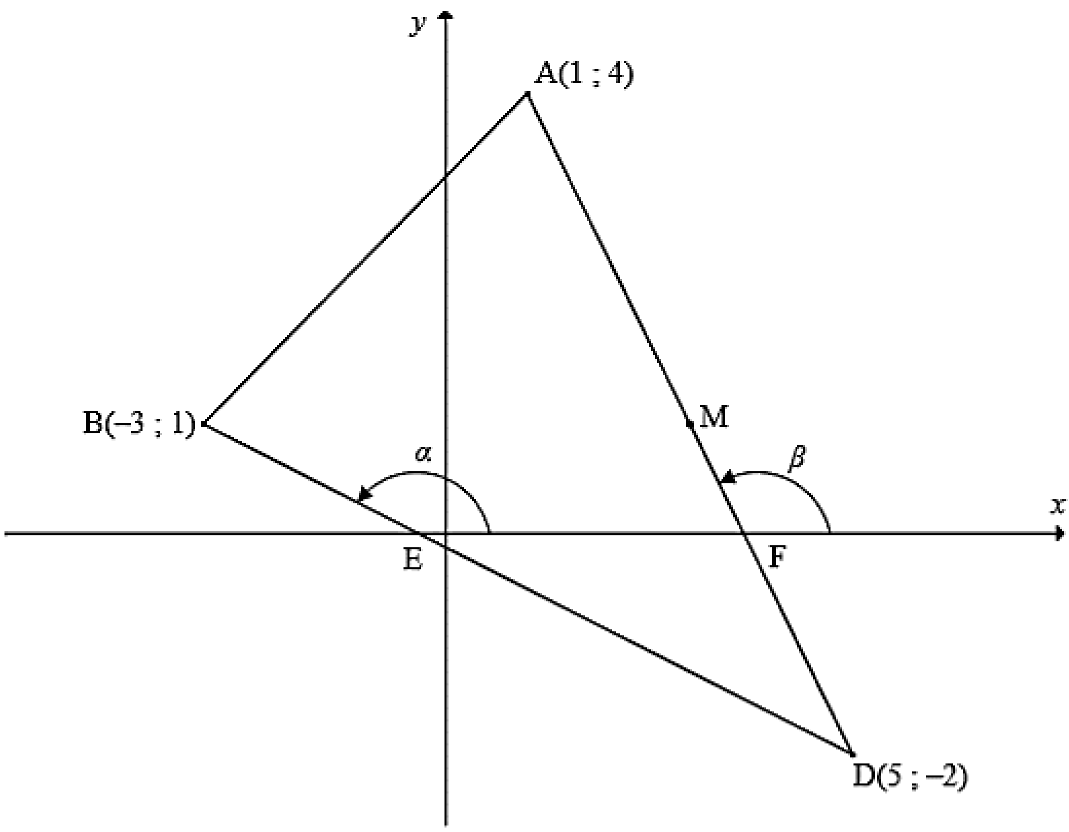
$$\tan\beta = \frac{2-(-2)}{-1-(-2)}$$

$\tan\beta = 4$

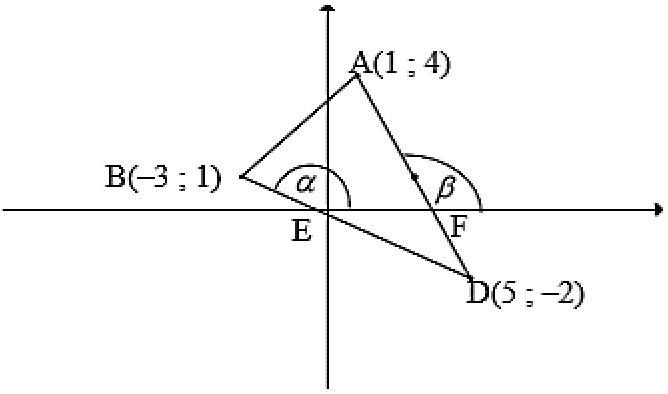
$\beta = 75{,}96°$

13.2.	$M\left(\frac{-1+3}{2};\frac{2+0}{2}\right)$ $M(1;1)$
13.3.	$PQ=\sqrt{(-1+2)^2+(2+2)^2}$ $=\sqrt{17}$ $PR=\sqrt{(-1-3)^2+(2-0)^2}$ $=\sqrt{20}$ $QR=\sqrt{(0-(-2))^2+(3-(-2))^2}$ $=\sqrt{29}$ Perimeter $=\sqrt{29}+\sqrt{20}+\sqrt{17}$ $=13{,}98$ units $=14$ to the nearest whole number
13.4.	$y-1=4(x-1)$ $y=4x-3$

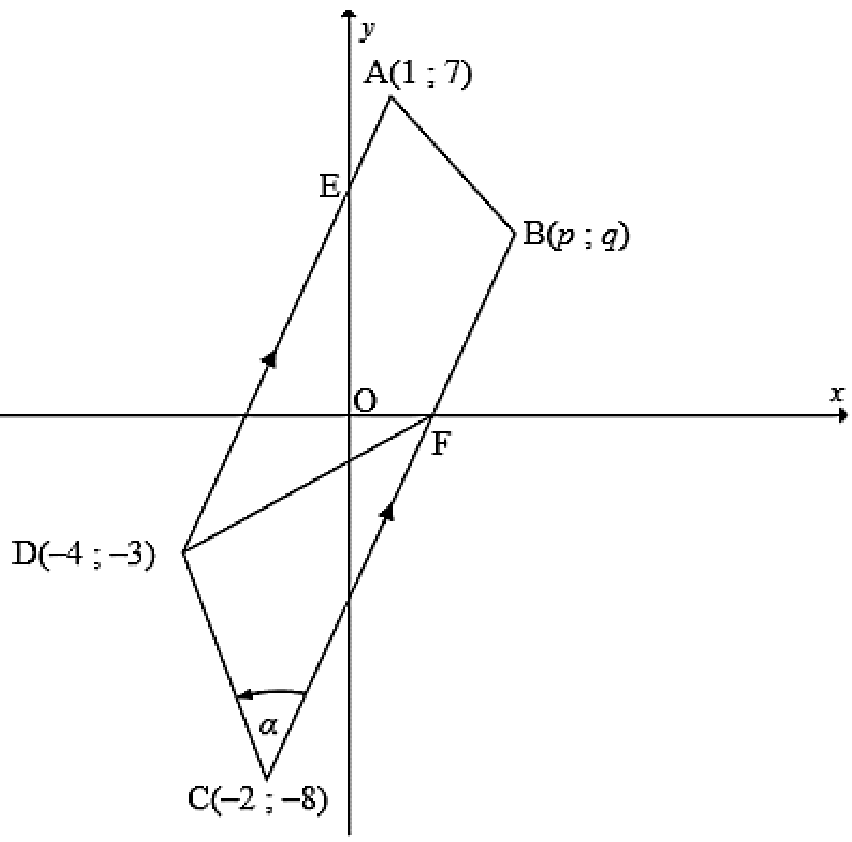
Activity 14	



14.1.	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2-4}{5-1}$ $= -\frac{6}{4} \quad = -\frac{3}{2}$
14.2.	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5-1)^2 + (-2-4)^2}$ $= \sqrt{16+36}$ $= \sqrt{52}$
14.3.	$M = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $M = \left(\frac{1+5}{2}; \frac{4-2}{2}\right)$ $M = (3\,;1)$
14.4.	$m_{BC} = m_{AD}$ Lines are parallel $= -\frac{3}{2}$ $y - y_1 = m\,(x - x_1)$ $y - 1 = -\frac{3}{2}(x+3)$ $2y - 2 = -3x - 9$ $3x + 2y + 7 = 0$ **OR** $y = -\frac{3}{2}x + c$ $1 = -\frac{3}{2}(-3) + c$ $c = -\frac{7}{2}$ $y = -\frac{3}{2}x - \frac{7}{2}$ $3x + 2y + 7 = 0$

14.5.1.	$m_{AD} = -\frac{3}{2}$ $\tan\beta = -\frac{3}{2}$ $\beta = 180° - 56{,}31°$ $\beta = 123{,}69$ 
14.5.2.	$m_{BD} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\tan\alpha = -\frac{3}{8}$ $\alpha = 180° - 20{,}56°$ $\alpha = 159{,}44°$ $F\hat{E}D = 180° - 159{,}44° = 20{,}56°$ $E\hat{F}D = 123{,}69°$ $F\hat{D}E = 180° - (20{,}56° + 123{,}69°) = 35{,}75°$
14.6.	Co-ordinates of centre M (3 ; 1) Radius of circle: $\frac{1}{2}$ of AD = $\frac{1}{2}(2\sqrt{13}) = \sqrt{13} = \frac{1}{2}\sqrt{52}$ Equation of the circle is: $(x-3)^2 + (y-1)^2 = 13$ **OR** $r^2 = (3-1)^2 + (1-4)^2 = 13$ Equation of the circle is: $(x-3)^2 + (y-1)^2 = 13$
14.7.	M(3 ; 1) B(– 3 ; 1) $MB = \sqrt{(3+3)^2 + (1-1)^2}$ $MB = 6$ Point B lies outside the circle because MB > radius **OR** M(3 ; 1) B(– 3 ; 1) $MB = 3 + 3 = 6$ Radius of the circle $= \sqrt{13} < 6$ Point B lies outside the circle because MB > radius

Activity 15



15.1.	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{7-(-3)}{1-(-4)}$ $= 2$
15.2.	AD//BC $m_{AD} = m_{BC} = 2$ $y - y_1 = m(x - x_1)$ $y-(-8) = 2(x-(-2))$ $\therefore y = 2x - 4$
15.3.	At F: $y = 0$ $0 = 2x - 4$ $x = 2$ F(2 ; 0)

15.4.

D is translated C according to the rule:

$D(x;y) \rightarrow C(x+2\,;y-5)$

A must also be translated according to this rule to B$^{/}$.

$\therefore \mathrm{A}(1\,;\,7) \rightarrow \mathrm{B}^{/}\,(3\,;\,2)$

OR

$x_{B^{/}} = -2+(1+4) = 3$

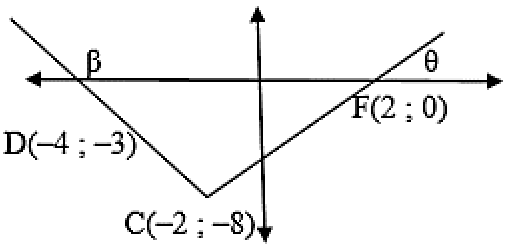
$y_{B^{/}} = -8+(7+3) = 5$

15.5.

$m_{BC} = 2$

$\tan\theta = 2$

$\theta = 63{,}43°$



$m_{DC} = \dfrac{-8-(-3)}{-2-(-4)} = -\dfrac{5}{2}$

$\tan\beta = -\dfrac{5}{2}$

$\beta = 180° - 68{,}20° = 111{,}80°$

$\alpha = 111{,}80° - 63.43° = 48{,}37°$

OR

$\mathrm{DC} = \sqrt{(-4+2)^2 + (-3+8)^2}$

$= \sqrt{29}$

$\mathrm{CF} = \sqrt{(-2-2)^2 + (-8-0)^2}$

$= \sqrt{80}$

$\mathrm{DF} = \sqrt{(2+4)^2 + (0+3)^2}$

$= \sqrt{45}$

$\cos\alpha = \dfrac{29+80-45}{2(\sqrt{29})(\sqrt{80})}$

$= 0{,}6643\ldots$

$\alpha = 48{,}37°$

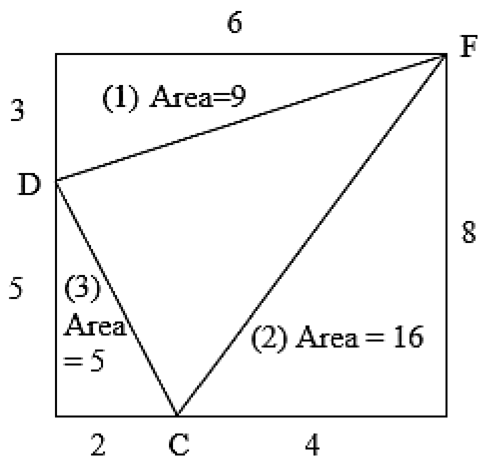
OR

$$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$$
$$= \sqrt{29}$$
$$DB = \sqrt{(3+4)^2 + (2+3)^2}$$
$$= \sqrt{74}$$
$$BC = \sqrt{(3+2)^2 + (2+8)^2}$$
$$= \sqrt{125}$$
$$\cos\alpha = \frac{29 + 125 - 74}{2(\sqrt{29})(\sqrt{125})}$$
$$= 0{,}6643\ldots$$
$$\alpha = 48{,}37°$$

15.6.

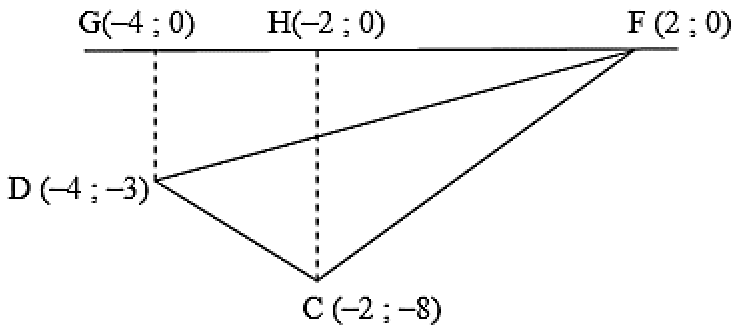
$$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$$
$$= \sqrt{29}$$
$$CF = \sqrt{(-2-2)^2 + (-8-0)^2}$$
$$= \sqrt{80}$$
$$\text{Area } \Delta DCF = \frac{1}{2}.DC.CF.\sin\alpha$$
$$= \frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37°$$
$$= 18 \text{ units}^2$$

OR



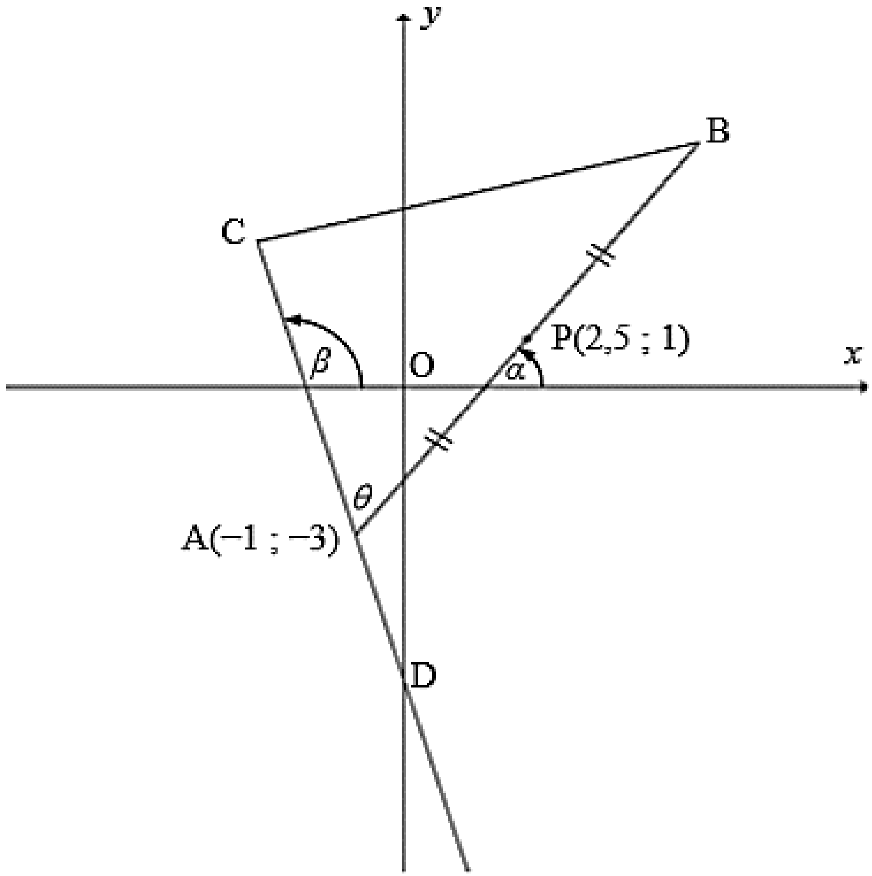
$$\text{Area } \Delta DCF = \text{Area of rectangle} - (1) - (2) - (3)$$
$$= 48 - 9 - 5 - 16$$
$$= 18 \text{ sq units}$$

OR



$$\text{Area CDF} = \text{Area CHF} + \text{Area CDGH} - \text{Area DGF}$$
$$= \frac{1}{2}\times 4\times 8 + 2\times\frac{1}{2}(3\times 8) - \frac{1}{2}\times 6\times 3$$
$$= 16 + 11 - 9$$
$$= 18$$

Activity 16



16.1.	$y = -3x + k$ $-3 = (-3)(-1) + k$ $k = -6$ **OR** By inspection, using the gradient: $k = -6$
16.2.	$\frac{x_A + x_B}{2} = x_P$ and $\frac{y_A + y_B}{2} = y_P$ $\frac{-1 + x_B}{2} = \frac{5}{2}$ and $\frac{-3 + y_B}{2} = 1$ $x_B = 6$ and $y_B = 5$ $\therefore$ B $(6\,;5)$ **OR** By using translation: B$(6\,;5)$
16.3.	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-(-3)}{6-(-1)}$ $= \frac{8}{7}$ **OR** $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-(-3)}{2{,}5-(-1)}$ $= \frac{8}{7}$
16.4.	$\tan\beta = m_{AD} = -3$ $\beta = 108{,}43^\circ$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81^\circ$ $\theta = 108{,}43^\circ - 48{,}81^\circ$ $\theta = 59{,}62^\circ$ **OR** $\tan\beta = m_{AD} = -3$ $\beta = 108{,}43^\circ$ $\text{C}\hat{\text{D}}\text{O} = 18{,}43^\circ$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81^\circ$ $\theta = 18{,}43^\circ + (90^\circ - 48{,}81^\circ)$ $\theta = 59{,}62^\circ$
16.5.	$\text{AD} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+1)^2 + (-6+3)^2}$ $= \sqrt{10}$

16.5.	$AC = 2\ AD$ $= 2\sqrt{10}$ $CB^2 = AC^2 + AB^2 - 2AC.AB.\cos\theta$ $= (2\sqrt{10})^2 + (\sqrt{113})^2 - 2(2\sqrt{10})(\sqrt{113})\cos 59{,}62°$ $= 84{,}998...$ $CB = 9{,}22$ units. **OR** $D(0\ ;-6)$, $A(-1\ ;-3)$, $AC = 2AD$ So $x_c - x_A = 2(x_A - x_D)$ $x_C + 1 = 2(-1-0)$, $x_C = -3$ $y_c - y_A = 2(y_A - y_D)$ $y_C + 3 = 2(-3+6)$, $y_C = 3$ The coordinates of C are $(-3\ ;3)$. $CB = \sqrt{(6-(-3))^2 + (5-3)^2}$ $= 9{,}22$ units

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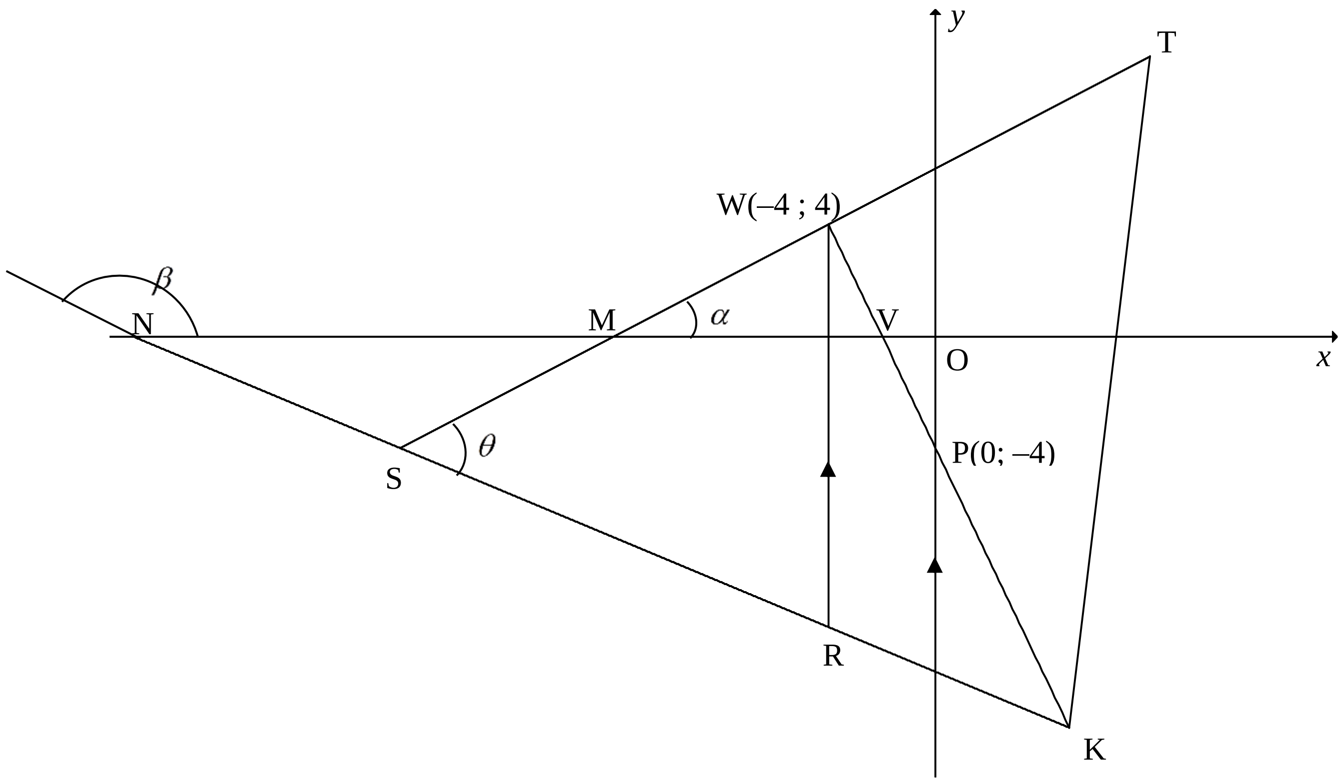

