



# JENN

**Training and Consultancy**

**The path to enlightened education**

**SUBJECT: MATHEMATICS**

**CONTENT: ALGEBRA, EQUATIONS, AND INEQUALITIES**

**ACTIVITY MANUAL**

**LEARNER/TEACHER MANUAL**

**Algebra, equations, and  
inequalities**



**JENN TRAINING: CONTENT MANUAL TEACHER/LEARNERS:**

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# Algebra, Equations and Inequalities

## Outline :

1. Simplify expressions and solve equations using the laws of exponents for rational exponents where

$$x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0, q > 0$$

2. Add, subtract, multiply and divide simple surds.
3. Quadratic equations (by factorisation and by using the quadratic formula)
4. Quadratic inequalities in one unknown (Interpret solutions graphically.)

NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs

5. Equations in two unknowns, one of which is linear and the other quadratic
6. Nature of roots

# Parts

Part 1: Quadratic equations

Part 2: Quadratic Inequalities

Part 3: Exponential equations

Part 4: Surd equations

Part 5: Simultaneous equations

Part 6: Nature of roots

Part 7: Fusion

Part 8: More on Algebra, equations and inequalities

Formulae to be used in this section are not limited to the following:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac \quad (\text{not provided in the information sheet})$$

# Part 1

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $x^2 = 5x - 4$

2.  $x(3 - x) = -3$

3.  $x(x - 4) = 5$

4.  $4x^2 - 20x + 1 = 0$

5.  $x(x - 1) = 30$

6.  $3x^2 - 5x + 1 = 0$

7.  $3x + \frac{1}{x} = 4$

8.  $x = \frac{5}{3x - 2}$

# Part 2

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $3 - x < 2x^2$

2.  $15x - 4 < 9x^2$

3.  $4 + 5x > 6x^2$

4.  $4x^2 + 1 \geq 5x$

5.  $x^2 + 7x - 8 < 0$

6.  $(x + 1)(x - 3) > 12$

7.  $(x + 1)(4 - x) > 0$

8. Given:  $(x + 2)(x - 3) < -3x + 2$

8.1 Solve for  $x$  if:  $(x + 2)(x - 3) < -3x + 2$

8.2 Hence or otherwise, determine the sum of all the integers satisfying the expression  $x^2 + 2x - 8 < 0$ .

## Part 3

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $2.3^x = 81 - 3^x$

4.  $27^{x^2+x} = 3^{3x^2} \times 9$

2.  $9.2^{x-1} = 2.3^x$

5.  $5^x = \frac{1}{125}$

3.  $2^{x\sqrt{x}} = 2^{27}$

## Part 4

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $\sqrt{2x+1} = x-1$

2.  $2x-3 = \sqrt{2x-3}$

3.  $2\sqrt{x-3} = x-3$

4.  $x - \sqrt{x} = 6$

## Part 5

1. Determine the values of  $x$  and  $y$  if they satisfy both the following equations simultaneously:

$$2x + y = 3$$

$$x^2 + y + x = y^2$$

2. Solve simultaneously for  $x$  and  $y$ :

$$y - x + 3 = 0$$

$$x^2 - x = 6 + y$$

3. Consider the equation:  $x^2 + 5xy + 6y^2 = 0$

3.1 Calculate the values of the ratio  $\frac{x}{y}$ .

3.2 Hence, calculate the values of  $x$  and  $y$  if  $x + y = 8$ .

4. Solve for  $x$  and  $y$  if:  $3^{x-10} = 3^{3x}$  and  $y^2 + x = 20$ .

5. Given:  $2^x + 2^{x+2} = -5y + 20$

5.1 Express  $2^x$  in terms of  $y$ .

5.2 How many solutions for  $x$  will the equation have if  $y = -4$ ?

5.3 Solve for  $x$  if  $y$  is the largest possible integer value for which  $2^x + 2^{x+2} = -5y + 20$  will have solutions.

## Part 6

1. Determine the nature of the roots of  $x^2 + 9 = 0$ .
2. Determine the values of  $k$  for which the equation  $x^2 - 3x + (k + 1) = 0$  has:
  - 2.1 Real roots
  - 2.2 One root equal to 1
3. Show that the equation  $x^2 - px - p^2 = 2$  has two real and unequal roots for all real values of  $p$ .
4. The coefficients of a quadratic equation are rational numbers, and its discriminant is  $\Delta = 2k(2k - 9)$ .

