JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

CONTENT: ALGEBRA, EQUATIONS, AND INEQUALITIES

SOLUTIONS MANUAL

LEARNER/TEACHER MANUAL

Algebra, equations, and inequalities

CONTENTS **PAGE**

PART 1: Quadratic equations	**1-2**
PART 2: Quadratic inequalities	**2-4**
PART 3: Exponential equations	**5**
PART 4: Surd equations	**6**
PART 5: Simultaneous equations	**7-8**
PART 6: Nature of roots	**9-11**
PART 7: Fusion	**12-14**

PART 1

1 $x^2 = 5x - 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4$ or $x = 1$

2 $x(3-x) = -3$

$3x - x^2 = -3$

$x^2 - 3x - 3 = 0$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{21}}{2}$$

$x = 3{,}79$ or $x = -0{,}79$

3 $x(x-4) = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$

4 $4x^2 - 20x + 1 = 0$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{20 \pm \sqrt{384}}{8}$$

$x = 4{,}95$ or $x = 0{,}05$

5 $x(x-1) = 30$

$x^2 - x = 30$

$x^2 - x - 30 = 0$

$(x-6)(x+5) = 0$

$x = 6$ or $x = -5$

6 $3x^2 - 5x + 1 = 0$

$a = 3$ $b = -5$ $c = 1$

$$x = \frac{-(-5) \pm \sqrt{25 - 4(3)(1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{13}}{6}$$

$x = 1{,}4$ or $x = 0{,}2$

7 $3x + \frac{1}{x} = 4$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ or $x = 1$

9 $x = \frac{5}{3x-2}$

$\therefore 3x^2 - 2x - 5 = 0$

$\therefore (3x-5)(x+1) = 0$

$\therefore x = \frac{5}{}$ or/of $x = -1$

PART 2

1 $3 - x < 2x^2$

$-2x^2 - x + 3 < 0$

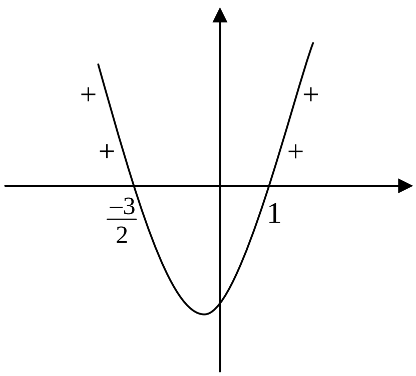
$2x^2 + x - 3 > 0$

$(2x+3)(x-1) > 0$

$x < -\frac{3}{2}$ or $x > 1$

OR

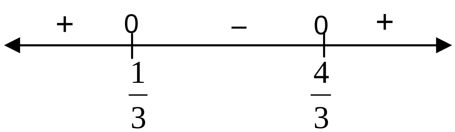
$x \in (-\infty; -\frac{3}{2}) \cup (1; \infty)$



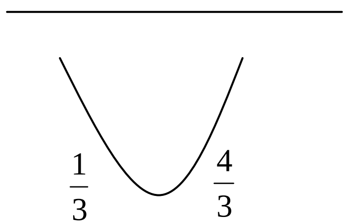
2 $-9x^2 + 15x - 4 < 0$

$9x^2 - 15x + 4 > 0$

$(3x-4)(3x-1) > 0$



OR



$x < \frac{1}{3}$ or $x > \frac{4}{3}$

Answer can be given as: $x \in \left(-\infty; \frac{1}{3}\right) \cup \left(\frac{4}{3}; \infty\right)$

3 $4+5x>6x^2$ OR $-6x^2+5x+4>0$

$0>6x^2-5x-4$ OR $6x^2-5x-4<0$

$0>(3x-4)(2x+1)$ OR $(3x-4)(2x+1)<0$

critical values: $x=\dfrac{5\pm\sqrt{121}}{12}$

$x=-\dfrac{1}{2}$ or $\dfrac{4}{3}$

$-\dfrac{1}{2}<x<\dfrac{4}{3}$ **OR** $x\in\left(-\dfrac{1}{2};\dfrac{4}{3}\right)$ **OR** $-\dfrac{1}{2}<x$ and $x<\dfrac{4}{3}$

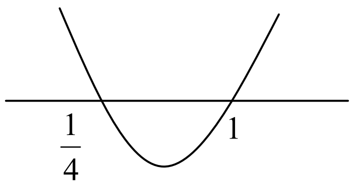
4 $4x^2+1\geq 5x$

$4x^2-5x+1\geq 0$

$(4x-1)(x-1)\geq 0$

+ 0 − 0 +

$\frac{1}{4}$ 1



$x\leq\dfrac{1}{4}$ or $x\geq 1$ **OR** $\left(-\infty;\dfrac{1}{4}\right]\cup[1;\infty)$