3.1	$m_{WP} = \dfrac{4-(-4)}{-4-0} = \dfrac{8}{-4}$ $m_{WP} = -2$	✓ substitution of W and P ✓ m_{WP} (2)
3.2	$m_{ST} = \dfrac{1}{2}$ (given) $(m_{WP})(m_{ST}) = (-2)\left(\dfrac{1}{2}\right)$ $= -1$ $\therefore ST \perp WP$	✓ $(m_{WP})(m_{ST})$ ✓ $(m_{WP})(m_{ST}) = -1$ (2)
3.3	$5y + 2x + 60 = 0$ $\therefore y = -\dfrac{2}{5}x - 12$ $-\dfrac{2}{5}x - 12 = \dfrac{1}{2}x + 6$ $-4x - 120 = 5x + 60$ $9x = -180$ $x = -20$ $\therefore y = -\dfrac{2}{5}(-20) - 12$ $\therefore y = -4$ $\therefore$ S(–20; –4) **OR**	✓ equating ✓ x value ✓ substitution ✓ y value (4)

3.4	$y=-\frac{2}{5}(-4)-12$ OR $5y+2(-4)+60=0$ $y=-\frac{52}{5}$ $\therefore R\left(-4;-\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore WR=4-\left(-\frac{52}{5}\right)$ OR $WR=\sqrt{(-4-(-4))^2+(4-\left(-\frac{52}{5}\right))^2}$ $\therefore WR=\frac{72}{5}$ units or $WR=14\frac{2}{5}$ units **OR** $WR=ST-SK$ $=\frac{1}{2}x+6-\left(-\frac{2}{5}x-12\right)$ $=\frac{9}{10}x+18$ $=\frac{9}{10}(-4)+18$ $=14{,}4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst x = –4 ✓ answer (4)
3.5	$m_{SK}=-\frac{2}{5}$ $\beta=158{,}19...^\circ$ (Ref. $\angle$ =21, 801…°) $M\hat{N}S=21{,}80...^\circ$ $m_{ST}=\frac{1}{2}$ $N\hat{M}S=26{,}56...^\circ$ $\theta=21{,}80...^\circ+26{,}56...^\circ$ [ext $\angle$ of Δ] $\theta=48{,}366...^\circ=48{,}37^\circ$	✓ m_{SK} ✓ size of β ✓ size of $N\hat{M}S$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h$ = –4 – (–20) $\perp h$ = 16units Area $\Delta SRW=\frac{1}{2}(\perp h)(WR)$ $=\frac{1}{2}(16)\left(\frac{72}{5}\right)$ $=115{,}2$ square units Area SWRL = 2Area ΔSRW =2(115,2) = 230,4square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

	In ΔSRW: $\perp h = -4-(-20)$ $\perp h = 16$units Area SWRL $= 16 \times \dfrac{72}{5}$ $= 230{,}40$ square units **OR** $\text{SW} = \sqrt{(-20+4)^2 + (-4-4)^2} = 8\sqrt{5} = 17{,}89$ $\text{SR} = \sqrt{(-20+4)^2 + \left(-4+10\dfrac{2}{5}\right)^2} = \dfrac{16\sqrt{29}}{5} = 17{,}23$ Area SWRL = $2 \times$ Area ΔSRW $= 2\left(\dfrac{1}{2}\text{SW} \times \text{SR} \sin\theta\right)$ $= 2\left(\dfrac{1}{2} 8\sqrt{5} \times \dfrac{16\sqrt{29}}{5} \sin 48{,}37°\right)$ $=230{,}41$square units	✓ $\perp h$ ✓ ✓ substitution ✓ answer (4) ✓SW $= 8\sqrt{5}$ ✓SR$= \dfrac{16\sqrt{29}}{5}$ ✓substitution ✓answer (4)
		[21]

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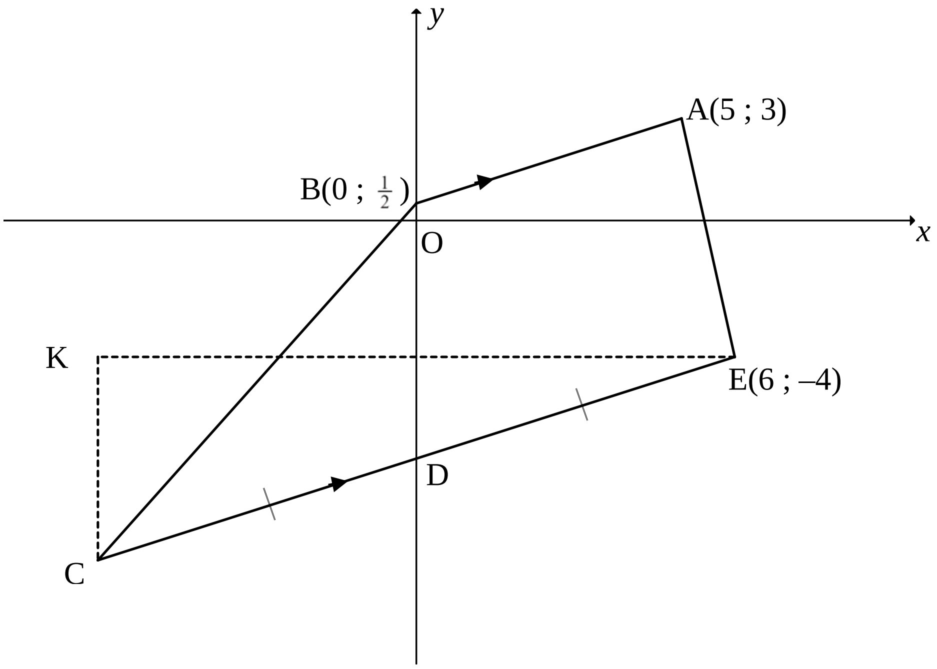

3.1	$m_{AB}=\dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB}=\dfrac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)
3.2	$m_{CE}=m_{BA}=\dfrac{1}{2}$ $-4=\dfrac{1}{2}(6)+c$ **OR/*OF*** $y-(-4)=\dfrac{1}{2}(x-6)$ $c=-7$ $y=\dfrac{1}{2}x-7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\dfrac{x_C+6}{2}=0$ $\dfrac{y_C+(-4)}{2}=-7$ $x_C=-6$ $y_C=-10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C=-6$ ✓ $y_C=-10$ (3)
3.3.2	Area ΔBCD $=\dfrac{1}{2}(7,5)(6)$ = 22,5 Area ΔABD $=\dfrac{1}{2}(7,5)(5)$ = 18,75 Area ABCD = 22,5 + 18,75 = 41,25 units^2	✓ subst of correct base and height into the area formula ✓ area ΔBCD = 22,5 ✓ area ΔABD = 18,75 ✓ answer (4)

[41 d]

3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter ΔKEC = 6 + 12 + $\sqrt{180}$ = 31,42 units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63{,}43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63{,}43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$ $K\hat{C}E = 90° - 26{,}57°$ $K\hat{C}E = 63{,}43°$ **OR/*OF*** $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63{,}43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3) ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}57°$ ✓ answer (3) ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)
		[21]

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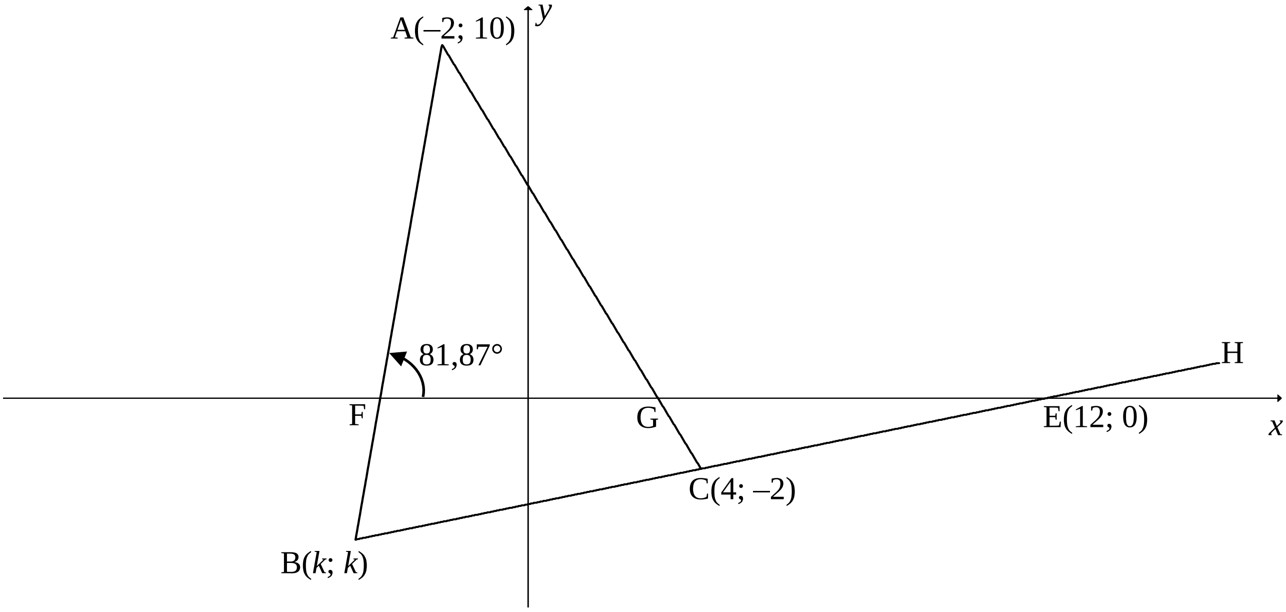


3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

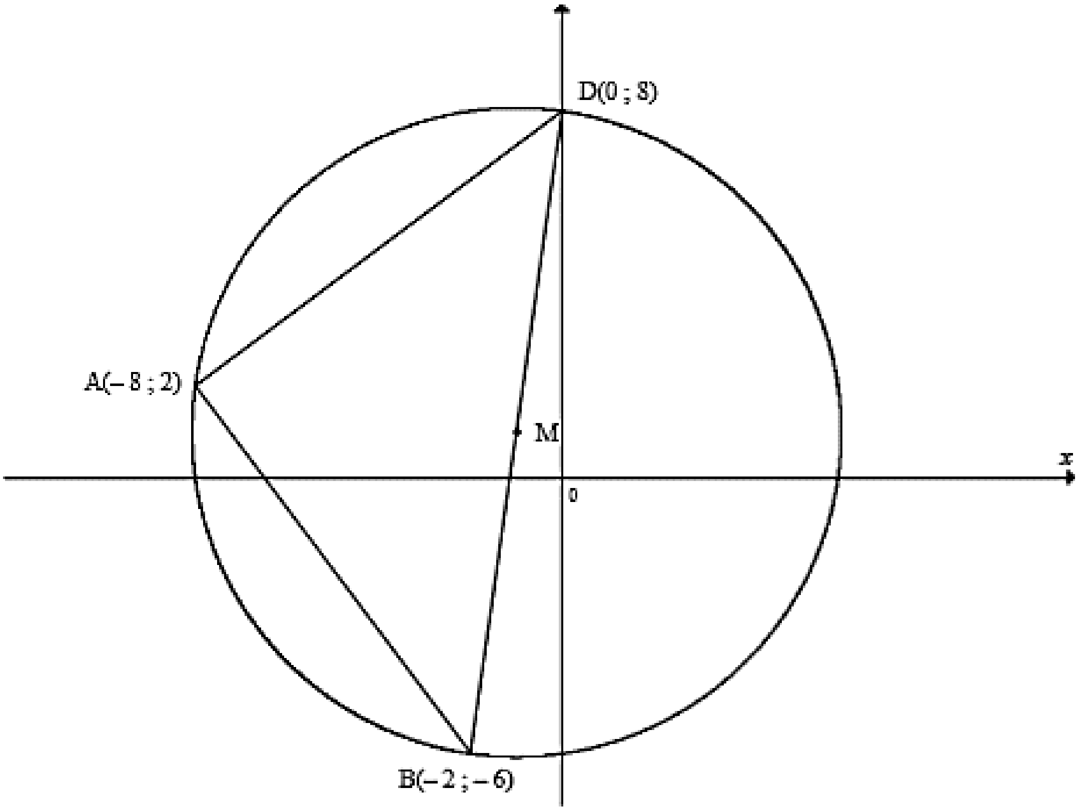
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore \text{B}(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{\text{BE}}=\frac{1}{4}$ **OR/*OF*** $m_{\text{BE}}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $-4k=12-k$ $4k=k-12$ $k=-4$ $k=-4$ $\therefore \text{B}(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{\text{AB}}=\tan 81{,}87^\circ$ $m_{\text{AB}}=7$ $m_{\text{AB}}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore \text{B}(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore \text{B}(-4;-4)$	✓ equating EB & AB ✓ answer (2)

[41 g]

3.3.2	In ΔAFG: $m_{AC} = \dfrac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34{,}70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17};\ b = AC = 6\sqrt{5};\ c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \dfrac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\dfrac{12+(-2)}{2}\ ;\dfrac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \dfrac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

TOPIC 2: Equation of a circle

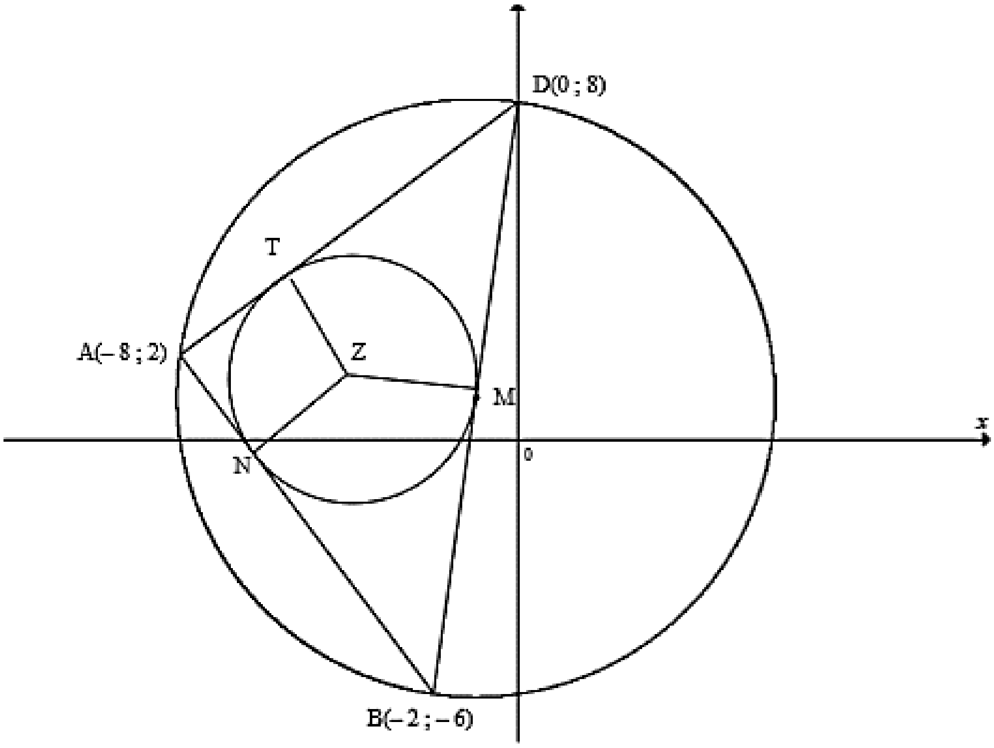
Activity 1



1.1.	Midpoint BD $\left(\frac{0-2}{2};\frac{8-6}{2}\right)$ $=(-1;\ 1)$
1.2.	$y=7(-8)+58$ $=2$ $\therefore$ A lies on the line.
1.3.	The line $y = 7x + 58$ is a tangent to the circle at A. $m_{line}=7$ $m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ $m_{line}\times m_{AM}=7\times-\frac{1}{7}=-1$ $\therefore$ AM $\perp$ to the line **OR**

	$m_{BD} = 7$ $m_{line} = 7$ $\therefore$ line // diameter **OR** $(x+1)^2 + (y-1)^2 = 50$ $x^2 + 2x + 1 + y^2 - 2y + 1 = 50$ $x^2 + 2x + 1 + (7x+58)^2 - 2(7x+58) + 1 = 50$ $x^2 + 2x + 1 + 49x^2 + 812x + 3364 - 14x - 116 + 1 = 50$ $50x^2 + 800x + 3200 = 0$ $x^2 + 16x + 64 = 0$ $(x+8)^2 = 0$ $x = -8$ $y = 2$ $y = 7x + 58$ is a tangent to the circle
1.4.	$\text{AD} = \sqrt{(8-2)^2 + (0+8)^2}$ $= \sqrt{36+64}$ $= 10$ $\text{AB} = \sqrt{(2+6)^2 + (-8+2)^2}$ $= \sqrt{64+36}$ $= 10$
1.5.	$m_{AD} = \dfrac{8-(2)}{0-(-8)}$ $m_{AD} = \dfrac{3}{4}$ $m_{AB} = \dfrac{2-(-6)}{-8-(-2)}$ $= -\dfrac{4}{3}$ $m_{AB}.m_{AD} = -\dfrac{4}{3} \times \dfrac{3}{4}$ $= -1$ $\text{D}\hat{\text{A}}\text{B} = 90°$ **OR**

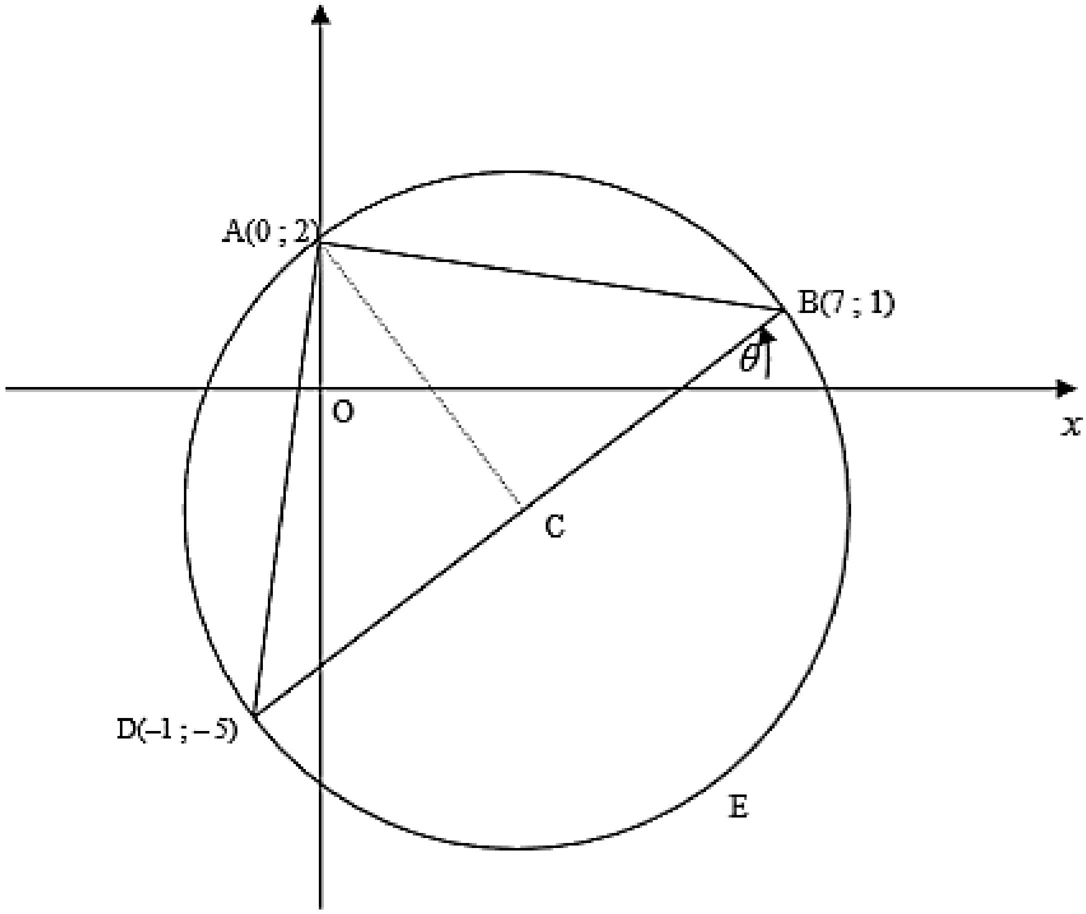
	$BD^2 = (8+6)^2 + (0+2)^2$ $= 200$ $= AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ **OR** $a^2 = b^2 + d^2 - 2(b)(d)\cos A$ $200 = 100 + 100 - 2(10)(10)\cos A$ $0 = -200\cos A$ $A = 90°$ **OR** $(AD)^2 = 100$ $(AB)^2 = 100$ $BD^2 = (-2-0)^2 + (-6-8)^2$ $= 4 + 196$ $= 200$ $\therefore BD^2 = AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ (Pyth) **OR** $\hat{A} = 90°$ (angles in semi - circle)
1.6.	$\theta = 45°$