Describe the nature of the roots of the equation if  $k = 6$ .
5. If 5 is one root of the equation  $x^2 + kx - 15 = 0$ , determine the value of  $k$ , and the other root.
6. Given:  $f(x) = 3x^2 - 6x + m$ 

Determine the values of  $m$  if  $f(x) = 0$  has non-real roots.
7. Given:  $f(x) = x^2 - 5x + c$ 

Determine the value of  $c$  if it is given that the solutions of  $f(x) = 0$  are  $\frac{5 \pm \sqrt{41}}{2}$ .
8. Given:  $f(x) = 5x^2 + 6x - 7$ 
  - 8.1 Solve for  $x$  if  $f(x) = 0$  (correct to TWO decimal places).
  - 8.2 Hence, or otherwise, calculate the value of  $d$  for which  $5x^2 + 6x - d = 0$  has equal roots.
9. The solutions of a quadratic equation are given by  $x = \frac{-2 \pm \sqrt{2p + 5}}{7}$ 

For which value(s) of  $p$  will this equation have:

  - 9.1 Two equal solutions
  - 9.2 No real solutions

## Part 7

- Given  $x = 999\,999\,999\,999$ , determine the exact value of  $\frac{x^2 - 4}{x - 2}$ .  
Show ALL your calculations.
- Explain why the equation  $\frac{x^4 + 1}{x^4} = \frac{1}{2}$  has no real roots.
- If  $m$  and  $n$  are rational numbers such that  $\sqrt{m} + \sqrt{n} = \sqrt{7 + \sqrt{48}}$ , calculate a possible value of  $m^2 + n^2$ .
- Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}} \quad (\text{Show ALL calculations.})$$

- Calculate the integer that is the closest approximation to:

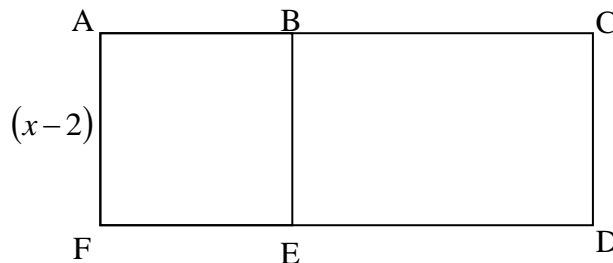
$$\frac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}} \quad (\text{Show ALL workings.})$$

- The volume of a box with a rectangular base is  $3\,072 \text{ cm}^3$ . The lengths of the sides are in the ratio  $1 : 2 : 3$ . Calculate the length of the shortest side.
- Mary gave one third of her money to Nazeem and one fifth of her money to Elwethu. Elwethu received R28 less than Nazeem. How much money did Mary have originally?
- Given:  $m + \frac{1}{m} = 3$

8.1 Determine the value of:  $m^2 - 1 + \frac{1}{m^2}$ .

8.2 Hence determine the value of:  $m^3 + \frac{1}{m^3}$ .

- ACDF is a rectangle with an area of  $x^2 + 2x - 8 \text{ cm}^2$ . B is a point on AC and E is a point on FD such that ABEF is a square with sides of length  $(x - 2)$  cm each.



Calculate the length of ED.

## QUESTION 1

- 1.1 Solve for  $x$ :
- 1.1.1  $(x-2)(4+x) = 0$  (2)
- 1.1.2  $3x^2 - 2x = 14$  (correct to TWO decimal places) (4)
- 1.1.3  $2^{x+2} + 2^x = 20$  (3)
- 1.2 Solve the following equations simultaneously:
- $$x = 2y + 3$$
- $$3x^2 - 5xy = 24 + 16y$$
- (6)
- 1.3 Solve for  $x$ :  $(x-1)(x-2) < 6$  (4)
- 1.4 The roots of a quadratic equation are:  $x = \frac{3 \pm \sqrt{-k-4}}{2}$   
 For which values of  $k$  are the roots real? (2)  
**[21]**

November 2015

## QUESTION 1

- 1.1 Solve for  $x$ :
- 1.1.1  $x^2 - 9x + 20 = 0$  (3)
- 1.1.2  $3x^2 + 5x = 4$  (correct to TWO decimal places) (4)
- 1.1.3  $2x^{\frac{-5}{3}} = 64$  (4)
- 1.1.4  $\sqrt{2-x} = x-2$  (4)
- 1.1.5  $x^2 + 7x < 0$  (3)
- 1.2 Given:  $(3x-y)^2 + (x-5)^2 = 0$   
 Solve for  $x$  and  $y$ . (4)
- 1.3 For which value of  $k$  will the equation  $x^2 + x = k$  have no real roots? (4)  
**[26]**



**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x(x-7) = 0$  (2)

1.1.2  $x^2 - 6x + 2 = 0$  (correct to TWO decimal places) (3)