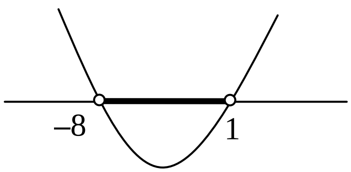
5 $x^2+7x-8<0$

$(x+8)(x-1)<0$

+ 0 − 0 +

–8 1

OR



6 $(x+1)(x-3)>12$

$x^2-2x-3>12$

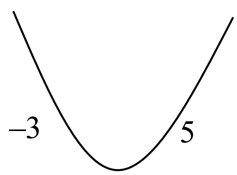
$x^2-2x-15>0$

$(x-5)(x+3)>0$

+ 0 − 0 +

– 3 5

OR



$x<-3$ or $x>5$

7 $(x+1)(4-x) > 0$

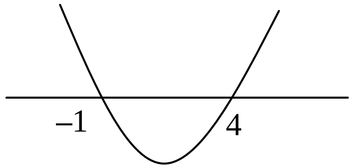
$(x+1)(x-4) < 0$

+ 0 − 0 +

———————————

−1 4

or



$-1 < x < 4$

8.1 $(x+2)(x-3) < -3x+2$

$x^2 - x - 6 + 3x - 2 < 0$

$x^2 + 2x - 8 < 0$

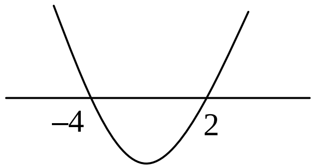
$(x+4)(x-2) < 0$

+ 0 − 0 +

———————————

−4 2

or



$-4 < x < 2$

8.2 $x^2 + 2x - 8 < 0$

$-4 < x < 2$

Sum of integers $= (-3) + (-2) + (-1) + (0) + (1)$

$= -5$

PART 3

1
$$2 \cdot 3^x = 81 - 3^x$$
$$2 \cdot 3^x + 3^x = 81$$
$$3^x(2+1) = 81$$
$$3^x = 27$$
$$3^x = 3^3$$
$$x = 3$$

2
$$9.2^{x-1} = 2.3^x$$
$$3^2.2^{x-1} = 2.3^x$$
$$2^{x-2} = 3^{x-2}$$
$$\left(\frac{2}{3}\right)^{x-2} = 1$$
$$\left(\frac{2}{3}\right)^{x-2} = \left(\frac{2}{3}\right)^0$$
$$x - 2 = 0$$
$$x = 2$$

3
$$\sqrt{2x+1} = x - 1$$
$$2x + 1 = (x-1)^2$$
$$2x + 1 = x^2 - 2x + 1$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x = 4$$
$$n/a$$

4
$$27^{x^2+x} = 3^{3x^2} \times 9$$
$$(3^3)^{x^2+x} = 3^{3x^2} \times 3^2$$
$$3^{3x^2+3x} = 3^{3x^2} \times 3^2$$
$$\therefore 3x^2 + 3x = 3x^2 + 2$$
$$3x = 2$$
$$x = \frac{2}{3}$$

5
$$5^x = \frac{1}{125}$$
$$5^x = 5^{-3}$$
$$x = -3$$

1 $\sqrt{2x+1} = x-1$

$2x+1 = (x-1)^2$

$2x+1 = x^2 - 2x + 1$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$

n/a

2 $2x-3 = \sqrt{2x-3}$

$4x^2 - 12x + 9 = 2x - 3$

$4x^2 - 14x + 12 = 0$ **OR**

$2(2x-3)(x-2) = 0$

$x = \frac{3}{2}$ or $x = 2$ x

3 $2\sqrt{x-3} = x-3$

$4(x-3) = x^2 - 6x + 9$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x = 7$ or $x = 3$

OR

Let $y = \sqrt{x-3}$

$\therefore 2y = y^2$

$y(y-2) = 0$

$y = 0$ or $y = 2$

$x - 3 = 0$ or $x - 3 = 4$

$x = 3$ or $x = 7$

---

4 $x - \sqrt{x} = 6$

$x - 6 = \sqrt{x}$

$x^2 - 12x + 36 = x$

$x^2 - 13x + 36 = 0$

$(x-4)(x-9) = 0$

$x = 4$ of $x = 9$

$\therefore x = 9$ alleenlik

PART 5

1 $y = 3 - 2x$

$x^2 + (3-2x) + x = (3-2x)^2$

$x^2 + 3 - 2x + x = 9 - 12x + 4x^2$

$3x^2 - 11x + 6 = 0$

$(3x-2)(x-3) = 0$

$x = \frac{2}{3}$ or $x = 3$

$\therefore y = \frac{5}{3}$ $\therefore y = -3$

2 $y = x - 3$

$x^2 - x = 6 + (x-3)$

$x^2 - 2x - 2 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

$y = 0$ or $y = -4$

Solutions are (x ; y) = (3 ; 0) or (– 1 ; – 4)