1.7.	Let the radius of circle TNM be r $NB = BM$ (properties of a kite) $AN = TZ = r$ (TZNA is a square) $NB = 10 - r$ $BD = 2MB$ $\sqrt{(8-(-6))^2 + (0-(-2))^2} = 2(10-r)$ $\sqrt{200} = 2(10-r)$ $10\sqrt{2} = 2(10-r)$ $r = 10 - 5\sqrt{2}$ $= 2{,}93$ **OR** $Z\hat{M}B = 90°$ $MB = \frac{1}{2}\sqrt{200}$ $= 7{,}07$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$ **OR**

	$MB^2 = (-1+2)^2 + (1+6)^2$ $= 1 + 49$ $= 50$ $MB = \sqrt{50}$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$ **OR** By a well known formula Area $\Delta ABD = r \times$ (semi—perimeter) $\frac{1}{2} \times 10 \times 10 = r \times \frac{1}{2}(20 + \sqrt{200})$ $50 = r(10 + 5\sqrt{2})$ $r = 2{,}93$ **OR** $MB = \sqrt{50}$ (radius of circle) $NB = \sqrt{50}$ (adjacent sides of kite) $AB = 10$ $AN = 10 - \sqrt{50}$ $= 2{,}93$ But TANZ is a square $\therefore AN = ZN$ $\therefore$ radius $= 2{,}93$

Activity 2

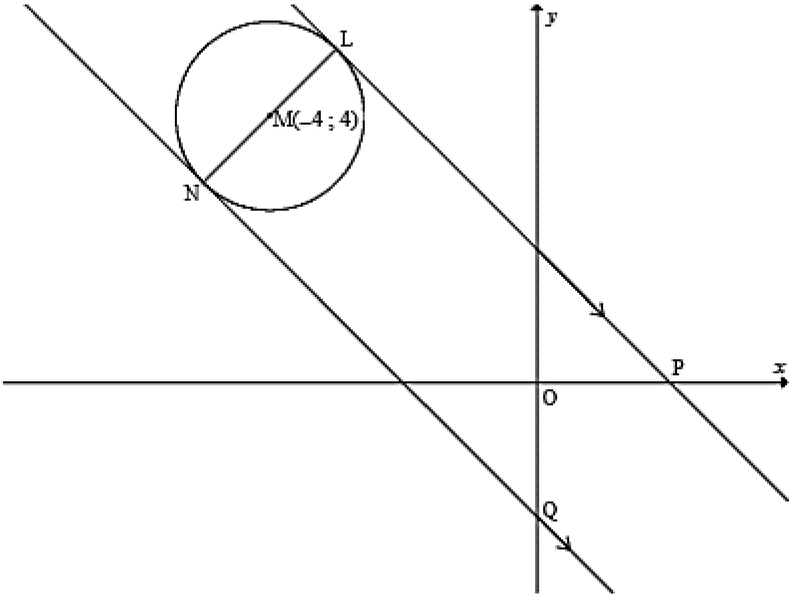


2.1.	Midpoint BD : $\left(\frac{7-1}{2};\frac{1-5}{2}\right)$ $= (3; -2)$
2.2.	$CA = \sqrt{(3-0)^2 + (-2-2)^2} = \sqrt{25} = 5$ $CB = \sqrt{(3-7)^2 + (-2-1)^2} = \sqrt{25} = 5$ $\therefore CA = CB = CD$
2.3.	$r = 5$ and centre$(3\ ;-2)$ Equation of circle is $(x-3)^2 + (y+2)^2 = 25$

2.4.	$m_{BD} = \frac{1-(-5)}{7-(-1)}$ $= \frac{3}{4}$ $m_{BD} = \tan\theta = \frac{3}{4}$ $\therefore \theta = 36{,}87°$
2.5.	Let E (x ; y) $\frac{x+0}{2} = 3$ $\quad$ $\frac{y+2}{2} = -2$ $\therefore x = 6$ $\quad$ $\therefore y = -6$ E(6 ; −6) **OR** E(3 + 3; −2 − 4) = (6 ; −6)
2.6.	The diagonals AE and BD bisect each other $m_{AB} \times m_{AD} = \frac{1-2}{7-0} \times \frac{2+5}{0+1} = \frac{-1}{7} \times 7 = -1$ $\therefore \hat{A} = 90°$ $\therefore$ ABED is a rectangle (diagonals bisect each other and adjacent sides are perpendicular) **OR** Show that $\hat{A} = \hat{B} = \hat{C} = \hat{D} = 90°$
2.7.	Gradient of tangent is $= -\frac{4}{3}$. Equation of a tangent at B is: $y - 1 = -\frac{4}{3}(x-7)$ $y = -\frac{4}{3}x + \frac{28}{3} + 1$ $y = -\frac{4}{3}x + \frac{31}{3}$ **OR**

	$y-6=-\frac{4}{3}(x)$ $1=-\frac{4}{3}(7)+c$ $c=\frac{31}{3}$ $\therefore y=-\frac{4}{3}x+\frac{31}{3}$
2.8.	$x^2+2x+\left(\frac{1}{2}(2)\right)^2+y^2-4y+\left(\frac{1}{2}(-4)\right)^2=5+1+4$ $(x+1)^2+(y-2)^2=10$ But $(x;y)\rightarrow(x-2;y+4)$ $\therefore(-1;2)\rightarrow(-1-2;2+4)$ $=(-3;6)$ $\therefore(x+3)^2+(y-6)^2=10$ **OR** $(x+2)^2+2(x+2)+(y-4)^2-4(y-4)-5=0$ $x^2+y^2+6x-12y+35=0$ $(x+3)^2+(y-6)^2=10$
2.9.	Distance from origin to centre $=\sqrt{(-3-0)^2+(6-0)^2}=\sqrt{45}$ Since $\sqrt{45}>\sqrt{10}$, the origin lies outside the circle.

Activity 3	

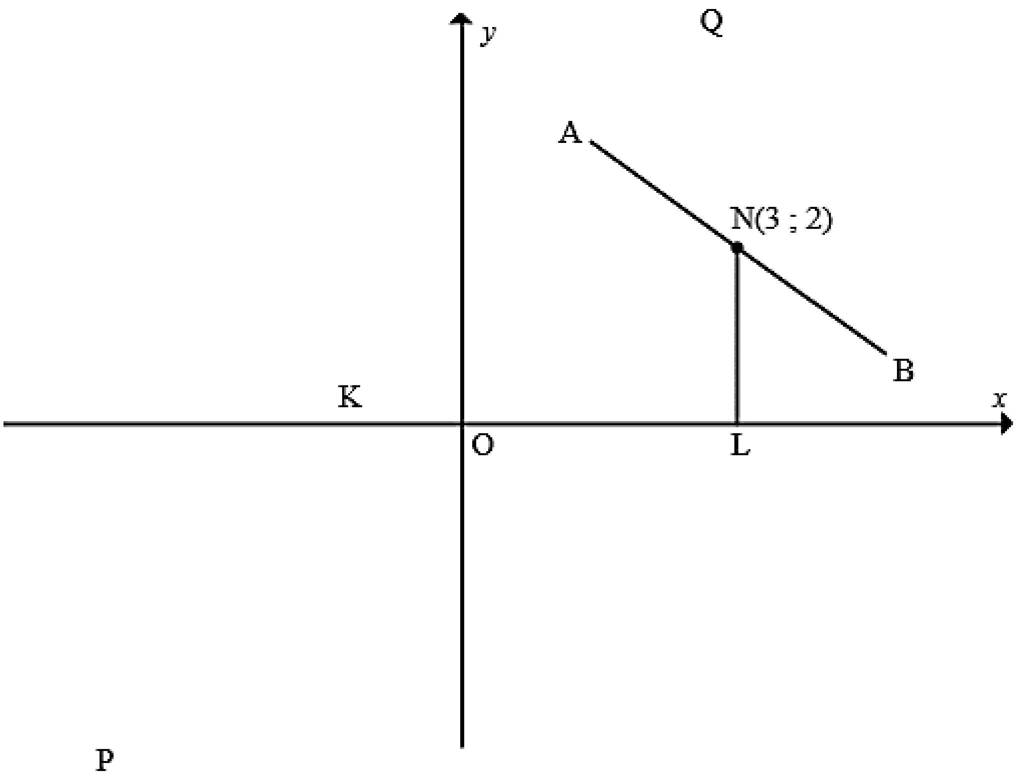


3.1.	$y=-x+2$ $m_{LP}=-1$ $\therefore m_{LN}=\frac{-1}{-1}=1$ $y=x+c$ $4=-4+c$ $\therefore c=8$ $y=x+8$ **OR** $y-4=1(x+4)$ $y=x+8$
3.2.	$x+8=-x+2$ $2x=-6$ $x=-3$ $y=-3+8$ $y=5$ L(–3;5) **OR** $y+x=2$............(1) $y-x=8$...........(2 $2y=10$ $\therefore y=5$ $\therefore x=-3$ L(–3;5)

3.3.	$(x+4)^2+(y-4)^2=r^2$ $(-3+4)^2+(5-4)^2=r^2$ $\therefore r^2=2$ $(x+4)^2+(y-4)^2=2$ Equation can be left as: $x^2+8x+y^2-8y+30=0$
3.4.	Let N(x, y). Since M(-4 ; 4) is the midpoint of LN and L(-3 ; 5) $\dfrac{x-3}{2}=-4$; $\dfrac{y+5}{2}=4$ $\therefore x=-5$; $y=3$ **OR** $y=x+8$ $(x+4)^2+(y-6)^2=2$ $(x+4)^2+(x+8-4)^2-2=0$ $x^2+8x+16+x^2+8x+16-2=0$ $2x^2+16x+30=0$ $x^2+8x+15=0$ $(x+5)(x+3)=0$ $x=-3$ or $x=-5$ $y=5$ $\quad$ $y=3$ $\therefore$ N(-5 ; 3)
3.5.	$m_{NQ}=-1$ $y=-x+c$ $3=-(-5)+c$ $c=-2$ $y=-x-2$ **OR** $m_{NQ}=-1$ $y-3=-(x+5)$ $y=-x-2$ **OR**

	Equation of LP is $x+y=2$ NQ || LP $\therefore$ equation of NQ is $x+y=k$ for some $k \in R$ But N$(-5\,;\,3)$ lies on NQ $\therefore x+y=-5+3=-2$ **OR** NQ is a reflection of LP $(y+x=2)$ in the line $y=x$ $\therefore$ equation of NQ is $x+y=-2$
3.6.	Let new radius of circle be R and centre be M$'$. $M'(-4+6\,;\,4)$ $=(2\,;\,4)$ $R=2r$ $R^2=4r^2$ $=4(2)$ $=8$ $\therefore (x-2)^2+(y-4)^2=(2\sqrt{2})^2$ $\therefore (x-2)^2+(y-4)^2=8$ **OR** Let R = new radius of circle $R^2=(2r)^2=4(2)=8$ $(x-6+4)^2+(y-4)^2=8$ $\therefore (x-2)^2+(y-4)^2=8$

Activity 4



4.1.	The radius (NL) of a circle is perpendicular to the tangent (OL) at the point of contact.
4.2.	L(3 ; 0)
4.3.	Centre N (3 ; 2) and $r = \text{NL} = 2$ Equation of the circle N: $(x-a)^2+(y-b)^2=r^2$ $(x-3)^2+(y-2)^2=4$

4.4.	Coordinates of K. K is the x-intercept of the tangent. $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ KL = 3 − (−1) **OR** KL = 3 + 1 KL = 4 **OR** $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KL=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $KL=\sqrt{(3+1)^2+(0-0)^2}$ $KL=\sqrt{16}$ $KL=4$ **OR** For AK, $m=\frac{4}{3}$, $c=\frac{4}{3}$ $\frac{\frac{4}{3}}{OK}=\tan A\hat{K}O=\frac{4}{3}$ $OK=1$ $\therefore KL=4$ **OR**

	$y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KN^2=NL^2+KL^2$ Theorem of Pythagoras $(-1-3)^2+(0-2)^2=4+KL^2$ $20=4+KL^2$ $16=KL^2$ $KL=4$
4.5.	$m_{AB}\times m_{AK}=-1$ tangent $\perp$ radius $m_{AK}=\frac{4}{3}$ $\therefore m_{AB}=-\frac{3}{4}$ $y-y_1=m(x-x_1)$ $y-2=-\frac{3}{4}(x-3)$ $y=-\frac{3}{4}x+\frac{9}{4}+\frac{8}{4}$ $y=-\frac{3}{4}x+\frac{17}{4}$ **OR**

$$m_{AB} \times m_{AK} = -1$$ tangent $\perp$ radius

$$m_{AK} = \frac{4}{3}$$

$$\therefore \; m_{AB} = -\frac{3}{4}$$

$$y = -\frac{3}{4}x + c$$

$$2 = \left(-\frac{3}{4}\right)(3) + c$$

$$c = \frac{8}{4} + \frac{9}{4}$$

$$c = \frac{17}{4}$$

$$y = -\frac{3}{4}x + \frac{17}{4}$$

4.6. Point A lies on PQ and AB. Therefore

$$\frac{4}{3}x + \frac{4}{3} = -\frac{3}{4}x + \frac{17}{4}$$

$$16x + 16 = -9x + 51$$

$$25x = 35$$

$$x = \frac{7}{5}$$

$$y = -\frac{3}{4}\left(\frac{7}{5}\right) + \frac{17}{4}$$

$$y = \frac{16}{5}$$

$$A\left(\frac{7}{5}\,;\,\frac{16}{5}\right)$$

OR

Point A lies on PQ and the circle. Therefore

$$(x-3)^2+(\frac{4}{3}x+\frac{4}{3}-2)^2=4$$

$$(x-3)^2+(\frac{4}{3}x-\frac{2}{3})^2=4$$

$$25x^2-70x+49=0$$

$$(5x-7)^2=0$$

$$x=\frac{7}{5}$$

$$y=-\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$$

$$y=\frac{16}{5}$$

OR

Point A lies on the circle and line AB

$$(x-3)^2+(y-2)^2=4 \quad ----------(1)$$

$$y=-\frac{3}{4}x+\frac{17}{4} \qquad ----------(2)$$

$$\text{Subs (2) in (1)}: x^2-6x+9+(-\frac{3}{4}x+\frac{17}{4}-2)^2=4$$

$$x^2-6x+9+(-\frac{3}{4}x+\frac{9}{4})^2=4$$

$$25x^2-150x+161=0$$

$$(5x-23)(5x-7)=0$$

$$x=\frac{7}{5}$$

$$y=-\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$$

$$y=\frac{16}{5}$$

OR

Using rotation:

Let $\theta = A\hat{K}N = L\hat{K}N$

Move diagram 1 unit to the right. Then A$^{/}$ is L$^{/}$ rotated through 2θ .

$$\tan\theta = \frac{AN}{KA} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \sin 2\theta = 2\sin\theta\cos\theta = 2(\frac{1}{\sqrt{5}})(\frac{2}{\sqrt{5}}) = \frac{4}{5}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = (\frac{2}{\sqrt{5}})^2 - (\frac{1}{\sqrt{5}})^2 = \frac{3}{5}$$

$$\therefore x_{A'} = x_{L'}\cos 2\theta - y_{L'}\sin 2\theta = 4(\frac{3}{5}) - (0)(\frac{4}{5}) = \frac{12}{5}$$

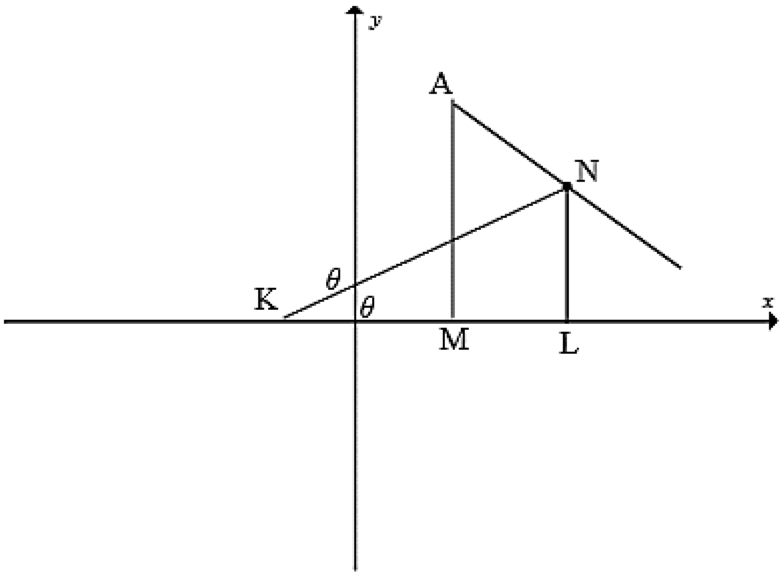
$$y_{A'} = x_{L'}\sin 2\theta + y_{L'}\cos 2\theta = 4(\frac{4}{5}) - (0)(\frac{3}{5}) = \frac{16}{5}$$

$$A'(\frac{12}{5}\ ;\ \frac{16}{5})$$

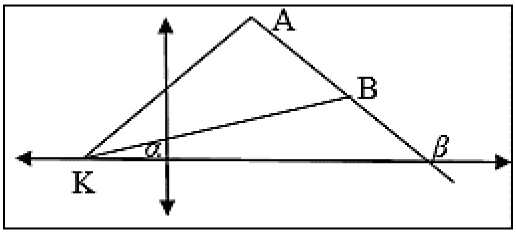
Now to get back to A, move back 1 unit to the left.