1.1.3  $\sqrt{x-1} + 1 = x$  (5)

1.1.4  $3^{x+3} - 3^{x+2} = 486$  (4)

1.2 Given:  $f(x) = x^2 + 3x - 4$ 

1.2.1 Solve for  $x$  if  $f(x) = 0$  (2)

1.2.2 Solve for  $x$  if  $f(x) < 0$  (2)

1.2.3 Determine the values of  $x$  for which  $f'(x) \geq 0$  (2)

1.3 Solve for  $x$  and  $y$ :  $x = 2y$  and  $x^2 - 5xy = -24$  (4)**[24]**

Feb.–Mar. 2015

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - x - 20 = 0$  (2)

1.1.2  $2x^2 - 11x + 7 = 0$  (correct to TWO decimal places) (3)

1.1.3  $5x^2 + 4 > 21x$  (5)

1.1.4  $2^{2x} - 6 \cdot 2^x = 16$  (4)

1.2 Solve for  $x$  and  $y$  simultaneously:

$$\begin{aligned} y + 1 &= 2x \\ x^2 - xy + y^2 &= 7 \end{aligned} \quad (6)$$

1.3 The roots of a quadratic equation are given by  $x = \frac{-5 \pm \sqrt{20 + 8k}}{6}$ , where  $k \in \{-3; -2; -1; 0; 1; 2; 3\}$ .1.3.1 Write down TWO values of  $k$  for which the roots will be rational. (2)1.3.2 Write down ONE value of  $k$  for which the roots will be non-real. (1)1.4 Calculate  $a$  and  $b$  if  $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a(7^b)$  and  $a$  is not a multiple of 7. (4)**[27]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 + 5x - 6 = 0$  (3)

1.1.2  $4x^2 + 3x - 5 = 0$  (correct to TWO decimal places) (3)

1.1.3  $4x^2 - 1 < 0$  (3)

1.1.4  $\left(\sqrt{\sqrt{32} + x}\right)\left(\sqrt{\sqrt{32} - x}\right) = x$  (4)

1.2 Solve simultaneously for  $x$  and  $y$ :

$y + x = 12$  and  $xy = 14 - 3x$  (5)

1.3 Consider the product  $1 \times 2 \times 3 \times 4 \times \dots \times 30$ .Determine the largest value of  $k$  such that  $3^k$  is a factor of this product. (4)**[22]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - 6x = 0$  (2)

1.1.2  $x^2 + 10x + 8 = 0$  (correct to TWO decimal places) (3)

1.1.3  $(1-x)(x+2) < 0$  (3)

1.1.4  $\sqrt{x+18} = x-2$  (5)

1.2 Solve simultaneously for  $x$  and  $y$ :

$x + y = 3$  and  $2x^2 + 4xy - y = 15$  (6)

1.3 If  $n$  is the largest integer for which  $n^{200} < 5^{300}$ , determine the value of  $n$ . (3)**[22]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - 6x - 16 = 0$  (3)

1.1.2  $2x^2 + 7x - 1 = 0$  (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to  $x^2 - 25 < 0$ . (4)1.3 Solve for  $x$  and  $y$ :

$$-2y + x = -1 \quad \text{and} \quad x^2 - 7 - y^2 = -y$$
 (6)

1.4 Evaluate:  $\frac{3^{2018} + 3^{2016}}{3^{2017}}$  (2)1.5 Given:  $t(x) = \frac{\sqrt{3x-5}}{x-3}$ 1.5.1 For which values of  $x$  will  $\frac{\sqrt{3x-5}}{x-3}$  be real? (3)1.5.2 Solve for  $x$  if  $t(x) = 1$ . (4)**[26]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 + 2x - 15 = 0$  (3)

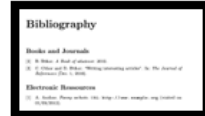
1.1.2  $5x^2 - x - 9 = 0$  (Leave your answer correct to TWO decimal places.) (3)

1.1.3  $x^2 \leq 3x$  (4)

1.2 Given:  $a + \frac{64}{a} = 16$ 1.2.1 Solve for  $a$ . (3)1.2.2 Hence, solve for  $x$ :  $2^x + 2^{6-x} = 16$  (3)1.3 **Without using a calculator**, calculate the value of  $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$  (4)1.4 Solve for  $x$  and  $y$  simultaneously:

$$2x - y = 2 \quad \text{and} \quad \frac{1}{x} - 3y = 1$$
 (6)  
**[26]**

## Bibliography



1. FET CAPS document
2. Mathematics Grade 12 Examination Guidelines
3. Grade 11 and Grade 12 previous year papers from national and different provinces