3.1 $x^2 + 5xy + 6y^2 = 0$

$(x+3y)(x+2y) = 0$

$x + 3y = 0$ $+ 2y = 0$

$x = -3y$ OR $x = -2y$

$\frac{x}{y} = -3$ $\frac{x}{y} = -2$

3.2 $x + y = 8$ $x + y = 8$

$-3y + y = 8$ $-2y + y = 8$

$-2y = 8$ OR $-y = 8$

$y = -4$ $y = -8$

$x = 12$ $x = 16$

4. $3^{x-10} = 3^{3x}$

$x - 10 = 3x$

$2x = -10$

$x = -5$

$y^2 + x = 20$

$y^2 - 5 = 20$

$y^2 = 25$

$y = -5$ or $y = 5$

5.1 $2^x + 2^{x+2} = -5y + 20$

$2^x(1 + 2^2) = -5y + 20$

$$2^x = \frac{-5y+20}{5}$$

OR

$2^x = -y + 4$

5.2 If $y = -4$,

$2^x + 2^{x+2} = -5y + 20$

$2^x + 2^{x+2} = 40$

$2^x\left(1 + 2^2\right) = 40$

$2^x = 8$

$2^x = 2^3$

$x = 3$

5.3 $-y + 4 > 0$

$y < 4$

Largest integer value of y is 3

$2^x = -3 + 4$

$2^x = 1$

$x = 0$

1 $x^2+9=0$

$\Delta=b^2-4ac$

$=0^2-4(1)(9)=-36<0$

$\therefore$ The roots are non – real

2.1 $x^2-3x+(k+1)=0$

For real roots : $\Delta\geq 0$

$b^2-4ac\geq 0$

$(-3)^2-4(1)(k+1)\geq 0$

$9-4k-4\geq 0$

$-4k\geq -5$

$k\leq \frac{5}{4}$

2.2 Put $x=1$ into equation:

$1^2-3(1)+(k+1)=0$

$k-1=0$

$\therefore k=1$

3 $x^2-px-p^2=2$

$x^2-px-p^2-2=0$

$\Delta=b^2-4ac$

$=p^2-4(-p^2-2)$

$=p^2+4p^2+8$

$=5p^2+8$

$\geq 8>0$ for all values of p.

$\therefore$ The roots are real and unequal

4 $\Delta=2k(2k-9)$

For $k=6$:

$\Delta=(12)(3)=36$

which is a perfect square

$\therefore$ The roots are real, **rational** and unequal

5 $x = 5$ is die wortel van

$x^2 + kx - 15 = 0$

$\therefore 5^2 + 5k - 15 = 0 \Rightarrow 25 + 5k - 15 = 0$

$5k = -10$

$k = -2$

$\therefore x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$x = -3$ of $x = 5$

$\therefore$ die ander wortel is -3.

6 $f(x) = 3x^2 - 6x + m$

$f(x) = 0$ het nie-reële wortels as

$\Delta < 0$

$b^2 - 4ac < 0$

$(-6)^2 - 4(3)(m) < 0$

$-12m < -36$

$\therefore m > 3$

7 $f(x) = x^2 - 5x + c$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(c)}}{2}$$

$25 - 4c = 41$

$-4c = 16$

$c = -4$

8.1 $$5x^2+6x-7=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-6\pm\sqrt{(6)^2-4(5)(-7)}}{2(5)}$$

$$=0{,}73 \quad or \quad -1{,}93$$

8.2 $$5x^2+6x-d=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-6\pm\sqrt{(6)^2-4(5)(-d)}}{2(5)}$$

$$36+20d=0$$

$$d=-\frac{9}{5}$$

OR/OF For equal roots/ *vir gelyke wortels* : $\Delta=0$

$$\Delta=b^2-4ac$$

$$=(6)^2-4(5)(-d)$$

$$36+20d=0$$

$$d=-\frac{9}{5}$$

9.1 $$\sqrt{2p+5}=0$$

$$2p+5=0$$

$$2p=-5$$

$$p=-\frac{5}{2}$$

9.2 $$2p+5<0$$

$$p<-\frac{5}{2}$$

1 $$\frac{x^2-4}{x-2}=\frac{(x+2)(x-2)}{(x-2)}=x+2$$

Therefore when x = 999 999 999 999, the value is
999 999 999 999 +2 = 1 000 000 000 001.