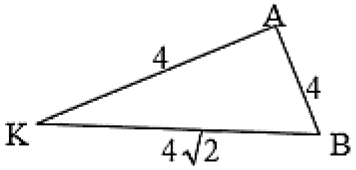
$$\therefore A(\frac{7}{5}\ ;\ \frac{16}{5})$$

OR

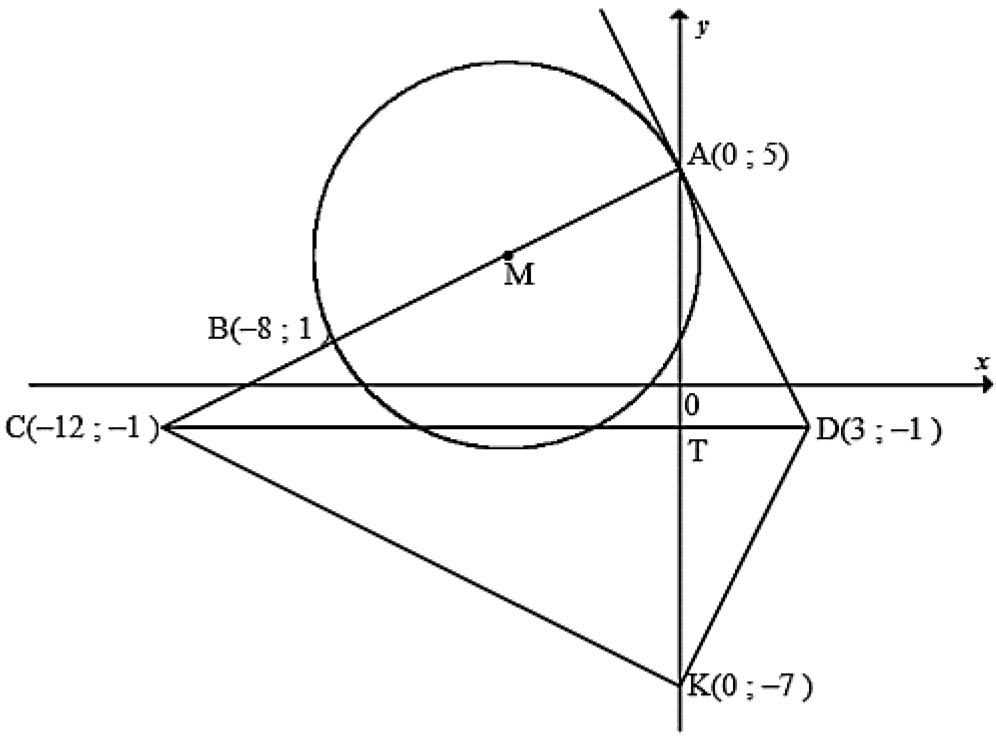


	Let $N\hat{K}L = \theta$. So, $\tan\theta = \frac{NL}{KN} = \frac{2}{4} = \frac{1}{2}$. Hence $\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$ Let $AM \perp x-$axis with M on x - axis $\Delta NAK \equiv \Delta NLK$ $A\hat{K}N = N\hat{K}L = \theta$ $\therefore A\hat{K}L = 2\theta$ $y_A = AM = AK\sin 2\theta = KL\sin 2\theta = 4\sin 2\theta$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$ $y_A = 4\left(\frac{4}{5}\right) = \frac{16}{5}$ $x_A = OL - NA\sin M\hat{A}N$ $= 3 - 2\sin(90° - M\hat{A}K)$ $= 3 - 2\sin 2\theta$ $= 3 - \frac{8}{5}$ $= \frac{7}{5}$
4.7.	$KA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{7}{5}+1\right)^2 + \left(\frac{16}{5} - 0\right)^2}$ $= 4$ **OR** $KN = \sqrt{4^2 + 2^2} = \sqrt{20}$ $KA^2 = KN^2 - AN^2$ $= 20 - 4$ $= 16$ $KA = 4$ **OR** KA = KL Tangents from a common point are equal KA = 4

4.8.	AN = NL Radii are equal KA = KL $\therefore$ KLNA is a kite two pairs of adjacent sides are equal.
4.9.	AB = AN + NB = 2 + 2 = 4 AK = 4= AB $K\hat{A}B = 90°$ tangent $\perp$ radius $\therefore \Delta$AKB is a right – angled isosceles triangle $A\hat{K}B + A\hat{B}K = 90°$ $2A\hat{B}K = 90°$ $\therefore A\hat{B}K = 45°$ **OR** N is midpoint of AB Let B be $(x_B; y_B)$ $\dfrac{x_B + \frac{7}{5}}{2} = 3$ $\dfrac{y_B + \frac{16}{5}}{2} = 2$ $\therefore x_B = \dfrac{23}{5}$ $\therefore y_B = \dfrac{4}{5}$ $\therefore B\left(\dfrac{23}{5}; \dfrac{4}{5}\right)$  $\tan\beta = m_{AB} = -\dfrac{3}{4}$ $\beta = 180° - 36{,}87°$ $\beta = 143{,}13°$ $\tan\alpha = m_{KB} = \dfrac{\frac{4}{5} - 0}{\frac{23}{5} + 1} = \dfrac{1}{7}$ $\alpha = 8{,}13°$ $A\hat{B}K = \alpha + (180° - \beta)$ $= 8{,}13° + 36{,}87°$ $= 45°$ **OR**

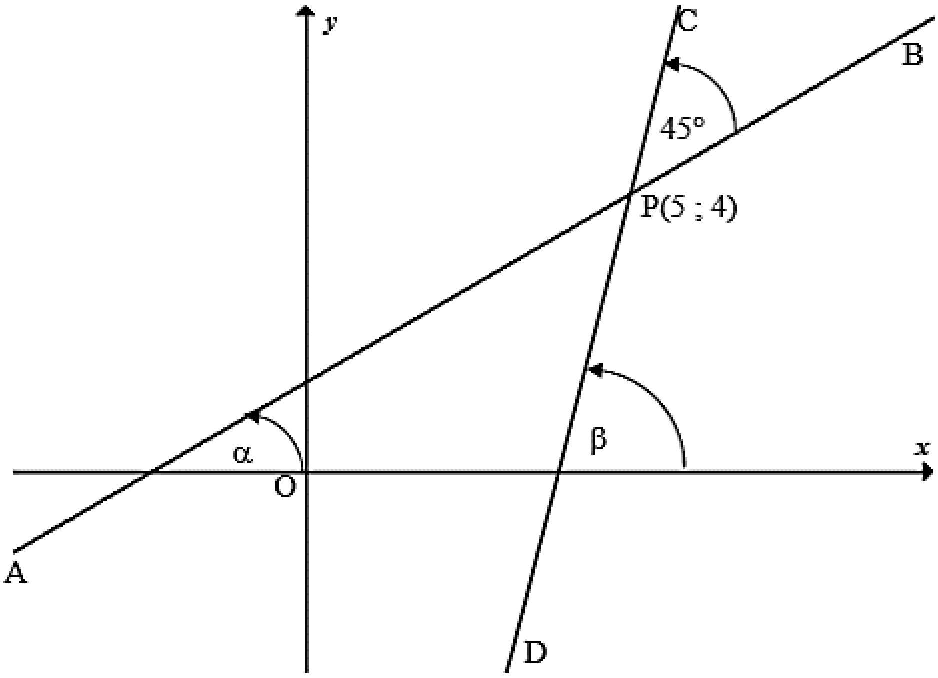
	N is midpoint of AB Let B be $(x_B;y_B)$ $\frac{x_B+\frac{7}{5}}{2}=3$ $\qquad$ $\frac{y_B+\frac{16}{5}}{2}=2$ $\therefore x_B=\frac{23}{5}$ $\qquad$ $\therefore y_B=\frac{4}{5}$ $\therefore B\left(\frac{23}{5};\frac{4}{5}\right)$  $KB=\sqrt{(\frac{23}{5}+1)^2+(\frac{4}{5})^2}=4\sqrt{2}$ $4^2=4^2+(\sqrt{32})^2-2(4)(\sqrt{32})\cos\theta$ $\cos\theta=\frac{\sqrt{2}}{2}$ $\therefore\theta=45°$
4.10.	$N'(3;-2)$

Activity 5	



5.1.	Midpoint AB $\left(\frac{-8+0}{2};\frac{1+5}{2}\right)$ $=(-4\,;3)$
5.2.	$M_{AD}=\frac{5+1}{0-3}=\frac{-2}{1}$ $y-y_1=m(x-x_1)$ $y-5=-2(x-0)$ $y=-2x+5$
5.3.	$AM^2=(5-3)^2+(0+4)^2$ $\mathrm{AM}^2=2^2+4^2$ $AM=\sqrt{20}$
5.4.	$(x+4)^2+(y-3)^2=\left(\sqrt{20}\right)^2$ $(x+4)^2+(y-3)^2=20$ $x^2+y^2+8x-6y+5=0$
5.5.	$\mathrm{AT}=\mathrm{TK}=6$ $\mathrm{CD}\perp\mathrm{AK}$ Therefore, ACKD is a kite since diagonal CD bisects diagonal AK at right angles. **OR** $C\hat{A}D=90°$ $M_{KC}.M_{KD}=\frac{6}{-12}.\frac{6}{3}=-1$ $\therefore C\hat{K}D=90°$ Δ CAD & Δ CKD are right angles & congruent ACKD is a kite

Activity 6	



6.1.

AB is defined as $5y - 3x - 5 = 0$ which can be written as $y = \frac{3}{5}x + 1$

$m_{AB} = \frac{3}{5}$

Let α be the inclination of AB.

$\tan \alpha = \frac{3}{5}$

$\alpha = 30.96°$.

Let β be the inclination of CD

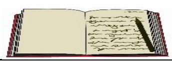
$\beta = 45° + 30,96°$

$= 75,96°$

Gradient of CD = tan 75,96° = 4.

OR

	$\tan\beta = \tan(\alpha + 45°)$ $= \dfrac{\tan\alpha + \tan 45°}{1 - \tan\alpha.\tan 45°}$ $= \dfrac{\frac{3}{5} + 1}{1 - \frac{3}{5} \times 1}$ $= 4$ $m_{CD} = \tan\beta$ $m_{CD} = 4$
6.2.	Equation of CD is $y = 4x + c$ $\therefore\ 4 = 4(5) + c$ $c = -16$ Equation of CD is $y = 4x - 16$. **OR** $y - 4 = 4(x - 5)$ $y - 4 = 4x - 20$ $y = 4x - 16$

Activity 7	

7.1.	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x+4)^2 + (y+2)^2 = 58$ Centre is $(-4\ ;\ -2)$ and the radius is $\sqrt{58}$
7.2.	Centre of second circle is $(4\ ;\ 6)$ Distance between centres is $\sqrt{(4+4)^2 + (6+2)} = \sqrt{128} = 11{,}31$
7.3.	Sum of radii $= \sqrt{58} + \sqrt{26} = 12{,}71$ Distance between centres is 11,31. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.

7.4.

Equation of second circle:

$$(x-4)^2+(y-6)^2=26$$
$$x^2-8x+16+y^2-12y+36=26$$
$$x^2-8x+y^2-12y+26=0$$

Let $(x\,;y)$ be either of the two points on intersection.
Then

$$x^2+y^2+8x+4y-38=0$$

and

$$x^2+y^2-8x-12y+26=0$$

Subtract

$$16y+16x-64=0$$
$$y=-x+4$$

Both points of intersection lie on this line.
$\therefore\ y=-x+4$ is the equation of the common chord.

OR

Check that the line $y=-x+4$ cuts the two circles at the same points:

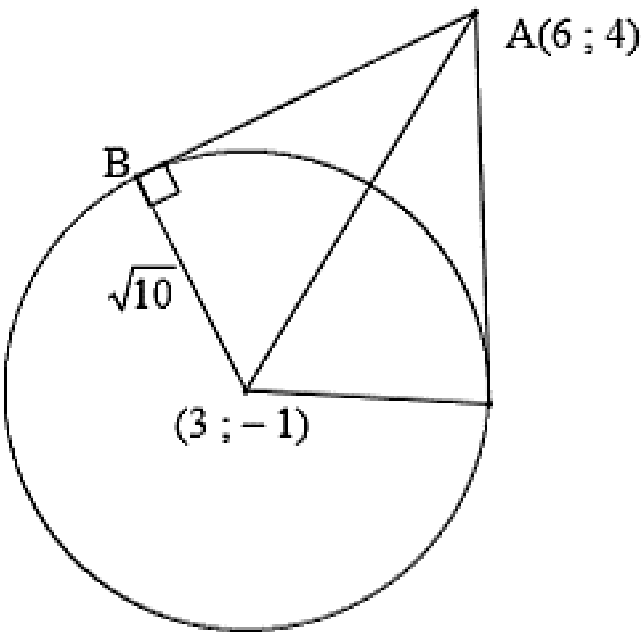
$$(x-4)^2+(-x-2)^2=26$$
$$x^2-8x+16+x^2+4x+4=26$$
$$2x^2-4x-6=0$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x=3\ \text{ or }\ x=-1$$

$$x^2+y^2+8x+4y-38=0$$
$$x^2+(4-x)^2+8x+4(4-x)-38=0$$
$$x^2+16-8x+x^2+8x+16-4x-38=0$$
$$2x^2-4x-6=0$$
$$x^2-2x-3=0$$
$$x=3\quad\text{or}\quad x=-1$$

Activity 8

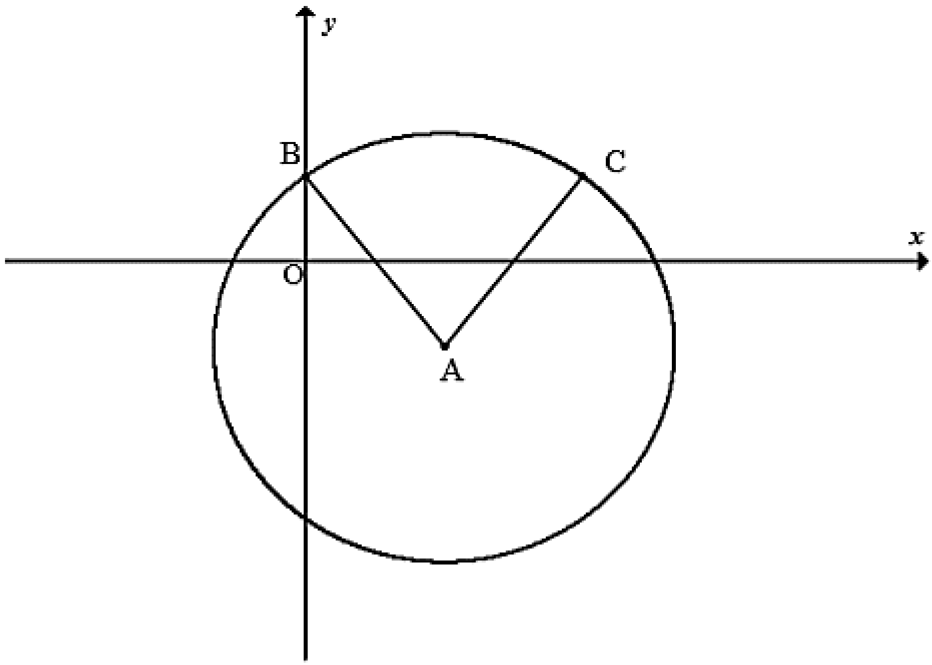
$$x^2+y^2-8x+6y=15$$

8.1.1.	$x^2+y^2-8x+6y$ $=(2)^2+(-9)^2-8(2)+6(-9)$ $=4+81-16-54$ $=15$ Hence, the point lies on the circumference of the circle. **OR** $x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ $(x-4)^2+(y+3)^2$ $=(2-4)^2+(-9+3)^2$ $=2^2+6^2$ $=40$ $\therefore$ The point lies on the circumference of the circle.
8.1.2.	$x^2+y^2-8x+6y=15$ $(x-4)^2+(y+3)^2=15+16+9$ $(x-4)^2+(y+3)^2=40$ Circle centre $(4\,;-3)$ $m_{rad}=\dfrac{-3-(-9)}{4-2}$ $m_{rad}=3$ $m_{\tan}=-\dfrac{1}{3}$ $y+9=-\dfrac{1}{3}(x-2)$ $y=-\dfrac{1}{3}x-\dfrac{25}{3}$



8.2	Radius AB $=\sqrt{10}$ Distance from A to centre of circle is $=\sqrt{(6-3)^2+(4+1)^2}$ $=\sqrt{9+25}$ $=\sqrt{34}$ $AB^2=34-10$ $AB^2=24$ $AB=\sqrt{24}$ $AB=2\sqrt{6}$ $AB=4{,}90$ **OR** $r^2=10$ $r=\sqrt{10}$ Radius $\perp$ tangent By Pythagoras $\mathrm{AB}^2=(6-3)^2+(4+1)^2-10$ $=24$ $\mathrm{AB}=4{,}90$

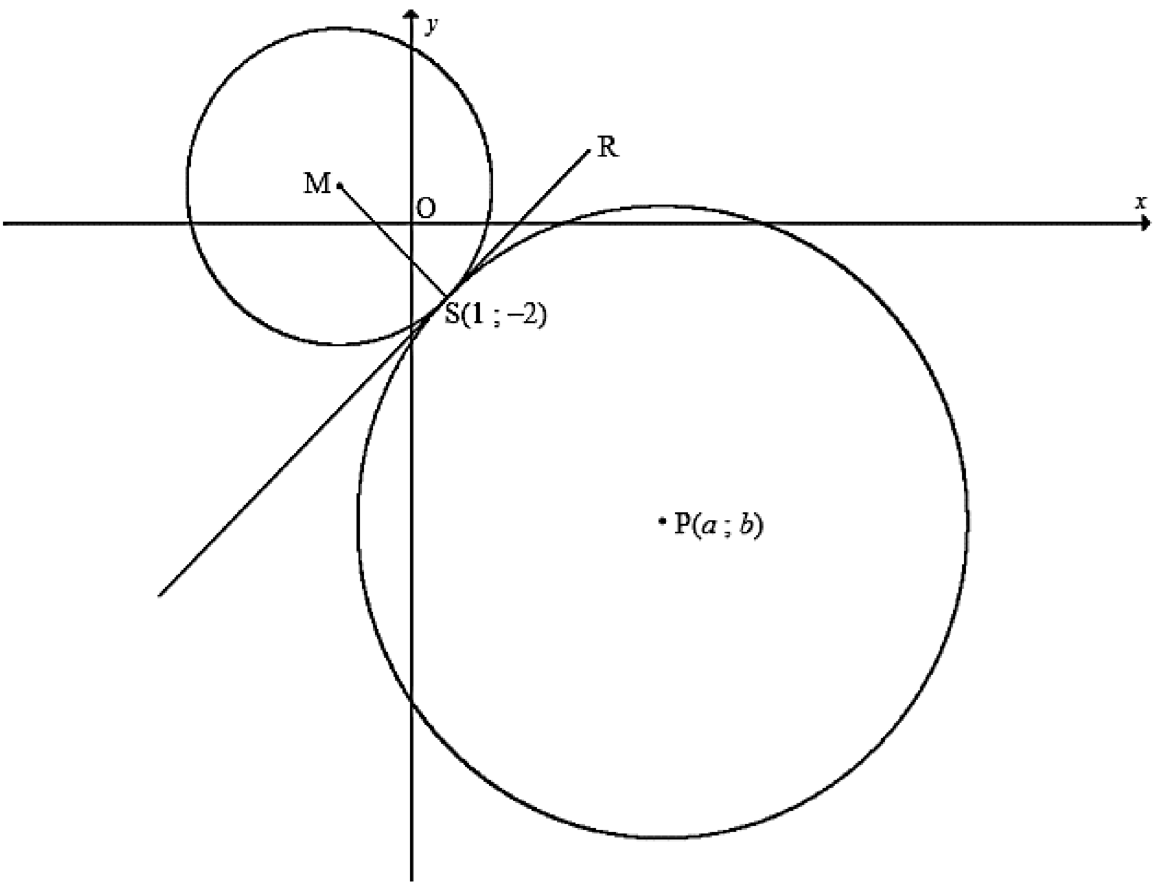
Activity 9



9.1.	$9+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm4$ $y=2$ or $y=-6$ B(0 ; 2) **OR** $x=0$ $(0)^2-6(0)+y^2+4y=12$ $y^2+4y-12=0$ $(y+6)(y-2)=0$ $y=-6$ or $y=2$ B(0 ; 2)
9.2.	C(6 ; 2)
9.3.	$\left(x-3\times\frac{3}{2}\right)^2+\left(y+2\times\frac{3}{2}\right)^2=\left(5\times\frac{3}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=\left(\frac{15}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=56{,}25$

9.4.1.	$AB = \sqrt{(12-3)^2 + (10-(-2))^2}$ $= \sqrt{9^2 + 12^2}$ $= 15$
9.4.2.	The radii are 5 and 10. $r_A + r_B = 5 + 10$ $= 15$ $= AB$ The circles will only intersect at one point.

Activity 10	



10.1.	Coordinates of centre M (–2 ; 1) $(1+2)^2 + (-2-1)^2 = 18 = r^2$ Radius = $\sqrt{18}$ or $3\sqrt{2}$

10.2.	$m_{MS} = \frac{-3}{3} = -1$ $m_{MS} x m_{RS} = -1$ OR tangent $\perp$ radius $m_{RS} = 1$ $y - y_1 = m(x - x_1)$ $y + 2 = 1(x-1)$ $y = x - 3$ **OR** $m_{MS} = \frac{-3}{3} = -1$ $m_{MS} x m_{RS} = -1$ $m_{RS} = 1$ $y = x + c$ $-2 = 1 + c$ $c = -3$ $y = x - 3$
10.3.	$\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162$ (1) $MS \perp SR$ and $PS \perp SR$ $\therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b + 2 = -a + 1$ $b = -a - 1$ (2) Subst (2) into(1) $(a+2)^2 + (-a-1-1)^2 = 162$ $(a+2)^2 + (a+2)^2 = 162$ $2(a+2)^2 = 162$ $(a+2)^2 = 81$ $a + 2 = 9$ or -9 $a = 7$ or -11 $b = -a - 1 = -8$ $P(7; -8)$ **OR**

	$\frac{MS}{MP}=\frac{1}{3}$ $\therefore MP=3MS$ $MP^2=9MS^2$ $(a+2)^2+(b-1)^2=9(3^2+3^2)=162 \qquad (1)$ $MS\perp SR \text{ and } PS\perp SR \qquad \therefore m_{PS}=m_{MS}$ $\frac{b+2}{a-1}=\frac{3}{-3}=-1$ $b+2=-a+1$ $b=-a-1 \qquad (2)$ Subst (2) into(1) $a^2+4a+4+a^2+4a+4=162$ $2a^2+8a-154=0$ $a^2+4a-77=0$ $(a+11)(a-7)=0$ $a=7 \text{ or } -11$ But $a>0$ $\therefore a=7$ $b=-a-1=-8$ $P(7;-8)$ **OR**

P(a ; b)

MSP is a straight line (MS ⊥ SR)

$m_{PM} = -1$

$\frac{b-1}{a+2} = -1$

$b-1 = -a-2$

$b = -a-1$.......(1)

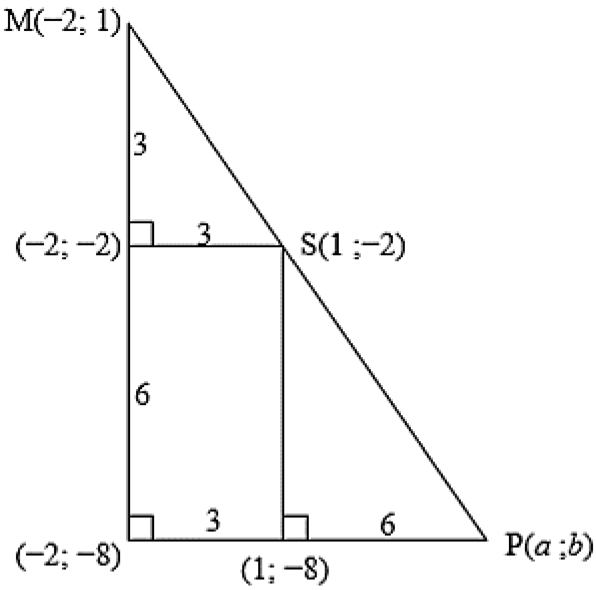
$PS = 2MS = 2\sqrt{9+9} = 2\sqrt{18}$

$PS^2 = 4(18) = 72$

$(a-1)^2 + (b+2)^2 = 72$.......(2)

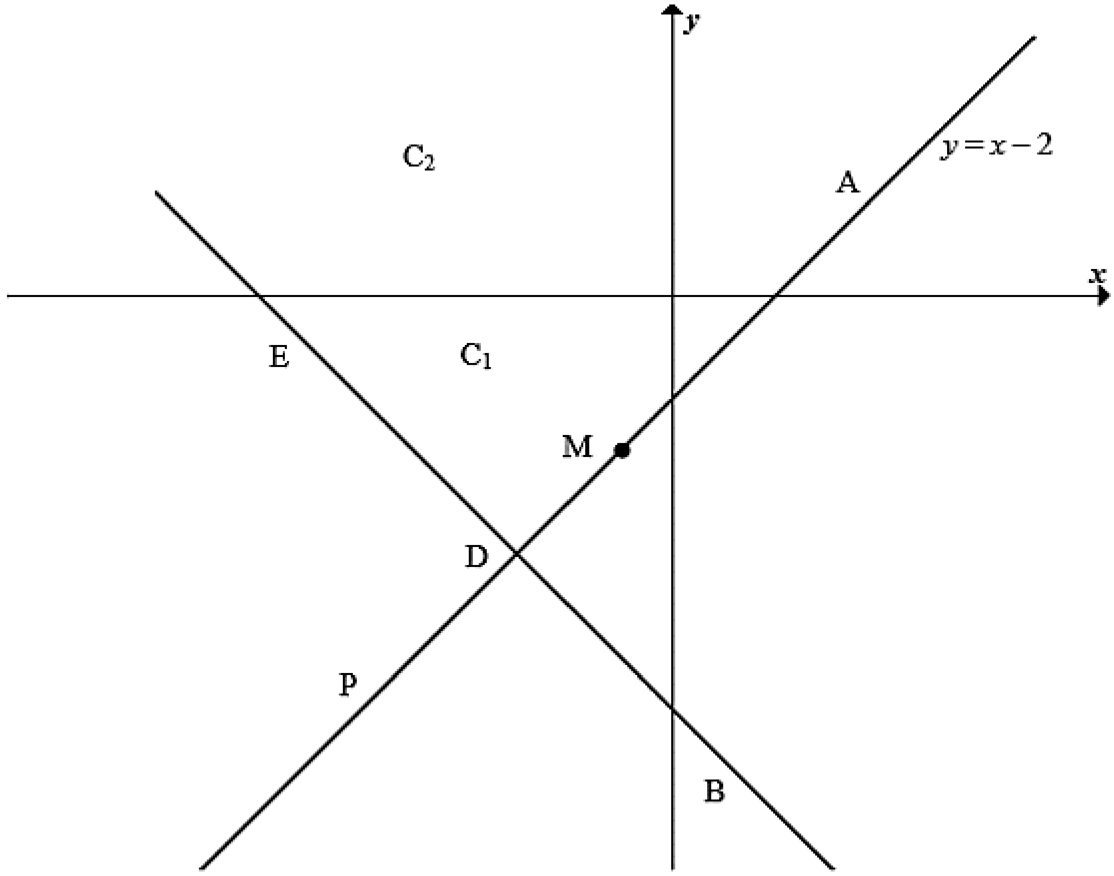
$(a-1)^2 + (-a-1+2)^2 = 72$		$2(a-1)^2 = 72$
$2a^2 - 4a - 70 = 0$		$(a-1)^2 = 36$
$a^2 - 2a - 35 = 0$		$a-1 = 6$ or -6
$(a-7)(a+5) = 0$	**OR**	$a = 7$ or -5
$a = 7$ or $a \neq -5$		$a = 7$
$b = -7-1 = -8$		$b = -8$
$P(7 ;-8)$		$P(7 ;-8)$

OR



	P(a ; b) $\frac{x_S - x_M}{x_P - x_M} = \frac{y_S - y_M}{y_P - y_M} = \frac{1}{3}$ $\frac{-3}{b-1} = \frac{3}{a+2} = \frac{1}{3}$ $-9 = b - 1$ $b = -8$ $9 = a + 2$ $a = 7$ $P(7;-8)$

Activity 11

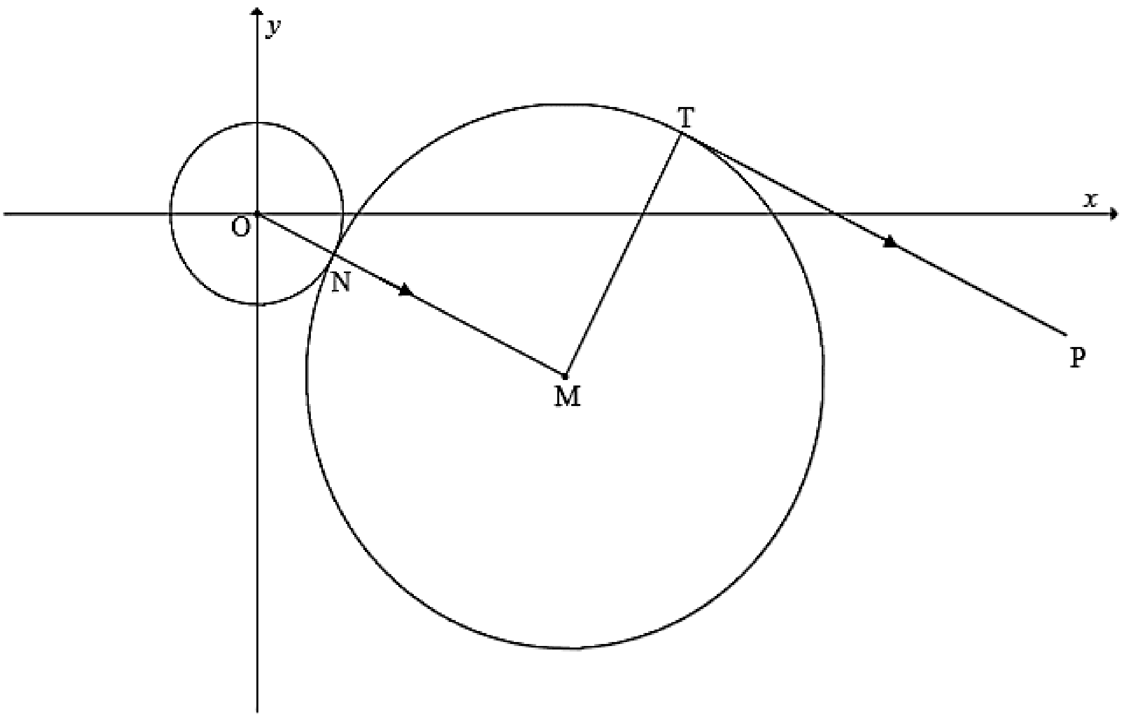


11.1.1.	$x^2 + y^2 + 2x + 6y + 2 = 0$ $x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 10$ $(x+1)^2 + (y+3)^2 = 8$ $M(-1;-3)$

11.1.2.	radius of circle $C_1 = \sqrt{8}$
11.2.	$x^2 + (x-2)^2 + 2x + 6(x-2) + 2 = 0$ $x^2 + x^2 - 4x + 4 + 2x + 6x - 12 + 2 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \ or \ \ x \neq 1$ $y = -3 - 2 = -5$ $\therefore D(-3;-5)$ **OR** $(x+1)^2 + (y+3)^2 = 8$ $subst. \ y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \ or \ \ x \neq 1$ $y = -3 - 2 = -5$ **OR** $(x+1)^2 + (y+3)^2 = 8$ $subst. \ y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $(x+1)^2 = 4$ $x + 1 = \pm 2$ $x = -3 \ or \ \ x \neq 1$ $y = -3 - 2 = -5$ **OR** PM makes 45° with the x-axis. $\sqrt{8} = \sqrt{2^2 + 2^2}$ Therefore: $x_D = x_M - 2 = -1 - 2 = -3$ $y_D = -3 - 2 = -5$

11.3.	$MD \perp DB$ (tangent $\perp$ radius) $MB^2 = MD^2 + DB^2$ (Pythagoras) $= (\sqrt{8})^2 + (4\sqrt{2})^2$ $= 40$ MB is the radius of C_2 $MB = \sqrt{40}$
11.4.	$(x+1)^2 + (y+3)^2 = 40$
11.5.	Distance from $(2\sqrt{5}\ ;\ 0)$ to centre $= \sqrt{(2\sqrt{5}+1)^2 + (0+3)^2}$ $= 6{,}24$ $6{,}24 < 6{,}32\ (\sqrt{40})$ Distance from $(2\sqrt{5}\ ;\ 0)$ to centre < radius of circle. $(2\sqrt{5}\ ;\ 0)$ lies inside the circle.

Activity 12

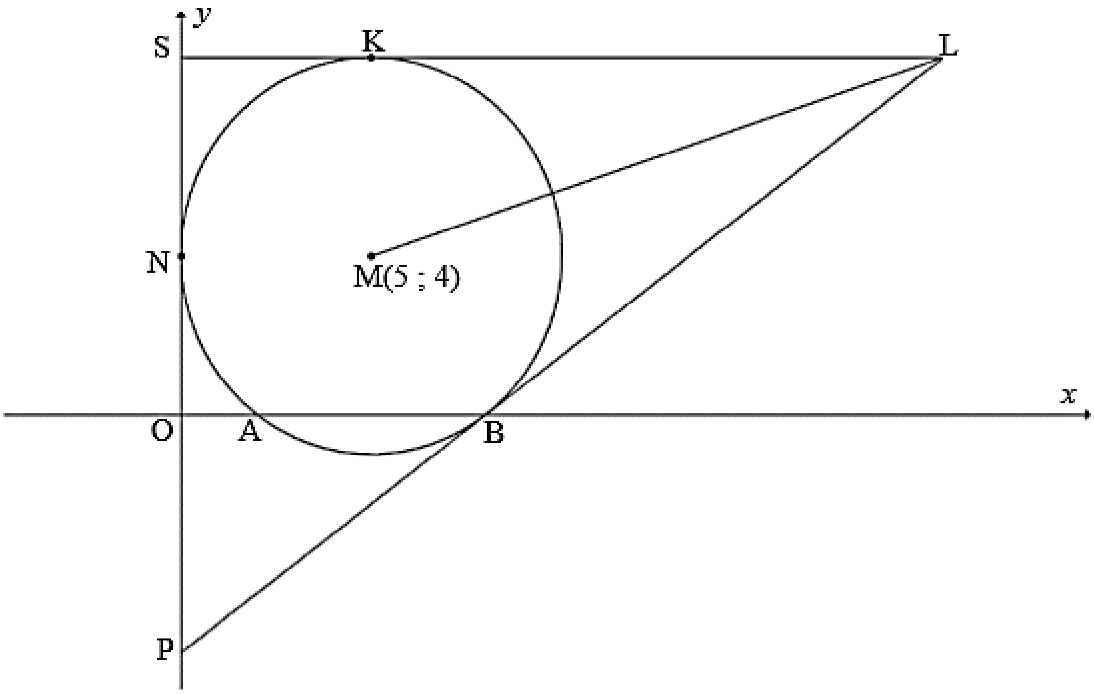


12.1.	$M(8\ ;\ -4)$

12.2.	$\text{OM} = \sqrt{(8-0)^2 + (-4-0)^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ units
12.3.	$ON = OM - NM$ $= \sqrt{80} - \sqrt{45}$ $= 4\sqrt{5} - 3\sqrt{5}$ $= \sqrt{5}$ *units*
12.4.	$\text{M}\hat{\text{T}}\text{P} = 90°$ (tangent/*raaklyn* $\perp$ radius) $\therefore\ \text{O}\hat{\text{M}}\text{T} = 90°$ (alternate $\angle$'s /*verwissellende* $\angle$'*e* ; TP | | OM)
12.5.	$m_{MT} \cdot m_{OM} = -1$ $m_{OM} = \frac{-4-0}{8-0} = -\frac{1}{2}$ $m_{MT} = 2$ $y + 4 = 2(x-8)$ $y = 2x - 20$ **OR** $y = 2x + c$ $-4 = 2(8) + c$ $c = -20$ $y = 2x - 20$
12.6.	$(x-8)^2 + (y+4)^2 = 45$ $(x-8)^2 + (2x - 20 + 4)^2 = 45$ $(x-8)^2 + (2x-16)^2 = 45$ $x^2 - 16x + 64 + 4x^2 - 64x + 256 - 45 = 0$ $5x^2 - 80x + 275 = 0$ $x^2 - 16x + 55 = 0$ $(x-11)(x-5) = 0$ $x = 11$ $y = 2(11) - 20$ $y = 2$ $\therefore \text{T}(11\ ;\ 2)$

TOPIC 3: Mixed questions

Activity 1

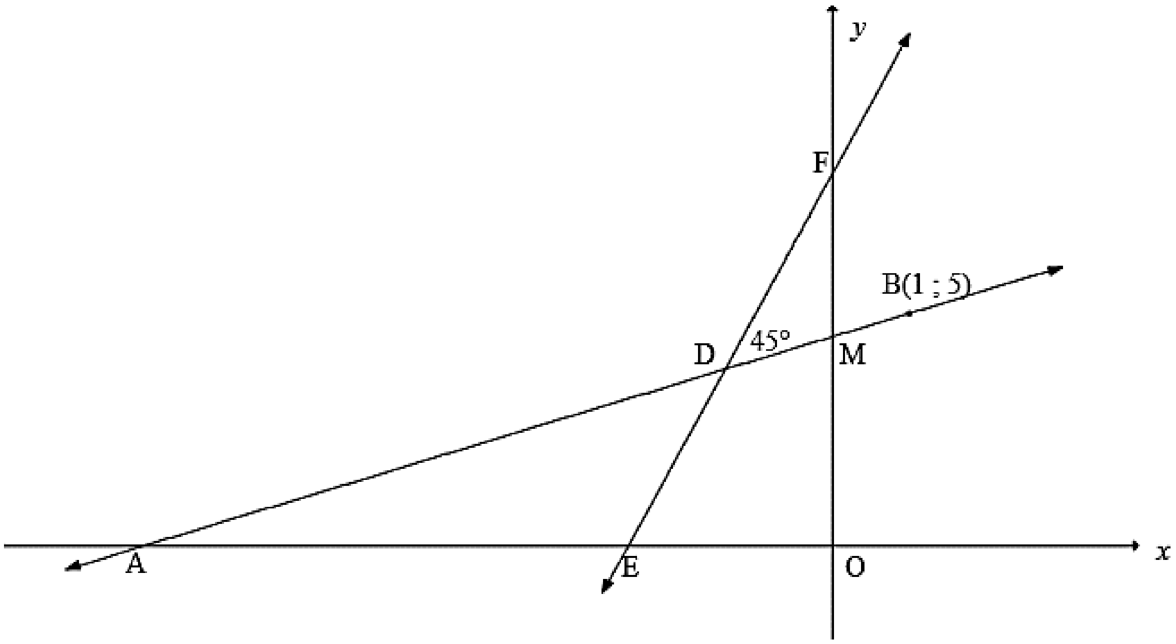


1.1.	$r = \text{MN} = 5$
1.2.	$(x-5)^2 + (y-4)^2 = 25$
1.3.	A$(x\,;0)$ $(x-5)^2 + (0-4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x-8)(x-2) = 0$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A$(2\,;0)$ **OR/*OF*** $(x-5)^2 + (0-4)^2 = 25$ $(x-5)^2 + 16 = 25$ $(x-5)^2 = 9$ $(x-5) = \pm 3$ $\therefore x = 8$ or/*of* $x = 2$ $\therefore$ A$(2\,;0)$
1.4.1.	$m_{\text{MB}} = \frac{4-0}{5-8}$ $= -\frac{4}{3}$

1.4.2.	$m_{MB} \times m_{PB} = -1$ (tangent ⊥ radius/ *rkl ⊥ radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$
1.5.	$y_K = y_M + r = 4 + 5$ $y = 9$
1.6.	At point L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)
1.7.	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$
1.8.	**MK ⊥ KL OR/OF** $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) $\therefore$ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR**

	MK ⊥ KL OR/OF $M\hat{K}L = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $(x-12{,}5)^2 + (y-6{,}5)^2 = r^2$ subst (5 ; 4): $(5-12{,}5)^2 + (4-6{,}5)^2 = r^2$ $62{,}5 = r^2$ $\therefore (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR** By symmetry about LM/*deur simmetrie om LM*: MK ⊥ KL OR/OF $M\hat{K}L = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* ML is a diameter /*ML is 'n middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$

Activity 2



2.1.	$y=0$: $3x+8=0$ $x=-\frac{8}{3}$ $\therefore$ E$\left(-2\frac{2}{3};0\right)$ **OR/*OF*** E$\left(-\frac{8}{3};0\right)$
2.2.	$\tan D\hat{E}O = m_{DE} = 3$ $\therefore D\hat{E}O = 71{,}565... = 71{,}57°$ $D\hat{A}E = 71{,}565...° - 45°$ $= 26{,}57°$
2.3.	$m_{AB} = \tan 26{,}57°$ $= \frac{1}{2}$ $y=\frac{1}{2}x+c$ **OR/*OF*** $y-y_1=\frac{1}{2}(x-x_1)$ $5=\frac{1}{2}(1)+c$ $\quad$ $y-5=\frac{1}{2}(x-1)$ $y=\frac{1}{2}x+4\frac{1}{2}$ $\quad$ $y=\frac{1}{2}x+\frac{9}{2}$

2.4.	Solve $x-2y+9=0$ and $y=3x+8$ simultaneously: $x-2(3x+8)+9=0$ $x-6x-16+9=0$ $-5x=7$ $x=-1\frac{2}{5}$ $\therefore y=3(-1\frac{2}{5})+8$ **OR/*OF*** $-1\frac{2}{5}-2y+9=0$ $y=3\frac{4}{5}$ $\quad$ $y=3\frac{4}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR** $x=2y-9$ $y=3(2y-9)+8$ $y=6y-27+8$ $\therefore y=3\frac{4}{5}$ $x=2(3\frac{4}{5})-9$ **OR/OF** $3\frac{4}{5}=3x+8$ $x=-1\frac{2}{5}$ $\quad$ $x=-1\frac{2}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR** $3x+8=\frac{1}{2}x+4\frac{1}{2}$ $6x+16=x+9$ $5x=-7$ $\therefore\quad x=-1\frac{2}{5}$ $\therefore y=3(-1\frac{2}{5})+8$ **OR/*OF*** $y=\frac{1}{2}(-1\frac{2}{5})+4\frac{1}{2}$ $y=3\frac{4}{5}$ $\quad$ $y=3\frac{4}{5}$ $\therefore$ D$(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR**

$x - 2y = -9$ ……(1)

$-6x + 2y = 16$ ……(2)

(1) + (2):

$-5x \quad = 7$

$\therefore x = -1\frac{2}{5}$

$\therefore -1\frac{2}{5} - 2y = -9$ **OR/*OF*** $y = 3(-1\frac{2}{5}) + 8$

$y = 3\frac{4}{5}$ $\qquad$ $y = 3\frac{4}{5}$

$\therefore \mathrm{D}(-1\frac{2}{5} \ ; \ 3\frac{4}{5})$

OR

$y = 3x + 8$ ……….(1)

$6y = 3x + 27$ …….(2)

(1) – (2):