OR

$$\frac{x^2-4}{x-2}=\frac{(x+2)(x-2)}{(x-2)}=x+2$$

999 999 999 999 = $10^{12}-1$
x + 2 = 999 999 999 999 + 2
= $10^{12}+1$

2 $$2x^4+2=x^4$$

$$\therefore x^4=-2$$

Which has no real solution since $x^4 \geq 0$ for all $x \in R$

3
$$\begin{aligned}
\sqrt{m}+\sqrt{n} &= \sqrt{7+\sqrt{48}} \\
m+2\sqrt{mn}+n &= 7+\sqrt{48} \\
m+2\sqrt{mn}+n &= 7+2\sqrt{12} \\
m+n &= 7 \\
mn &= 12 \\
(m+n)^2 &= 7^2 \\
m^2+2mn+n^2 &= 49 \\
m^2+n^2 &= 49-2mn \\
&= 49-2(12) \\
&= 25
\end{aligned}$$

OR

$$\sqrt{m}+\sqrt{n}=\sqrt{7+\sqrt{48}}$$
$$m+2\sqrt{mn}+n=7+\sqrt{48}$$
$$m+2\sqrt{mn}+n=7+2\sqrt{12}$$

$m+n=7$ and $mn=12$ are possible solutions

$m=7-n$		$n=7-m$
$n(7-n)=12$		$m(7-m)=12$
$n^2-7n+12=0$	or	$m^2-7m+12=0$
$(n-4)(n-3)=0$		$(m-4)(m-3)=0$
$n=4$ or $n=3$		$m=4$ or $m=3$
$m=3$ or $m=4$		$n=3$ or $n=4$

$$\therefore m^2+n^2=3^2+4^2$$
$$=25$$

4 $$\frac{\sqrt{10^{2007}}.\sqrt{10^2}}{\sqrt{10^{2007}.10^4}-\sqrt{10^{2007}}}$$
$$=\frac{10\sqrt{10^{2007}}}{\sqrt{10^{2007}}(\sqrt{10^4}-1)}$$
$$=\frac{10}{100-1}$$
$$=\frac{10}{99}$$

5 $$\frac{5^{2007}+5^{2010}}{5^{2008}+5^{2009}}$$
$$=\frac{5^{2007}+5^{2007}.5^3}{5^{2008}+5^{2008}.5}$$
$$=\frac{5^{2007}(1+5^3)}{5^{2008}(1+5)}$$
$$=\frac{126}{5\times 6}$$
$$=\frac{126}{30}$$
$$=\frac{21}{5}$$
$$=4\frac{1}{5}$$
$$\approx 4$$

6 Let the shortest side be x

Sides of the prism: x ; $2x$; $3x$

Volume = lbh

$(x)(2x)(3x) = 3\ 072$

$6x^3 = 3\ 072$

$x^3 = 512$

$x = \sqrt[3]{512}$

$x = 8$

7 Let the amount of money Mary had be Rx/*Laat die bedrag geld wat Mary gehad het x wees*.

$$\frac{1}{5}x = \frac{1}{3}x - 28$$

$$3x + 420 = 5x$$

$$2x = 420$$

$$x = 210$$

Mary had R210/*Mary het R210 gehad*.

8.1

$$m + \frac{1}{m} = 3$$

$$\left(m + \frac{1}{m}\right)^2 = 9$$

$$m^2 + 2 + \frac{1}{m^2} = 9$$

$$m^2 + 2 - 3 + \frac{1}{m^2} = 9 - 3$$

$$m^2 - 1 + \frac{1}{m^2} = 6$$

8.2

$$m^3 + \frac{1}{m^3} = \left(m + \frac{1}{m}\right)\left(m^2 - 1 + \frac{1}{m^2}\right)$$

$$= (3)(6)$$

$$= 18$$

9

$$\text{AC}.(x-2) = x^2 + 2x - 8$$

$$\text{AC}.(x-2) = (x+4)(x-2)$$

$$\text{AC} = (x+4)\text{ cm}$$

$$\therefore \text{FD} = (x+4)\text{ cm}$$

$$\therefore \text{ED} = x + 4 - (x-2)$$

$$\text{ED} = 6\text{ cm}$$