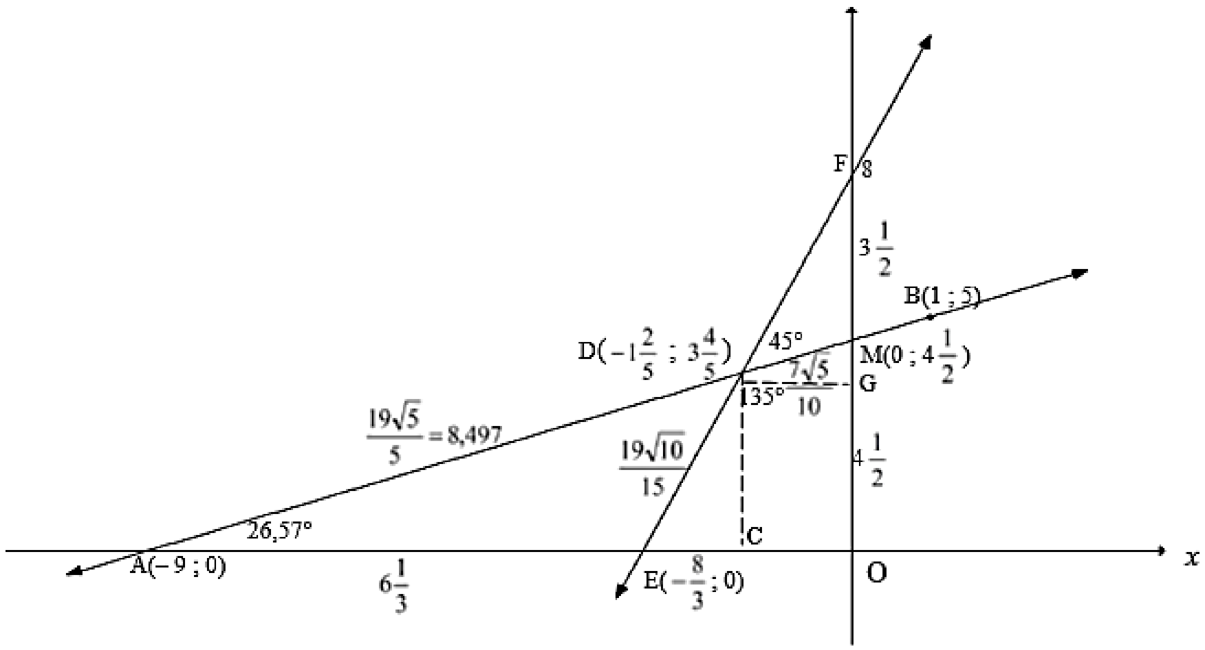
$-5y = -19$

$\therefore y = 3\frac{4}{5}$

$3\frac{4}{5} = 3x + 8$ **OR/*OF*** $x = 2(3\frac{4}{5}) - 9$

$x = -1\frac{2}{5}$ $\qquad$ $x = -1\frac{2}{5}$

$\therefore \mathrm{D}(-1\frac{2}{5} \ ; \ 3\frac{4}{5})$



2.5.	area DMOE = area ΔAMO − area ΔADE $x_A = 2(0) - 9 \quad \therefore A(-9\,;\,0)$ area ΔAMO $= \frac{1}{2}.\ AO.\ OM$ $= \frac{1}{2}(9)(4\frac{1}{2})$ $= 20{,}25$ area ΔADE $= \frac{1}{2}.\ AE.\ y_D$ $= \frac{1}{2}.\ (AO - EO).\ y_D$ $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ $= 12{,}03$ **OR/OF** area ΔADE $= \frac{1}{2} AD.AE.\sin\ D\hat{A}E$ $= \frac{1}{2}\ (\frac{19\sqrt{5}}{5}).6\frac{1}{3}.\sin 26{,}57°$ $= 12{,}03$ $\therefore$ area DMOE = 8,22 square units/*vk eenh* **OR** area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC $= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$ = 8,22 square units/*vk eenh* **OR** area DMOE = area ΔEDO + area ΔODM $= \frac{1}{2}(EO \times y_D) + \frac{1}{2}(OM \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304+189}{30}\right)$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh* **OR**

$$\text{area DMOE} = \text{area } \Delta\text{EOF} - \text{area } \Delta\text{DMF}$$

$$= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$$

$$= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$$

$$= \frac{1}{2}\left(\frac{640 - 147}{30}\right)$$

$$= \frac{493}{60} \quad \text{or} \quad 8\frac{13}{60} \quad \text{or} \quad 8{,}22 \text{ square units/}vk\ eenh$$

OR

$$\text{area } \Delta\text{EOM} = \frac{1}{2}(\text{EO} \times \text{OM})$$

$$= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$$

$$= 6 \text{ sq units/}vk\ eenh$$

$$\text{ED} = \sqrt{\left(-\frac{7}{5} + \frac{8}{3}\right)^2 + \left(\frac{19}{5}\right)^2} \quad \text{and DM} = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{9}{2} - \frac{19}{5}\right)^2}$$

$$= \frac{19\sqrt{10}}{15} \text{ or } 4{,}005... \qquad = \frac{7\sqrt{5}}{10} \text{ or } 1{,}565..$$

$$\text{area } \Delta\text{EDM} = \frac{1}{2}\left(\text{ED} \times \text{DM} \times \sin \text{E}\hat{\text{D}}\text{M}\right)$$

$$= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$$

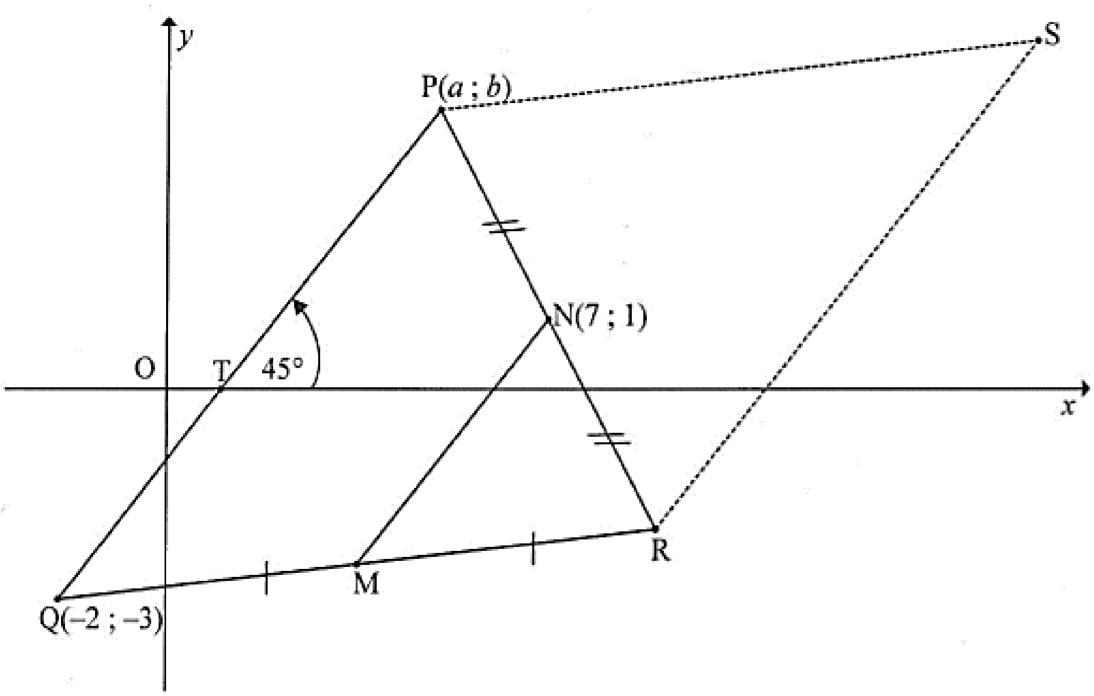
$$= \frac{133}{60} \quad \text{or } 2{,}216...$$

$$\therefore \text{area DMOE} = \text{area } \Delta\text{EOM} + \text{area } \Delta\text{EDM}$$

$$= 6 + 2{,}216...$$

$$= \frac{493}{60} \quad \text{or/}of \quad 8\frac{13}{60} \quad \text{or/}of \quad 8{,}22 \text{ square units/}eenh^2$$

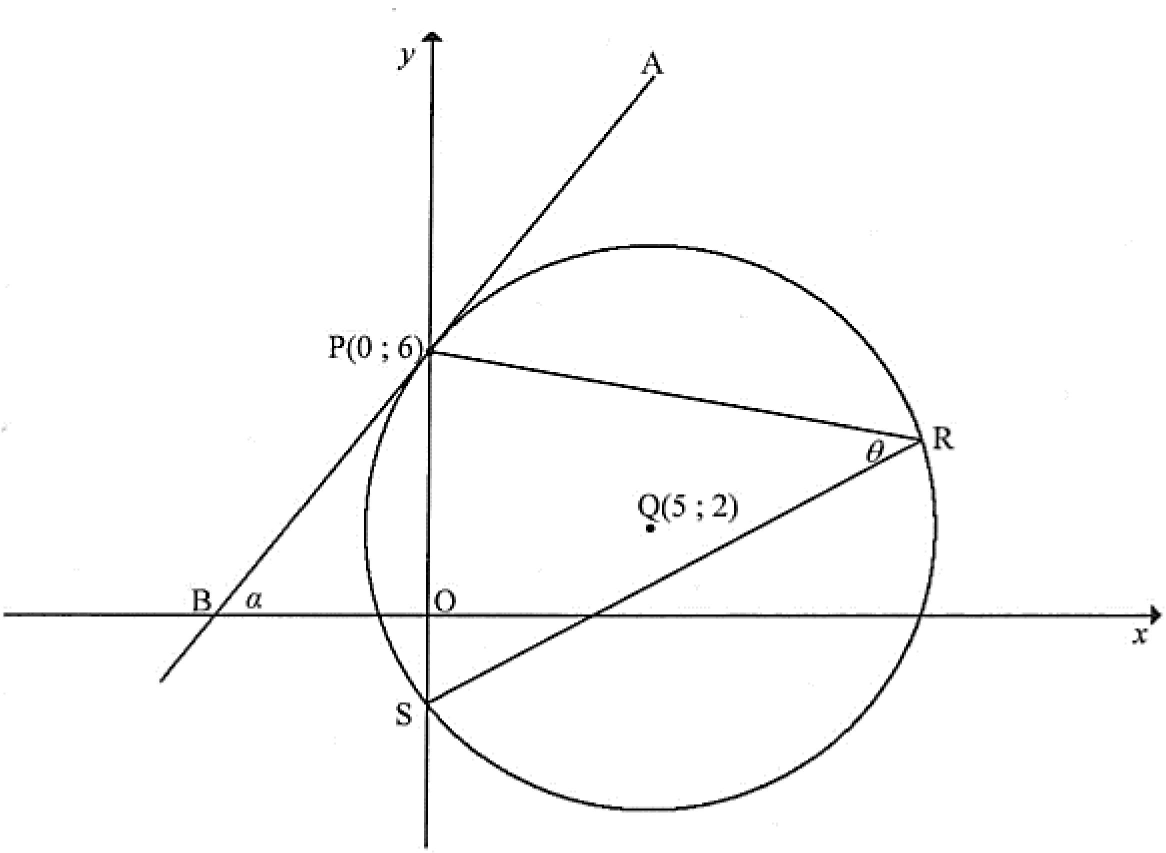
Activity 3	



3.1.	$m_{PQ} = \tan 45°$ $m_{PQ} = 1$
3.2.	In ΔRPQ PN=NR and QM=MR (Given) $\therefore$ QP//MN and QP=2MN (Midpoint Theorem) $\therefore m_{PQ} = m_{MN} = 1$ $y - 1 = 1(x - 7)$ $y = x - 6$
3.3.	QP=2MN $7\sqrt{2} = 2\text{MN}$ $\therefore \text{MN} = \dfrac{7\sqrt{2}}{2}$ *units*
3.4.	RS=QP $= 7\sqrt{2}$ *units*
3.5.	$\dfrac{x_S - 2}{2} = 7$ and $\dfrac{y_S - 3}{2} = 1$ $\therefore x_S = 16$ $\therefore y_S = 5$

	$\therefore S(16;5)$ (Diagonals bisect, share same midpoint N)
3.6.	$m_{PQ}=1$ $\dfrac{b+3}{a+2}=1$ $b+3=a+2$ $b=a-1$ ....(1) $7\sqrt{2}=\sqrt{(a+2)^2+(b+3)^3}$ $98=a^2+b^2+4a+6b+13$ $0=a^2+b^2+4a+6b-85$ ....(2) Substitution of (1) into (2) $0=a^2+(a-1)^2+4a+6(a-1)-85$ $0=2a^2+8a-90$ $a^2+4a-45=0$ $(a+9)(a-5)=0$ $a\neq-9$ or $a=5$ $\therefore a=5$ and $b=5-1=4$ P(5; 4)

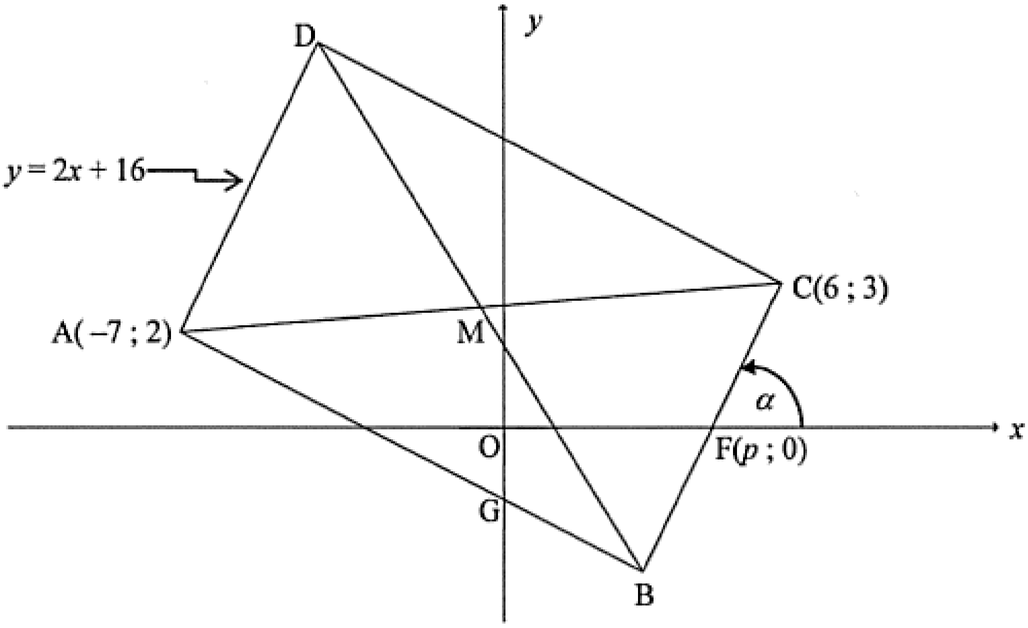
Activity 4	



4.1.	$(x-a)^2+(y-b)^2=r^2$ $(x-5)^2+(y-2)^2=r^2$ $r=d_{PQ}=\sqrt{(5-0)^2+(2-6)^2}$ $r=\sqrt{41}$ $\therefore (x-5)^2+(y-2)^2=41$
4.2.	$(0-5)^2+(y-2)^2=41$ $(y-2)^2=16$ $y-2=\pm 4$ $y=6$ or $y=-2$ $\therefore \text{S}(0;-2)$

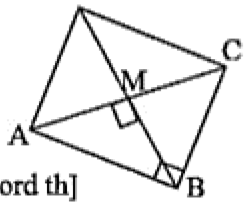
4.3.	$m_{PQ}=\frac{2-6}{5-0}=-\frac{4}{5}$ $m_{\tan}\times m_{rad}=-1$ (tan $\perp$ rad) $\therefore m_{\tan}=\frac{5}{4}$ $y-6=\frac{5}{4}(x-0)$ $y=\frac{5}{4}x+6$
4.4.	$\alpha=\tan^{-1}\left(\frac{5}{4}\right)$ $\alpha=51.34°$
4.5.	In ΔBOP $B\hat{P}O+\alpha+90°=180°$ (sum of $\angle$'s of Δ) $B\hat{P}O+51.34°+90°=180°$ $\therefore B\hat{P}O=38.66°$ but $B\hat{P}O=\theta$ (tan-chord theorem) $\therefore \theta$=38.66°
4.6.	base: $b=PS=y_P-y_S=6-(-2)=8$ *units* perpendicular hight: $h_{\perp}=5$ *units* Area $\Delta PQS=\frac{1}{2}\times b\times h_{\perp}=\frac{1}{2}(8)(5)=20\ unit^2$

Activity 5

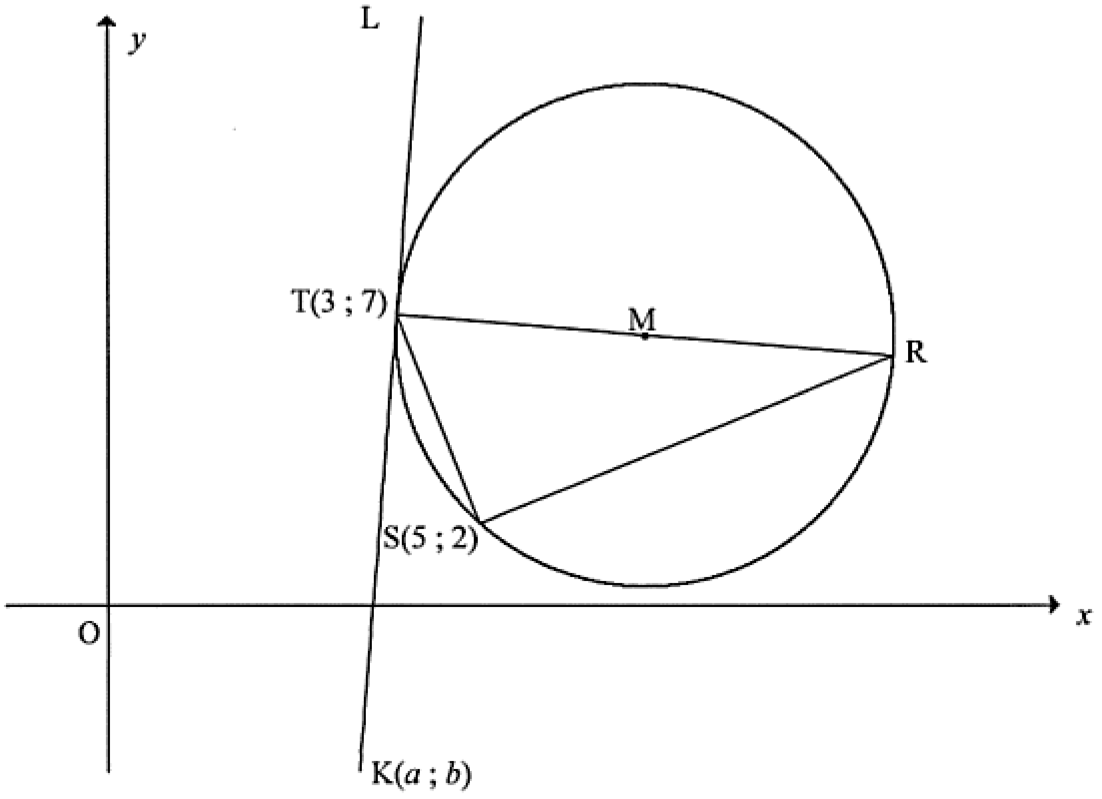


5.1.	M = Midpt of AC $= M\left(\frac{-7+6}{2};\frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2};\frac{5}{2}\right)$
5.2.	$m_{BC}=\frac{3-0}{6-p}=\frac{3}{6-p}$ **OR/OF** $m_{BC}=\frac{0-3}{p-6}=\frac{-3}{p-6}$
5.3.	$m_{AD}=m_{BC}$ [AD || BC] $m_{BC}=2$ $\frac{3}{6-p}=2$ $3=12-2p$ $p=4\frac{1}{2}$ **OR**

	$y-y_1=2(x-x_1)$ $C(6;3)$ $y-3=2(x-6)$ $\therefore y=2x-9$ $but\ y=0$ $\therefore x=4\frac{1}{2}=p$ **OR** $y=2x+c$ $3=12+c$ $-9=c$ $y=2x-9$ $0=2x-9$ $x=\frac{9}{2} \qquad \therefore p=\frac{9}{2}$
5.4.	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $AC=\sqrt{(6+7)^2+(3-2)^2}$ $AC=\sqrt{13^2+1^2}$ $AC=\sqrt{170}$ $\therefore DB=\sqrt{170}$ or 13,04
5.5.	$\tan\alpha=m_{BC}=2$ $\therefore \alpha=63,43°$
5.6.	In quadrilateral OFBG: $O\hat{F}B=63,43°$ [vert opp ∠s/*regoorst* ∠*e*] $F\hat{O}G=G\hat{B}F=90°$ $\therefore O\hat{G}B=360°-[90°+90°+63,43°]$ [sum ∠s quad/*som* ∠*e vierh* = 360°] $\therefore O\hat{G}B=116,57°$ **OR** $m_{AB}=-\frac{1}{2}$ $90°+O\hat{G}A=153,43°$ $\therefore O\hat{G}A=63,43°$ $O\hat{G}B=180°-63,43°$ $=116,57°$

	OR $\hat{FOG} = \hat{GBF} = 90°$ $\therefore$ GOFB is cyc quad $\hat{OGB} = 180° - 63,43°$ [$\angle$s of cyc quad = 180°] $= 116,57°$ **OR** $\hat{OFB} = 63,43°$ $\hat{XOG} = \hat{FBG} = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore \hat{OGB} = 180° - 63,43°$ $\hat{OGB} = 116,57°$
5.7.	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r = \frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{5}{2}\right)^2 = \left(\frac{\sqrt{170}}{2}\right)^2 = \frac{85}{2} = 42,5$
5.8.	$\hat{CBM} = \hat{BAM} = 45°$ [diag of square bisect $\angle$s/*hoekl v vierk halv* $\angle$*e*] $\therefore$ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF*** $\hat{AMB} = 90°$ [diag of square bisect $\perp$] $\therefore$ AB is diameter BC $\perp$ AB $\therefore$ BC is tangent [line $\perp$ radius *or* converse tan-chord th] 

Activity 6	



6.1.	∠ in semi circle/ ∠ at centre = 2∠ on circle
6.2.	$m_{TS}=\frac{7-2}{3-5}$ $=-\frac{5}{2}$
6.3.	$m_{TS}\times m_{RS}=-1$ $\therefore m_{RS}=\frac{2}{5}$ $y=\frac{2}{5}x+c$ $2=\frac{2}{5}(5)+c$ $c=0$ $y=\frac{2}{5}x$ **OR**

	$m_{TS} \times m_{RS} = -1$ [TS⊥SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$
6.4.1.	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$
6.4.2.	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR**

	$m_{TM}=\dfrac{9-3}{6\frac{1}{2}-7}=-\dfrac{1}{12}$ $TM:\quad 7=-\dfrac{1}{12}(3)+c\quad y=-\dfrac{1}{12}x+\dfrac{29}{4}$ ....(1) $SR:\quad y=\dfrac{2}{5}x$ ....(2) $\dfrac{2}{5}x=-\dfrac{1}{12}x+\dfrac{29}{4}$ $\dfrac{29}{60}x=\dfrac{29}{4}$ $\therefore x=15$ $\therefore y=\dfrac{2}{5}(15)=6$
6.4.3.	$\mathrm{ST}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{ST}=\sqrt{(5-3)^2+(2-7)^2}$ $\mathrm{ST}=\sqrt{4+25}=\sqrt{29}$ $\sin \mathrm{R}=\dfrac{\mathrm{TS}}{\mathrm{TR}}=\dfrac{\sqrt{29}}{\sqrt{145}}\ or\ \dfrac{\sqrt{5}}{5}\ or\ \dfrac{1}{\sqrt{5}}\ or\ 0{,}45$ **OR** $TS=\sqrt{29}$ $SR=2\sqrt{29}$ area of $\Delta\mathrm{TSR}=\dfrac{1}{2}(\sqrt{29})(2\sqrt{29})=29$ $29=\dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R=\dfrac{\sqrt{5}}{5}\ or\ \dfrac{1}{\sqrt{5}}$
6.4.4.	$m_{\mathrm{TR}}=\dfrac{7-6\frac{1}{2}}{3-9}=-\dfrac{1}{12}$ $m_{\mathrm{TR}}\times m_{\mathrm{KTL}}=-1$ $m_{\mathrm{KTL}}=12$ $y-y_1=12(x-x_1)$ $y-7=12(x-3)$ $y=12x-29$ substitute $\mathrm{K}(a;b)$: $b=12a-29$

	OR $m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a-29$ **OR** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$
6.4.5.	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\therefore K(2\,;-5)$ **OR**

$$TK = TR$$

$$\sqrt{(a-3)^2+(b-7)^2} = \sqrt{145}$$

$$(a-3)^2+(b-7)^2 = 145$$

Substitute $b = 12a - 29$ [from 4.4.4]

$$(a-3)^2+(12a-29-7)^2 = 145$$

$$(a-3)^2+(12a-36)^2 = 145$$

$$(a-3)^2+144(a-3)^2 = 145$$

$$(a-3)^2 = 1$$

$$a-3 = \pm 1$$

$$a = 2 \text{ or } 4$$

$\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$

$\therefore K(2;-5)$

OR

$$KR^2 = TR^2 + TK^2$$

$$(a-15)^2+(b-6)^2 = 145+145$$

$$(a-15)^2+(12a-29-6)^2 = 290$$

$$(a-15)^2+(12a-35)^2 = 290$$

$$a^2-30a+225+144a^2-840a+1225 = 290$$

$$145a^2-870a+1160 = 0$$

$$a^2-6a+8 = 0$$

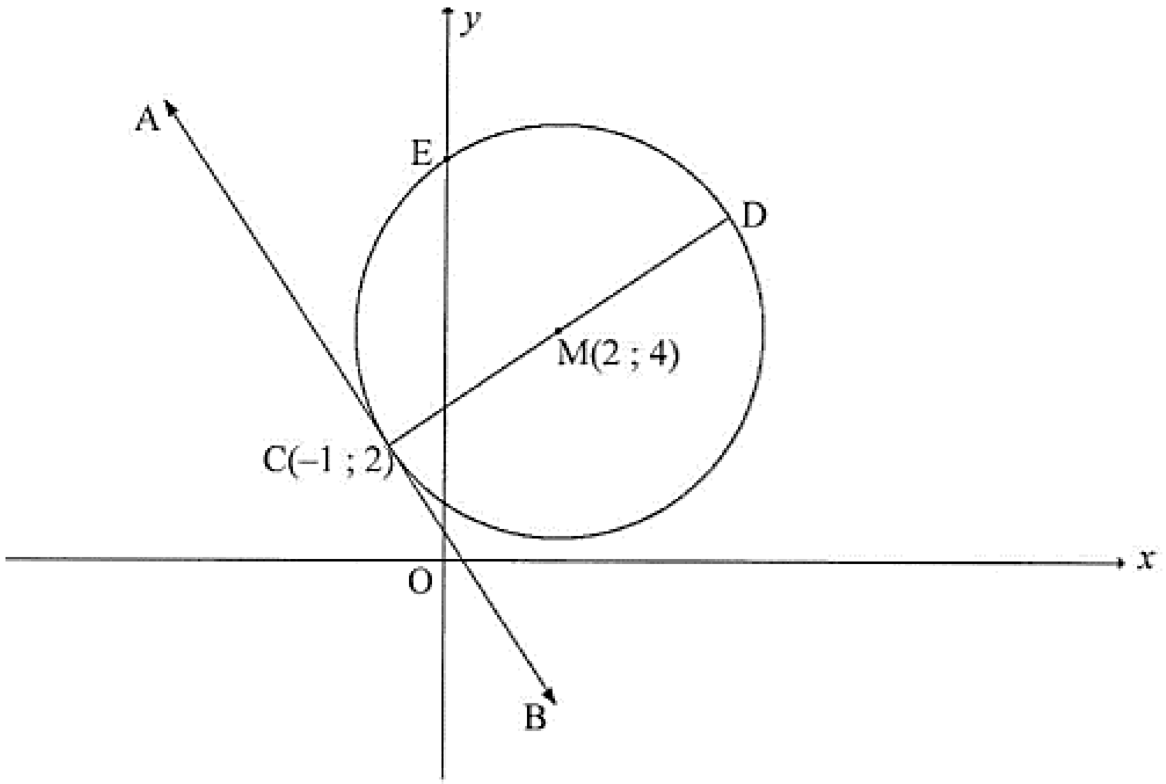
$$\therefore (a-2)(a-4) = 0$$

$a = 2$ or $a = 4$

$\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$

K(2;−5)

Activity 7	



7.1.	Radius = $\sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore\ (x-2)^2+(y-4)^2=13$
7.2.	At point A: $\frac{-1+x_D}{2}=2$ and/*en* $\frac{2+y_D}{2}=4$ $-1+x_D=4$ and/*en* $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR**

	By inspection/*deur inspeksie*: D(5 ; 6)
7.3.	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{3}{2}(x+1)$ $y = -\frac{3}{2}x + \frac{1}{2}$ **OR** $y = mx + c$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$
7.4.	At point E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR** $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)

7.5.	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore EM \,\|\|\, AB \qquad (m_{EM} = m_{AB})$

Activity 8

$(x+2)^2 + (y-4)^2 = 25$

centre: $P(-2;4)$ radius: $r_1 = 5$

$(x-5)^2 + (y+1)^2 = 9$

centre: $Q(5;-1)$ radius: $r_2 = 3$

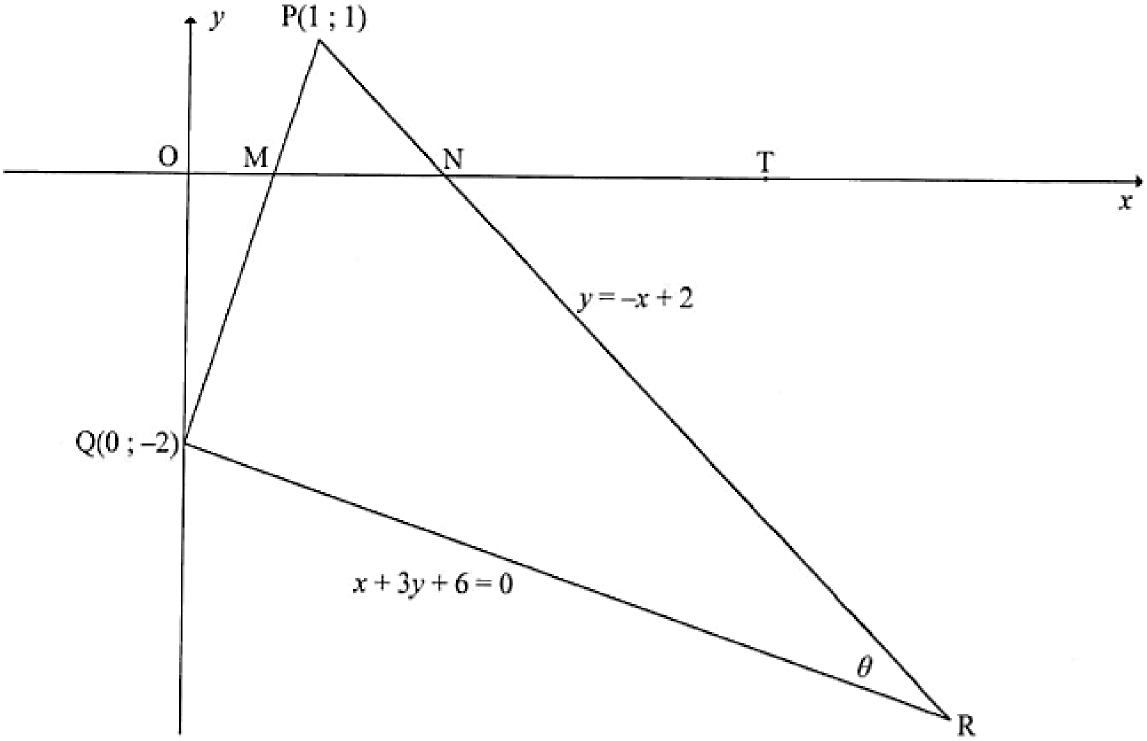
$d = d_{PQ} = \sqrt{(5-(-))^2 + (-1-4)^2} = \sqrt{74} \approx 8.6\ units$

$r_1 + r_2 = 5 + 3 = 8\ units$

$\therefore d > r_1 + r_2$

The circles do NOT intersection.

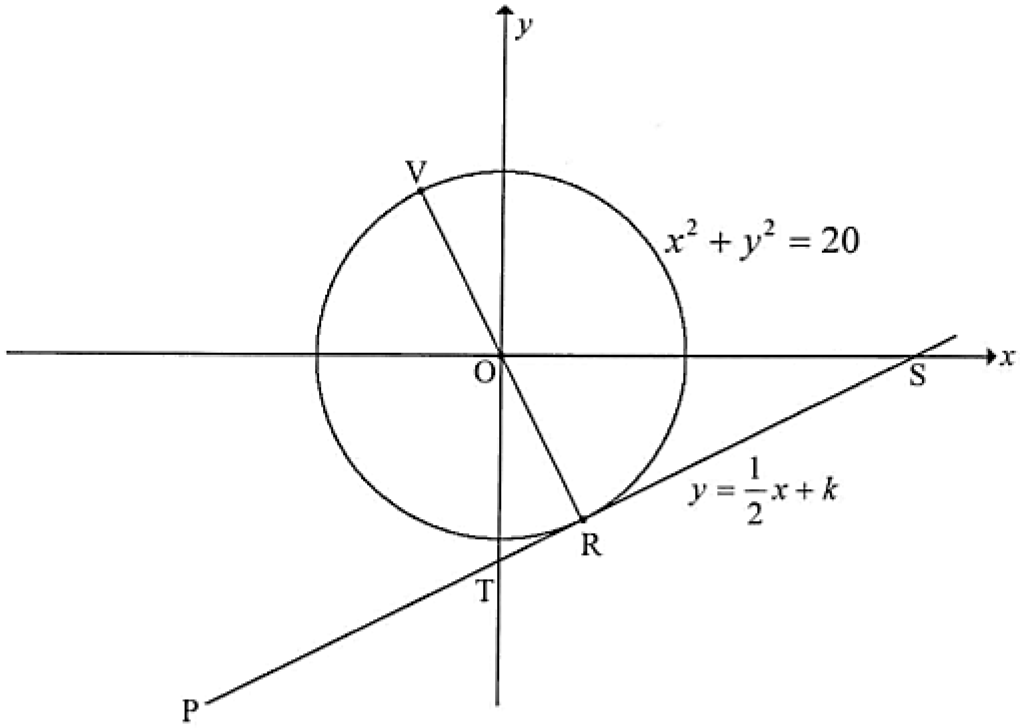
Activity 9



9.1.	$m_{QP} = \frac{1-(-2)}{1-0} = \frac{3}{1} = 3$
9.2.	QR: $x + 3y + 6 = 0$ $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{QP} \times m_{QR}$ $= 3 \times -\frac{1}{3}$ $= -1$ $\therefore$ QP $\perp$ QR and $P\hat{Q}R = 90°$

9.3.	$y_1 = y_2$ $-\frac{1}{3}x - 2 = -x + 2$ $\frac{2}{3}x = 4$ $\therefore x_R = 6$ and $y_R = -6 + 2 = -4$ $\therefore \text{R}(6;-4)$
9.4.	$d_{PR} = \sqrt{(1-6)^2 + (1-(-4))^2} = \sqrt{50} = 5\sqrt{2}$
9.5.	$mid_{PR}\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $mid_{PR}\left(\frac{7}{2};-\frac{3}{2}\right)$ $r = \frac{d_{PR}}{2} = \frac{5\sqrt{2}}{2}$ $\therefore r^2 = \frac{25}{2}$ $\left(x-\frac{7}{2}\right)^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{2}$
9.6.	$m_{PR} = \frac{1-(-4)}{1-6} = \frac{5}{-5} = -1$ $m_{\tan} \times m_{rad} = -1$ (tan $\perp$ rad) $\therefore m_{\tan} = 1$ $y - 1 = 1(x-1)$ $y = x$
9.7.	$d_{PQ} = \sqrt{(1-0)^2 + (1-(-2))^2} = \sqrt{10}$ $\frac{\sin\theta}{\text{PR}} = \frac{\sin 90^\circ}{\text{PR}}$ $\frac{\sin\theta}{\sqrt{10}} = \frac{\sin 90^\circ}{\sqrt{50}}$ $\theta = \sin^{-1}\left(\frac{\sqrt{5}}{5}\right)$ $\theta = 26.57^\circ$

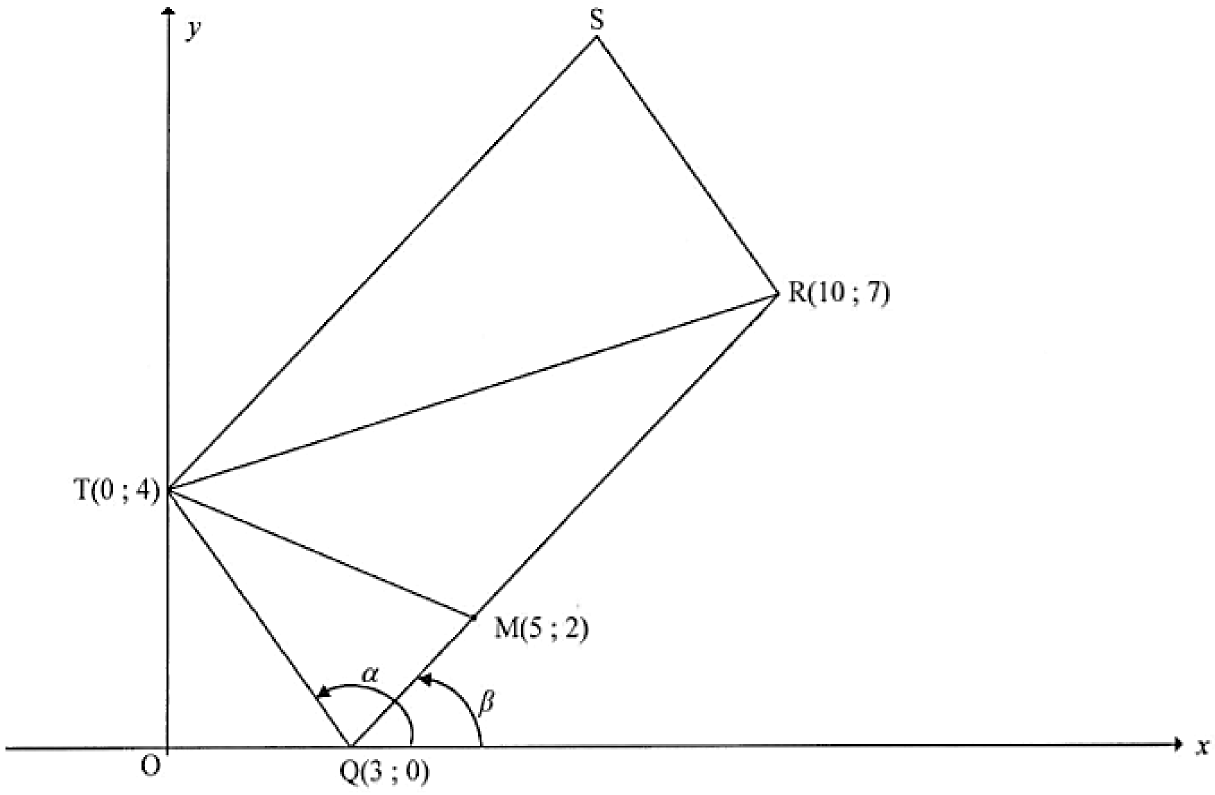
Activity 10



10.1.	$m_{\tan} \times m_{rad} = -1$ (tan ⊥ rad) $\frac{1}{2} \times m_{rad} = -1$ $\therefore m_{rad} = -2$ $y - 0 = -2(x - 0)$ $y = -2x$
10.2.	$y = -2x$ ....(1) $x^2 + y^2 = 20$ ....(2) Substitution of (1) into (2) $x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 = 20$ $x^2 = 4$ $x = \pm 2$ and $y = \pm 4$ $\therefore \text{R}(2; -4)$ and $\text{V}(-2; 4)$

10.3.	$y=\frac{1}{2}x+k$ passing through $R(2;-4)$ $-4=\frac{1}{2}(2)+k$ $\therefore k=-5$ $y=\frac{1}{2}x-5$ $\therefore T(0;-5)$ and $S(10;0)$ $OT=y_O-y_T=0-(-5)=5\ units$ $OS=x_S-x_O=10-0=10\ units$ $\text{Area }\Delta OTS=\frac{1}{2}(10)(5)=25\ unit^2$
10.4.	$d_{VT}=\sqrt{(0-(-2))^2+(-5-4)^2}=\sqrt{85}$

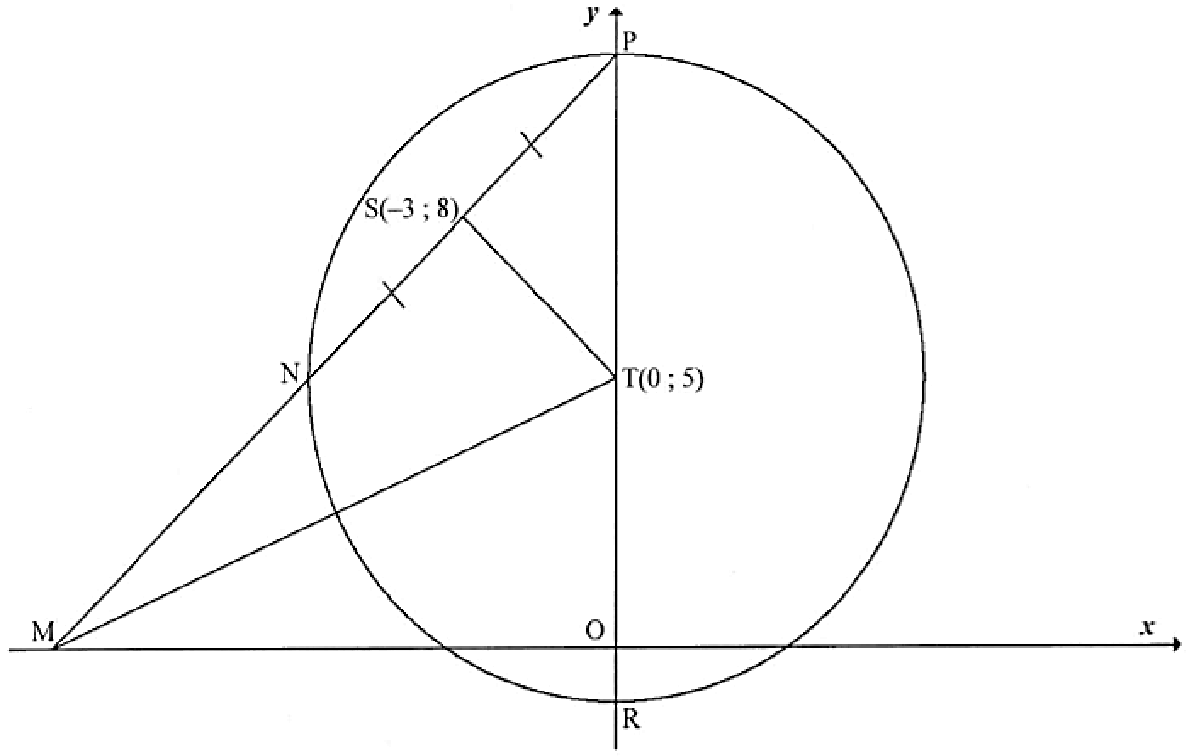
Activity 11



11.1.	$m_{TQ}=\frac{4-0}{0-3}=-\frac{4}{3}$
11.2.	$d_{RQ}=\sqrt{(10-3)^2+(7-0)^2}=\sqrt{98}=7\sqrt{2}$

11.3.	$m_{TQ} = m_{QF}$ $-\frac{4}{3} = \frac{0-(-8)}{3-k}$ $-12+4k = 24$ $4k = 36$ $k = 9$ $\therefore \text{F}(9;-8)$
11.4.	$mid_{TR}\left(\frac{0+10}{2};\frac{4+7}{2}\right)$ $mid_{TR}\left(5;\frac{11}{2}\right)$ Diagonals of a parallelogram bisect each other at one point $mid_{TR}\left(5;\frac{11}{2}\right)$ $\frac{x_S+3}{2} = 5$ and $\frac{y_S+0}{2} = \frac{11}{2}$ $x_S = 7$ $y_S = 11$ $\therefore \text{S}(7;11)$
11.5.	$m_{RQ} = \frac{7-0}{10-3} = \frac{7}{7} = 1$ $\beta = \tan^{-1}(1) = 45°$ $\alpha = \tan^{-1}\left(-\frac{4}{3}\right) + 180° = 126.87°$ $\text{T}\hat{\text{Q}}\text{R} = \alpha - \beta = 126.87° - 45° = 81.87°$ $\therefore \text{T}\hat{\text{S}}\text{R} = \text{T}\hat{\text{Q}}\text{R} = 81.87°$ (opp ∠'s of par'm equal)
11.6.1.	$d_{MQ} = \sqrt{(5-3)^2+(2-0)^2} = \sqrt{8} = 2\sqrt{2}$ $\frac{\text{MQ}}{\text{RQ}} = \frac{2\sqrt{2}}{7\sqrt{2}} = \frac{2}{7}$
11.6.2.	$\frac{\text{area of } \Delta\text{TQM}}{\text{area of } \Delta\text{TQR}} = \frac{\frac{1}{2}.\text{QM}.\perp \text{h}}{\frac{1}{2}.\text{QR}.\perp \text{h}}$ [$\perp$h same/*dieselfde*] $= \frac{\text{QM}}{\text{QR}} = \frac{2}{7}$ $\frac{\text{area of } \Delta\text{TQM}}{\text{area of parm RQTS}} = \frac{\text{area of } \Delta\text{TQM}}{2 \times \text{area of } \Delta\text{TQR}}$ $= \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{7}$

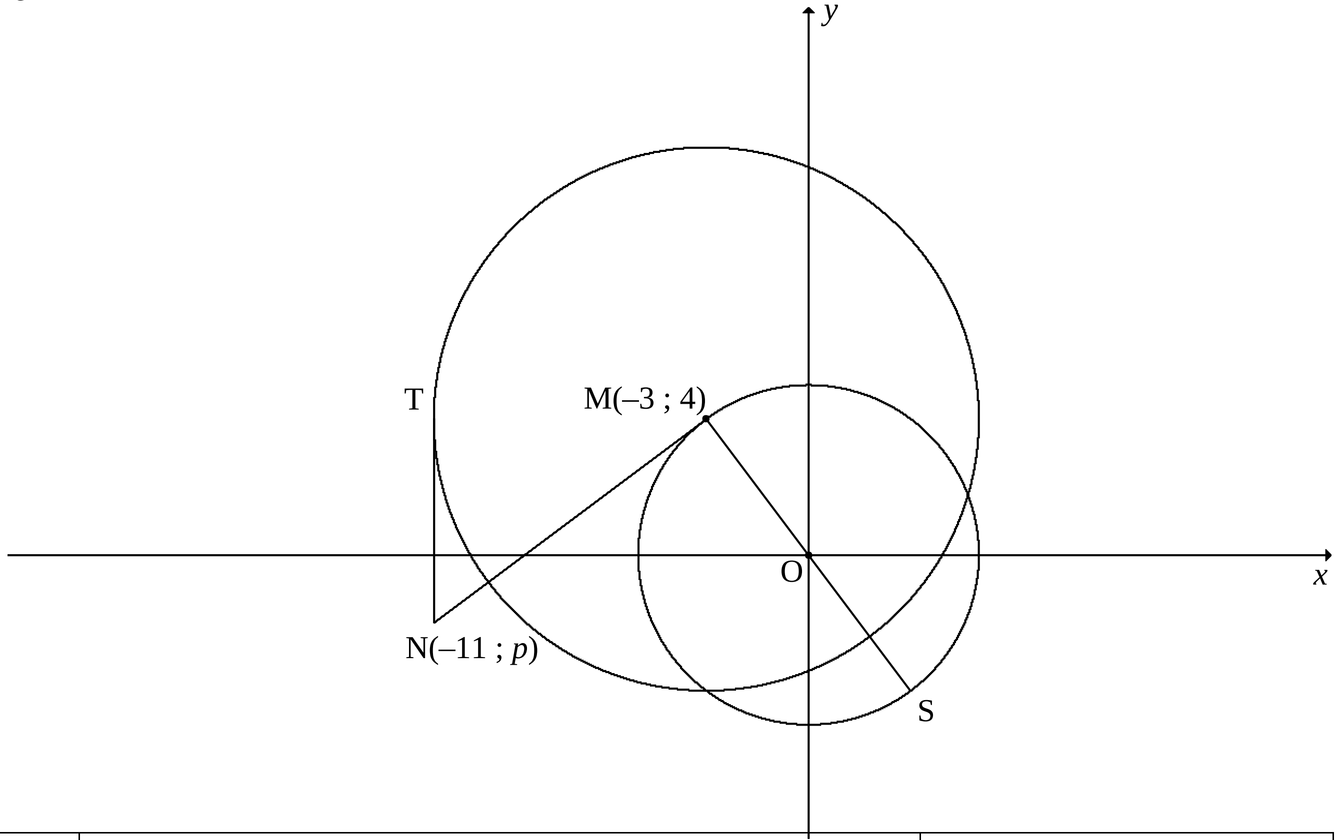
Activity 12



12.1.	Line from centre to midpoint chord
12.2.	$m_{TS} = \frac{8-5}{-3-0} = \frac{3}{-3} = -1$ $m_{TS} \times m_{NP} = -1$ (TS ⊥ NP) $\therefore m_{NP} = 1$ $y - 8 = 1(x - (-3))$ $y = x + 3 + 8$ $y = x + 11$
12.3.	$\text{P}(0; y_P)$ $y_P = 0 + 11 = 11$ $\therefore \text{P}(0;11)$ $\text{PT} = y_p - y_T = 11 - 5 = 6 \text{ } units$ $\text{TR} = y_T - y_R$ $6 = 5 - y_R$ $y_R = -1$ $\therefore \text{R}(0;-1)$ $\therefore y = 11$ and $y = -1$

12.4.	$M(x_M;0)$ $y = x + 11$ $0 = x + 11$ $x_M = -11$ $\therefore M(-11;0)$ $d_{MT} = \sqrt{(0-(-11))^2 + (5-0)^2} = \sqrt{146}$
12.5.	$M\hat{S}T = 90°$ is the angle in a semi-circle, and MT is the diameter of circle S, T and M. $mid_{MT}\left(\frac{0-11}{2};\frac{5+0}{2}\right)$ $mid_{MT}\left(-\frac{11}{2};\frac{5}{2}\right)$ is the centre of circle S, T and M. $r = \frac{d_{MT}}{2} = \frac{\sqrt{146}}{2}$ and $r^2 = \frac{73}{2}$ $\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{73}{2}$

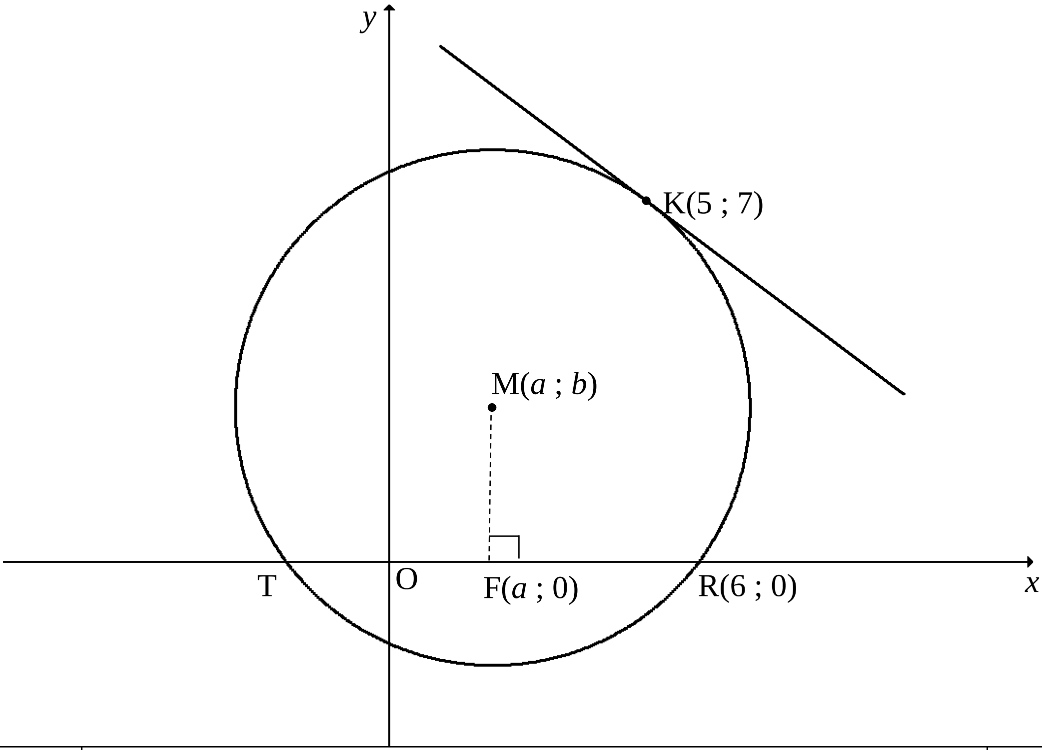
QUESTION/*VRAAG* 4



4.1	$x^2 + y^2 = r^2$ $\therefore r^2 = (-3)^2 + (4)^2 = 25$ $x^2 + y^2 = 25$	✓ substitution ✓ answer (2)
4.2	TM ⊥ TN [tangent ⊥ radius] T(–11 ; 4) $r = -3 - (-11) = 8$ $(x+3)^2 + (y-4)^2 = 64$	✓ $x_T = -11$ ✓ LHS ✓ RHS (3)
4.3	O (0 ; 0) and M(–3; 4) $m_{\text{OM}} = \frac{4-0}{-3-0} = -\frac{4}{3}$ OR $\frac{0-4}{0-(-3)} = -\frac{4}{3}$ $m_{\text{NM}} = \frac{3}{4}$ $y - 4 = \frac{3}{4}(x-(-3))$ **OR** $y = \frac{3}{4}x + c$ $y - 4 = \frac{3}{4}x + \frac{9}{4}$ $4 = \frac{3}{4}(-3) + c$ $\therefore y = \frac{3}{4}x + \frac{25}{4}$ $c = \frac{25}{4}$ $y = \frac{3}{4}x + \frac{25}{4}$	✓ $m_{\text{OM}} = -\frac{4}{3}$ ✓ $m_{\text{NM}} = \frac{3}{4}$ ✓ substitution of m and M ✓ equation (4)

4.4	N(–11 ; p) $y=\frac{3}{4}x+\frac{25}{4}$ $p=\frac{3}{4}(-11)+\frac{25}{4}$ OR $\frac{4-p}{-3-(-11)}=\frac{3}{4}$ $p=-2$ $\therefore \text{N}(-11;-2)$ $\frac{-3+x_S}{2}=0$ and $\frac{4+y_S}{2}=0$ $\therefore \text{S}(3;-4)$ $\text{SN}=\sqrt{(-11-3)^2+(-2-(-4))^2}$ $=10\sqrt{2}$ units or 14,14 units	✓ subst x = –11 into eq or gradient ✓ $p=-2$ ✓ x_S ✓ y_S ✓ answer (CA) (5)
4.5	$\text{B}(-2;\ 5)$ $\text{BM}=\sqrt{2}$ units Radius of circle centred at M = 8 units $k=8-\sqrt{2}$ or $k=8+\sqrt{2}$ = 6,59 units = 9,41 units = 6,6 units = 9,4 units	✓ $\sqrt{2}$ ✓✓ $k=6{,}6$ ✓✓ $k=9{,}4$ (5)
		[19]

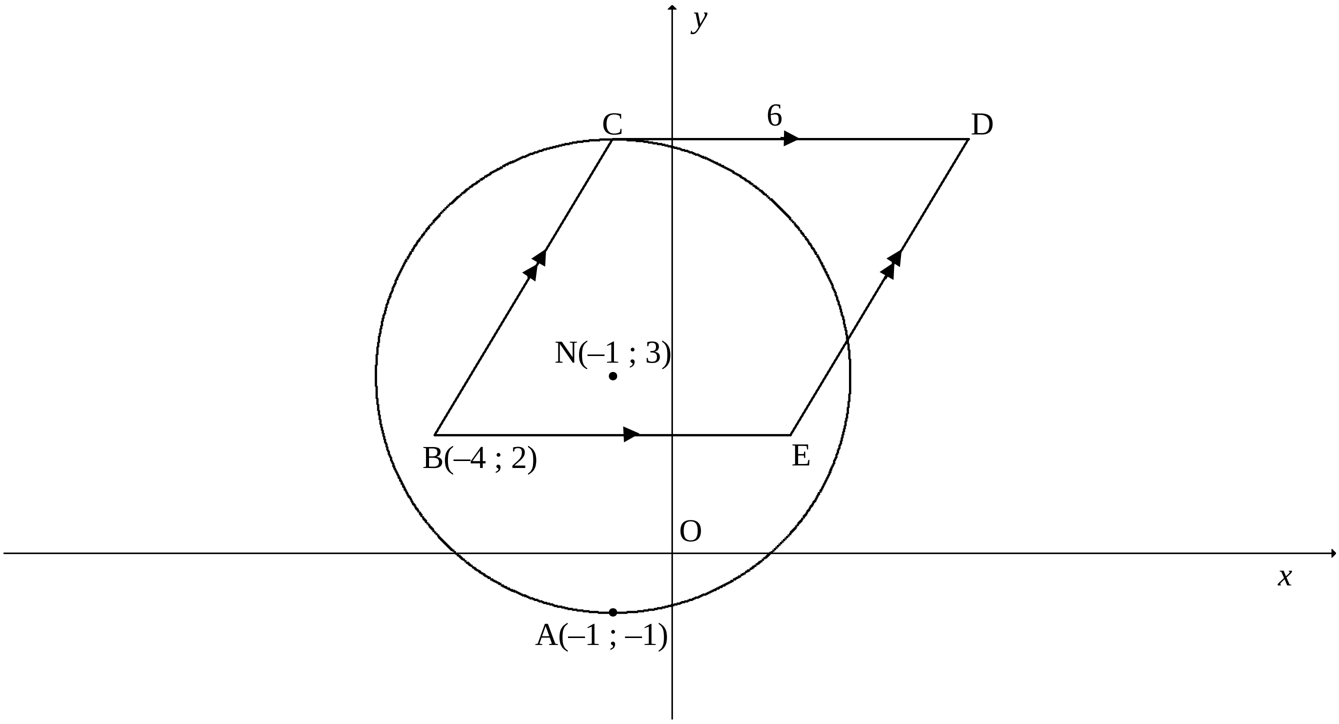
QUESTION/*VRAAG* 4



4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2 ; 3)$	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

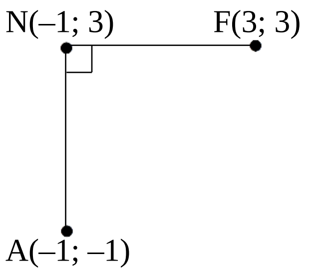
4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(*a* ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ *x* values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

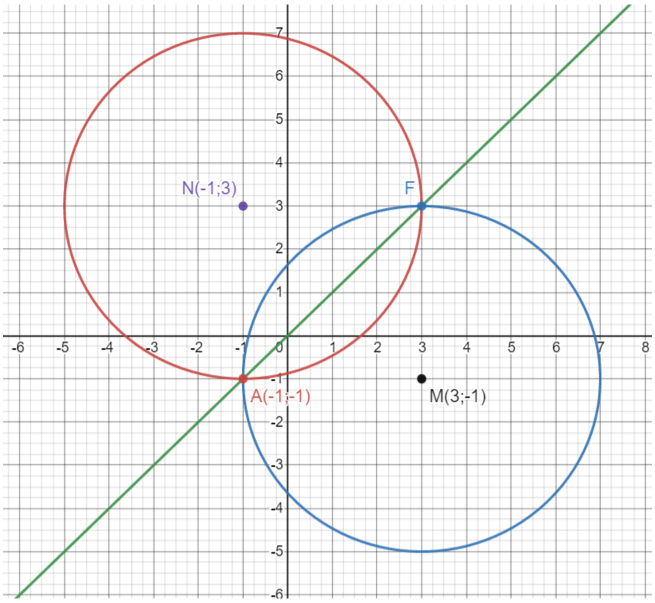
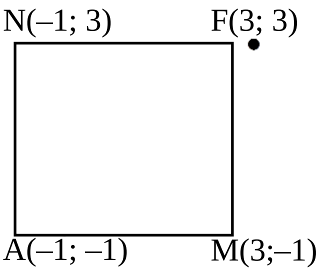
QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\ ;\ 7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\ ;\ 7)$	✓x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

[106 e]

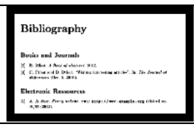
	OR/*OF* Let angle of inclination of BC = α $\tan\alpha=\frac{5}{3}$ $\alpha=59{,}036...^\circ$ $B\hat{C}D=180^\circ-\alpha$ $B\hat{C}D=180^\circ-59{,}036...^\circ$ $B\hat{C}D=120{,}96^\circ$ Area $\Delta BCD=\frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96^\circ$ $= 15$ units^2	✓ $B\hat{C}D=120{,}96^\circ$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN=\sqrt{(3-(-1))^2+(-1-3)^2}$ $MN=\sqrt{32}=4\sqrt{2}=5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN}=\frac{3-(-1)}{-1-3}=-1$ MN: $-1=-(3)+c$ or $y-3=-1(x+1)$ $c=2$ $y-3=-x-1$ $\therefore y=-x+2$ $y=-x+2$ $x=-x+2$ $2x=2$ $x=1$ $\therefore y=1$ midpoint $(1\,;1)$ **OR/*OF***  N(–1 ; 3) $y_F=y_N=3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2};\frac{-1+3}{2}\right)$ = (1 ; 1)	 ✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4) ✓✓ coordinates of F ✓x value ✓ y value (4)

	OR/*OF*   NAMF is a square (NA=NF=AM=MF and NA $\perp$ AM) Midpoint NM = (1 ; 1) = Midpoint of AF	✓ NAMF = square ✓x ✓ y of midpt NM ✓ midpt AF (4)
		[15]

[106 g]

Bibliography

Marking Guidelines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